ON THE OPTIMAL FOREST HARVESTING DECISION

WILLIAM J. STRANG*

This paper deals with the problems of when and whether to harvest a stand of timber which provides valuable services intact as well as when harvested. The analysis extends a previous treatment with the result that it is optimal not to harvest more often and for a greater number of reasons than had been realized.

INTRODUCTION

This note is a correction and extension of an analysis by Hartman (1976) which considers the optimal harvesting strategy for a forest that provides value while standing as well as when harvested. Standing timber may provide wildlife habitat, flood control, and recreational services which should be taken into account when the harvesting decision is made.[1] Two new points are advanced. First, even though a finite, local maximum for optimal rotation time may exist, a corner solution involving never cutting the forest may be the global maximum. Second, previous treatments of the harvesting problem have taken the tract in question as initially bare. For many stands of timber, a climax or old growth forest initially occupies the land. The age of timber at harvest in the first rotation, then, may exceed that for subsequent harvesting cycles. This consideration leads to a different objective function to be maximized. It is shown that it might be preferable never to cut an old growth forest even if it is optimal eventually to harvest the same land if initially barren. This is due simply to the considerable standing values available from a climax forest.

BACKGROUND

A number of simplifying assumptions are employed. First, a given plot of land is considered, with all trees harvested simultaneously. This treatment thus does not deal explicitly with a selective cutting procedure.[2] Further, lumber prices and the discount rate are assumed constant over time.[3] For example, the cyclical behavior of the forest products industry due to fluctuations in market interest rates is ignored. Finally, perfect certainty is assumed (see footnote 1).

*University of Washington. I am indebted to Dick Hartman and Evan Koenig for helpful comments, demonstration of some mathematical results, and correction of errors. Thanks also to Gardner Brown and an anonymous referee for useful suggestions. I am responsible for any errors which may remain.

1. To the extent that the objective functions of private timber companies neglect standing values, there can exist incentives for private agreements and/or federal regulation to guarantee optimality. The latter solution may be more desirable due to the public goods nature of many recreational areas, which may explain the large acreages under the jurisdiction of the U.S. Forest Service and other federal agencies. Note also that, under uncertainty, a decision *not* to harvest may be more easily reversible than a decision to cut, implying a further imputed value to a standing forest; however, I do not consider this problem explicitly. See Arrow and Fisher (1974) for a general treatment of this example of "quasi-option value."

2. Clearcutting is far more common than selective cutting in the Northwest.

3. Samuelson (1976) considers the effects of changes in these variables. First, an increase in lumber prices acts to lower the optimum cycle length by lowering wages relative to timber prices (*i.e.*, lowering wages in terms of timber). Firms thus substitute labor for tree capital in the only way they can — by harvesting sooner and more often. Second, an increase in the discount rate also lowers optimum cycle length, since shorter growth cycles imply a shorter time until the trees are first cut and a shorter time until the next cycle begins, so that the return on tree capital is brought forward in time. Samuelson neglected standing values, which were then considered by Hartman.

Economic Inquiry
Vol. XXI, October 1983

Following Hartman, let $G(t)$ denote the stumpage value in a forest of age t. This can be thought of as the value of the timber minus the costs of harvest. $G(t)$ is assumed to have the following growth curve shape: rising at an increasing rate, then at a decreasing rate, reaching a maximum, falling, and finally levelling off. The flow of services (such as viewing, hunting, etc.) from a standing forest of age t is $F(t)$, which is taken to rise at an increasing rate, then at a decreasing rate, asymptotically approaching a maximum. Thus, the older the forest, the more valuable the flow of standing services from that forest. The general shapes of $G(t)$ and $F(t)$ are shown in figures 1 and 2, respectively. The present is taken as t or $T = 0$ throughout, and the functions G and F are assumed twice differentiable.

