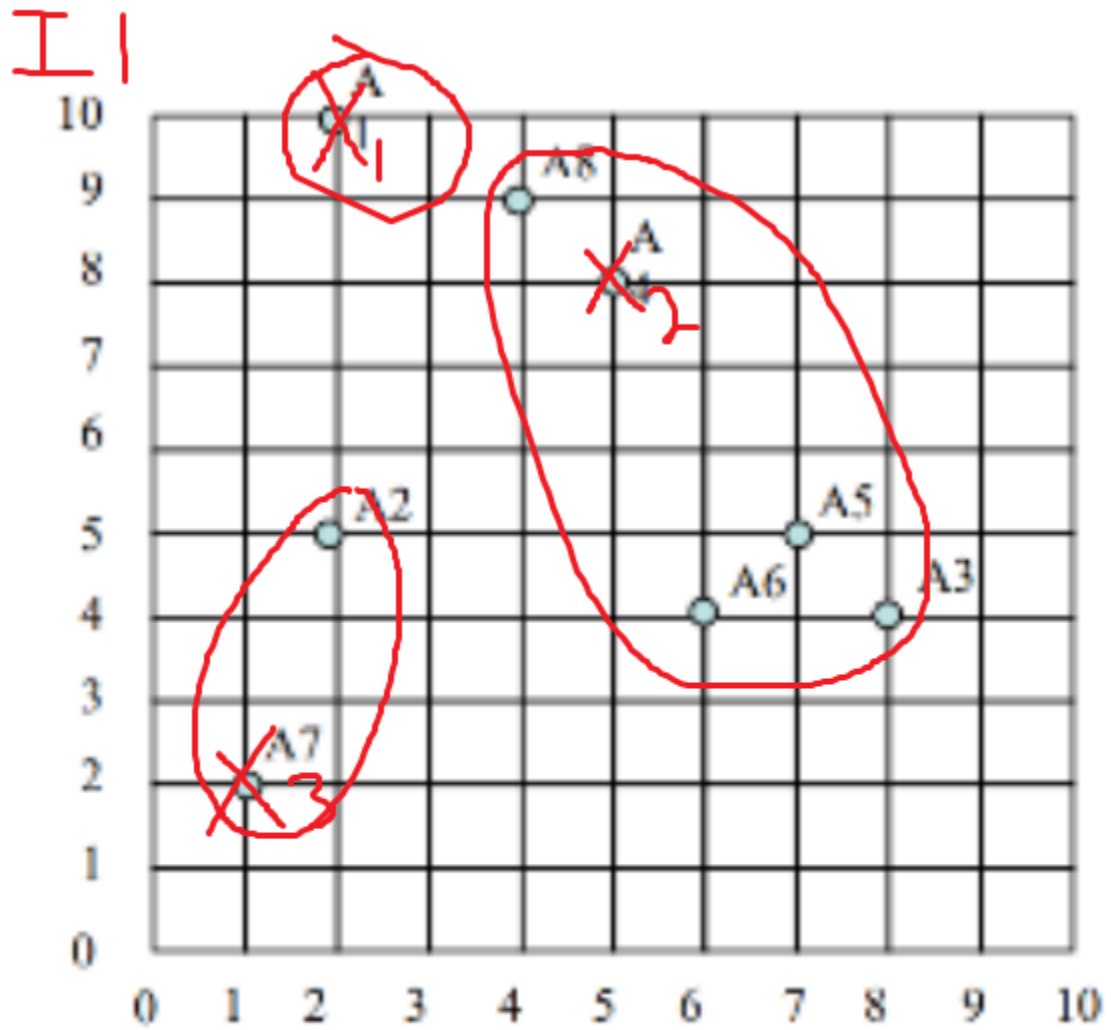


1. a)



