Biodiversity Conservation in Commercial Boreal Forestry: The Optimal Rotation Age and Retention Tree Volume

Erkki Koskela, Markku Ollikainen, and Timo Pukkala

Abstract: This article examines biodiversity conservation in commercial boreal forests. The means of promoting biodiversity are green tree retention and prolonged rotation age, which create dead and decaying wood artificially and via natural mortality, respectively. We extend the Hartman model to cover biodiversity benefits and to allow for leaving retention trees standing at the final felling. We first characterize qualitatively the socially optimal choice of the harvest volume and rotation age. We then assess empirically the optimal solution in a simulation model calibrated to the Finnish forestry for a pine stand. We find that biodiversity conservation increases the socially optimal rotation age beyond the Faustmann rotation age. The optimal volume of retention trees increases (decreases) with biodiversity valuation (timber price). The optimal retention volume is higher than suggested by the current forest management recommendations or certification systems. FOR. SCI. 53(3): 443–452.

Keywords: rotation, green tree retention, decaying and dead wood

IN COMMERCIAL BOREAL FORESTS, forest management has replaced natural disturbances in forest renewal. Unlike disturbance dynamics, prevailing even-aged management with clearcutting destroys the whole generation of old trees to create a new even-aged stand. As a result, the volume of dead and decaying wood has considerably decreased in Scandinavian boreal forests (Siitonen 2001). This threatens severely the viability of many old-growth species that depend directly or indirectly on dead wood (Similä et al. 2003).

Since the late 1990s, biodiversity maintenance in commercial boreal forests is increasingly based on creating uneven-aged structural elements in even-aged stands. Especially, the aim is to increase volumes of dead and decaying wood both artificially and via prolonged rotation ages. Moreover, multiple harvesting methods are favored to mimic more closely forest structure and disturbance dynamics. Selective harvesting and designing small clearing areas are the means of keeping spatially heterogeneous and uneven-aged forestry structures and ensuring forest cover continuum (see e.g., Kuuluvainen 2002).

One of the key means of promoting biodiversity in Scandinavia is green tree retention (GTR). GTR refers to trees that are permanently left uncut at final felling. Retention trees are left to grow, die, and decay in the new replanted stand to provide dead and decaying wood as habitats to endangered old-growth species. In Finland and Sweden leaving retention trees (5–10 tall, old trees per hectare) are recommended by national forest laws. Moreover, forest certification systems (Forest Stewardship Council, FSC, and the Finnish Forest Certification System, FFCS) in these countries require the same. Some provinces in Canada have similar requirements, and similar plans exists in the United States (for a survey, see Vanha-Majamaa and Jalonen 2001).

In this article we examine how biodiversity conservation affects the traditional even-aged forest management. More specifically, we ask: What is the socially optimal rotation age and retention trees volume when society values both harvest revenue and biodiversity benefits? Answering this question requires extending the existing rotation models to cover the possibility of leaving trees standing at the final felling, because in the traditional Faustmann (1849) and Hartman (1976) models, clearcutting is always optimal. For this purpose we extend the Hartman rotation framework to include biodiversity benefits from retention trees and analyze how the rotation age and retention volume are chosen simultaneously [1].

We first characterize a theoretical solution for the problem. Finnish data are then used to investigate empirically the socially optimal volumes of retention trees and biodiversity-adjusted rotation ages, and compare them to the conventional Faustmann harvesting solution. Using a forestry simulation model allows us to provide some additional insights beyond the theoretical model. The forestry simulation model includes the death process of trees and also natural regeneration by seeds from surrounding stands (for which no analytical solution can be provided). The former process provides additional dead wood during the rotation period and the latter brings an admixture of several species to an area that is planted or sowed using one tree species only. Therefore, we can assess empirically how much these natural processes augment forest management

Erkki Koskela, Department of Economics, PO Box 17 (Arkadiankatu 7), Fin-00014 University of Helsinki, Finland—erkki.koskela@helsinki.fi. Markku Ollikainen, Department of Economics and Management, PO Box 27, Fin-00014 University of Helsinki, Finland—markku.ollikainen@helsinki.fi. Timo Pukkala, Faculty of Forestry, University of Joensuu, PO Box 111, FIN-80101 Joensuu, Finland—timo.pukkala@joensuu.fi.

Acknowledgments: The authors gratefully acknowledge useful comments by three referees and an associate editor. Koskela thanks the Research Unit of Economic Structures and Growth (RUESG), the Yrjö Jahnsson Foundation and the Academy of Finland (Grant No. 1109089) for financial support. Ollikainen thanks Academy of Finland for Grant No. 204476 for the position of Senior Researcher.

Manuscript received October 31, 2006, accepted November 14, 2006

in biodiversity conservation and, thereby, affect costs and benefits of forest management.

Economic analyses of biodiversity conservation in forestry are sparse. Most often, biodiversity is analyzed in terms of species preservation using site selection models or ecosystem management and other interdependence frameworks [2]. Closest to our stand-based rotation analysis are empirical studies by Ranius et al. (2005), Jonsson et al. (2006), and Wikström and Erikson (2000). These studies analyze the cost-effective means of creating decaying and dead wood in commercial boreal forests. They include GTR as one option to produce dead wood. They do not, however, provide any specific elaboration of the rotation framework, nor do they focus on the socially optimal solutions.

To anticipate our results, the socially optimal rotation age is longer and the volume of unharvested trees larger than in the private optimum, in which retention trees are not left. In our empirical application to Finnish forestry we demonstrate that biodiversity conservation increases the optimal rotation age relative to the Faustmann age. The difference in rotation ages depends on the chosen interest rate, varying from 35 years for a 1% interest rate to 3 years for a 3% interest rate. Under our Finnish estimate of biodiversity valuation in commercial forests, i.e., outside forest conservation areas, the volume of retention trees per hectare is 10 cubic meters and consists of about 30 trees. The optimal green tree retention increases (decreases) rapidly with biodiversity valuation (timber price).

The rest of the article is structured as follows. The next section deals with our description of the retention trees and their decaying process and provides an analysis of both the socially optimal choice of rotation age and retention trees. Subsequent sections provide a numerical application for the case of Finnish forestry and a brief concluding discussion.

Socially Optimal Tree Retention and Biodiversity

In this section we introduce biodiversity benefits into the Hartman model (for its general properties, see Hartman 1976, Strang 1983, Koskela and Ollikainen 2001a). Despite the fact that research knowledge on the development of biodiversity benefits is still scarce, we provide a steady-state analysis following the logic of conventional rotation models.

