

Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

Nick Grunloh

UCSC :: CSTAR :: SWFSC :: NMFS

30 July 2018

Diagnostic

Show Example

Beta-Binomial Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}(\mu_{ijklm\eta}, \sigma_{ijklm\eta}^2)$$

$$\mu_{ijklm\eta} = n \text{ logit}^{-1}(\theta_{ijklm\eta})$$

$$\sigma_{ijklm\eta}^2 = \mu_{ijklm\eta} \left(1 - \frac{\mu_{ijklm\eta}}{n}\right) \left(1 + (n-1) \rho\right)$$

$$\theta_{ijklm\eta} = \beta_0 + \beta_j^{(s)} + \beta_k^{(p)} + \beta_l^{(g)} + \beta_{m\eta}^{(t)}$$

$y_{ijklm\eta}$: i^{th} sample of the j^{th} species',
integer weight, in the k^{th} port, caught
with the l^{th} gear, in the η^{th} quarter,
of year m , for a particular market
category.

$j \in \{1, \dots, J\}$ Species

$k \in \{1, \dots, K\}$ Ports

$l \in \{1, \dots, L\}$ Gears

$m \in \{1, \dots, M\}$ Years

$\eta \in \{1, \dots, H\}$ Quarters

Time Models

(M1)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)}$$

$$\beta_m^{(y)} \sim N(0, 32^2)$$

$$\beta_\eta^{(q)} \sim N(0, 32^2)$$

(M2)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_\eta^{(q)} \sim N(0, v^{(q)})$$

(M3)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)} + \beta_{m\eta}^{(y:q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_\eta^{(q)} \sim N(0, v^{(q)})$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M4)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M5)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_\eta)$$