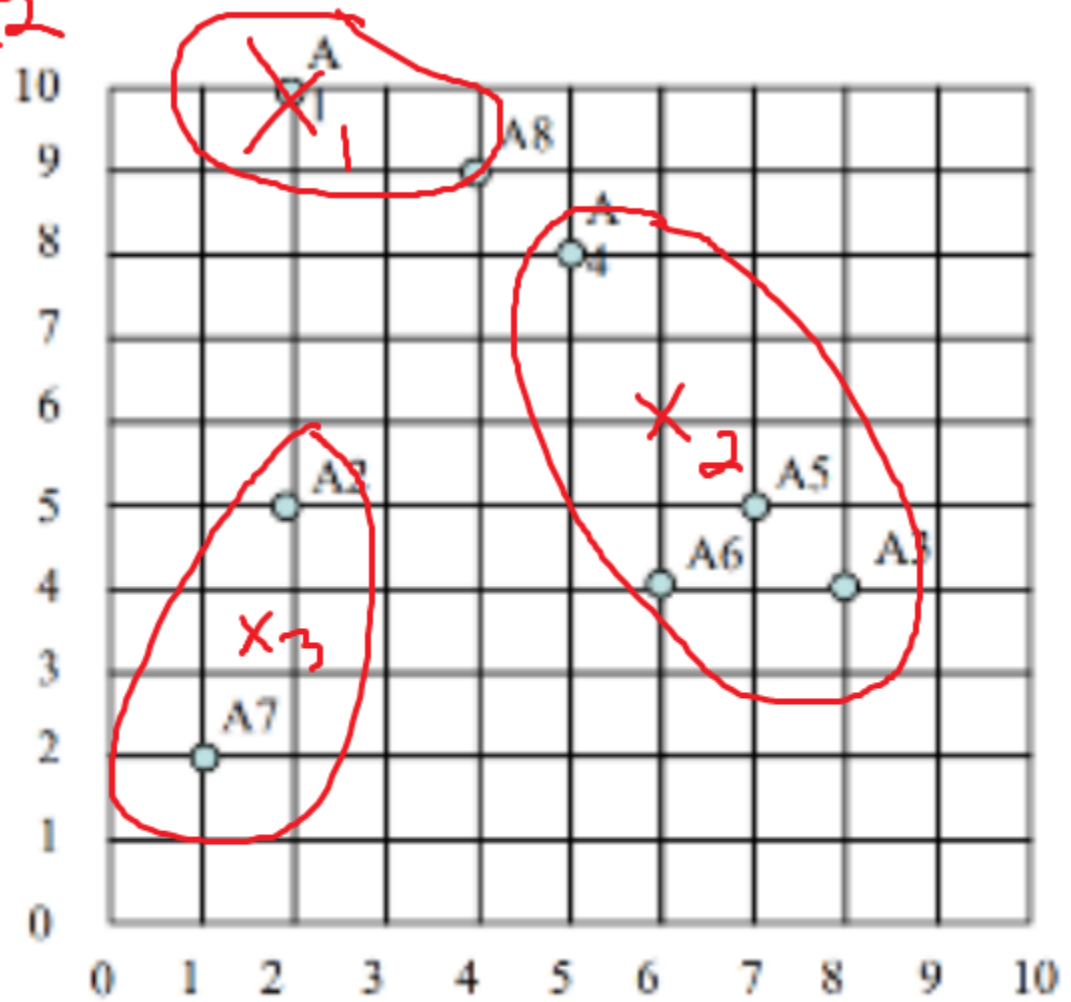
$$\text{Mean1} = (2, 10)/1 = (2, 10)$$

$$\text{Mean2} = (4+5+6+7+8)/5, (9+8+5+4+4)/5 = (6, 6)$$

$$\text{Mean3} = (1+2)/2, (2+5)/2 = (1.5, 3.5)$$

Move centroids to means. (movement will be shown at start of next iteration for simplicity of plot.)

