Homework 1
Miguel Garza Robledo

1. When using the original equation
$$a = b$$
The assumption is that
$$a - b = 0$$
Therefore, in the step after the equation, when you get the equation
$$(a+b)(a-b) = b(a-b)$$
The proof states that you divide by
$$(a-b)$$
which would be the same as dividing by zero. And since dividing by zero would cause the quotient
$$\frac{(a+b)(a-b)}{0} = \frac{b(a-b)}{0}$$
This step is would not make sense in correspondence to the proof. Thus, you have an error.

2. Basis: Prove that the formula is true for C_1

For $n = 1$, it is true for C_1
$$1 = \frac{1(1)^2(1+1)^2}{4}$$
$$= \frac{1(1)(4)}{4}$$
$$= 1$$

Assuming $n = k$ is true, we can use this assumption to prove $n = k+1$. That is,
$$1^3 + 2^3 + ... + k^3 = \frac{1k^2(k+1)^2}{4}$$
Starting from the equation above, we prove
$$1^3 + 2^3 + ... + k^3 + (k+1)^3 = \frac{1(k+1)^2(k+2)^2}{4}$$
$$= \frac{1(k)^2(k+1)^2}{4} + (k+1)^3$$

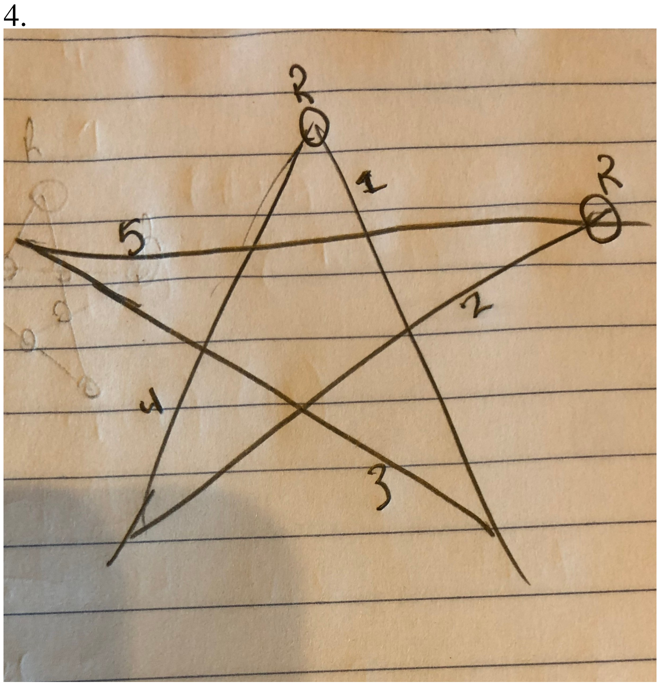
$$= \frac{(k+1)^2(k^2+4k+4)}{4}$$
$$= \frac{(k+1)^2(k+2)^2}{4}$$

Seeing how we are able to convert the following question by adding $(k+1)^3$ to the right side of the equation. We are able to see that the given equality for the sum of the cube of n=k natural numbers is also true for n+1.

3. The error has occured in the last sentence where the statement “all horses in $H_1$are the same or identical in color” and “all horses in $H_2$are identical or the same color” for the choice of $H_1$and H_2which has taken.

Moreover, the single horse in $H_1$and the single horse H_2could have a distinct color, therefore, it would not make sense to conclude that the horses have the same color since H=H_1^ H_2would be empty if the horses have distinct color.

4.

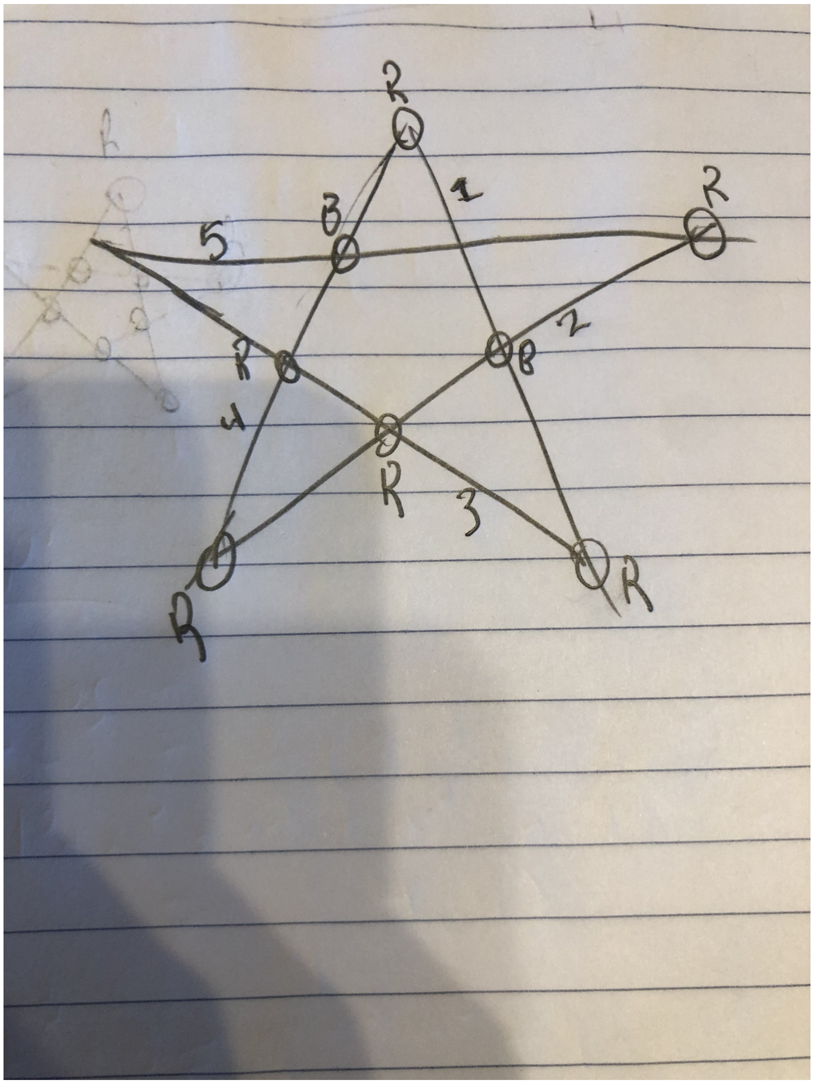


Let us attempt filling each line with one red shown in the picture above:

Filling one endpoint as red places two lines with an odd number of red nodes.

Filling another endpoint that does not have an odd number of red fills two more lines with red. Now, there is one line left without an odd number of red nodes.

Now, we need to fill 1 red in last line remaining. We cannot fill red at any of the other nodes because it makes 2 red in a line so we need to fill line such that 3 red can be seen in a line except 2.



We continuously fill every line such that every line has 3 reds so finally we need to fill the last node. If we fill last node with red. This line gets 4 reds. If we fill the last node with blue, then this line gets 2 reds. So, finally we can say that it is not possible to fill every line with an odd number of red nodes.