(M6)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_m)$$

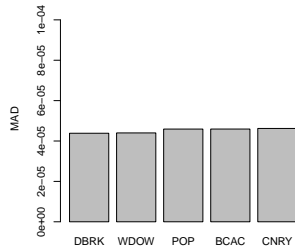
MCAT 250

	M1	M2	M3	M4	M5	M6
Δ DIC	6448.98	0.33	0	4.45	9.3	7.42
Δ WAIC	6421.5	0.37	0	4.52	8.25	6.55
$pr(M y)$	0	0	0	0.03	0	0.97

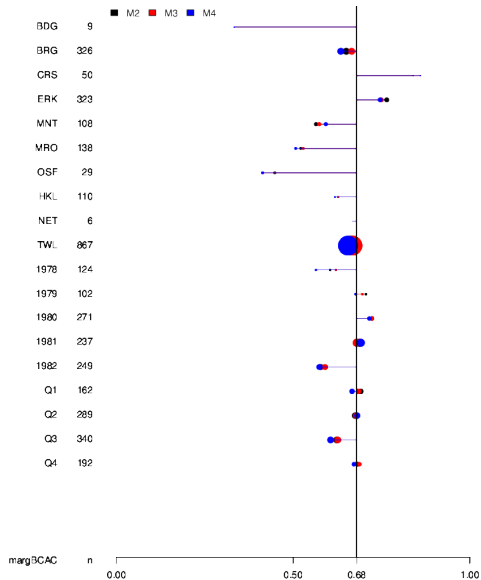
M3

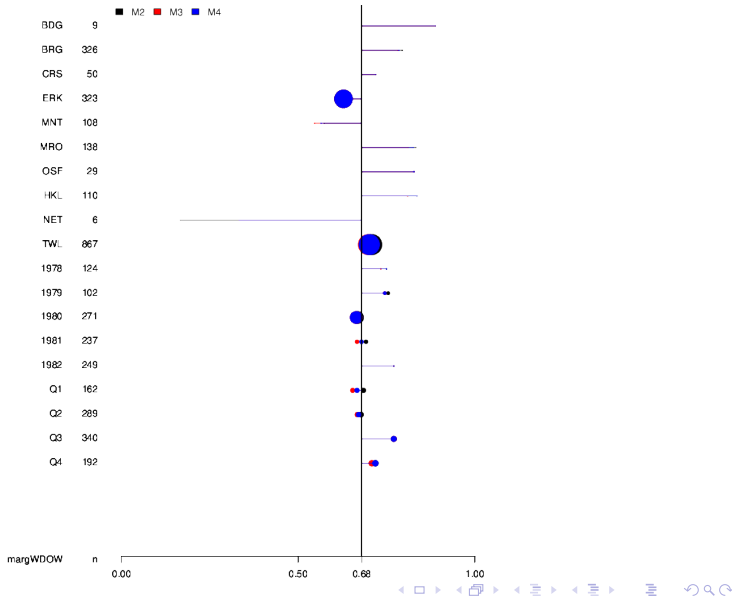
M4

MAD Ordered by Species



Combined



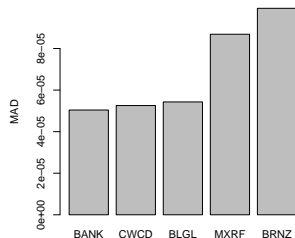


M2

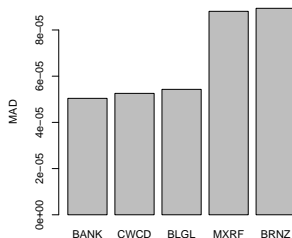
M3

M4

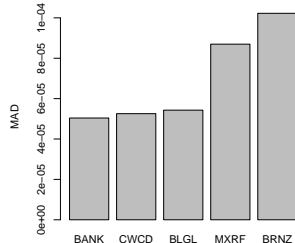
MAD Ordered by Species



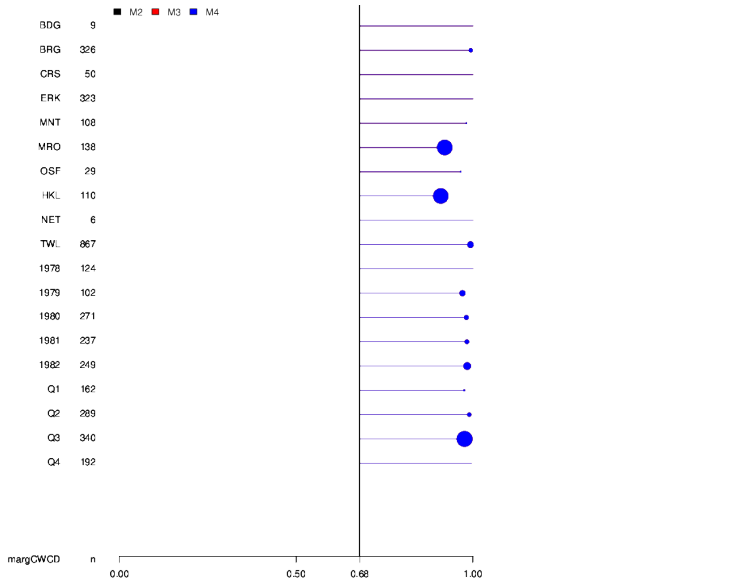
MAD Ordered by Species

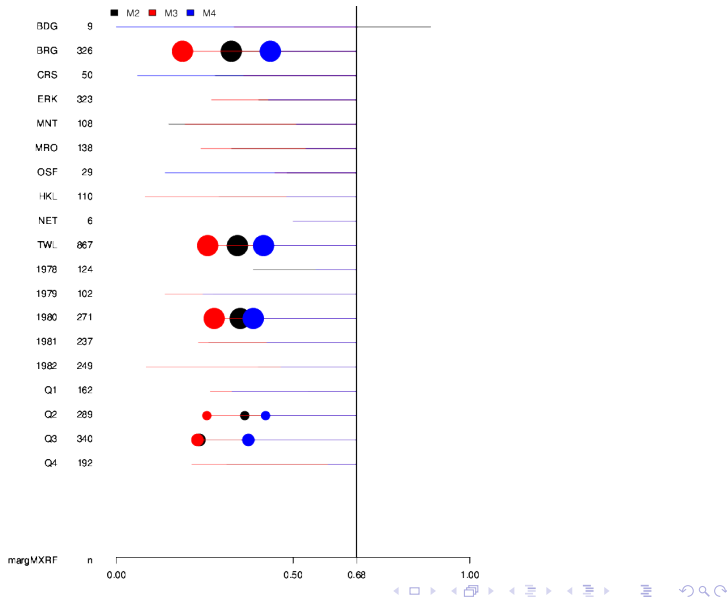


MAD Ordered by Species



Combined





MCAT 253

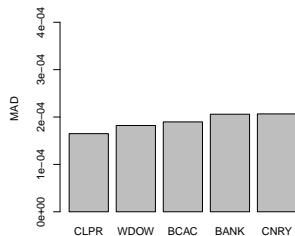
	M1	M2	M3	M4	M5	M6
Δ DIC	1409.81	0.09	0.1	0.07	0.05	0
Δ WAIC	1391.66	0.16	0.18	0	0.13	0.08
$pr(M y)$	0	0	0	1	0	0