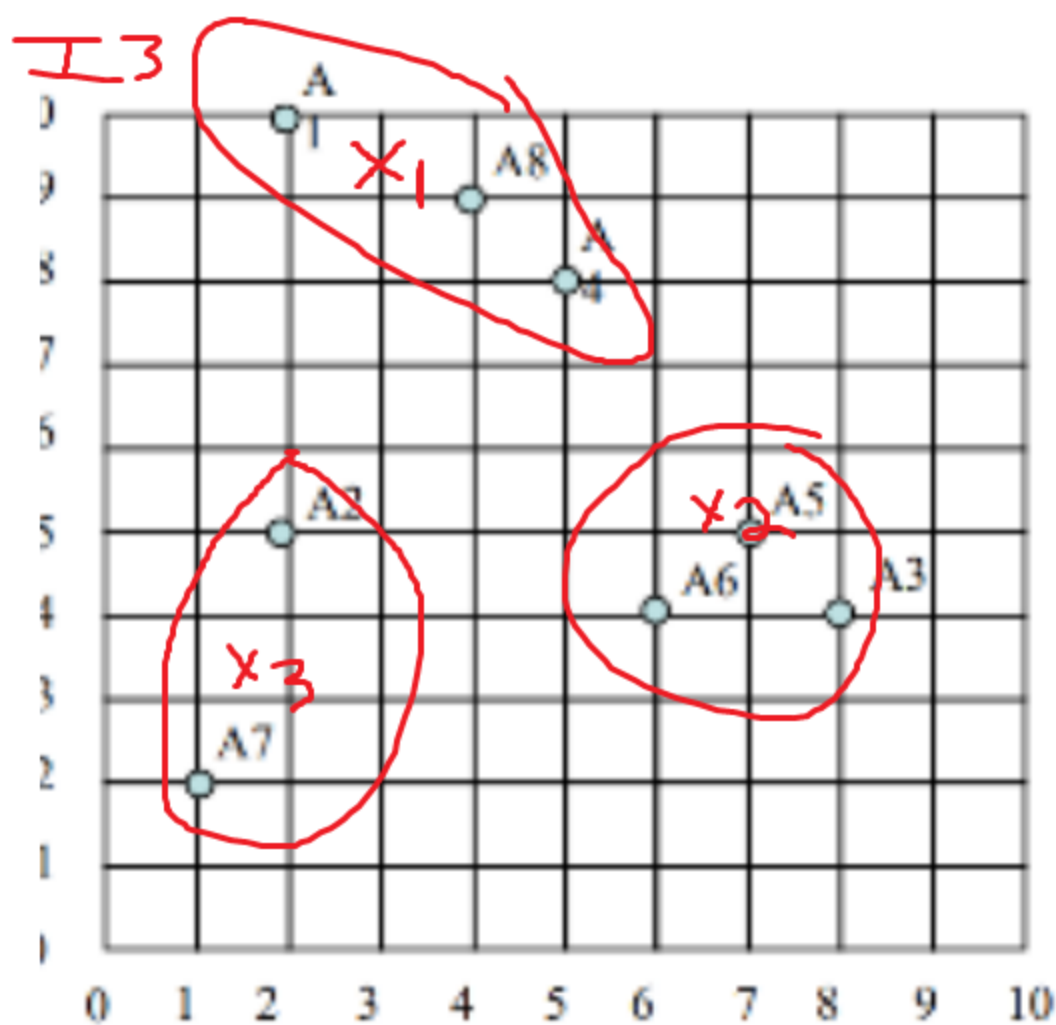
I2



$$\text{Mean1} = (2+4)/2, (9+10)/2 = (3, 9.5)$$

$$\text{Mean2} = (5+6+7+8)/4, (4+4+5+8)/4 = (6.5, 5.25)$$

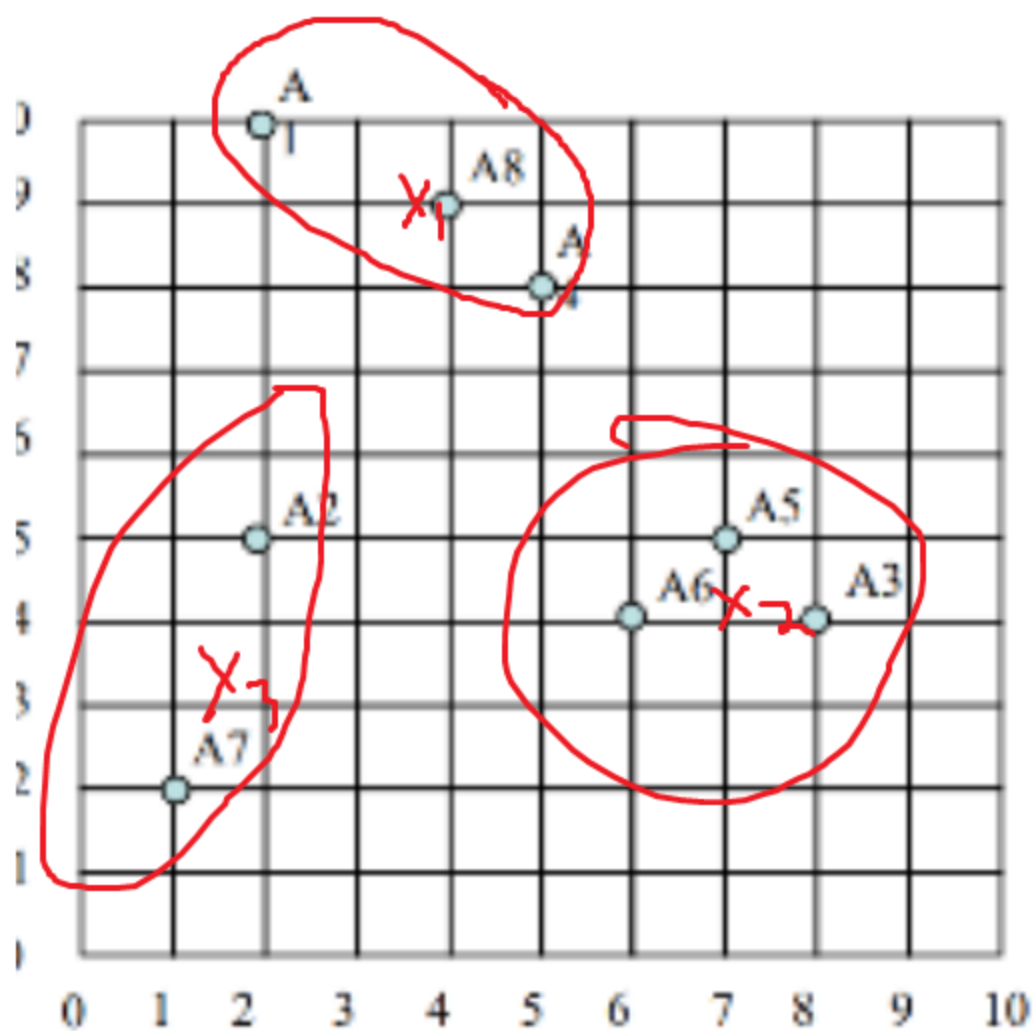
$$\text{Mean3} = (1+2)/2, (2+5)/2 = (1.5, 3.5)$$