Green Tree Retention, Rotation Age, and Biodiversity in Boreal Forests

The notion of green tree retention is of relatively young origin. It dates back to the 1990s and emerged from the need to develop forest management methods that are more capable of sustaining biodiversity than conventional clearcutting. GTR differs from conventional shelterwood or seed systems in that retention trees are left permanently unharvested. GTR attempts to mimic and restore natural disturbance regimes in commercial boreal forests.

In general, GTR has the following objectives (Franklin et al. 1996). First, by creating uneven-aged structures and increasing the long-run amount of dead wood, it increases structural variation in the tree stand. Second, it enhances connectivity on a landscape level. There are generally two basic ways of implementing GTR, either dispersed or aggregated retention. The former means that trees are distributed evenly over the clearcutting area and the latter implies that trees are left in groups. Aggregated retention is typically recommended in Finland, Sweden, and Norway. Moreover, the most frequently saved trees are aspen, birch, elm, and related hardwood tree species, which sustain higher numbers of species than softwood tree species and on which most of the red-list species rely.

The main biodiversity benefit from retention trees is a steady flow of deciduous trees and dead wood, which provides habitat for many threatened old-growth species, especially many red-listed beetles (Ehnström 2001). Even individual trees will increase the amount of snags and downwood logs, which are important habitats for many saproxylic species (Jalonen and Vanha-Majamaa 2001). Recent studies in Scandinavian boreal forests indicate that abundance of beetles and fungi is on average the same in harvested areas with retention trees and dead and decaying wood as in old-growth forests, which contain similar dead and decaying wood (see, e.g., Jonsell et al. 2004, Junninen et al. 2006).

GTR can be used to promote understory vegetation, especially vascular plants, although their relative abundance may change, provided that the retention volume is big enough (see, e.g., Jalonen and Vanha-Majamaa 2001, Koivula 2002). There is also evidence that GTR actually promotes the lifeboating of species and processes and is beneficial to species that are sensitive to forest management operations. For instance, some lichens (such as *Lobaria pulmonaria*) can sustain in living retention trees over a long time (see, e.g., Hazell and Gustafsson 1999 and Hilmo 2002).

Relative to GTR, prolonged rotation ages produce dead and decaying wood via natural mortality of trees. In a recent study, Ranius et al. (2005) found that by increasing the rotation age of Norway spruce in three experimental areas by 10, 25, and 50% raises the amount of dead and decaying wood on average by 1.5, 4, and 9 m^3, respectively. Thus, producing greater amounts of dead and decaying wood requires rather long rotation ages, which is more costly than creating dead wood artificially. Longer rotation ages serve, however, for many other purposes than dead wood creation. Prolonged rotation ages promote other habitats by creating older shaded stands with their specific microclimate promoting the abundance of more common old-growth species.

A natural way of modeling biodiversity benefits from the rotation age and retention trees is to modify the felicity function of the basic Hartman model. We assume that biodiversity benefits can be expressed as a sum of the benefits accruing from the age of the stand becoming harvested, and the retention trees, which reach their biological maturity and decay during the next rotation period. Additive form reflects the fact that prolonged rotation ages and artificial creation of dead wood serve two different purposes, that is, they increase habitats for two different sets of old-growth species.

We denote the rotation age and retention tree volume to be chosen in the current rotation period by T and G, respectively. Let $\bar{G}$ indicate inherited volume of retention trees from the previous rotation period. We use $\bar{T}$ to denote the rotation age that is chosen in the next rotation period. Using this notation, biodiversity benefits, BB, from retention trees can be expressed as

$$\begin{aligned}\mathrm{BB} &= a(T) + \nu(T, G)\\ &= \int_0^T \mathrm{F}(x)e^{-rx}\,\mathrm{d}x + \int_0^T \mathrm{B}(\bar{G}, x)e^{-rx}\,\mathrm{d}x\\ &\quad + e^{-rT}\int_0^{\bar{T}} \mathrm{B}(x, G)e^{-rs}\,\mathrm{d}x. \end{aligned} \tag{1}$$

The first term, $a(T) = \int_0^T \mathrm{F}(x)e^{-rx}\,\mathrm{d}x$, is the conventional amenity valuation function but applied here to biodiversity. The properties of this amenity valuation function have been thoroughly discussed in the previous literature. Because our focus is on biodiversity, we legitimately assume that older stands yield higher biodiversity benefits, $\mathrm{F}'(T) > 0$ [3]. Recall, this reflects the above-mentioned fact that some old-growth species (such as lichens) require living old trees.

The latter term, $\nu(T, G)$, is a product of two terms. The first term describes the biodiversity benefits (accruing in the current rotation period) from retention trees inherited from the previous harvest. The second term includes benefits accruing during the next rotation period from the retention trees left at the end of the current rotation period.

We make the following assumptions concerning biodiversity benefits

$$\begin{aligned}\nu_T &= \hat{\mathrm{B}}(T, G, r)\\ &\equiv e^{-rT}\left[\mathrm{B}(\bar{G}, T) - r\int_0^{\bar{T}} \mathrm{B}(x, G)e^{-rx}\,\mathrm{d}x\right] > 0, \end{aligned} \tag{2a}$$

$$\nu_{TT} = \hat{\mathrm{B}}_T(T, G, r) = -r\nu_T + e^{-rT}\,\mathrm{B}_T(T, \bar{G})) < 0, \tag{2b}$$

$$\nu_G = e^{-rT}\int_0^T \mathrm{B}_G(x, G)e^{-rx}\,\mathrm{d}x > 0,$$

$$\nu_{GG} = e^{-rT}\int_0^T \mathrm{B}_{GG}(x, G)e^{-rx}\,\mathrm{d}x < 0, \tag{2c}$$

$$\nu_{TG} = \nu_{GT} = -re^{-rT}\int_0^{\bar{T}} \mathrm{B}_G(x, G)e^{-rx}\,\mathrm{d}x < 0. \tag{2d}$$

Thus, marginal biodiversity benefit in (2a) is a positive difference in biodiversity benefit from green retention between the beginning and the end of the second rotation period. According to (2b), marginal biodiversity benefits are decreasing in the age of retention trees. The same is assumed to hold true for marginal benefits from the volume of green tree retention, G, by (2c). Finally, Equation 2d indicates that the cross-derivative between T and G is negative.

