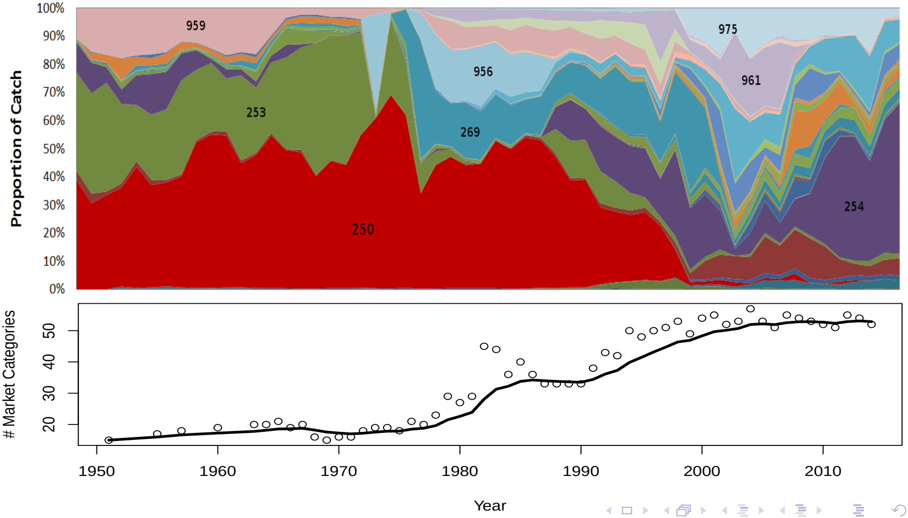
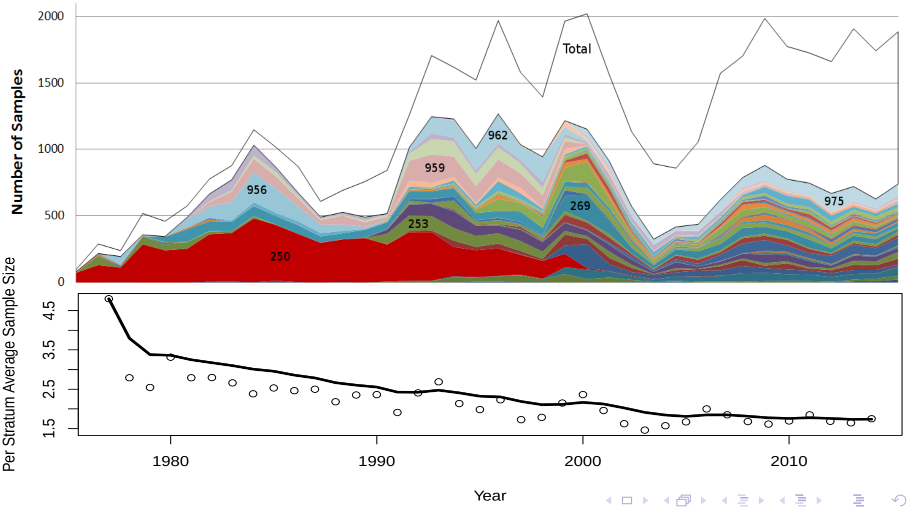
Market Category Stratification



Sampling Effort



Goal:

- Model Based Estimation
- Complete Inference = Point Estimation + Uncertainty

Issues:

- Overdispersion
- Sparse Sampling $\longrightarrow$ Degenerate Asymtoptics
  - $\hookrightarrow$ Stratum Pooling
  - $\hookrightarrow$ Hierarchical Modeling

$$
\begin{array}{ccc}
\text{Binomial} & \longleftrightarrow & \text{Beta-Binomial} \\
\uparrow & & \uparrow \\
\text{Multinomial} & \longleftrightarrow & \text{Dirichelet-Multinomial} \\
\uparrow\uparrow\uparrow & & \\
\text{Poisson} & \longleftrightarrow & \text{Negative-Binomial}
\end{array}
$$

Data Generating Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}\Big(\mu_{jklm\eta},\ \sigma^2_{jklm\eta}\Big)$$

$$\mu_{jklm\eta} = n\ \text{logit}^{-1}(\theta_{jklm\eta})$$

$$\sigma^2_{jklm\eta} = \mu_{jklm\eta}\left(1 - \frac{\mu_{jklm\eta}}{n}\right)\Big(1 + (n-1)\ \rho\Big)$$

$$\theta_{jklm\eta} = \beta_0 + \beta^{(s)}_j + \beta^{(p)}_k + \beta^{(g)}_l + \beta^{(y)}_m + \beta^{(q)}_\eta + \beta^{(y:q)}_{m\eta}$$

$y_{ijklm\eta}$: i^{th} sample of the j^{th} species' integer weight, in the k^{th} port, caught with the l^{th} gear, in the η^{th} quarter, of year m, for a particular market category.