$$\text{Mean1} = (2+4+5)/3, (8+9+10)/3 = (3.67, 9)$$

$$\text{Mean2} = (6+7+8)/3, (4+4+5)/3 = (7, 4.33)$$

$$\text{Mean3} = (1.5, 3.5), \text{ unchanged}$$



Mean1 = (3.67, 9), unchanged  
Mean2 = (7, 4.33), unchanged  
Mean3 = (1.5, 3.5), unchanged  
Terminate K means algorithm.

```

iteration    = 1
Centroid    = C1 = (2, 10), C2 = (5, 8), C3 = (1, 2)
instances   = [' A1 ', ' A2 ', ' A3 ', ' A4 ', ' A5 ', ' A6 ', ' A7 ', ' A8 ']
C1 distances = [' 0.00', ' 5.00', ' 8.49', ' 3.61', ' 7.07', ' 7.21', ' 8.06', ' 2.24']
C2 distances = [' 3.61', ' 4.24', ' 5.00', ' 0.00', ' 3.61', ' 4.12', ' 7.21', ' 1.41']
C3 distances = [' 8.06', ' 3.16', ' 7.28', ' 7.21', ' 6.71', ' 5.39', ' 0.00', ' 7.62']
iteration 2 centroids = C1 = (2.0, 10.0), C2 = (6.0, 6.0), C3 = (1.5, 3.5)

iteration    = 2
Centroid    = C1 = (2.0, 10.0), C2 = (6.0, 6.0), C3 = (1.5, 3.5)
instances   = [' A1 ', ' A2 ', ' A3 ', ' A4 ', ' A5 ', ' A6 ', ' A7 ', ' A8 ']
C1 distances = [' 0.00', ' 5.00', ' 8.49', ' 3.61', ' 7.07', ' 7.21', ' 8.06', ' 2.24']
C2 distances = [' 5.66', ' 4.12', ' 2.83', ' 2.24', ' 1.41', ' 2.00', ' 6.40', ' 3.61']
C3 distances = [' 6.52', ' 1.58', ' 6.52', ' 5.70', ' 5.70', ' 4.53', ' 1.58', ' 6.04']