Socially Optimal Biodiversity Management at the Stand Level

Consider now a social planner who replants bare land and decides how many retention trees to leave at the final felling in a steady state [4]. The bare land contains a given amount of retention trees $\bar{G}$ from the previous harvest. The new stand to be planted grows according to a growth function, $f(T, \bar{G})$, which has the exogenous inherited volume of retention trees as an additional argument. The growth function exhibits conventional properties in rotation age; $f_{\mathrm{T}} > 0$ and $f_{\mathrm{TT}} < 0$ over the relevant range of the rotation age.

In addition to the rotation age, the planner has to decide how many retention trees to leave at the end of the rotation period. Therefore, the planner must know how retention trees affect the conventional even-aged forest management. Beyond producing biodiversity benefits, retention trees affect profitability of forestry via two channels. First, retention trees decrease the growth of the new stand planted at the beginning of the next rotation period, $f(\bar{T}, G)$, in an increasing fashion, so that $f_{\mathrm{G}} < 0$ and $f_{\mathrm{GG}} < 0$ [5]. Second, regeneration costs of the new stand are affected by the volume of retention trees. We postulate a regeneration cost function, c, such that $c = \varepsilon c(G)$, with $c' > 0$ and $c'' > 0$, where $\varepsilon \geq 1$ is a cost-shifting parameter to facilitate comparative statics. During the basic analysis we normalize $\varepsilon = 1$.

Under a constant timber price p and the real interest rate r, the planner's economic problem is to maximize social welfare from forestry by choosing the optimal rotation age and optimal retention tree volume, that is,

$$\begin{aligned}\max_{T,G} \mathrm{SW} = &\left[pe^{-rT}[f(T, \bar{G}) - G] - c + \int_0^T \mathrm{F}(x)e^{-rx}\,\mathrm{d}x\right.\\ &\left.\quad + \int_0^T \mathrm{B}(\bar{G}, x)e^{-rx}\,\mathrm{d}x\right](1 - e^{-rT})^{-1}\\ &+ e^{-rT}\left[\int_0^{\bar{T}} \mathrm{B}(x, G)e^{-rx}\,\mathrm{d}x\right.\\ &\left.\quad + pf(\bar{T}, G)e^{-r\bar{T}} - c(G)\right](1 - e^{-rT})^{-1}. \end{aligned} \tag{3}$$

To make the linkage between the current and next rotation period more transparent in Equation 3, we have collected the effects of retention trees extending to the next rotation period into the second bracket term. These effects include biodiversity benefits, reduced growth of the new stand, and increased regeneration costs.

The first-order conditions for this problem can be expressed as

$$\mathrm{SW}_G = 0 \Leftrightarrow -p + \int_0^{\bar{T}} \mathrm{B}_G(x, G)e^{-rx}\,\mathrm{d}x + pf_G e^{-r\bar{T}} - c'(G) = 0, \tag{4a}$$

$$\mathrm{SW}_T = 0 \Leftrightarrow pf_T - rp[f(T, \bar{G}) - G] + \mathrm{F}(T) + \mathrm{B}(\bar{G}, T),$$

$$- r\left[\int_0^{\bar{T}} \mathrm{B}(x, G)e^{-rx}\,\mathrm{d}x + pf(\bar{T}, G)e^{-r\bar{T}} - c(G)\right]$$

$$- r\mathrm{SW} = 0. \quad (4b)$$

Economic interpretation of the first-order conditions is as follows. From (4a), the optimal volume of retention trees is chosen to equate the present value of the sum of the marginal utility from retention trees accruing during the next rotation period with the sum of the loss of the harvest revenue, decreased future growth, and increased regeneration costs. Equation (4b) requires that the optimal rotation age is chosen so that the marginal return of delaying the harvest by one unit of time equals the opportunity cost of delaying harvesting. In our case both the marginal return and the opportunity cost contain new terms. While the former is defined by the sum of the harvest revenue and biodiversity benefits during the first and the second rotation period, the latter includes the interest cost both on standing timber and land but also the interest cost on future growth and regeneration costs.

Using the notation for marginal biodiversity benefits from green retention adopted in Equation 2a, the second-order conditions can be expressed as

$$\mathrm{SW}_{GG} = \int_0^{\bar{T}} \mathrm{B}_{GG}(x, G)e^{-rx}\,\mathrm{d}x + pf_{GG}e^{-r\bar{T}} - c''(G) < 0, \quad (5a)$$

$$\mathrm{SW}_{TT} = pf_{TT} - rpf_T + \mathrm{F}'(T) + \hat{B}_T < 0, \quad (5b)$$

$$\mathrm{D} = \mathrm{SW}_{GG}\mathrm{SW}_{TT} - \mathrm{SW}_{TG}^2 > 0, \quad (5c)$$

where $\mathrm{SW}_{TG} = \mathrm{SW}_{GT} = 0$. Given our assumptions, condition (5a) holds automatically. Condition (5b) holds under the conventional Hartman model requirement according to which biodiversity benefits must not be too big relative to other terms of (5b).

We next analyze how the volume of retention trees and rotation age depend on exogenous parameters. Comparative statics will differ partly from the conventional rotation analysis, as now we have 2×2 equation system. However, given that the cross-derivative between the rotation age T and the volume of retention trees G is equal to zero, comparative statics of T remains conventional. We allocate the details in the appendix and just indicate here the signs for changes in timber price p, the real interest rate r, and regeneration cost c (calculated as a change of the cost-shift parameter ε).

$$T = T(\underset{-}{p}, \underset{-}{r}, \underset{+}{c}), \quad G = G(\underset{-}{p}, \underset{-}{r}, \underset{-}{c}). \quad (6)$$

From (6), the rotation age shortens and the amount of retention trees decreases when timber price increases. The former effect is well known from the basic Hartman model. The latter effect results from the fact that a higher timber price increases the profitability of harvesting, making it more costly to leave retention trees for biodiversity conservation purposes. A higher interest rate works qualitatively in a similar fashion by decreasing the opportunity cost of both delaying harvesting and leaving retention trees. Finally, while a higher regeneration cost lengthens the rotation age in a conventional way, it decreases the optimal retention volume.

How do equations (4a) and (4b) governing the socially optimal rotation age and green tree retention relate to the behavior of private landowners? The answer depends on how private landowners value amenity benefits. Traditionally, two hypotheses have been used: landowner maximizes either the present value of harvest revenue from timber production (the Faustmann rotation model), or the present value of the sum of harvest revenue and amenity services (the Hartman model). There is abundant evidence that landowners value amenities [6]. However, it is uncertain to what extent this valuation covers the relevant aspects of forest biodiversity. Although landowners may sometimes put value on some species or land areas, it is plausible to think that they do not take into account the whole spectrum of biodiversity. In the empirical part we use the Faustmann model as the benchmark of the private solution.

