Simulation of Mammalian Sperm in Viscous Media

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I. Overview

A. Project Goal

The overarching goal of this project is to develop a MATLAB simulation of an actuated mammalian sperm-inspired robot moving through a viscous fluid. Since our initial project proposal, we have identified that the primary focus of our simulation will be effectively capturing sperm locomotion and analyze the characteristics of progressive motility for sperm. We believe a simulation designed in 2D space will allow us to pay specific attention to the lateral, whip-like motion of a sperm flagellum.

B. Assumptions

The simulation will be constructed based on a discrete elastic beam model assuming a low Reynolds number environment and a Newtonian fluid. These are standard assumptions pertaining to microorganisms in a viscous fluid where the inertial effects are negligible compared to the viscous effects and will allow for simpler modeling. It will be useful to simulate the complex dynamics of the sperm motion within a viscous medium using a Discrete Elastic Rod (DER) approach since it will allow for a more detailed examination of the sperm's behavior, such as the flagellar waveform and how it interacts with the surrounding fluid [8].

II. Flow of MATLAB Code

In this section we will go through the flow of our MATLAB script and explain the steps taken that lead us to our result. The following is a high-level schematic of the code structure:

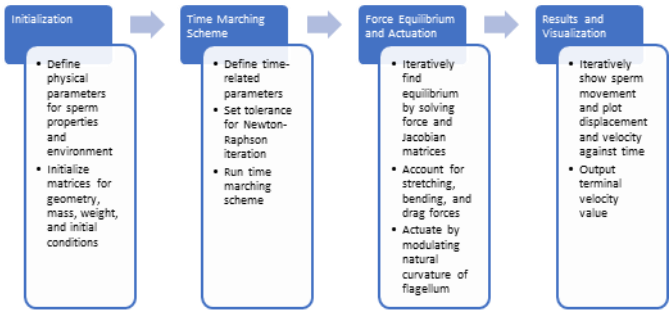

Fig. 1: High-level schematic of code structure.

A. Initialization

The code starts by creating variables that define the physical parameters of the environment and sperm properties, such as fluid viscosity, number of nodes, sperm length, radii, and Young's modulus. Next, matrices for initial geometry, mass, viscous damping, and initial conditions are initialized.

B. Time Marching Scheme

The time marching scheme in this code utilizes the Newton-Raphson method, which iteratively solves for the equilibriums, until the error falls below a pre-defined tolerance value. For easier coding, the Newton-Raphson method is contained within the swimming function, which forms the mechanism for sperm actuation.

C. Force Equilibrium and Actuation

Within the swimming function, equilibriums are iteratively solved and updated for the force and Jacobian matrices, accounting for stretching, bending, and drag forces. The actuation occurs by the code modulating the natural curvature of the flagellum, which creates a time-dependent waveform for the sperm flagellum.

D. Results and Optimization

Throughout the Newton-Raphson iteration, the code plots the motion of the sperm. After exceeding the simulation run-time, the code plots the displacement and velocity of the sperm against time, as well as outputs the terminal velocity value.

III. Geometry Modeling

To create an accurate depiction with our simulation, we modified the nodes in the DER model to resemble the geometry of actual sperm. We divided the rod into three sections: the head, midpiece, and flagellum. The head is in the shape of a sphere with a radius three times larger than the midpiece. And both the midpiece and the flagellum are long round rods, while the flagellum has half the radius of the midpiece. The flagellum is also 2.3 times longer compared to the midpiece. This created a simplified model that captures the profile of sperm and can accurately depict its dynamic characteristics in our simulation environment. With the nonuniform geometry of our model, the rigidity also changes accordingly. This also improved the authenticity of our simulation [4].

Fig. 2: Simplified sperm structure.

IV. Environment Structuring

A. Simulation Environment Overview

To ensure a realistic depiction of sperm motion through a viscous fluid, it is necessary to include the influence of hydrodynamic forces acting on the sperm. As such, the code implements drag coefficients based on the Gray and Hancock model (1) (2), which are used to approximate the drag forces acting on microorganisms in a viscous environment [3].

B. Gray and Hancock's Drag Coefficients

With the previous DER model, we assume all nodes in the rod to be independent spheres. This meant that that all nodes experience the same amount of drag force no matter what the shape of the rod is. Including this assumption in a simulation of sperm moving in a viscous fluid would lead to inaccuracies since a flagellum has a streamline shape and should be able to move freely in its tangential direction. By introducing Gray and Hancock's drag coefficients, the code splits the viscous drag force into tangential and normal components. These equations are dependent on the fluid viscosity, flagellar wavelength, and characteristic length (specifically, the sperm's diameter). The Gray and Hancock equations are based on resistive force theory and ensure that twice as much force is required for the sperm to move radially than tangentially [5]. The inclusion of the tangential and normal drag within the simulation dampens the flagellar dynamics slightly, reducing unwanted chaotic movement. It should be noted that the equations are used to describe microorganisms specifically with helical locomotion, however, for the purposes of the sperm simulation, they allow for a useful approximation of the sperm flagellar dynamics [9].

$$C_t = \frac{2\pi\mu}{\ln\frac{2\lambda}{a} - 1/2} \quad \text{and} \quad C_n = \frac{4\pi\mu}{\ln\frac{2\lambda}{a} + 1/2} \qquad (1), (2)$$

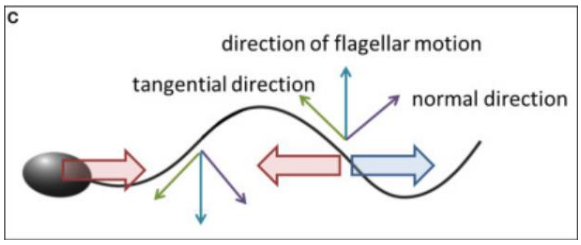

Fig. 3: Illustration of tangential and normal directions of flagellar motion [5].