iteration 3 centroids = C1 = (3.0, 9.5), C2 = (6.5, 5.25), C3 = (1.5, 3.5)

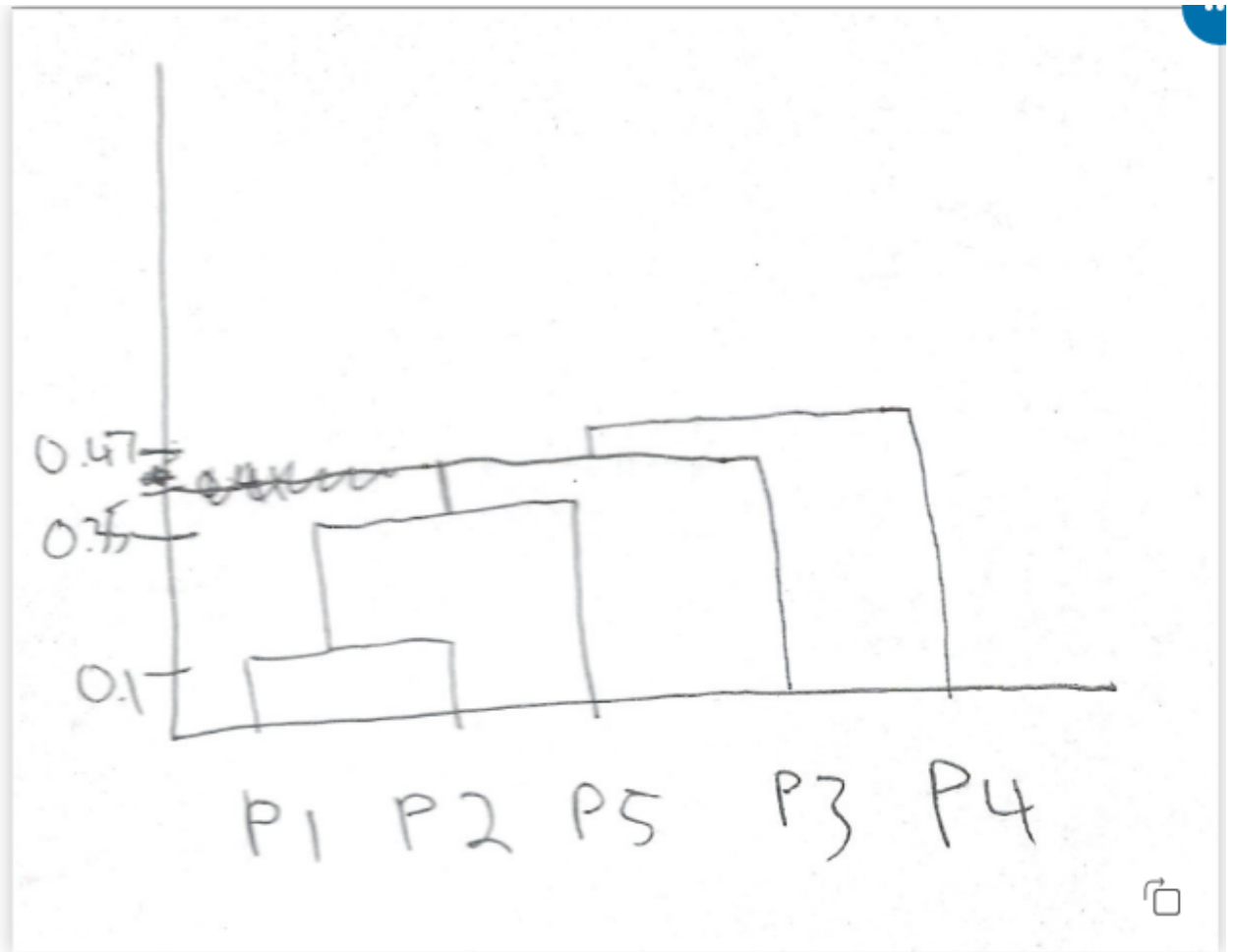
iteration    = 3
Centroid    = C1 = (3.0, 9.5), C2 = (6.5, 5.25), C3 = (1.5, 3.5)
instances   = [' A1 ', ' A2 ', ' A3 ', ' A4 ', ' A5 ', ' A6 ', ' A7 ', ' A8 ']
C1 distances = [' 1.12', ' 4.61', ' 7.43', ' 2.50', ' 6.02', ' 6.26', ' 7.76', ' 1.12']
C2 distances = [' 6.54', ' 4.51', ' 1.95', ' 3.13', ' 0.56', ' 1.35', ' 6.39', ' 4.51']
C3 distances = [' 6.52', ' 1.58', ' 6.52', ' 5.70', ' 5.70', ' 4.53', ' 1.58', ' 6.04']
iteration 4 centroids = C1 = (3.6666666666666665, 9.0), C2 = (7.0, 4.333333333333333), C3 = (1.5, 3.5)

iteration    = 4
Centroid    = C1 = (3.6666666666666665, 9.0), C2 = (7.0, 4.333333333333333), C3 = (1.5, 3.5)
instances   = [' A1 ', ' A2 ', ' A3 ', ' A4 ', ' A5 ', ' A6 ', ' A7 ', ' A8 ']
C1 distances = [' 1.94', ' 4.33', ' 6.62', ' 1.67', ' 5.21', ' 5.52', ' 7.49', ' 0.33']
C2 distances = [' 7.56', ' 5.04', ' 1.05', ' 4.18', ' 0.67', ' 1.05', ' 6.44', ' 5.55']
C3 distances = [' 6.52', ' 1.58', ' 6.52', ' 5.70', ' 5.70', ' 4.53', ' 1.58', ' 6.04']
iteration 5 centroids = C1 = (3.6666666666666665, 9.0), C2 = (7.0, 4.333333333333333), C3 = (1.5, 3.5)