FIGURE 1

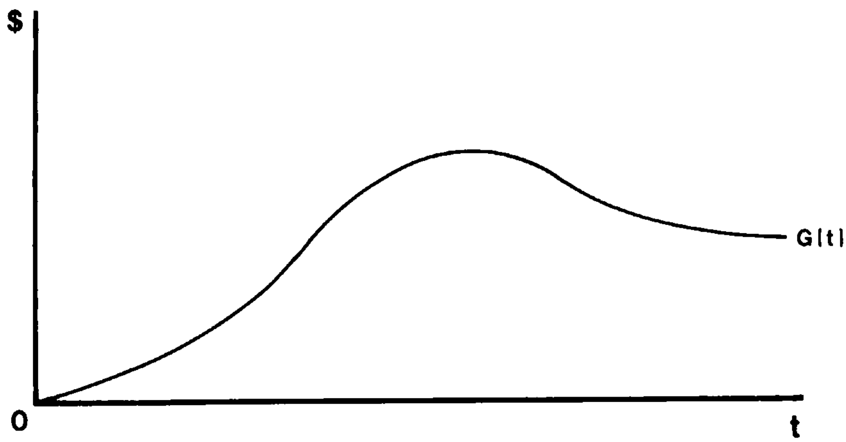
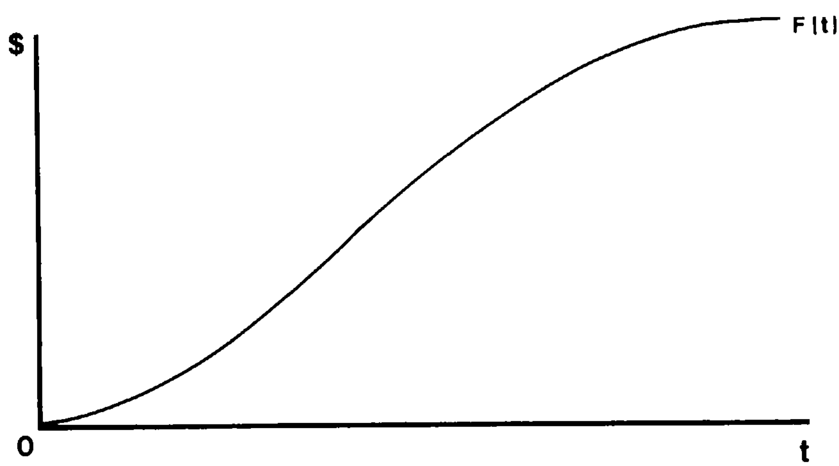


FIGURE 2



The Fisher (One Cycle) Solution

The generalized Fisher (one growth cycle) solution to the problem of optimum rotation length is to choose the harvest age t so as to maximize

$$V(t) = \int_0^t e^{-rx} F(x)dx + e^{-rt} G(t), \tag{1}$$

where r is the discount rate. This is just the integral of discounted standing values received up until the time cutting occurs, plus discounted stumpage value at the time of harvest. The first order condition for an interior maximum simplifies to

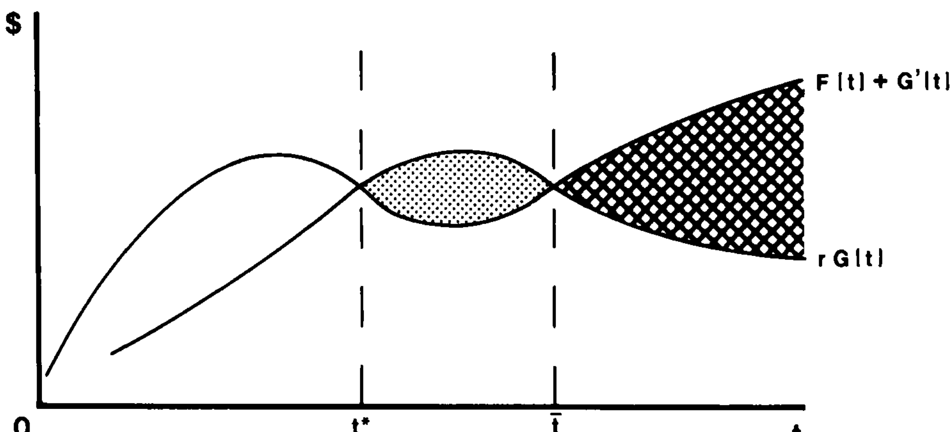
$$F(t) + G'(t) = rG(t), \tag{2}$$

and the second order condition, to

$$F'(t) + G''(t) < rG'(t). \tag{3}$$

Let an interior maximum (if one exists) satisfying these conditions be denoted t^*. At this point, the flow of standing-timber services from the forest plus the change in stumpage value or capital gain (the sum of which represents the marginal value of not harvesting) just equals the interest lost on the stumpage value (the marginal cost of not harvesting). This is seen in figure 3. The second order condition requires that the slope of the marginal cost, $rG'(t)$, exceed the slope of the marginal value, $F'(t) + G''(t)$. Thus, in figure 3, t^* is a local maximum and $\overline{t}$, a local minimum.