We ask next: How great is the optimal number of retention trees and how does the socially optimal rotation age relate to the privately optimal rotation age under a plausible description of actual forestry and social valuation of biodiversity? These are the issues we study in the next section by using a forestry simulation model.

An Empirical Application to Finnish Forestry

Drawing on the theoretical framework outlined above we develop in this section a forestry simulation model. We use the model to assess empirically the length of the rotation period and the volume of retention trees, and their behavior in terms of exogenous parameters.

The Numerical Simulation-Optimization System

A forestry simulation model was developed for numerical optimization of the rotation length and amount of retention trees. The simulation system calculates the value of the objective function with the combination of our decision variables, while the optimization system gradually modifies the values of decision variables based on the feedback from the simulation system, and eventually finds the optimal rotation length and basal area of retention trees. The algorithm developed by Hooke and Jeeves (1961), and adopted from Osyczka (1984), for nonlinear derivative-free optimization was used (see Pukkala and Miina 1997 for more details).

Simulation of stand development is based on individual trees. The simulation begins with bare land with no retention trees and no deadwood. The stand establishment is predicted with the models of Miina and Saksa (unpublished). The models predict the number of surviving planted trees per hectare, as well as the amount of naturally regenerated pine, spruce, birch, and hardwood coppice. Stand development is simulated in 5-year time steps. The models of Björkdahl (1973), Mielikäinen and Valkonen (1995), and

Valkonen et al. (2002) are used to predict the juvenile height growth of seedlings up to the sapling stage (dbh 5 cm), after which the individual-tree growth models of Nyyssönen and Mielikäinen (1978) are used. A tending treatment is simulated at a stand age of 5 to 20 years (depending on site and planted tree species). It removes all coppices and regulates the frequencies of other trees. The stand establishment and tending costs, used in the simulator, are based on cost statistics (Finnish Forest Research Institute 2002). In the simulations of this study, which pertain to pine stands on rather poor sites, the stand establishment cost was 702 €/ha. The tending cost (TC) depends on tree size and amount of removed trees, which jointly determine the payment for the worker. We approximate this cost by the following quadratic equation

$$\mathrm{TC} = 187.252 - 53.827D_{\mathrm{stump}} + \frac{2.472D_{\mathrm{stump}}N_{\mathrm{removed}}}{1000} + 11.190\,D_{\mathrm{stump}}^2, \qquad (7)$$

where D_{stump} is the mean stump diameter (cm) and N_{removed} is the number (trees per hectare) of removed trees. Thus, increasing the number and size of removed trees increases the labor costs, and thereby the tending cost [7].

The self-thinning models of Pukkala and Miina (1997) are used to calculate the maximum stand density for a given mean tree diameter. Mortality occurs when this limit is passed, creating one or several cohorts of standing deadwood (snags). During a time step, a part of a snag cohort forms a downwood cohort, its relative frequency being equal to the probability of falling down (Pukkala et al. 1997). Both snag and downwood cohorts decompose with time, the decomposition rate being higher for downwood than for snags.

Stand development is simulated until the rotation age is reached, after which a final cut is simulated. Retention trees may be left to continue growing, depending on the current input value of the retention tree parameter. The roadside value of the removed volume (gross income) is calculated using user-supplied unit prices of different timber assortments. This study used the mean roadside prices on Jan. 31, 2006 (www2.mhy.fi/mhy/puumarkkinat; 31.1.2006), which were as follows (sawlog/pulpwood): pine 46.26/23.97 €/m^3, spruce 47.91/30.36 €/m^3, and birch 43.67/23.88 €/m^3). The assortment volumes are calculated using the taper functions of Laasasenaho (1982) and the harvesting cost is calculated with the models of Valsta (1992).

Simulation is continued for three additional rotations, keeping the deadwood cohorts and retention trees of the previous rotation(s). The simulation is otherwise similar as during the first rotation except that there are now initial retention tree cohorts and initial deadwood. The growth of retention trees is simulated using the growth models of Nyyssönen and Mielikäinen (1978). A part of a retention tree cohort is wind-thrown and another part may die of senescence during a time step, the relative frequencies of these new cohorts depending on the probabilities of these events (Pukkala et al. 1997). Dead retention tree cohorts decompose with the same rate as the other deadwood cohorts. A standing deadwood cohort originating from a retention tree cohort falls down with the same probability as other snags.

As in the theoretical model, retention trees are assumed to reduce the growing space that is available to the other trees: Their effect to the other growing stock is simulated through an area multiplier. The share of growing space taken by retention trees is equal to the ratio of the basal area of retention trees to the maximum stand basal area that the site can sustain. If the basal area of retention trees decreases because of mortality, the growing space available to other trees increases creating accelerated growth. It is assumed that the other trees can fully use the growing space left by dead retention trees. This kind of simulation is reasonable when retention trees occur in dense and small groups, which is the current practice.

Retention trees do not affect stand establishment and tending costs. They may cause physical obstacles, but they reduce the treated area, which means that a nil net effect is a justified assumption. Retention trees increase harvesting cost per cubic meter, because harvesting cost per cubic meter increases with decreasing harvested volume (Valsta 1992).

In addition to costs and incomes, the simulator calculates a biodiversity index for the stand at every time point. The biodiversity index is as a weighted sum of scaled values of various structural elements present in the stand. The structural elements are volumes of different tree species, timber volumes in 10 cm-diameter classes, and volumes of deadwood components (standing deadwood and downwood of different tree species). Each element increases the index fast up to a certain level ("satisfactory amount"), after which its additional contribution becomes very small.

The monetary value (€ $\mathrm{ha}^{-1}\ a^{-1}$) of the maximum biodiversity index is a user-supplied parameter. We used Finnish estimates for valuation of biodiversity conservation as a part of normal practices in commercial forestry. A contingent valuation study by Pouta (2005) suggests that the mean of the value of willingness to pay (WTP) for an increase of retention trees from the current 15 to 30 would be €40 [8]. We calibrate our quadratic biodiversity valuation function to reflect this estimate as follows. The value (VAL) of the biodiversity index was calculated from the equation $\mathrm{VAL}=\mathrm{WTP}(\mathrm{BD}/\mathrm{BD}_{\max})$, where WTP is the value of the maximum biodiversity index ($\mathrm{BD}_{\max}$) of the stand [9].

