Problem 1

$$\sigma_{ji,j} + F_i = 0$$

Problem 3

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f = 0 \quad , \left(u\frac{du}{dx}\right)\Big|_{x=0} = 0 \quad , u(L) = \sqrt{2}$$

WR: $\int_0^L w\left[-\frac{d}{dx}\left(u\frac{du}{dx}\right) + f\right] dx = 0$ $\quad\left(\int \frac{d}{dx}\left(uw\frac{du}{dx}\right) - \int \frac{dw}{dx}\left(u\frac{du}{dx}\right) = \int w\frac{d}{dx}\left(u\frac{du}{dx}\right)\right)$

$$\Rightarrow \int_0^L \left[\frac{dw}{dx} u \cdot \frac{du}{dx} + wf\right] dx - uw\frac{du}{dx}\Big|_0^L = 0$$

plug in BCs

$$\Rightarrow \int_0^L \left[\frac{dw}{dx} u \cdot \frac{du}{dx} + wf\right] dx - 2\frac{du}{dx}\Big|_{x=L} = 0$$

Problem 6

$$-\rho\nu \cdot v_{i,jj} + \rho v_j \cdot v_{i,j} + P_{,i} = 0 \leftarrow \text{test function } w_i$$

$$v_{k,k} = 0 \leftarrow \text{test function } \tau$$

BCs: $v = \bar{v}$, $p = \bar{p}$ on $\partial\Omega_D$

$\rho\nu(v_{i,j} \cdot n_j) - p n_i = \rho \bar{t}_i$ on $\partial\Omega_N$

Weak form ①

$$\int_\Omega w_i [-\rho\nu \cdot v_{i,jj} + \rho v_j \cdot v_{i,j} + P_{,i}]\, dv = 0$$

$$\Rightarrow \int_\Omega [\rho\nu w_{i,j} \cdot v_{i,j} + w_i(\rho v_j \cdot v_{i,j} + P_{,i})]\, dv + \underbrace{\int_{\partial\Omega} (-\rho\nu \cdot w_i v_{i,j} \cdot n_j)\, ds}_{} = 0$$

$$\int_{\partial\Omega} (-\rho\nu \cdot w_i v_{i,j} n_j)\, ds = \left(\int_{\partial\Omega_D} + \int_{\partial\Omega_N}\right)(-\rho\nu \cdot w_i \cdot v_{i,j} n_j)\, ds$$

$$= \underbrace{\int_{\partial\Omega_D} (-\rho\nu \cdot w_i \cdot v_{i,j} n_j)\, ds}_{=0} + \underbrace{\int_{\partial\Omega_N} (-\rho\nu \cdot w_i \cdot v_{i,j} n_j)\, ds}_{}$$

$$= \int_{\partial\Omega_N} [-w_i(p n_i + \rho \bar{t}_i)]\, ds$$

$$\Rightarrow \int_\Omega [\rho\nu w_{i,j} v_{i,j} + w_i(\rho v_j v_{i,j} + P_{,i})]\, dv + \int_{\partial\Omega_N} [-w_i(p n_i + \rho \bar{t}_i)]\, ds$$

Weak form ②

$$\int_\Omega \tau \cdot v_{k,k}\, dv = 0$$

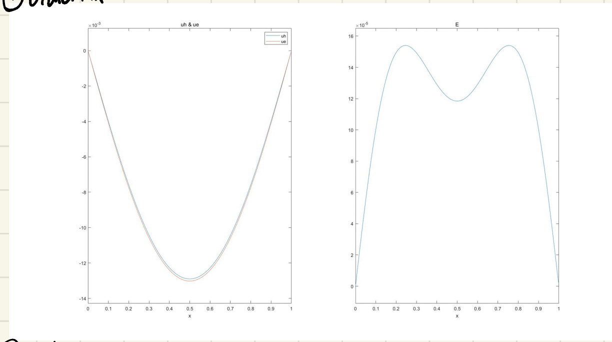
$$\Rightarrow \int_\Omega (-\tau_{,k} v_k)\, dv + \int_{\partial\Omega} \tau \cdot v_k n_k\, ds = 0$$

