University of Notre Dame

Aerospace and Mechanical Engineering

AME 30314: Differential Equations, Vibrations and Controls I

Examination 1 Supplemental Problem

Due: Beginning of Class Friday October 14

- This problem is worth 20 exam points
- List all assumptions and clearly show all steps in your solution
- *Hint:* Make sure your governing equation is dimensionally consistent
- *There is to be no collaboration with classmates on this problem*.

A lake of volume ∀ contains an amount of pollutant $Q(t)$ at time t. The pollutant is distributed uniformly throughout the lake, yielding a pollutant concentration $C(t)$, where $C(t) = Q(t)/\forall$. Now assume that water containing a concentration k of pollutant enters the lake at a flow rate r and that water leaves the lake at the flow same flow rate. Additional pollutant is added into the lake at a constant rate P.

[a] If the concentration of pollutant is C_o at $t = 0$, determine the expression for $C(t)$ and then the limiting concentration, C_∞, as $t \to \infty$.

[b] If the addition of pollutants into the lake is terminated (k = 0 and P = 0 for $t > 0$), determine the times that it takes for the pollutant concentration to be reduced to 50 % and to 10% of its original value.

[c] The volume of Lake Michigan is 4900 $km^3$and its flow rate is 158 km^3/yr. Determine how long it will take for the pollutant concentration in Lake Michigan to reach 10 % of its original value (like in part [b]).