$j \in \{1, ..., J\}$ Species

$k \in \{1, ..., K\}$ Ports

$l \in \{1, ..., L\}$ Gears

$m \in \{1, ..., M\}$ Years

$\eta \in \{1, ..., H\}$ Quarters

Heirarchical Prior Structure

$$\text{logit}(\rho) \sim N(0, 2^2)$$

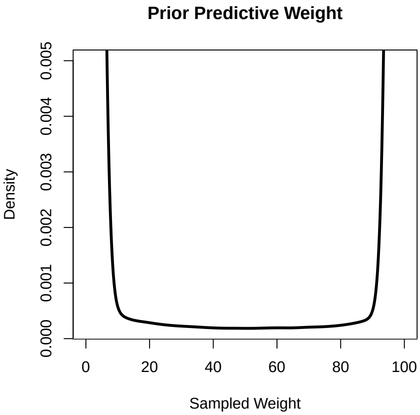
$$\left\{\beta_j^{(s)}, \beta_k^{(p)}, \beta_l^{(g)}\right\} \sim N(0, 32^2)$$

$$\beta_0 \propto 1 \qquad \beta_m^{(y)} \sim N\left(0,\ v^{(y)}\right)$$

$$\beta_\eta^{(q)} \sim N\left(0,\ v^{(q)}\right)$$

$$\beta_{m\eta}^{(y:q)} \sim N\left(0,\ v^{(y:q)}\right)$$

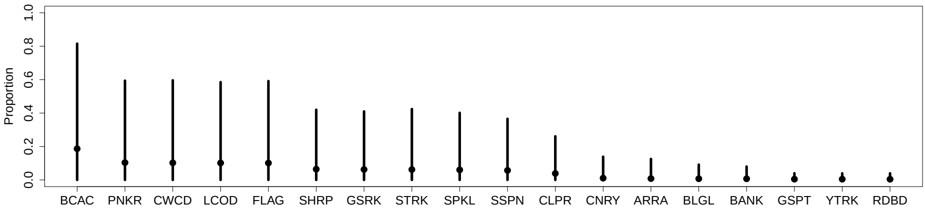
$$v \sim IG(1, 2x10^3) \quad \forall \quad v$$



Species Composition Prediction

$$p(y^*_{jklm\eta}|\mathbf{y}) = \iint \mathrm{BB}\left(y^*_{jklm\eta}|\mu_{jklm\eta}, \sigma^2_{jklm\eta}\right) P\left(\mu_{jklm\eta}, \sigma^2_{jklm\eta}|\mathbf{y}\right) d\mu_{jklm\eta} d\sigma^2_{jklm\eta}$$

$$\pi^*_{jklm\eta} = \frac{y^*_{jklm\eta}}{\sum_j y^*_{jklm\eta}} \quad \mathbf{y}^*_{klm\eta} \neq \mathbf{0}$$



Port Pooling

$$\mathrm{MSE}(\hat{\theta}) = \mathbb{E}\left[\ (\hat{\theta}-\theta)^2\ \right] = \overbrace{\mathbb{E}\left[\ \left(\hat{\theta}-\mathbb{E}(\hat{\theta})\right)^2\ \right]}^{\mathrm{Var}(\hat{\theta})} + \overbrace{\left(\ \mathbb{E}(\hat{\theta})-\theta\ \right)^2}^{\mathrm{Bias}(\hat{\theta},\theta)^2}$$

Bayesian Model Averaging (BMA)

Consider $j \in \{1, ..., \mathcal{M}\}$ Models ($\mathbb{M}$):

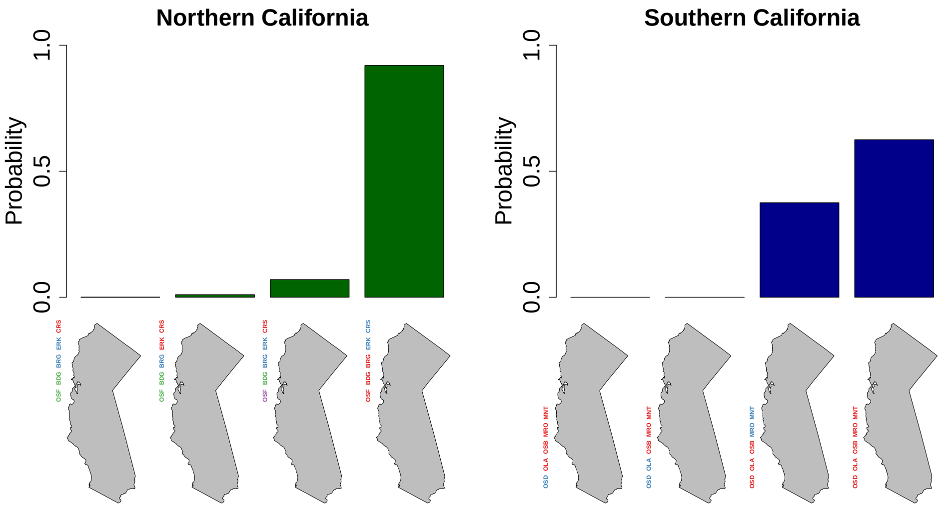
$$\omega_j = Pr(\mathbb{M}_j|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbb{M}_j)\cancelto{\propto 1}{p(\mathbb{M}_j)}}{\sum_i p(\mathbf{y}|\mathbb{M}_i)\cancelto{\propto 1}{p(\mathbb{M}_i)}} = \frac{p(\mathbf{y}|\mathbb{M}_j)}{\sum_i p(\mathbf{y}|\mathbb{M}_i)}$$

