In [3]:
```
import pandas as pd
import numpy as np
from sklearn import linear_model, cross_validation
from sklearn.linear_model import LinearRegression, Ridge, Lasso, ElasticNet
from sklearn.metrics import r2_score, mean_squared_error
from sklearn.model_selection import train_test_split
from sklearn import linear_model, cross_validation
from sklearn.cross_validation import cross_val_score
from sklearn.model_selection import GridSearchCV
import matplotlib.pyplot as plt
```

Formatting the raw data

In [63]:
```
raw_data = pd.read_csv('winequality-red.csv')
raw_data = pd.read_csv('winequality-red.csv', sep = ';', header = None)
new_header = raw_data.iloc[0] #grab the first row for the header
df = raw_data[1:].astype(float) #take the data less the header row
df.columns = new_header #set the header row as the df header

df.head()
```

Out[63]:

	fixed acidity	volatile acidity	citric acid	residual sugar	chlorides	free sulfur dioxide	total sulfur dioxide	density	pH	sulphates	alcohol	qu
1	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	
2	7.8	0.88	0.00	2.6	0.098	25.0	67.0	0.9968	3.20	0.68	9.8	
3	7.8	0.76	0.04	2.3	0.092	15.0	54.0	0.9970	3.26	0.65	9.8	
4	11.2	0.28	0.56	1.9	0.075	17.0	60.0	0.9980	3.16	0.58	9.8	
5	7.4	0.70	0.00	1.9	0.076	11.0	34.0	0.9978	3.51	0.56	9.4	

Splitting data into training and testing (80/20) sets

In [48]:
```
# Next, we set the quality of the wine as the y column, and split the data i

target_column = ['quality']
predictors = list(set(list(df.columns))-set(target_column))
X = df[predictors].values
y = df[target_column].values

X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, rar
```

Ordinary Least Squares, with intercept and

normalizing

In [6]:
```
lr = LinearRegression(fit_intercept=True, normalize=True).fit(X_train, y_tra
pred_train_lr= lr.predict(X_train)
pred_test_lr= lr.predict(X_test)
```

In [7]:
```
print("Training Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_train,pred_train_lr))
print("R Squared: %s" % r2_score(y_train, pred_train_lr))
```

```
Training Data Results:
Mean squared error: 0.41914135600144115
R Squared: 0.3435138143942261
```

In [8]:
```
print("Testing Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_test,pred_test_lr))
print("R Squared: %s" % r2_score(y_test, pred_test_lr))
```

```
Testing Data Results:
Mean squared error: 0.40938401510422534
R Squared: 0.4183385391256862
```

Note: The model performed well on the testing data, indicating that there is no overfitting and our model is good.

Coefficients of OLS model

In [19]:
```
coeff_lr = pd.DataFrame(np.append(lr.intercept_,lr.coef_.transpose()), ['Int
coeff_lr
```

Out[19]:

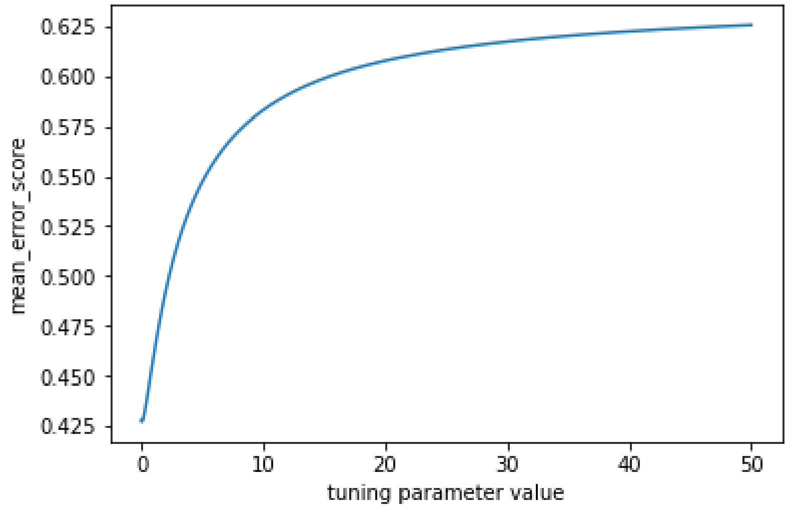
	Coefficients
Intercept	21.201403
citric acid	-0.174033
chlorides	-1.810628
alcohol	0.271437
free sulfur dioxide	0.004230
volatile acidity	-1.034351
density	-17.129773
pH	-0.393833
total sulfur dioxide	-0.003296
sulphates	0.833811
residual sugar	0.015139
fixed acidity	0.027243