$$\Rightarrow \int_\Omega (-\tau_{,k} \cdot v_k)\, dv + \int_{\partial\Omega_D} \tau v_k n_k\, ds + \int_{\partial\Omega_N} \tau v_k n_k\, ds = 0$$

$$\Rightarrow \int_\Omega (-\tau_{,k} \cdot v_k)\, dv + \int_{\partial\Omega_D} \bar{v}_k n_k\, ds + \int_{\partial\Omega_N} \tau v_k n_k\, ds = 0$$

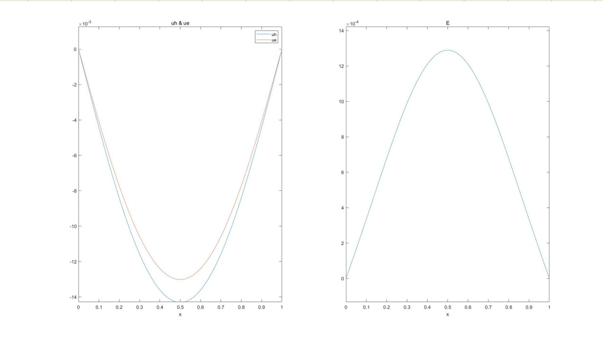
Problem 7

① Galerkin



$E = 1.25 \times 10^{-4}$

② Collocation



$E = 8.77 \times 10^{-4}$

Weak Form

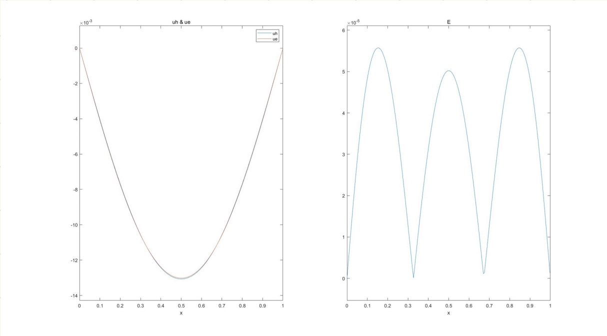
$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right) = q_0$$

$$\Rightarrow \int_0^L v\left(EI\frac{d^4w}{dx^4} - q_0\right)dx = \int_0^L \left(-\frac{dv}{dx}EI\cdot\frac{d^3w}{dx^3}dx - vq_0\right)dx + EIv\frac{d^3w}{dx^3}\Big|_0^L$$

$$= \int_0^L \left(\frac{d^2v}{dx^2}EI\cdot\frac{d^2w}{dx^2} - vq_0\right)dx + EIv\frac{d^3w}{dx^3}\Big|_0^L - EI\cdot\frac{dv}{dx}\cdot\frac{d^2w}{dx^2}\Big|_0^L$$

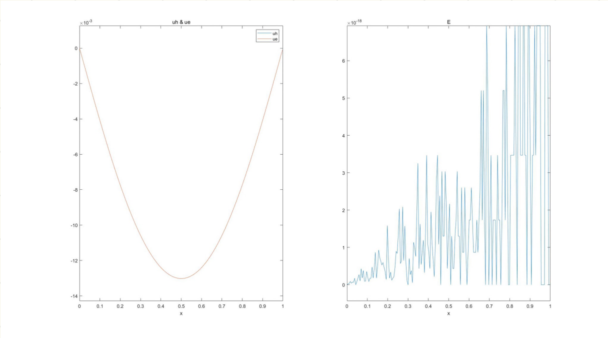
$$\xrightarrow{\text{Plug in BCs}} \underline{\int_0^L \left(EI\frac{d^2v}{dx^2}\frac{d^2w}{dx^2} - vq_0\right)dx = 0}$$

③ Ritz Method 1



$E = 3.82 \times 10^{-5}$

④ Ritz Method 2



$E = 0$ (small enough to be 0 in matlab)