M4

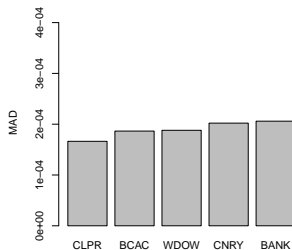
M5

M6

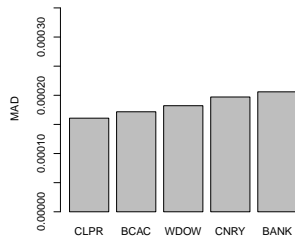
MAD Ordered by Species



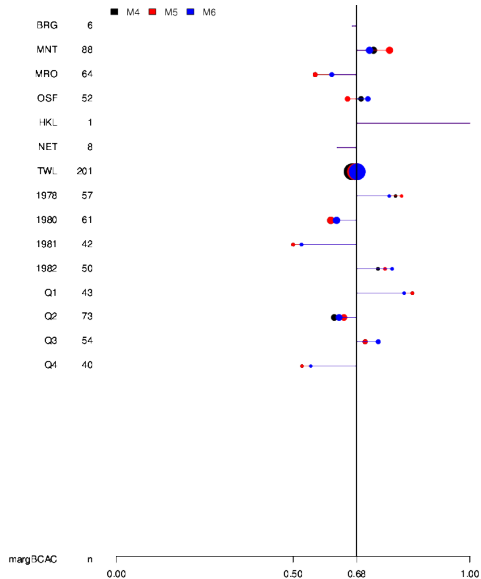
MAD Ordered by Species

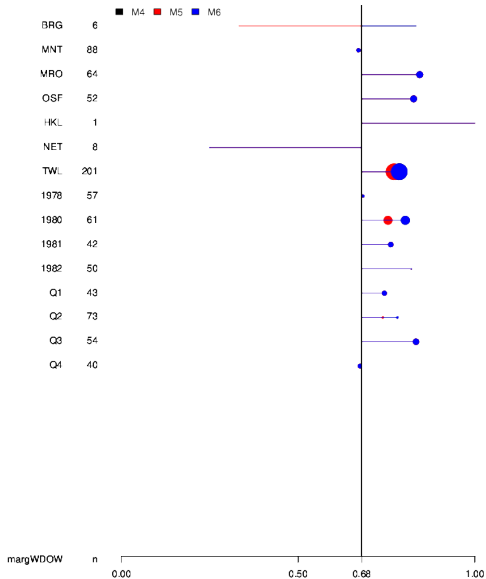


MAD Ordered by Species



Combined



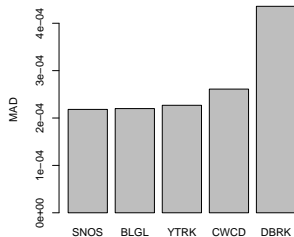


M4

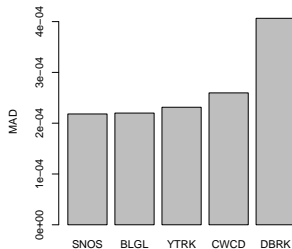
M5

M6

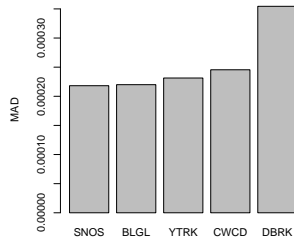
MAD Ordered by Species



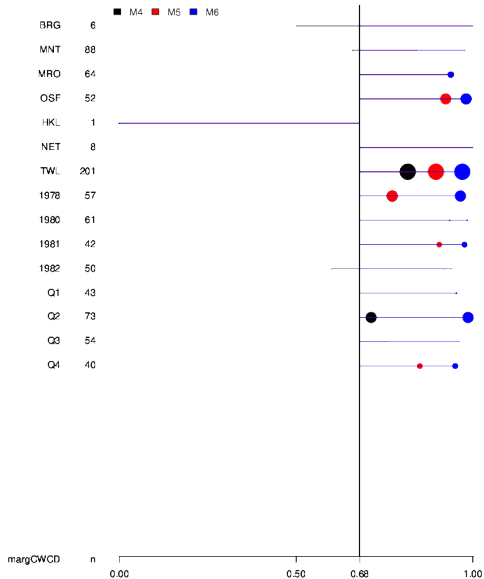
MAD Ordered by Species

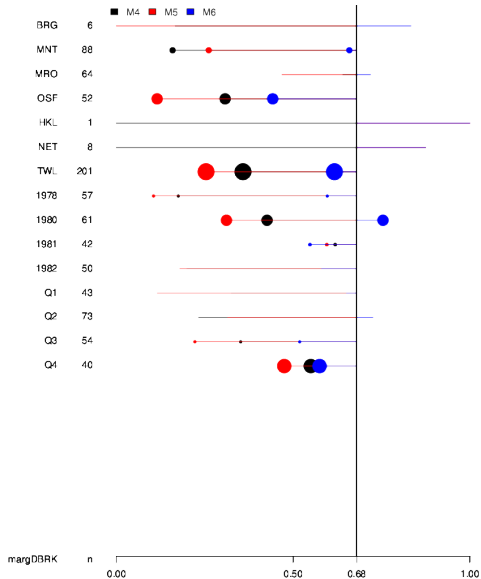


MAD Ordered by Species



Combined





MCAT 269