FIGURE 3

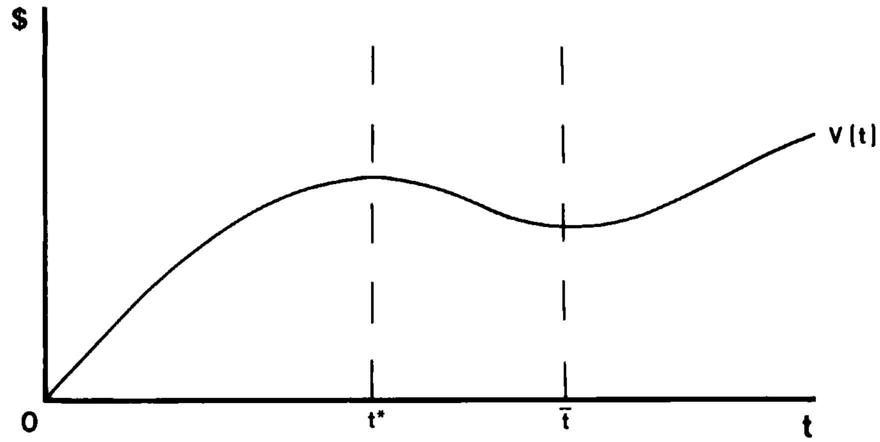


But this analysis is incomplete. These are sufficient conditions for an *interior* maximum only. The objective function $V(t)$ must be checked at the endpoints $t = 0$ and $\lim_{t\to\infty}$ of the relevant time horizon. The former endpoint implies immediate cutting. The latter represents the case of allowing the forest to stand unharvested forever. Since a previously cut forest is being considered, $V(0) = G(0) = 0$ (assuming nonexistent trees are costless to harvest). But

$$\lim_{t\to\infty} V(t) = \lim_{t\to\infty} \int_0^t e^{-rx} F(x)dx,$$

which exceeds zero due to positive standing values, so the $t = 0$ solution will always be dominated by the $\lim_{t\to\infty}$ solution. In other words, an interior maximum t^* obtained by solving (2) will be optimal for the problem specified in (1) if and only if $V(t^*)$ exceeds $\lim_{t\to\infty} V(t)$. Otherwise, the forest's greatest value will be realized if and only if it is never cut. The possibility of a corner maximum at $\lim_{t\to\infty}$ which is also the global maximum can be seen in figure 3. If the (continuously discounted) cross-hatched area (where the marginal value of not harvesting exceeds the marginal cost) is greater than the discounted stippled area (where marginal cost exceeds marginal value), the forest should not be cut.[4] The shape of $V(t)$ in this case might appear as in figure 4.

FIGURE 4



Intuitively, forest aging at some point causes a decline in the stumpage value of timber. This tends to favor harvesting before the senescence takes place. Indeed, the occurrence of our local maximum at t^* was facilitated by this. Eventually, however, the decline in value slows, and the resulting climax forest may provide valuable environmental services. Acquisition of these positive net benefits from a standing forest may necessitate incurring an initial cost, yet this may be desirable. Like any investment project, if the eventual benefits exceed the initial costs in present value terms, the project should be undertaken.

The Faustmann (Many Cycle) Solution

The Fisher solution may be suboptimal in that earlier harvesting allows a new rotation to begin sooner than if cutting were delayed. In other words, an infinite number of finite-length growth cycles is the relevant alternative to the never-cut decision. This is the generalized Faustmann problem.[5] Saying the same thing in a different way, we should maximize discounted net benefits over *time* (which may involve many cycles) rather than over *one cycle*.

The well known solution to this problem is obtained by maximizing the returns $V(t)$ at intervals t periods long,[6] where t is again the choice variable, or

4. Hartman infers that the never-cut decision is optimal only when marginal value *always* exceeds marginal cost, so that no interior maximum exists. See his figure 5.