$$\bar{p}(\boldsymbol{\theta}|\mathbf{y}) = \sum_{j=1}^{\mathcal{M}} \omega_j p(\boldsymbol{\theta}|\mathbf{y}, \mathbb{M}_j)$$

if f only depends on $\mathbb{M}$ thru $\boldsymbol{\theta}$, then

$$\bar{p}(y^*|\mathbf{y}) = \int f(y^*|\boldsymbol{\theta})\bar{p}(\boldsymbol{\theta}|\mathbf{y})\mathbf{d}\boldsymbol{\theta}$$

* Hoeting, J. A., Madigan, D., Raftery, A. E., and Volinsky, C. T. (1999). Bayesian model averaging: a tutorial. *Statistical science*, 382-401.


Northern California
Southern California
Probability
0.0
0.5
1.0
OSF BDG BRG ERK CRS
OSD OLA OSB MRO MNT

Expansion and Integration

If $\lambda_{\cdot klm\eta\omega}$ is the observed landings of **all species** in the k^{th} port, caught with the l^{th} gear, in the η^{th} quarter, of year m, in market category ω^{th}. Then,

$$\lambda^*_{jklm\eta\omega} = \lambda_{\cdot klm\eta\omega}\pi^*_{jklm\eta\omega}$$

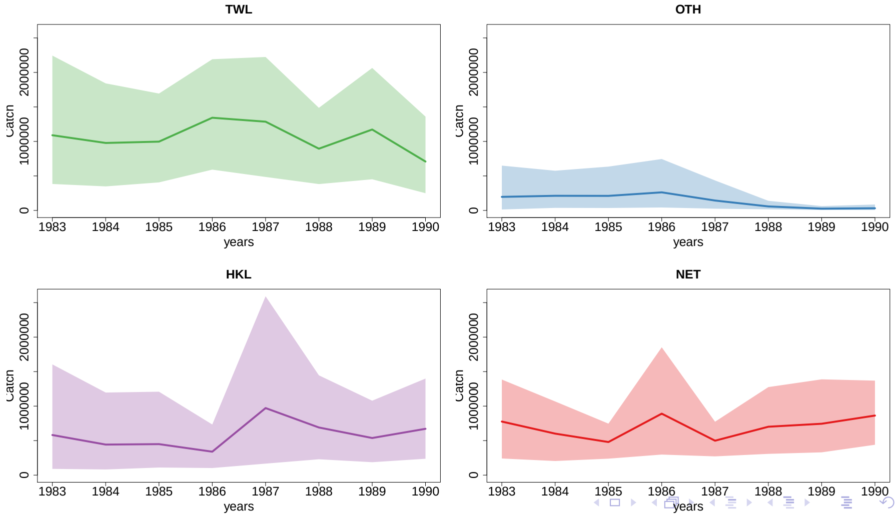
$$\lambda^*_{jklm\eta\cdot} = \sum_{\omega} \lambda^*_{jklm\eta\omega}$$

$$\lambda^*_{jklm\cdot\cdot} = \sum_{\eta}\sum_{\omega} \lambda^*_{jklm\eta\omega}$$