Using k-fold CV to find alpha parameter for ridge regression

In [69]:
```
rr = Ridge(fit_intercept=True, normalize=True)
parameters = np.arange(0, 50, 0.01)

ridge_regressor = GridSearchCV(rr, {'alpha': parameters} , scoring="neg_mear
                               cv=10,return_train_score = True)

ridge_regressor.fit(X_train,y_train)

cve_error = -1* ridge_regressor.best_score_ # error is negative score
print("After k-fold cv, we found the best alpha value to be %s with error of
      % (ridge_regressor.best_params_['alpha'], cve_error) )
```

```
After k-fold cv, we found the best alpha value to be 0.05 with error of
0.42719659336479276
```

In [70]:
```
plt.xlabel("tuning parameter value")
plt.ylabel("mean_error_score")
plt.plot(parameters, -1*ridge_regressor.cv_results_["mean_test_score"])
plt.show()
```

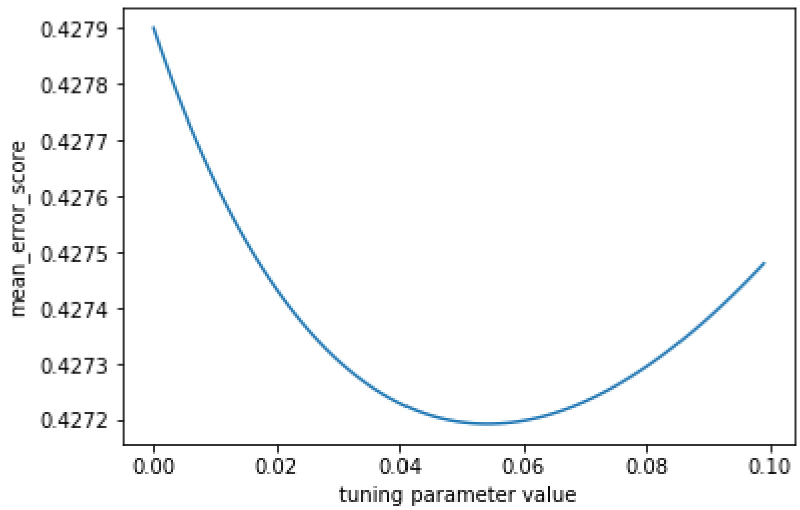


Let's try using a more specific range of values to fine tuning our parameter

In [24]:
```
fine_rr = Ridge(fit_intercept=True, normalize=True)
parameters = np.arange(0, .1, 0.001)

ridge_regressor = GridSearchCV(fine_rr, {'alpha': parameters} , scoring="neg
                               cv=10,return_train_score = True)

ridge_regressor.fit(X_train,y_train)

cve_error = -1* ridge_regressor.best_score_ # error is negative score
print("After a finer tune, we found the best alpha value to be %s with error
      % (ridge_regressor.best_params_['alpha'], cve_error) )
```

```
After a finer tune, we found the best alpha value to be 0.054 with error
of 0.42719360628905756
```

In [25]:
```
plt.xlabel("tuning parameter value")
plt.ylabel("mean error score")
plt.plot(parameters, -1*ridge_regressor.cv_results_["mean_test_score"])
plt.show()
```