5. See Gaffney (1960) for a discussion of alternative models, most of which he shows to be incorrect.

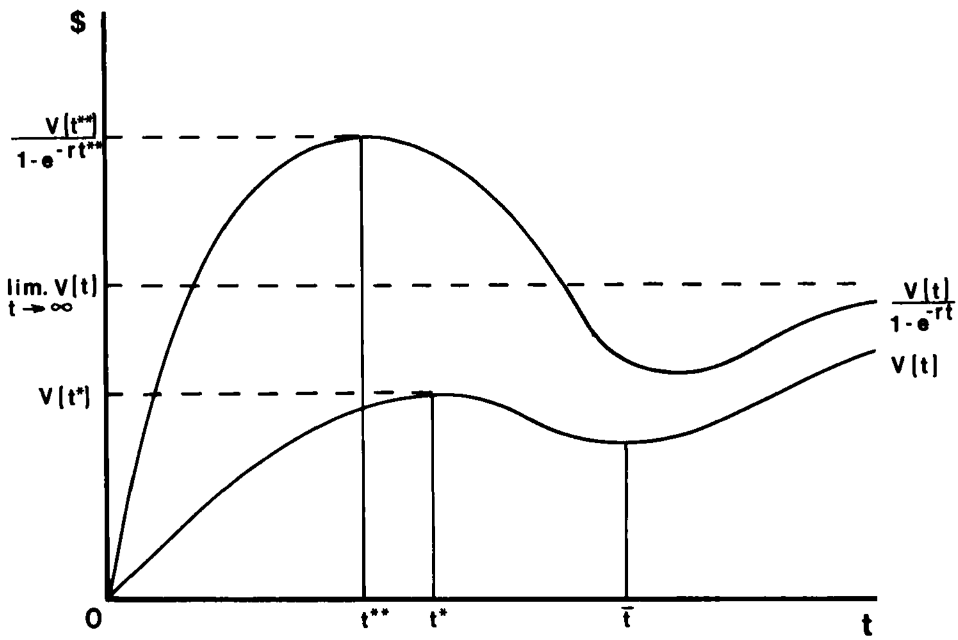
6. The V function (thus, presumably, the F and G functions) is assumed not to change over time.

$$\begin{aligned} & V(t) + e^{-rt}V(t) + e^{-r(2t)}V(t) + \ldots \\ = \; & V(t) + e^{-rt}V(t) + (e^{-rt})^2 V(t) + \ldots \\ = \; & V(t)/(1 - e^{-rt}), \text{ since } e^{-rt} < 1. \end{aligned}$$

As Hartman shows, the interior maximum (referred to as t^{**} here) for this problem will be smaller than t^*, since earlier cutting is more attractive in this case. Again, however, the solution $V(t^{**})/(1 - e^{-rt^{**}})$ is only a *local* maximum and should be checked against $\lim_{t\to\infty} V(t)/(1 - e^{-rt}) = \lim_{t\to\infty} V(t)$. Note the $\lim_{t\to\infty}$ solution is more likely to be optimal in the one-cycle problem, as the alternative (*viz.*, cutting the trees) to it is more limited in value in the one-cycle than in the many-cycle case. Mathematically, $V(t)/(1 - e^{-rt})$ exceeds $V(t)$ for any finite t, but each has an identical limit as $t \to \infty$.

Consider figure 5. It plots the Fisher and Faustmann objective functions ($V(t)$ and $V(t)/(1 - e^{-rt})$, respectively) against time. The specific case drawn is one where never cutting is superior to the interior, local maximum for the Fisher problem, but inferior to that for the Faustmann problem. Of course, it is optimal never to harvest in this case, since the Faustmann problem is the relevant one.

FIGURE 5



Optimal Harvesting Given an Initially Standing Forest

Even the Faustmann specification is unrealistic if the forest is not previously cleared. Note that t^{**}, $2t^{**}$, $3t^{**}$, etc., are the optimal times to harvest starting at any $t < t^{**}$. But if the optimal time of the first cut, t^{**}, has passed without the cut being made, the problem is a different one altogether. Now the costs incurred in letting the forest grow more than t^{**} years are sunk, and a considerable flow of standing value services is being derived from the forest. For example, old-growth (virgin) stands still exist, having begun growth, perhaps, after a centuries-old fire.[7] Harvesting at the "optimal" time t^{**} will no longer be a possibility for these tracts, unless t^{**} is very large.

