In this problem, I correctly completed the variable substitution and in the final step correctly integrated both sides and swapped back the x and y variables. I admit that I made a very low-level error in the calculation, but I actually wrote all the steps needed to solve the problem completely and correctly, and thus I do not think that a score of 2/10 is justified.

Also I think the TA is assuming that I made a mistake in the variable substitution step shown in the green pen in the diagram, but in fact I did not, so I am requesting that this problem be re-corrected.

Thank you for your help!

42/50

Homework #3

2/10

Promblem ①

$2xy + (x^2+y^2)\frac{dy}{dx} = 0$

$\Rightarrow 2\frac{y}{x} + [1+(\frac{y}{x})^2]\frac{dy}{dx} = 0$ $= \frac{dy}{dx} = \frac{-dv}{1+v^2}$

let $v = \frac{y}{x}$, so $y = vx \Rightarrow \frac{dy}{dx} = \frac{d(vx)}{dx} = x\frac{dv}{dx} + v$ $+2$

substitude $\longrightarrow 2v + (1+v^2)(x\frac{dv}{dx}+v) = 0$

$\Rightarrow 2v + \cancel{x\frac{dv}{dx} + v^2x\frac{dv}{dx}}\,(1+v^2)x\frac{dv}{dx} + v + v^3 = 0$

$\Rightarrow x(1+v^2)\frac{dv}{dx} + 3v + v^3 = 0$ $\Rightarrow -\frac{1+v^2}{v^3+3v}dv = \frac{dx}{x}$

$\Rightarrow (1+v^2)(3v+v^3)dv = -\frac{dx}{x}$ $\Rightarrow \ln x = -\frac{1}{3}\ln(v^2+3) - \frac{1}{3}\ln v + C$

$\Rightarrow \frac{v^5}{5} + \frac{v^4}{4} + v^3 + \frac{3}{2}v^2 = -\ln x + C$ $= -\frac{1}{3}\ln(v^3+3v) + C$

$\xrightarrow{v=\frac{y}{x}} \frac{y^5}{5x^5} + \frac{y^4}{4x^4} + \frac{y^3}{x^3} + \frac{3y^2}{2x^2} + \ln x = C$

$\Rightarrow x = C(v^3+3v)^{-\frac{1}{3}}$

$\xrightarrow{v=\frac{y}{x}} x = C(\frac{y^3}{x^3} + 3\frac{y}{x})^{-\frac{1}{3}}$

$\Rightarrow x(\frac{y^3}{x^3} + 3\frac{y}{x})^{\frac{1}{3}} = C$

Problem ②

$(x+y) + (y-x)\frac{dy}{dx} = 0$

$\Rightarrow (1+\frac{y}{x}) + (\frac{y}{x}-1)\frac{dy}{dx} = 0$

let $v = \frac{y}{x}$

$\Rightarrow (1+v) + (v-1)(x\frac{dv}{dx}+v) = 0$

$\Rightarrow x(v-1)\frac{dv}{dx} + v^2 + 1 = 0$

$\Rightarrow \frac{v-1}{v^2+1}dv = -\frac{dx}{x}$ $\Rightarrow \frac{1}{2}\ln(v^2+1) - \tan^{-1}(v) = -\ln x + C$

$\xrightarrow{v=\frac{y}{x}} \frac{1}{2}\ln(\frac{y^2}{x^2}+1) - \tan^{-1}(\frac{y}{x}) + \ln x = C$