```

- b)
- SSE of cluster1 = 6.67
  - SSE of cluster2 = 2.67
  - SSE of cluster3 = 5
  - SSE of final clustering = 14.33

2. a)



- b)  $k = 2$ , cluster1 = {p1, p2, p5, p3}, cluster2 = {p4}  
 $k = 3$ , cluster1 = {p1,p2,p5}, cluster2 = {p3}, cluster3 = {p4}  
 $k = 4$ , cluster1 = {p1, p2}, cluster2 = {p5}, cluster3={p3}, cluster4={p4}

3. <https://github.com/garyjsk271/Machine-Learning.git>

4. a)

$$\cos(\text{John}, \text{Fred}) = 0.85$$

$$\cos(\text{John}, \text{Lilian}) = 0.99$$

$$\cos(\text{John}, \text{Cathy}) = 0.95$$

Compute weighted combination of Lilian, Cathy's ratings.

For kiss:

$$2.25 + (0.99 \cdot (3 - 2.125) + 0.95 \cdot (1.5 - 2.25)) / (0.99 + 0.95) = 2.33$$

For Guns n Roses:

$$2.25 + (0.99 \cdot (1 - 2.125) + 0.95 \cdot (2 - 2.25)) / (0.99 + 0.95) = 1.55$$

For Kiss, weighted combination of ratings  $> 2.0$ , recommend to John.

For Guns n' Roses, weighted combination of ratings  $< 2.0$ , do not recommend to John.

b)

For Kiss, find all cosine similarities.

$$\cos(\text{Kiss}, \text{BonJovi}) = 0.99$$

$$\cos(\text{Kiss}, \text{Metallica}) = 0.77$$

$$\cos(\text{Kiss}, \text{Scorpions}) = 0.94$$

$$\cos(\text{Kiss}, \text{ACDC}) = 0.82$$

Use BonJovi and Scorpions to compute rating.

$$1.83 + (0.99 \cdot (3 - 2) + 0.94 \cdot (2 - 1.83)) / (0.99 + 0.94) = 2.43$$

Since rating is  $> 2.0$ , recommend Kiss to John.

For Guns n Roses, find all cosine similarities.

$$\cos(\text{Guns n Roses}, \text{BonJovi}) = 0.71$$

$$\cos(\text{Guns n Roses}, \text{Metallica}) = 0.99$$

$$\cos(\text{Guns n Roses}, \text{Scorpions}) = 0.88$$

$$\cos(\text{Guns n Roses}, \text{ACDC}) = 0.97$$

Use Metallica, ACDC to compute rating.

$$2 + (0.99 \cdot (2 - 2.17) + 0.97 \cdot (2 - 2.67)) / (0.99 + 0.97) = 1.59$$

Since rating is  $< 2.0$ , do not recommend Guns n Roses to John.