C. Implementation of Drag Forces

The computation for drag forces is tailored to node-specific calculations for the head, midpiece, and flagellum nodes based on the different diameters of each segment. This ensures that more drag acts on the head and midpiece nodes compared to the flagellum nodes. The Gray and Hancock model for tangential and normal drag is implemented into the Newton-Raphson iteration, where the force and Jacobian matrices are updated with the discrete contributions of tangential and normal drag (3) (4). Within the equation for normal drag, the term ($\mathbf{I}$ – $tang \cdot tang^T$) is a projection matrix that projects the normal vector of the sperm motion onto a plane orthogonal to the tangent vector. By incorporating these drag forces, the code realistically simulates sperm flagellar dynamics, propelling itself forward by exerting forces on the surrounding fluid.

$$dF_{normal} = -dl \cdot C_n(\boldsymbol{I} - tang \cdot tang^T) \cdot \frac{q_i(t_{k+1}) - q_i(t_k)}{\Delta t} \qquad (3)$$

$$dF_{tangential} = -dl \cdot C_t(tang \cdot tang^T) \cdot \frac{q_i(t_{k+1}) - q_i(t_k)}{\Delta t} \qquad (4)$$

V. Self-Actuation

A. Actuation Method

From microscopy data of actual sperm movement, we see that the flagellum is actuated by oscillatory behavior created by the sperm's midpiece. By controlling the frequency and the magnitude of this oscillation, we will have full control of its movement and can create the desired motility. We implement this by determining the period (time constant) and the amplitude of a sinusoidal function. To make the movement identical to the real-world counterpart, we also include a linear incrementation of the oscillating amplitude throughout the length of the flagellum. The resulting equation used for calculating the actuating curvature is shown in (5). This creates the constant varying curvature that is then plugged into the gradient and Hessian equation for calculating the bending force in Newton-Raphson method.

$$\kappa = F \times \sin\left(2\pi dt \times \frac{n}{T}\right) \qquad (5)$$

B. Simulation Test and Validation

To test out the feasibility of our actuation method and find optimal input configuration, we started off by fixing the head of the sperm and seeing the resulting waveform created by the actuation. From this experiment we were able to find the natural frequency of our model and the best amplitude value that leads to efficient beating movement while still maintaining the resemblance of real sperm movement. With the midpiece beating at the natural frequency, the amplitude of the flagellum builds up and creates a uniform and stable sinusoidal waveform that can lead to efficient locomotion.

C. Implementing Actuation in a Free Model

After we obtained the optimal input for efficient oscillation and verified the feasibility of our method, we implemented this into a free moving sperm model in viscous fluid. The flagellum waveform efficiently propels the sperm forward without any turbulent movement. The waveform created by the sperm flagellum shows the accuracy of our simulation, with slight flexing at the end of the midpiece, which generates the large sideways swiping motion of the flagellum [1]. We found that using an oscillation with amplitude of 0.2 and frequency of 10 Hz yields the best swimming velocity, approximately 0.42 body length per second.

VI. Simulating and Controlling Motility

A. Motility Analysis

With the implementation of the oscillating curvature, we found that the inclusion of nonuniform geometry and oscillation amplitude discussed in previous paragraphs play substantial roles in the stability of sperm movement. The rigidity and profile of the head and midpiece reduces sideways motion and acts as a dart head that significantly stabilizes the direction the sperm is moving in. This phenomenon is unprecedented in studies in bacteria using helical flagellum, since they do not have forces acting normal to its moving direction. Without the midpiece structure and the varying actuation amplitude within. The sperm ended up only swiveling its head and is unable to produce positive motility. From this result we can further understand the reason behind the shape of sperm and why they are structured so differently from other bacteria with flagellum. There is much more disturbance and turbulent movement in flagellum moving sideways, which requires precise actuation control and specific streamline shape [7]. However, such locomotion does come with some crucial benefits, which will be discussed in the following paragraph.

B. Swimming Direction Control

Because of the nature of our oscillating actuation input, we can add a phase shifting value to change the base that the curvature fluctuates from. We can modify (5) by including the phase shift like shown in (6).

$$\kappa = F \times \left(Phase\ shift + \sin\left(2\pi dt \times \frac{n}{T}\right)\right) \tag{6}$$

With this new equation for actuating the curvature of the midpiece, we can efficiently control the moving direction of the sperm. We found that because of the natural lateral motion, shifting the phase of the curvature is an outstandingly effective way of changing swimming direction, that does not require much excessive energy. From our simulation we found that we are easily able to change the swimming direction with fine precision and can even perform a U-turn on a dime. This kind of maneuver is essentially impossible for bacteria with a spiral flagellum since their orientation is constantly changing [6]. The reason this characteristic is present in sperm may be due to its purpose of finding and reaching eggs for fertilization. Whereas bacteria do not require a method for steering but only an efficient locomotion.