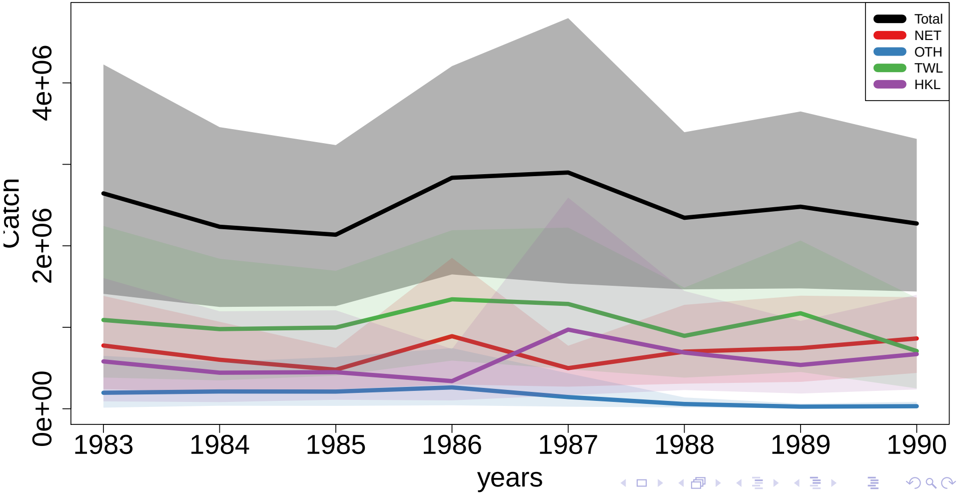
$$\lambda^*_{j\cdot lm\cdot\cdot} = \sum_{k}\sum_{\eta}\sum_{\omega} \lambda^*_{jklm\eta\omega}$$

$$\lambda^*_{j\cdot\cdot m\cdot\cdot} = \sum_{l}\sum_{k}\sum_{\eta}\sum_{\omega} \lambda^*_{jklm\eta\omega}$$

$\lambda^*_{j \cdot lm \cdot \cdot}$: $j =$ Boccaccio



$\lambda^{*}_{j \cdot \cdot m \cdot \cdot} \; : \; j = \text{Boccaccio}$

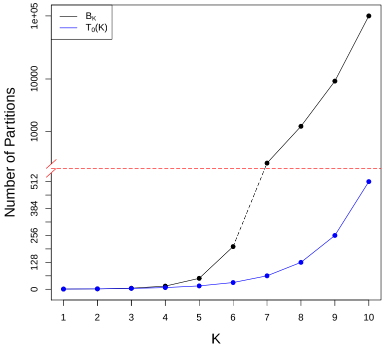


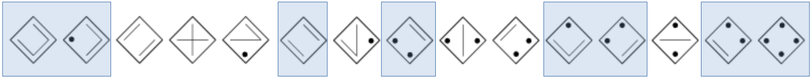
Combinatorics

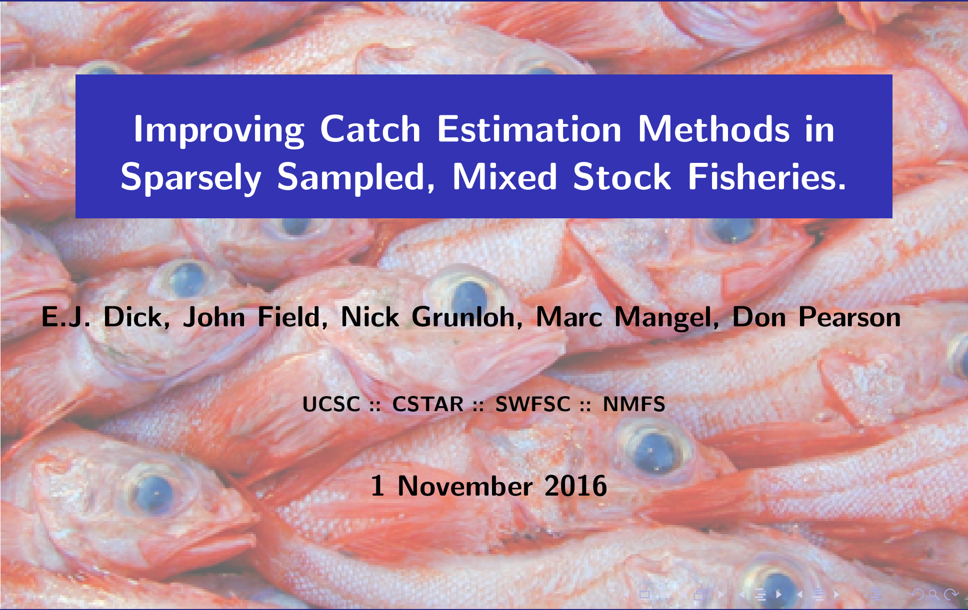
Bell Number: Total number of ways to partition K items.

$$B_K = \sum_{\hat{k}=0}^{K} \frac{1}{\hat{k}!} \left(\sum_{j=0}^{\hat{k}} (-1)^{\hat{k}-j} \binom{\hat{k}}{j} j^K \right)$$

Define $T_0(K)$: Number of ways to partition K items, such that, partitions are adjacent.







Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

E.J. Dick, John Field, Nick Grunloh, Marc Mangel, Don Pearson

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