1 Distribution for the sample mean when the underlying distribution is lognormal

In Lab 5 we have a problem that delt with the distribution for $\overline{X}$ when the underlying distribution for generating individual values is lognormal with parameters μ and σ.

This situation is extra confusing because μ and σ are used in multiple places.

First, for a log-normal with parameters μ and σ, we have that the mean of a log-normal is $e^{\mu+\sigma^2/2}$ and the standard deviation of a log-normal is $\sqrt{\left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}}$.

Lets write this as:

$$\mu_X = \mu_{logn} = e^{\mu+\sigma^2/2}$$

$$\sigma_X = \sigma_{logn} = \sqrt{\left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}}$$

These are the mean and standard deviation of the underlying distribution.

Next we want the mean and standard deviation for the sampling distribution. We called these values $\mu_{\overline{X}}$ and $\sigma_{\overline{X}}$. The formulas we have are:

$$\mu_{\overline{X}} = \mu_X$$

and

$$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}}$$

In the two formulas above, μ_X and σ_X represent the mean and standard deviation for the **underlying distribution** they are not the μ and σ that are the *parameters* of the lognormal. If we are dealing with log-normal with parameters μ and σ then the mean and standard deviation for the underlying distribution are the values calculated above as μ_X, and σ_X.

Thus, if we are dealing with a sampling distribution for $\overline{X}$ and the underlying distribution is lognormal then we get:

$$\mu_{\overline{X}} = \mu_X = e^{\mu+\sigma^2/2}$$

and

$$\sigma_{\overline{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{\sqrt{\left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}}}{\sqrt{n}}$$