C. Path Planning

We arranged the code that controls the swimming of the sperm into a function, which allowed us to plan out a series of movements with different input. With the addition of our ability to control its movement direction, we can make the sperm follow a desired path. As a result, by setting the input difference and duration, we can precisely control the sperm to perform a complex set of maneuvers. This really simplified the process of data collection and motion analysis for our simulation and can give us insight into how we can possibly control a similar soft robot in this form factor.

VII. Model Validation

A. Reynolds Number Approximation

As mentioned at the beginning of the paper, the assumption of low Reynolds number is essential for accurate simulations of microorganisms within a viscous fluid. The assumption (Re << 1) simplifies the implementation of a realistic model, as the inertial forces can be considered negligible compared to viscous forces. To validate the accuracy of the sperm simulation, we calculate a Reynolds number (7) as roughly Re = 0.5 based on a set of values for density, fluid velocity, diameter, and viscosity that qualitatively best represent the movement of sperm. The simulation value of Re = 0.5 is within the region considered to be low Reynolds, validating our simulation environment as reasonably comparative to the real world, however, in the context of life at micrometer scale, an optimal Reynolds number would be even lower than that [10].

$$Re = \frac{\rho v L}{\mu} \tag{7}$$

VIII. Results and Discussion

The MATLAB script outputs the resulting displacement and velocity over time as shown in Fig.3 and Fig.4. The significant noise in both these graphs were caused by the oscillating movements of the sperm head. In spite of that, we can see that the trend of the velocity slowly increases and reaches a steady oscillating value. Indicating that as the swimming speed increases, the drag force caused by viscosity increases too. They eventually reach an equilibrium where the sperm swims at its terminal velocity. This further verifies the authenticity of our simulation and backs up our results.

By taking the average of the oscillation we can obtain the terminal velocity = 0.21 m/s, which corresponds to 0.42 body length per second. Actual sperm moves at 0.1~2 body length per second. Proving that our simulation's capability of accurately depicting sperm's dynamic characteristic.

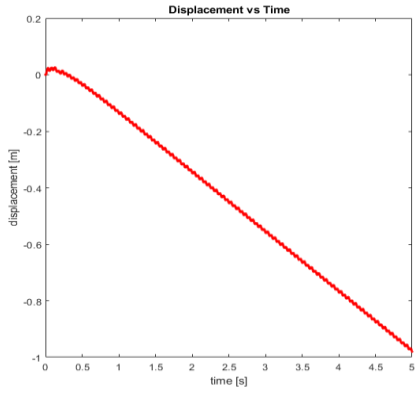

Fig. 4: Displacement of head node over time.

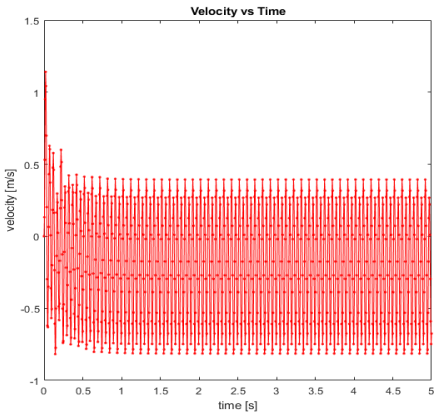

Fig. 5: Velocity of head node over time.

With the results from analyzing the motility of sperm, we were able to obtain a deeper understanding of reasons behind sperm's structure and their method of locomotion. To be specific. The lateral motion that sets them apart from other organisms allows them to control their moving direction much more efficiently, perfectly fitting their goal of reaching the egg to begin the process of fertilization. The shape of sperm act as a stabilizing dart head that minimize unwanted swiveling that can increase drag and turbulent movement. With these characteristics, the sperm's method of locomotion is efficient and capable of adapting to different scenarios, making it an interesting case study for implementation in soft robot locomotion and design.

IX. Future Work

A. *Dimensionless Quantities Analysis*

In order to reduce the complexity of scaling parameters for the sperm simulation, dimensionless quantities may be implemented to better parameterize the system. By using Buckingham Pi theorem, we can create dimensionless groups around Reynolds number-related quantities. Integrating dimensionless quantities into the sperm simulation code would help the system remain on the micrometer scale and simplify future experimentation.

B. *Effects of Flagellar Buckling*

In our sperm simulation, the sperm changes direction via a coded phase shift in the natural curvature of the flagellum, however, it would be interesting to exploit the phenomenon of flagellar buckling for changing the sperm's direction. Buckling occurs when angular velocity reaches a critical threshold, which leads to a significant change in the flagellum's waveform. Through precise control input, a sperm-inspired robot, such as the one simulated in this project, can effectively navigate a pre-designated path [2].

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