In [26]:
```
# test cv ridge regression model on training and test set
cv_rr = Ridge(fit_intercept=True, normalize=True, alpha = 0.054).fit(X_trair

pred_train_rr= cv_rr.predict(X_train)
pred_test_rr= cv_rr.predict(X_test)
```

In [27]:
```
print("Training Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_train,pred_train_rr))
print("R Squared: %s" % r2_score(y_train, pred_train_rr))
```

```
Training Data Results:
Mean squared error: 0.41966708492918714
R Squared: 0.34269038388922635
```

In [28]:
```
print("Testing Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_test,pred_test_rr))
print("R Squared: %s" % r2_score(y_test, pred_test_rr))
```

```
Testing Data Results:
Mean squared error: 0.4127119537574192
R Squared: 0.41361013355219545
```

Note: again the model fits the testing data well. Using all the data, we will proceed to train a ridge regression model with the optimal alpha and display the weights of the features in a table.

Coefficients of Ridge Regression model

In [29]:
```
coeff_rr = pd.DataFrame(np.append(cv_rr.intercept_,cv_rr.coef_.transpose()),
coeff_rr
```

Out[29]:

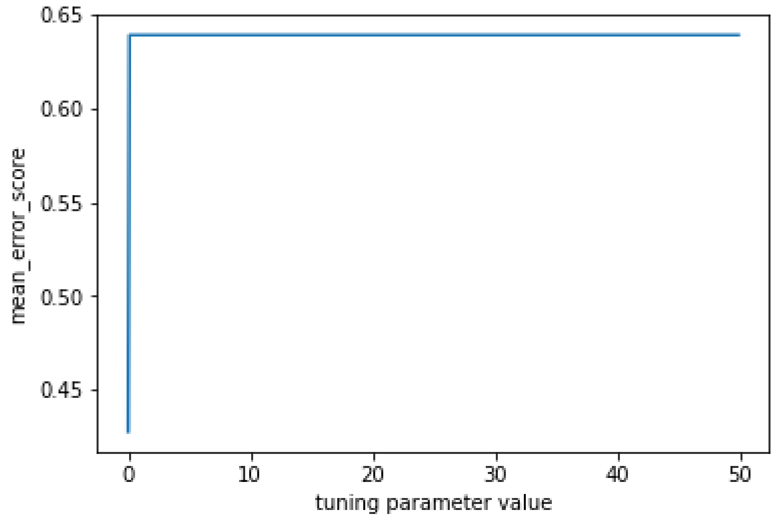
	Coefficients
Intercept	30.979037
citric acid	-0.074723
chlorides	-1.745374
alcohol	0.248620
free sulfur dioxide	0.003576
volatile acidity	-0.968760
density	-27.127704
pH	-0.293352
total sulfur dioxide	-0.003106
sulphates	0.801991
residual sugar	0.017597
fixed acidity	0.032257

Using k-fold CV to find alpha parameter for lasso regression

In [74]:
```
lar = Lasso(fit_intercept=True, normalize=True)
parameters = np.arange(0, 50, 0.1)

lasso_regressor = GridSearchCV(lar, {'alpha': parameters} , scoring="neg_mea
                               cv=10,return_train_score = True)

lasso_regressor.fit(X_train,y_train)

cv_error_lar = -1* lasso_regressor.best_score_ # error is negative score
print("After k-fold cv, we found best alpha value to be %s with error of %s'
      % (lasso_regressor.best_params_['alpha'], cv_error_lar) )
```

```
After k-fold cv, we found best alpha value to be 0.0 with error of 0.4279
0045491771045
```

In [75]:
```
plt.xlabel("tuning parameter value")
plt.ylabel("mean_error_score")
plt.plot(parameters, -1*lasso_regressor.cv_results_["mean_test_score"])
plt.show()
```