In this case, the age of timber at the time of the first cut may be larger than that for subsequent cuts, so the problem becomes asymmetrical. If the timber is ever harvested once, however, succeeding returns can be modelled as in the Faustmann problem, for in that case the forest would become barren after the first cut. Thus, the post-first cut solution t^{**} should be calculated, since a stream of value $V(t^{**})/(1-e^{-rt^{**}}) \equiv K$ can be derived after the forest is cut for the first time (if ever). I have defined K as the maximum value of the many-cycle (Faustmann) objective function for ease of notation in what is to follow. Also, let A denote the age of the forest at present, $T = 0$. The objective here is to maximize

$$(4) \qquad U(T) = \int_0^T e^{-rx}F(A + x)dx + e^{-rT}G(A + T) + e^{-rT}K,$$

with respect to T (large case), the length of time until the first cut (all subsequent cuts being made at intervals of t^{**} years). $U(T)$ is simply the value of the forest derived until the first cut, represented by the first two terms on the right hand side of (4), plus the value derived after the first cut, $e^{-rT}K$. The value obtained until the first cut is just the Fisher (one cycle) objective function $V(t)$ with the F and G functions updated by A, the initial age of the forest. After this cut is made, we revert to our original problem, which was concerned with a previously-cleared forest. The solution to this problem was K, but we must discount K by e^{-rT}, T being the length of time until we first obtain a cleared forest (*i.e.*, the time until the first cut).

The first order condition for an interior maximum of (4) simplifies easily to

$$(5) \qquad F(A + T) + G'(A + T) = r(G(A + T) + K).$$

In words, an interior maximum requires that the marginal value of postponing the initial harvest just equals the marginal cost of not harvesting: the interest lost on the first $(r(G(A + T))$ and all subsequent (rK) harvests.[8]

Note that if $A = 0$ (*i.e.*, the forest is initially bare), our objective function collapses to that for the Faustmann problem. If A is nonzero, but less than t^{**}, the optimal time T to cut the forest is simply t^{**}-A. In words, t^{**} is the optimal forest age to make the first cut *up until* the forest has actually aged that long (after which we can no longer cut at this forest age). If A exceeds t^{**}, of course, T cannot be chosen equal to t^{**}-A (since T must be nonnegative), and our new problem is relevant. This is discussed at length in what follows.

Now we will perform a transformation of $U(T)$ which will facilitate comparison with our previous objective functions. $U(T)$ differs from the Fisher and Faustmann objective functions in that, first, it *begins* with trees of age A (instead of zero) and it discounts *future* benefits back to the time when the trees are at age A (again instead

7. Indeed, most forests encountered by original American settlers were old-growth stands, and such stands still exist.

8. The second order condition for an interior maximum is identical to that for the Fisher problem, namely equation 3 evaluated at $t = A + T$. This is due to the fact that K is constant for all t.

of zero). Consider, then, discounting $U(T)$ back to age zero and adding the integral of the flow of standing value services from the forest between ages zero and A, also discounted back to age zero. This yields[9]

$$e^{-rA}U(t) + \int_0^A e^{-rw}F(w)dw = \int_0^{A+T} e^{-rw}F(w)dw + e^{-r(A+T)}G(A+T) + e^{-r(A+T)}K$$

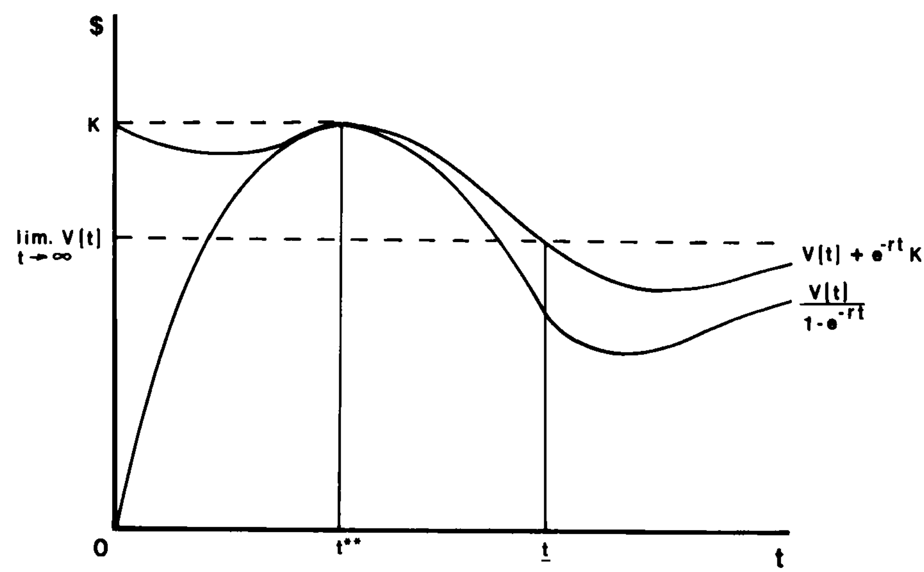
$$= V(A+T) + e^{-r(A+T)}K.$$

We have simply altered $U(T)$ by a linear transformation, $aU(T) + b$, with $a = e^{-rA}$ and $b = \int_0^A e^{-rw}F(w)dw$. Note that a and b are both *constants* given A. Since a is positive, maximizing $aU(T) + b$ is equivalent to maximizing $U(T)$.