The forestry simulation model was operated as follows. In a steady-state optimization, the objective function value was calculated from the last simulated rotation, which was assumed to be repeated to infinity. The other rotations were used to initialize the steady-state amounts of deadwood and retention tree cohorts present in the beginning of the last rotation.

Simulation Results

Our results are solved for pine under typical growth conditions in southern Finland without thinning treatment. In Table 1 we report three alternative harvesting solutions in terms of the rotation age, harvested timber volume, retention volume, net income, and biodiversity benefits from

Table 1. The optimal harvesting solution in the privately and socially optimal solutions

	$r = 1\%$	$r = 2\%$	$r = 3\%$
Private optimum			
Rotation length	76	71	64
Retention volume, m^3/ha	0	0	0
Mean annual harvest, m^3/ha	4.41	4.41	4.30
Mean annual net income, €/ha	117	110	100
Timber benefit, €/ha	6,989	1,681	288
Biodiversity benefit, €/ha	0	0	0
Total benefit, €/ha	6,989	1,681	288
Social optimum 1 (valuable trees left)			
Rotation length	101	73	67
Retention volume, m^3/ha	10.5	9.0	0
Mean annual harvest, m^3/ha	4.19	4.12	4.21
Mean annual net income, €/ha	123	109	104
Timber benefit, €/ha	6,279	1,585	263
Biodiversity benefit, €/ha	2,752	667	286
Total benefit, €/ha	9,031	2,252	549
Social optimum 2 (poor trees left)			
Rotation length	96	73	66
Retention volume, m^3/ha	15.4	24.5	21.3
Mean annual harvest, m^3/ha	4.66	3.90	4.05
Mean annual net income, €/ha	121	102	99
Timber benefit, €/ha	6,400	1,436	236
Biodiversity benefit, €/ha	3,402	1,468	832
Total benefit, €/ha	9,802	2,904	1,069

retention trees. The real interest rate is assumed to vary between 1 and 3%.

The first solution is the private optimum, defined by the Faustmann model. The second solution is the social optimum reflecting our theoretical model. Here we assume that retention trees are economically valuable trees. The third solution represents a minor modification of the social optimum. We now assume that via natural regeneration from surrounding forest, the stand also contains other tree species, notably broadleaf trees. We postulate that a maximum of 20% of stand volume has a lower economic value than the mean timber price. The choice of retention trees is made among these trees, so that economically more valuable trees can be harvested. Provided that they are about the same size as the economically more valuable trees, these trees perform equally well in biodiversity maintenance, sometimes even better [10].

Using the value of 0.02 as our benchmark interest rate, the privately optimal rotation age is 71 years. Given our growth function, this implies 4.4 m^3 as the mean annual harvest and 110 €/ha as the respective net income. The total site expectation value is 1,681 €/ha. Naturally, the volume of retention trees is zero in the Faustmann model. The privately optimal rotation age decreases in the real interest rate, as the model suggests.

In the social optimum 1 with economically valuable retention trees, the optimal rotation age is 73 years, being only slightly longer than the privately optimal age. The most important difference between these two solutions is, naturally, that the amount of retention trees is now positive, being 9.0 m^3/ha. Assuming that the breast height diameter of retention trees is 22 cm, this gives 27 retention trees per hectare. Thus, given that at the final felling there are 800–900 trees/ha, this means that 3% of wood biomass is left standing in small groups in the area. The mean annual harvest slightly decreases because of retention trees, but the mean annual net income remains almost the same, because the size of harvested timber increases with the longer rotation age (recall, the Faustmann solution lies below the MSY, so that a longer rotation implies a higher volume of timber).

The site expectation value is higher in the private optimum reflecting the fact that the social value of forests is higher than private valuation. The difference to the private solution is 571 €/ha. Biodiversity benefits account for about 30% of the total benefit. As in the private solution, the optimal rotation age is decreasing in the interest rate as long as the value of land remains positive. Also, retention tree volume is decreasing in the interest rates, as was demonstrated in the theoretical model. Note, finally, that both the privately and socially optimal rotation ages are slightly shorter than those under current forestry practice. One reason for this is that, following our theoretical models, we omit commercial thinning, which tends to postpone the optimal age for final felling (see, e.g., Pukkala et al. 1998).

The social optimum 2 with poor trees left as retention trees has the same rotation age but a much higher retention volume than in the social optimum 1. The retention volume is expanded up to 24.5 m^3 (73 trees/ha). Now both mean annual harvest and net income are reduced below the social optimum 1, because under the given biodiversity valuation, lower prices from these trees imply that a greater share of forest benefits will be derived as biodiversity benefits. There are now €1,468 accounting for slightly over 50% of the site expectation value.

We conducted a sensitivity analysis of the socially optimal solutions to see how they change when biodiversity (BD) valuation, timber price, and regeneration costs change. Their values were changed by 20% below and above the basic run estimates used in Table 1. The results are collected in Table 2; we calculated them using 2% real interest rate as our reference point.

The rotation age in the social optimum 1 is very insensitive to changes in exogenous variables. Only a lower timber price affects the rotation age by lengthening it. In contrast to the rotation age, the optimal retention volume varies with exogenous parameters and in the directions pointed out by the theoretical model. Increasing biodiversity valuation by 20% increases the retention volume by 1 m^3. This means that three additional trees (0.5% more of the wood biomass) are left standing at the final felling. Interestingly, the retention tree volume is very sensitive to biodiversity valuation; lowering valuation by 20% reduces it to zero. Changes in the timber price clearly affect the retention volume, while the regeneration costs have no effect on it.

Mean annual harvest and income behave accordingly. Thus, we find that under a higher biodiversity valuation both the mean annual harvest and mean annual net income decrease relative to Table 1, because a considerable part of the growing space is used by retention trees. Biodiversity benefits account for half of the overall benefits. Hence, we can conclude that the higher biodiversity valuation, the more forest management approaches selective harvesting and small amounts of harvested timber. Finally, the site expectation value exceeds (falls short of) the basic run under higher (lower) biodiversity valuation and timber prices lower (higher).

In the social optimum 2, the rotation age turns out more sensitive to changes in the exogenous parameters. While a higher biodiversity valuation has no impact on the rotation age, the price and regeneration costs affect it just like the theory predicts. The impacts of the exogenous parameters on the retention volume are strong. A higher biodiversity valuation increases this volume by 1.5 m^3, leading to 78 trees left standing. In accordance with the theory, higher regeneration clearly decreases the optimal retention volume.