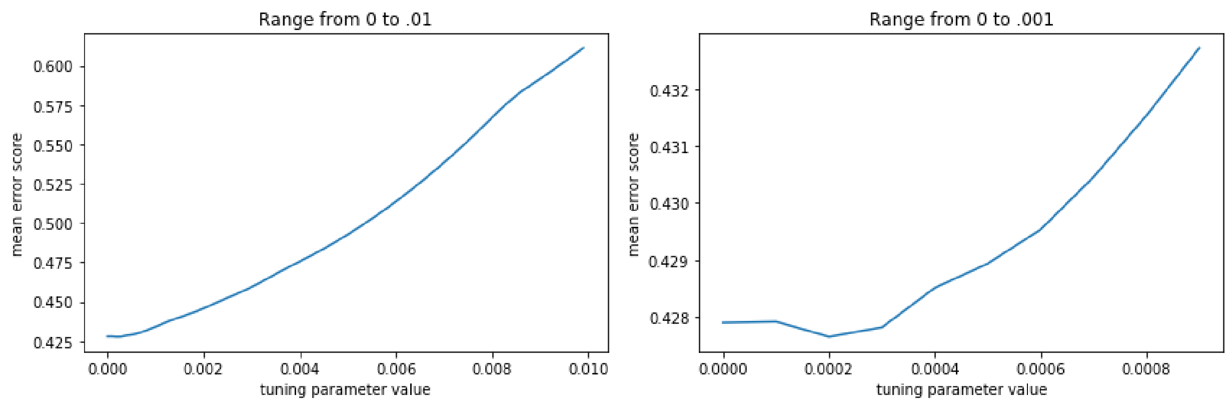


Note: Again, let us do a finer tune

In [80]:
```
lar = Lasso(fit_intercept=True, normalize=True)
parameters = np.arange(0, .01, 0.0001)

lasso_regressor = GridSearchCV(lar, {'alpha': parameters} , scoring="neg_mea
                               cv=10,return_train_score = True)

lasso_regressor.fit(X_train,y_train)

cv_error_lar = -1* lasso_regressor.best_score_ # error is negative score
print("After k-fold cv, we found best alpha value to be %s with error of %s'
      % (lasso_regressor.best_params_['alpha'], cv_error_lar) )
```

```
After k-fold cv, we found best alpha value to be 0.0002 with error of 0.4
276526827775983
```

In [102]:
```
x1 = np.arange(0, .01, 0.0001)
x2 = np.arange(0, .001, 0.0001)

y1 = -1*lasso_regressor.cv_results_["mean_test_score"]
y2 = -1*lasso_regressor.cv_results_["mean_test_score"][:10]

plt.figure(figsize=(12, 4))

plt.subplot(1, 2, 1)
plt.plot(x1, y1)
plt.title('Range from 0 to .01')
plt.ylabel("mean error score")
plt.xlabel("tuning parameter value")

plt.subplot(1, 2, 2)
plt.plot(x2, y2)
plt.title('Range from 0 to .001')
plt.xlabel("tuning parameter value")
plt.ylabel("mean error score")
plt.tight_layout()

plt.show()
```



In [115]:
```
# test cv lasso regression model on training and test set

cv_lar = Lasso(fit_intercept=True, normalize=True, alpha = .0002).fit(X_trai
pred_train_la= cv_lar.predict(X_train)
pred_test_la= cv_lar.predict(X_test)
```

In [116]:
```
print("Training Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_train,pred_train_la))
print("R Squared: %s" % r2_score(y_train, pred_train_la))
```

```
Training Data Results:
Mean squared error: 0.42033505336426963
R Squared: 0.34164416870722736
```

In [117]:
```
print("Testing Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_test,pred_test_la))
print("R Squared: %s" % r2_score(y_test, pred_test_la))
```

```
Testing Data Results:
Mean squared error: 0.41303925872643255
R Squared: 0.41314509173472425
```