In what follows, let $t = A + T$, the age of the forest at the time of the first cut, as before. Now, of course, the *time* until the first cut (T) is different from the *age* of the forest at the first cut (t). In the Fisher and Faustmann problems, they were identical and denoted by t (small case).

Consider $V(t) + e^{-rt}K$ in the context of figure 5, redrawn as figure 6. This function reaches maxima (of K) at $t = 0$ and $t = t^{**}$. $t = 0$ implies that $T = -A$. This is possible only if $A = 0$, and choosing $T = 0$ just means that we cut our initially bare forest immediately (which yields zero net value by assumption) and wait t^{**} years until the next cut, so we simply have the Faustmann problem and attain its maximum, K. If A exceeds zero (which is the case being considered), however, t must be positive, and the $t = 0$ solution is unattainable.[10] If A is positive but less than t^{**}, it is optimal to choose T to be $t^{**}-A$ (so $t = t^{**}$), yielding a value of K for our transformed objective function.

FIGURE 6



9. The key step in this derivation is to let $w = A + x$ in the first term of $e^{-rA}U(T)$, namely $\int_0^T e^{-r(A+x)}F(A+x)dx$, and change the variable of integration to w.

For any attainable level of t other than t^{**}, the value of $V(t) + e^{-rt}K$ must be less than K. This is because in these cases, the first cut will not occur when the forest is of optimum age t^{**}, although all subsequent cuts will. In the Faustmann objective function, however, *all* cuts would be of suboptimal length, so $V(t)/(1 - e^{-rt})$ falls short of $V(t) + e^{-rt}K$ at all non-optimal t (except at $\lim_{t\to\infty}$).

For the example drawn, optimal behavior may be summarized in terms of three cases. First, if A is less than or equal to t^{**}, it is best to cut at $T = t^{**}-A$ (*i.e.*, $t = t^{**}$). Second, if the forest has aged past t^{**} years, but less than $\underline{t}$ (see figure 6) years, it is optimal to cut immediately (at $T = 0$ or $t = A$), since the highest value of the objective function attainable from then on can be had only by immediate harvest. Lastly, if $A > \underline{t}$ years, it is preferable never to cut. This is because the objective function reaches a maximum at $\lim_{t\to\infty} V(t)$ on the relevant interval $\underline{t} < t$.[11] Thus, even if regular clearcutting is desirable for an initially barren tract, optimal behavior may imply never harvesting the same land occupied by an old growth forest. This is because in the former case, standing values are low whereas in the latter, standing values are high enough to exceed the gains from cutting the trees.

REFERENCES

Arrow, K., and Fisher, A., "Preservation, Uncertainty, and Irreversibility," *Quarterly Journal of Economics*, 1974, *87*, 312-319.

Gaffney, M., "Concepts of Financial Maturity of Timber and Other Assets," A.E. Information Series No. 62, Dept. of Agricultural Economics, North Carolina State College, 1960.

Hartman, R., "The Harvesting Decision When a Standing Forest Has Value," *Economic Inquiry*, 1976, *14*, 52-58.

Samuelson, P., "Economics of Forestry in an Evolving Society," *Economic Inquiry*, 1976, *14*, 466-492.

10. Of course, $A > 0$ implies that the maximum value of $U(T)$ exceeds that of the other two objective functions, since, for example, we could cut immediately (which provides positive stumpage value) and obtain K in addition to this, so that the maximum of $U(T)$ exceeds K. In words, it is preferable to start out with standing timber instead of barren land, since the value from the first cycle is obtained earlier and additional cycles can be started sooner.

11. The problem of an initially standing forest can also be considered in the context of figure 3, with rK (a constant) added to the $rG(t)$ function and the vertical axis shifted to the right by the initial age for the forest, A (see equation 5). This is left as an exercise for the interested reader.