5. a)

$$|T| = 10$$

$$\text{Support}(a) = 5/10 = 0.5$$

$$\text{Support}(b) = 7/10 = 0.7$$

$$\text{Support}(c) = 5/10 = 0.5$$

$$\text{Support}(d) = 8/10 = 0.8$$

$$\text{Support}(e) = 6/10 = 0.6$$

no pruning as all supports  $> 0.3$

$$F1 = \{ \{a\}, \{b\}, \{c\}, \{d\}, \{e\} \}$$

$$\text{Support}(ab) = 3/10 = 0.3$$

$$\text{Support}(ac) = 2/10 = 0.2 \quad \text{*prune}$$

$$\text{Support}(ad) = 4/10 = 0.4$$

$$\text{Support}(ae) = 4/10 = 0.4$$

$$\text{Support}(bc) = 2/10 = 0.2 \quad \text{*prune}$$

$$\text{Support}(bd) = 5/10 = 0.5$$

$$\text{Support}(be) = 4/10 = 0.4$$

$$\text{Support}(cd) = 4/10 = 0.4$$

$$\text{Support}(ce) = 2/10 = 0.2 \quad \text{*prune}$$

$$\text{Support}(de) = 6/10 = 0.6$$

$$F2 = \{ \{ab\}, \{ad\}, \{ae\}, \{bd\}, \{be\}, \{cd\}, \{de\} \}$$

$$F2 \times F2 \text{ after merge} = \{ \{abd\}, \{abe\}, \{bde\}, \{ade\} \}$$

$$\text{Support}(abd) = 2/10 = 0.2 \quad \text{*prune}$$

$$\text{Support}(abe) = 2/10 = 0.2 \quad \text{*prune}$$

$$\text{Support}(bde) = 4/10 = 0.4$$

$$\text{Support}(ade) = 4/10 = 0.4$$

$$F3 = \{ \{bde\}, \{ade\} \}$$

Rank by all frequent items by support.

$$d = 0.8$$

$$b = 0.7$$

$$e, de = 0.6$$

$$a, c, bd = 0.5$$

$$ad, ae, be, cd, bde, ade = 0.4$$

$$ab = 0.3$$



b)  $F3 = \{ \{bde\}, \{ade\} \}$

$$\text{Support}(bde) = 4/10 = 0.4$$

$$\text{Support}(ade) = 4/10 = 0.4$$

$b \rightarrow de$

$$\text{Support}(b) = 7/10 = 0.7$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(b) = 4/7 = 0.57$$

does not satisfy the confidence requirement.

$d \rightarrow be$

$$\text{Support}(d) = 8/10 = 0.8$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(d) = 4/8 = 0.50$$

does not satisfy the confidence requirement.

$e \rightarrow bd$

$$\text{Support}(e) = 6/10 = 0.6$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(e) = 4/6 = 0.67$$

satisfies the confidence requirement.

$bd \rightarrow e$

$$\text{Support}(bd) = 5/10 = 0.5$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(bd) = 4/5 = 0.8$$

satisfies the confidence requirement.

$be \rightarrow d$

$$\text{Support}(be) = 4/10 = 0.4$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(be) = 4/4 = 1.0$$

satisfies the confidence requirement.

$de \rightarrow b$

$$\text{Support}(de) = 6/10 = 0.6$$

satisfies the support requirement.

$$\text{confidence} = \text{Support}(bde) / \text{Support}(de) = 4/6 = 0.67$$

satisfies the confidence requirement.

$a \rightarrow de$

$\text{Support}(a) = 5/10 = 0.5$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(a) = 4/5 = 0.8$

satisfies the confidence requirement.

$d \rightarrow ae$

$\text{Support}(d) = 8/10 = 0.8$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(a) = 4/8 = 0.50$

satisfies the confidence requirement.

$e \rightarrow ad$

$\text{Support}(e) = 6/10 = 0.6$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(e) = 4/6 = 0.67$

satisfies the confidence requirement.

$ad \rightarrow e$

$\text{Support}(ad) = 4/10 = 0.4$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(ad) = 4/4 = 1.0$

satisfies the confidence requirement.

$ae \rightarrow d$

$\text{Support}(ae) = 4/10 = 0.4$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(ae) = 4/4 = 1.0$

satisfies the confidence requirement.

$de \rightarrow a$

$\text{Support}(de) = 6/10 = 0.6$

Satisfies support requirement.

$\text{confidence} = \text{Support}(ade) / \text{Support}(de) = 4/6 = 0.67$

satisfies the confidence requirement.

c)  $F2 = \{ab\}, \{ad\}, \{ae\}, \{bd\}, \{be\}, \{cd\}, \{de\}$

$F2 \times F2 = \{$   
     $(ab, ad), (ab, ae), (ab, bd), (ab, be), (ab, cd), (ab, de),$   
     $(ad, ae), (ad, bd), (ad, be), (ad, cd), (ad, de),$   
     $(ae, bd), (ae, be), (ae, cd), (ae, de),$   
     $(bd, be), (bd, cd), (bd, de),$   
     $(be, cd), (be, de),$   
     $(cd, de)$   
 $\}$

Merge all pairs with matching prefix size of 1.