Coefficients of Lasso model

In [118]:
```
coeff_lar = pd.DataFrame(np.append(cv_lar.intercept_,cv_lar.coef_.transpose(
coeff_lar
```

Out[118]:

	Coefficients
Intercept	4.200448
citric acid	-0.000000
chlorides	-1.677439
alcohol	0.280748
free sulfur dioxide	0.002716
volatile acidity	-0.981772
density	-0.000000
pH	-0.379928
total sulfur dioxide	-0.002836
sulphates	0.743150
residual sugar	0.001634
fixed acidity	0.002439

Note: a slight change in alpha value drasitcally changes our feature weights. Consider, the case when we use alpha = 0 instead of .0002. We get a table that is much more similar to our previous models.

In [120]:
```
cv_lar0 = Lasso(fit_intercept=True, normalize=True, alpha = 0).fit(X_train,

coeff_lar0 = pd.DataFrame(np.append(cv_lar0.intercept_,cv_lar0.coef_.transpo
coeff_lar0
```

Out[120]:

	Coefficients
Intercept	21.201403
citric acid	-0.174033
chlorides	-1.810628
alcohol	0.271437
free sulfur dioxide	0.004230
volatile acidity	-1.034351
density	-17.129773
pH	-0.393833
total sulfur dioxide	-0.003296
sulphates	0.833811
residual sugar	0.015139
fixed acidity	0.027243

Using k-fold CV to find alpha and l1_ratio parameters for ElasticNet

In [227]:
```
er = ElasticNet(fit_intercept=True, normalize=True)

alphas1 = np.arange(0,5,.1)
l1_ratio1 = np.arange(0.0, 1, 0.1)
parametersGrid = {"alpha": alphas1,
                  "l1_ratio": l1_ratio1}

er_regressor = GridSearchCV(er, parametersGrid, scoring="neg_mean_squared_er
er_regressor.fit(X_train,y_train)

best_alpha = er_regressor.best_params_['alpha']
best_l1_ratio = round(er_regressor.best_params_['l1_ratio'],3)

print("After k-fold cv, we found the best alpha value to be %s, and l1_ratio
      % (best_alpha,best_l1_ratio, -1* er_regressor.best_score_) )
```

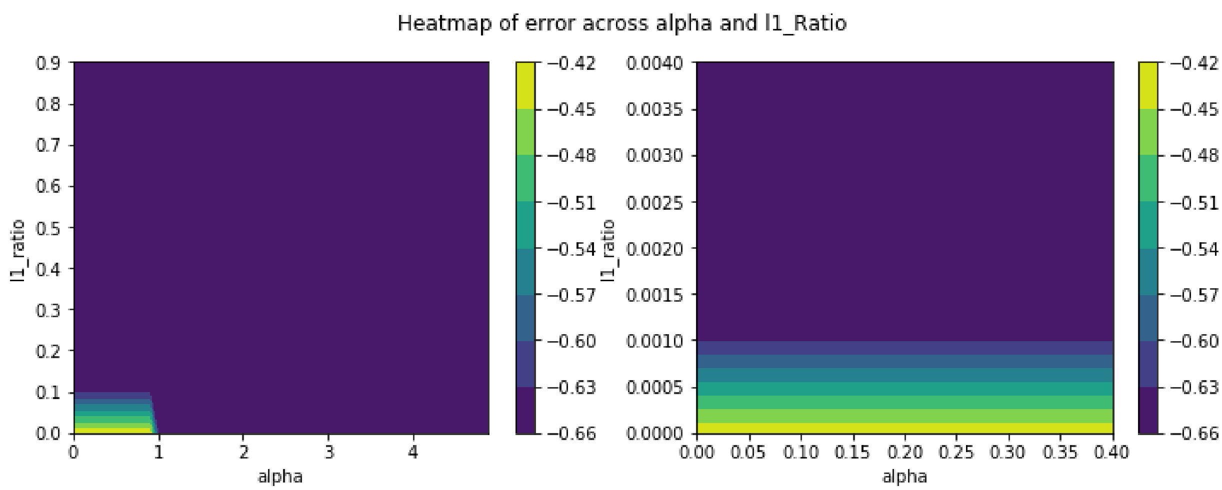
```
After k-fold cv, we found the best alpha value to be 0.0, and l1_ratio to
be 0.0 with error of 0.4301277784918643
```

