Fall 2023 MAE 263F MS Comprehensive Exam

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I. Introduction

In the report, a DER model is used to simulate a beam fixed-free beam's deformation under load. First, I will go over how the environment is set up and explain the flow of my MATLAB script. Second, I will discuss the measured deformation over time at constant load P = 10N. Next, the simulation will be running multiple times to measure the maximum deformation as load gradually increases. We will also compare the result to that of Euler-Bernoulli theory and see how it deviates from our simulations.

II. Flow of Code

Below I will provide the flow of my code and explain each step taken throughout the whole script.

A. Initialization

I started off by clearing plots and variables that may affect values in my script. Then several parameters are defined, including the number of nodes in my DER model, time frame, beam length, and cross section dimensions b and h. I also defined some environment parameters such as gravitational acceleration g.

B. Creating Physical Matrices

A mass matrix M is created based on the geometry of the cantilever beam and its density. A weight vector W is also initialized to house the gravitational force exserted on the entirety of the beam nodes.

C. Material Properties

I then defined material properties according to the question description. The Young's modulus YM is initialized and was used to calculate the bending stiffness EI and axial rigidity EA.

D. Loop to Calculate Force and Deflection

I then created a for-loop to go over all the P value starting from 1 to 200. For each force magnitude, the code starts off by creating a force matrix according to the P value. Then I initialize the geometry of the beam by setting q0. Next a second loop is created to run the simulation for one second by time stepping. A Newton-Raphson method is used to solve the force equilibrium equation, where I calculated the elastic forces, weight, and external load. The position and velocity of the third to the last node is then updated from the result of the converged Newton-Raphson method equation. This can make sure that the first two nodes are fixed and maintain the same position and angle to meet the required boundary condition. The maximum deformation is then stored in a vector for later use.

E. Plotting P=10N

As P reaches a value of 10 N, I plotted its deflection over time with the time stepping loop. Giving us an insight into how the load affects the deformation of the beam over a set amount of time.

F. Comparison With Euler-Bernoulli Beam Theory

After the resulting data is collected, I then use a simple equation to calculate the result of Euler-Bernoulli beam theory of the same beam under the different external loads. And save the result into a vector for later comparison with the simulation.

G. Plotting Comparison Results

I plotted the maximum deflection under different load magnitude from our DER simulation against the results from the Euler-Bernoulli beam theory, allowing us to visualize the deviation of the two results as the load gradually increases.

H. Displaying Deviation Between Simulation and Theory

Finally, we calculated the P where the results from our simulation and theory deviate more than ten percent.

III. Deformation at Constant Load over Time

In part E of the flow of the code, we plotted the deflection of the end node at P=10 N as time goes from 0 to 1 second, as shown in Fig.1. The resulting figure is shown below, where we can see an immediate deformation at the beginning of time and overshoots before reaching steady value. However, since our system did not include viscosity, such damping characteristic isn't foreseen. This may be due to a numerical damping found in previous assignments, which may be caused by our calculations in Newton-Raphson method.

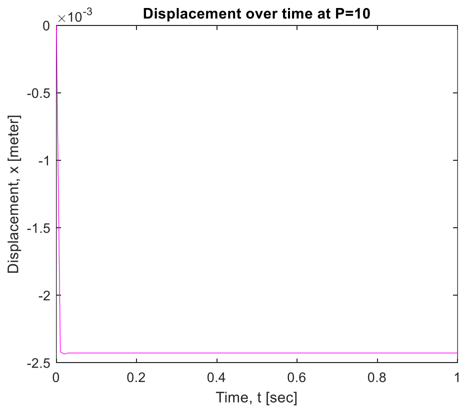

Figure 1. Deflection over time at constant load P = 10N

IV. DEFORMATION OF SIMULATION COMPARED TO EULER-BERNOULLI BEAM THEORY

As in part G of my code, I plotted the deflection of the beam from the range of load calculated using both DER simulation and Euler-Bernoulli beam theory. With a side-by-side comparison shown in Fig. 2, we can see that there is a significant deviation between the two result sets. From my calculations I found that the deviation exceeds 10 percent after the load P reaches around 110 Newton of force, 10.4% deviation to be exact. Therefore, instead of having $1 \leq P \leq 100$, I ranged the load from 1 to 200 in order to better demonstrate the increasing deviation. At 200 Newton external load, there is nearly a 25% deviation between our simulation and the beam theory. Which clearly indicated a noticeable difference in either method, which will be discussed in the next section.

V. DISCUSSION ON DEVIATION OF SIMULATION FROM EULER-BERNOULLI BEAM THEORY

Euler-Bernoulli beam theory is a widely used method of calculating deformation of beams under load. However, as shown in the section before, the calculated result deviated significantly from our simulation using discrete elastic beam algorithm. The main reason for this is one of the key assumptions that is used in the Euler beam theory, that is assuming small deformation and no horizontal deflection of the end of the beam. This means that it is assumed that the end of the beam only moves downward when under load. However, intuitively we know that as the beam bends, it deforms toward the bottom left corner. Therefore, as deformation increases, the error caused by this assumption becomes greater, which eventually leads to this drastic result. This explains why we see larger deviations as our external load increases.

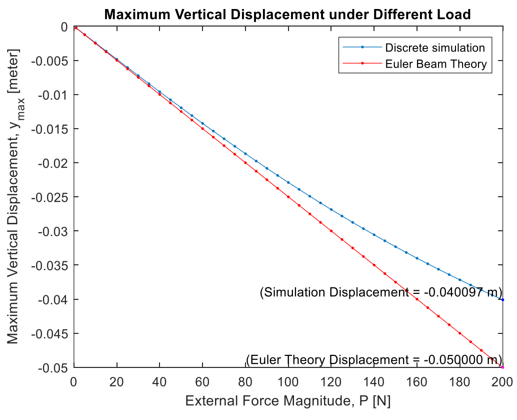

Figure 2. Deflection over time at constant load P = 10N