We finally compare our findings with the requirements of the current certification systems used in Finland. FSC requires that at the final felling the landowner must leave at least 10 tall trees with a diameter size of at least 20 cm (FSC Finland 2002) [11]. This amount does not include the volume of deadwood. Moreover, tall broadleaf trees (except birch, which is common in Finland) must be left uncut unless the overall retention volume exceeds 10–20 m^3. The alternative system, FFCS in turn requires that (in addition to preserving key biotopes required by the Finnish forest law) at least 5 to 10 trees with the diameter size at least 10 cm be left standing (www.ffcs-finland.org). Moreover, neither certification system requires prolonged rotation ages. Using our social optimum 1 (valuable trees left) and 2 (poor trees left) with 2% real interest rate as benchmarks, Table 3 gives the comparisons for FSC and FFCS in terms of rotation age, retention volume, and number of trees.

FSC provides retention volumes that come closer to our benchmark than FFCS. In cases where the stand contains a high broadleaf tree volume it achieves the optimum and may even exceed it. Requirements under FFCS imply a much lower retention tree volume than our social optimum. Finally, we want to emphasize that both certification systems have many other criteria that directly or indirectly affect biodiversity and that are not captured by GTR.

Conclusions

Prolonged rotation ages and green tree retention are becoming the key means of promoting biodiversity in commercial boreal forests in Scandinavia. While retention trees create decaying and deadwood artificially, prolonged rotation ages produce it via natural mortality. Both aim at providing habitats for endangered old-growth species living either in decaying wood or on old living trees. In this article we asked: What is the socially optimal rotation age and retention tree volume, when the society values both harvest revenue and biodiversity benefits?

Because traditional Faustmann and Hartman models imply clearcutting, we extended the Hartman model to allow for the case where it is optimal to leave some trees standing at the final felling. This was done via biodiversity benefit function, which depends on the rotation age and retention volume. We solved the socially optimal rotation age and the

Table 2. Sensitivity analysis: Biodiversity valuation, prices, and costs

	Basic run	BD valuation		Timber price		Regener. cost	
		0.8	1.2	0.8	1.2	0.8	1.2
Social optimum 1 (valuable trees left)							
Rotation length	73	73	73	97	73	73	73
Retention volume, m^3/ha	9.0	0	9.7	11.1	8.0	9.1	9.1
Mean annual harvest, m^3/ha	4.12	4.28	4.11	4.07	4.16	4.12	4.12
Mean annual net income, €/ha	109	113	108	84	144	111	106
Timber benefit, €/ha	1,585	1,671	1,579	522	2,367	1,819	1,352
Biodiversity benefit, €/ha	667	400	888	1,262	579	668	668
Total benefit, €/ha	2,252	2,071	2,467	1,784	2,946	2,487	2,487
Social optimum 2 (poor trees left)							
Rotation length	73	75	75	82	76	73	75
Retention volume, m^3/ha	24.5	18.0	26.1	27.1	19.8	24.5	20.5
Mean annual harvest, m^3/ha	3.90	4.12	4.00	3.87	4.12	3.90	4.09
Mean annual net income, €/ha	102	108	104	71	141	105	104
Timber benefit, €/ha	1,436	1,496	1,415	554	2,201	1,669	1,243
Biodiversity benefit, €/ha	1,468	1,082	1,818	1,709	1,387	1,468	1,422
Total benefit, €/ha	2,904	2,578	3,232	2,263	3,587	3,137	2,665

Table 3. Socially optimal GTR as related to the forest certification systems in Finland

	Social optimum 1 & 2 (real interest rate 2%)		FSC	FFCS
Number of trees	27	73	10	5–10
Retention volume	9.0	24.5	≥3.3	0.7–1.4
Rotation age	73	73	71	71

volume of retention trees, and compared them with the private solution when the landowner behaves according to the Faustmann model.

We assessed the optimal solution quantitatively in the empirical application to the Finnish Forestry. We used our forestry simulation model to provide some additional insights beyond the theoretical model by including natural mortality and regeneration from surrounding forests via seeds. Thus, we could assess how much these natural processes can augment forest management in biodiversity conservation. We demonstrated that biodiversity conservation increases the optimal rotation age relative to the Faustmann case, the difference varying from 35 to 3 years, depending on the interest rate. The optimal volume of retention trees per hectare is slightly over 9 m^3 and consists of about 27 trees, but increases to 24 m^3 (78 trees) if the stand has economically low-value trees.

Contrasting our findings to the existing forest management recommendations and certification systems is quite revealing. Most often they suggest lower retention volumes than we found to be socially optimal. Moreover, given that retention trees are economically valuable, leaving them standing implies a considerable cost burden to private landowners. Existing forestry laws and forest management recommendation systems do not provide any compensation for this cost. Thus, they do not provide proper incentives to private landowners to conserve biodiversity. Consequently, there is much scope for improving biodiversity policies in Scandinavia.

Our study raises new interesting research topics. First, drawing on what was said above, it is interesting to analyze what kind of policy instruments the social planner should use to promote biodiversity conservation. Second, extending the stand level analysis to cover biodiversity management at the landscape level is an important but demanding challenge. This extension could be made by introducing interdependency between stands.

Endnotes

[1] Biodiversity is a multi-faceted term comprising species diversity, ecological (habitat) diversity, and genetic diversity. In this paper retention trees provide habitats (dead and decaying wood) to threatened old-growth species. For example, in Finland there are about 4000–5000 saproxylic species (25% of all species) that are dependent on dead wood (Siitonen 2001). Yet, commercial forestry has decreased the volume of dead wood to one-tenth of the amount in virgin forests.

[2] Most of the stand-based literature focuses on species preservation in the site selection framework (see Ando et al. 1998 for a general approach, and Stockland 1997 and Juutinen et al. 2004 for applications to forestry). Another strand of the literature is ecosystem management in forestry, see for instance, Hopf and Raphael 1993, Haight 1995, Beavers and Hopf 1999. For stand interdependence literature, see Swallow and Wear 1993, Koskela and Ollikainen 2001b.

[3] In what follows, derivatives of a function with one argument are denoted with primes, while partial derivatives of functions with more than one argument are denoted by subscripts.

[4] Our assumption implies that the "initial" stand, which the first GTR comes from, has already been harvested. The analysis of this case is conventional, yielding qualitatively similar outcomes as our steady-state formulation, so that we omit it here.

[5] Using Finnish data for Scots pine, Valkonen et al (2002) find that retention trees decrease the growth of a new stand by 9–17% along a 10-meter-long ray and estimate the overall reduction in growth as 1–2%.

[6] See, e.g., Binkley 1981, Kuuluvainen et al. (1996), and Amacher et al. (2003).