Note: we fine tune some more:

In [226]:
```
er2 = ElasticNet(fit_intercept=True, normalize=True)

alphas2 = np.arange(0,.5,.1)
l1_ratio2 = np.arange(0.0, .005, .001)
parametersGrid = {"alpha": alphas2,
                  "l1_ratio": l1_ratio2}

er_regressor2 = GridSearchCV(er, parametersGrid, scoring="neg_mean_squared_e
er_regressor2.fit(X_train,y_train)

best_alpha2 = er_regressor2.best_params_['alpha']
best_l1_ratio2 = round(er_regressor2.best_params_['l1_ratio'],3)

print("After k-fold cv, we found the best alpha value to be %s, and l1_ratic
      % (best_alpha2,best_l1_ratio2, -1*er_regressor2.best_score_) )
```

```
After k-fold cv, we found the best alpha value to be 0.0, and l1_ratio to
be 0.0 with error of 0.4301277784918643
```

In [225]:
```
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12,4))
fig.suptitle('Heatmap of error across alpha and l1_Ratio ')

X1, Y1 = np.meshgrid(alphas1, l1_ratio1)
Z1 = np.reshape(er_regressor.cv_results_['mean_test_score'], X1.shape)

p_plot = ax1.contourf(X1, Y1, Z1)
fig.colorbar(p_plot, ax=ax1)
ax1.set(xlabel='alpha', ylabel='l1_ratio')

X2, Y2 = np.meshgrid(alphas2, l1_ratio2)
Z2 = np.reshape(er_regressor2.cv_results_['mean_test_score'], X2.shape)

run_plot = ax2.contourf(X2, Y2, Z2)
fig.colorbar(run_plot, ax=ax2)
ax2.set(xlabel='alpha', ylabel='l1_ratio')

plt.show()
```



Coefficients of the Elastic Net model

In [223]:
```
cv_en = ElasticNet(fit_intercept=True, normalize=True, alpha = 0, l1_ratio =

coeff_en = pd.DataFrame(np.append(cv_en.intercept_,cv_en.coef_.transpose()),
coeff_en
```

Out[223]:

	Coefficients
Intercept	21.201403
citric acid	-0.174033
chlorides	-1.810628
alcohol	0.271437
free sulfur dioxide	0.004230
volatile acidity	-1.034351
density	-17.129773
pH	-0.393833
total sulfur dioxide	-0.003296
sulphates	0.833811
residual sugar	0.015139
fixed acidity	0.027243

In [229]:
```
# test cv lasso regression model on training and test set

cv_en = ElasticNet(fit_intercept=True, normalize=True, alpha = 0, l1_ratio =
pred_train_en = cv_en.predict(X_train)
pred_test_en = cv_en.predict(X_test)
```

In [230]:
```
print("Training Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_train,pred_train_en))
print("R Squared: %s" % r2_score(y_train, pred_train_en))
```

```
Training Data Results:
Mean squared error: 0.419141356001441
R Squared: 0.34351381439422646
```

In [231]:
```
print("Testing Data Results:")
print("Mean squared error: %s" % mean_squared_error(y_test,pred_test_en))
print("R Squared: %s" % r2_score(y_test, pred_test_en))
```

```
Testing Data Results:
Mean squared error: 0.40938401510422373
R Squared: 0.4183385391256884
```

Conclusion, it seems like OLS and Elastic net barely have the best MSE and R^2 scores