$C3 = \{abd, abe, bde, ade\}$

$\text{Support}(abd) = 2/10 = 0.2$      **\*prune**

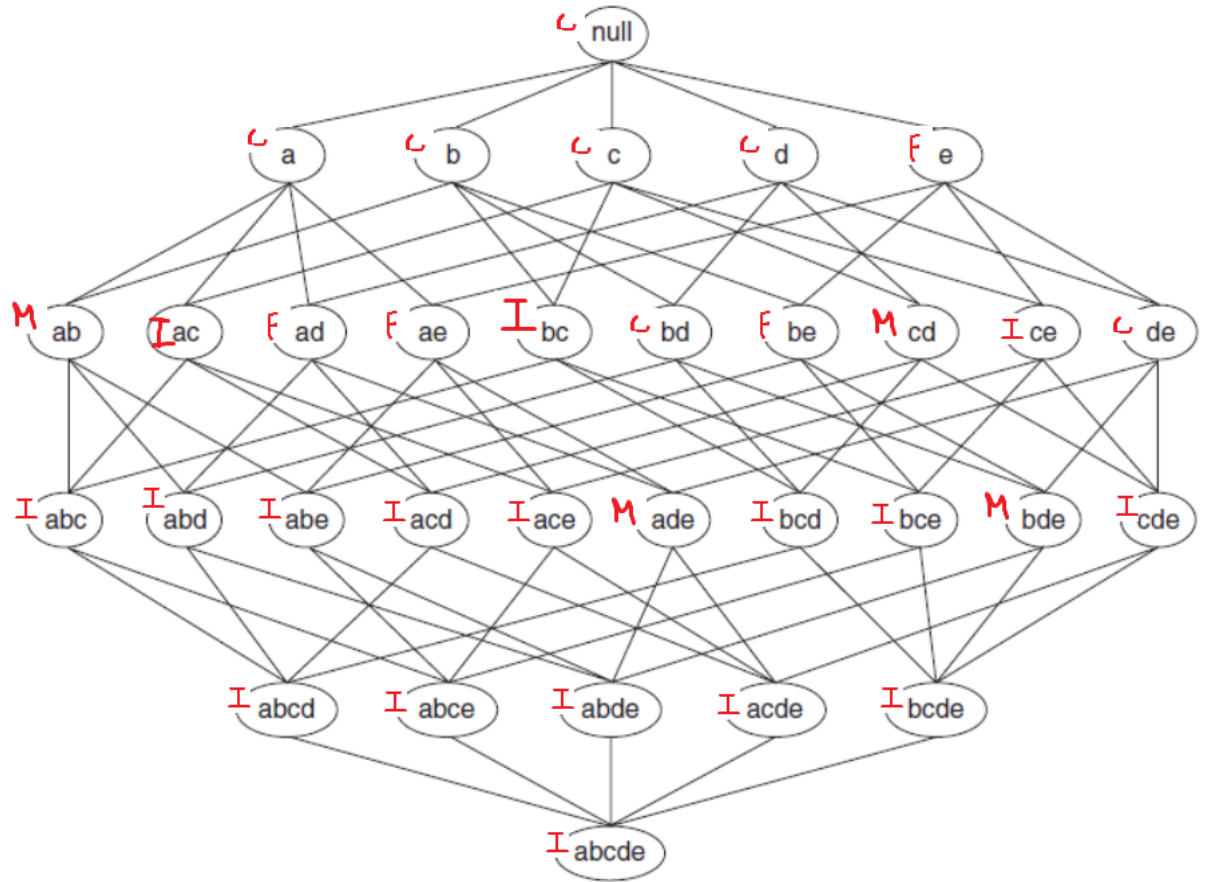
$\text{Support}(abe) = 3/10 = 0.3$      **\*prune**

$\text{Support}(bde) = 4/10 = 0.4$

$\text{Support}(ade) = 4/10 = 0.4$

$F3 = \{ade, bde\}$

d)



6. <https://github.com/garyjsk271/Machine-Learning.git>