[7] Arnott and Beese (1997) have focused more closely on harvesting costs under alternative harvesting methods, including green tree retention in British Columbia coastal montane forests. They found that on average, harvesting costs as a sum cost of felling and yarding were 12% higher for GTR relative to traditional clearcutting.

[8] To be more precise, Pouta (2005) examined peoples' willingness to pay for retention trees under two alternative forest-cutting programs by using a contingent valuation method in a random utility framework. The sample size, drawn randomly from the census of Finland, was 1,100 Finns from the ages of 18 to 70. The first program entails leaving retention trees (15 to 35 trees/ha) on the cutting area. The second program adds to the volume of retention trees some additional restrictions, such as a limit to the size of large cutting areas, scenery conditions, and so on. The sample size comprised 1,100 randomly chosen respondents. The respondents had to choose between either of the two programs or the currently recommended (not mandatory) cutting practice with 15 retention trees. The costs of increased retention volume were told to be collected by taxes. Using the logit model, Pouta found that the median willingness to pay for the first program was 40 €/ha and in the larger program 65 €/ha. As our model includes only retention trees, we apply €40 as the estimate.

[9] Unfortunately, there are no suitable valuation estimates available for the other component of biodiversity valuation function (1). Therefore, we omit this from our empirical analysis.

[10] Besides pines, the stands typically comprise a number of old aspens, willows, and other leave trees. These are very important for old-growth species, yet they are economically less valuable than pines. Thus, they provide a more efficient and cheaper means to promote biodiversity than leaving pines.

[11] Regarding biodiversity, we want to mention that in addition to the criteria governing end-cutting, the landowner must preserve permanently 5% of the overall forest land area from cutting. Naturally, this requirement is an important contribution to biodiversity maintenance in commercial forests.

Literature Cited

AMACHER, G.S., M.C. CONWAY, AND J. SULLIVAN. 2003. Econometric analyses of nonindustrial forest landowners: Is there anything left to study? *J. For. Econ.* 9:137–164.

ARNOTT, J., AND W. BEESE. 1997. Alternatives to clearcutting in BC coastal montane forests. *For. Chron.* 73:670–678.

BEAVERS, M., AND J. HOPF. 1999. Spatially optimizing wildlife habitat edge effects in forest management linear and mixed-integer programs. *For. Sci.* 45:249–258.

BINKLEY, C. 1981. Timber supply from private nonindustrial forests. Yale Univ. School of Forestry and Environmental Studies Bull. No. 92. New Haven, CT.

BJÖRKDAHL, G. 1983. Höjdutveckling hos stubbskott av vårt- och glasbjörk samt tall och gran efter mekanisk röjning. Sveriges Lantbruksuniversitet, Institutionen för skogsproduktion. *Stencil* 18:54. (In Swedish).

EHNSTRÖM, B. 2001. Leaving dead wood for insects in boreal forests—Suggestions for the future. *Scand. J. For. Res.* (Suppl. 3):91–98.

FAUSTMANN, M. 1849. Calculation of the value which forest land and immature stands possesses, Reprinted in *J. For. Econ.* 1995 (1):89–114.

FINNISH FOREST RESEARCH INSTITUTE. 2002. *Finnish statistical* yearbook of forestry. Gummerus, Jyväskylä. 352 p.

FRANKLIN, J., D. BERG, D. THORNBURGH, AND J. TAPPEINER. 1996. Alternative silvicultural approaches to timber harvesting: Variable retention harvest systems. P. 111–139 in *Creating a forestry for the 21st century: The science of ecosystem management*, Kohm, K., and J. Franklin (eds.). Island Press, Washington, DC.

FSC FINLAND. 2002. *Man and the forest.* The Finnish FSC standard. In Finnish.

HAIGHT, R. 1995. Comparing extinction risk and economic cost in wildlife conservation planning. *Ecol. Applic.* 5:767–775.

HARTMAN, R. 1976. The harvesting decision when a standing forest has value. *Econ. Inquiry* 14:52–58.

HAZELL, P., AND L. GUSTAFSSON. 1999. Retention of trees at final harvest—Evaluation of a conservation technique using epiphytic bryophyte and lichen transplants. *Biol. Conserv.* 90:133–142.

HILMO, O. 2002. Growth and morphological response of old-forest lichens transplanted into a young and an old *Picea abies* forest. *Ecography* 25:329–335.

HOOKE, R., AND T. JEEVES. 1961. "Direct search" solution of numerical and statistical problems. *J. Assoc. Comput. Mach.* 8:212–229.

HOPF, J., AND M. RAPHAEL. 1993. Some mathematical programming approaches for optimizing timber age-class distributions to meet multispecies wildlife population objectives. *Can. J. For. Res.* 23:828–834.

JALONEN, J., AND I. VANHA-MAJAMAA. 2001. Immediate effects of four different felling methods on mature boreal spruce forest understorey vegetation in southern Finland. *For. Ecol. Manag.* 146:25–34.

JONSELL, M., K. NITTÉRUS, AND K. STIGHÄLL. 2004. Saproxylic beetles in natural and man made deciduous high stumps retained for conservation. *Biol. Conserv.* 118:163–173.

JONSSON, M., T. RANIUS, H. EKVALL, G. BOSTED, A. DAHLBERG, B. EHNSTRÖM, B. NORDÉN, AND J. STOKLAND. 2006. Cost-effectiveness of silvicultural measures to increase substrate availability for red-listed wood-living organisms in Norway spruce forests. *Biol. Conserv.* 127:443–462.

JUNNINEN, K., R. PENTTILÄ, AND P. MARTIKAINEN. 2006. Fallen retention aspen trees on clear-cuts can be important habitats for red-listed polypores: A case study in Finland. *Biodiversity Conserv.* 16:475–490.

JUUTINEN, A., E. MÄNTYMAA, M. MÖNKKÖNEN, AND J. SALMI. 2004. Cost-effective selection of boreal old forest reserves: A case from northern Fennoscandia. *For. Sci.* 50:527–539.

KOIVULA, M. 2002. Alternative harvesting methods and boreal carabid beetles (Coleoptera, Carabidae). *For. Ecol. Manag.* 167:103–121.

KOSKELA, E., AND M. OLLIKAINEN. 2001a. Forest taxation and rotation age under private amenity valuation: New results. *J. Environ. Econ. Manag.* 42:374–384.

KOSKELA, E., AND M. OLLIKAINEN. 2001b. Optimal private and public harvesting under spatial and temporal interdependence. *For. Sci.* 47:484–496.

KUULUVAINEN, J., H. KARPPINEN, AND V. OVASKAINEN. 1996. Landowner objectives and nonindustrial private timber supply. *For. Sci.* 42:300–309.

KUULUVAINEN, T. 2002. Natural variability of forests as a reference for restoring and managing biological diversity in boreal Fennoscandia. *Silva Fennica* 36:97–125.

LAASASENAHO, J. 1982. Taper curve and volume functions for pine, spruce and birch. *Comm. Inst. For. Fennicum* 108:74.

MIELIKÄINEN, K., AND S. VALKONEN. 1995. Kaksijaksoisen Kuusikoivu-sekametsikön Kasvu. *Metsätieteen aikakauskirja* 1995(2):81–97. In Finnish.

NYYSSÖNEN, A., AND K. MIELIKÄINEN. 1978. Estimation of stand increment. *Acta For. Fennica* 163:40. In Finnish with an English summary.

OSYCZKA, A. 1984. Multicriterion optimization in engineering with FORTRAN programs. Ellis Horwood, Chichester. 178 p.

POUTA, E. 2005. Sensitivity to scope of environmental regulation in contingent valuation of forest cutting in Finland, *For. Policy Econ.* 7:539–550.

PUKKALA, T., J. KANGAS, M. KNIIVILÄ, AND A-M. TIAINEN. 1997. Integrating forest-level and compartment-level indices of species diversity with numerical forest planning. *Silva Fennica* 31(4):417–429.

PUKKALA, T., AND J. MIINA. 1997. A method for stochastic multiobjective optimization of stand management. *For. Ecol. Manag.* 98:189–203.

PUKKALA, T., J. MIINA, M. KURTTILA, AND T. KOLSTRÖM. 1998. A spatial yield model for optimizing the thinning regime of mixed stands of Pinus sylvestris and Picea abies. *Scand. J. For. Res.* 13:31–42.

RANIUS, T., H. EKVALL, M. JONSSON, AND G. BOSTEDT. 2005. Cost-efficiency of measures to increase the amount of coarse woody debris in managed Norway spruce forests. *For. Ecol. Manag.* 206:119–133.

SIITONEN, J. 2001. Forest management coarse woody debris and saproxylic organisms: Fennoscandian boreal forests as an example. *Ecol. Bull.* 49:11–41.

SIMILÄ, M., J. KOUKI, AND P. MARTIKAINEN. 2003. Saproxylic beetles in managed and seminatural Scots pine forests: Quality of dead wood matters. *For. Ecol. Manag.* 174:365–381.

STOCKLAND, J. 1997. Representativeness and efficiency of bird and insect conservation in Norwegian boreal forest reserves. *Conserv. Biol.* 11:101–111.

STRANG, W. 1983. On the optimal forest harvesting decision. *Econ. Inquiry* 21:576–583.

SWALLOW, S., AND D. WEAR. 1993. Spatial interactions in multiple-use forestry and substitution and wealth effects for the single stand. *J. Environ. Econ. Manag.* 25:103–120.

VALKONEN, S., J. RUUSKA, AND J. SIIPILEHTO. 2002. Effect of retained trees on the development of young Scots pine stands in southern Finland. *For. Ecol. Manag.* 166(1–3):227–243.

VALSTA, L. 1992. An optimization model for Norway spruce management based on individual-tree growth models. *Acta Forestalia Fennica* 232:20.

VANHA-MAJAMAA, I., AND J. JALONEN. 2001. Green tree retention in Fennoscandian forestry. *Scand. J. For. Res.* (Suppl. 3):79–90.

WIKSTRÖM, P., AND L. ERIKSON. 2000. Solving the stand management problem under biodiversity-related consideration. *For. Ecol. Manag.* 126:261–376.

APPENDIX

Appendix: Comparative Statics

In this appendix we characterize the comparative statics of the model in terms of timber price p, regeneration cost c, and interest rate r.

The effects of timber price:

$$\frac{\partial G}{\partial p} = D^{-1}\{-\mathrm{SW}_{Gp}\mathrm{SW}_{TT}\} < 0 \qquad \text{(A.1)}$$

$$\frac{\partial T}{\partial p} = D^{-1}\{-\mathrm{SW}_{Tp}\mathrm{SW}_{GG}\} < 0,$$

where $SW_{Gp} = -(1 - f_G e^{-r\bar{T}}) < 0$ and

$$SW_{Tp} = f_T - r[(f - \bar{G}] - rf(\bar{T}, G)e^{-r\bar{T}}$$
$$-re^{-rT}\lfloor (f(T, \bar{G}) - G) + f(\bar{T}, G)e^{-r\bar{T}} \rfloor (1 - e^{-rT})^{-1} < 0.$$

The sign of SW_{Tp} can be ascertained by rearranging the first-order condition (4b).

The effects of regeneration costs:

$$\frac{\partial G}{\partial \varepsilon} = D^{-1}\{-SW_{G\varepsilon}SW_{TT}\} < 0 \qquad \text{(A.2)}$$

$$\frac{\partial T}{\partial \varepsilon} = D^{-1}\{-SW_{T\varepsilon}SW_{GG}\} > 0,$$

where $SW_{G\varepsilon} = -c'(G)e^{-rT} < 0$ and $SW_{T\varepsilon} = r[c(G) + c(G)(1 - e^{-rT})^{-1} > 0$ and $c = \varepsilon c(G)$, with $c' > 0$ and $\varepsilon \geq 1$.

The effects of the real interest rate:

$$\frac{\partial G}{\partial r} = D^{-1}\{-SW_{Gr}SW_{TT}\} < 0 \qquad \text{(A.3)}$$

$$\frac{\partial T}{\partial r} = D^{-1}\{-SW_{Tr}SW_{GG}\} < 0$$

where

$$SW_{Gr} = -\int_0^{\bar{T}} xB_G(x, G)e^{-rx}\,dx - \bar{T}pf_G e^{-r\bar{T}} < 0$$

and

$$SW_{Tr} = -p(f(T, \bar{G}) - G)$$
$$-\left(\int_0^{\bar{T}} B(x, G)e^{-rx}\,dx + pf(\bar{T}, G)e^{-r\bar{T}} - c(G)\right)$$
$$+ r\left(\int_0^{\bar{T}} xB(x, G)e^{-rx}\,dx - \bar{T}f_G e^{-r\bar{T}}\right)$$
$$-\frac{d}{dr}rSW < 0.$$

The sign of SW_{Tr} can be shown to be negative despite that fact that it includes two positive terms.