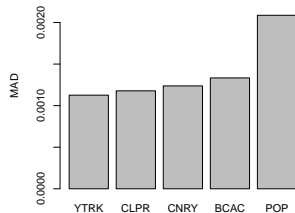
	M1	M2	M3	M4	M5	M6
Δ DIC	572.51	176.63	599.41	0.57	0	193.35
Δ WAIC	427.48	69.37	454.41	0.23	0	78.07
$pr(M y)$	0	0	0	0	0	1

M4

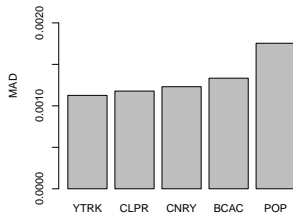
M5

M6

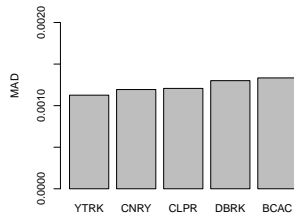
MAD Ordered by Species



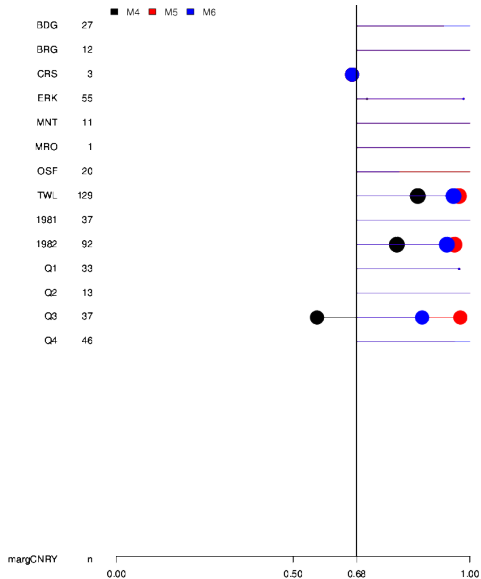
MAD Ordered by Species

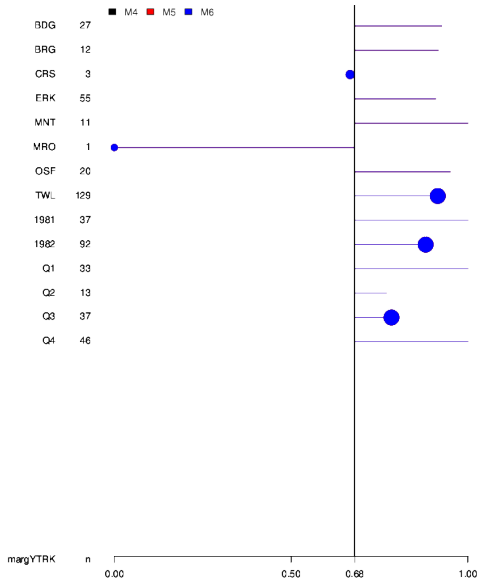


MAD Ordered by Species



Combined



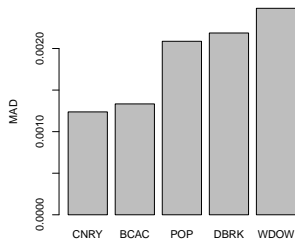


M4

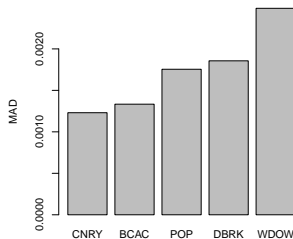
M5

M6

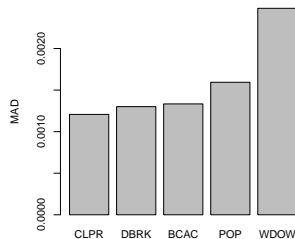
MAD Ordered by Species



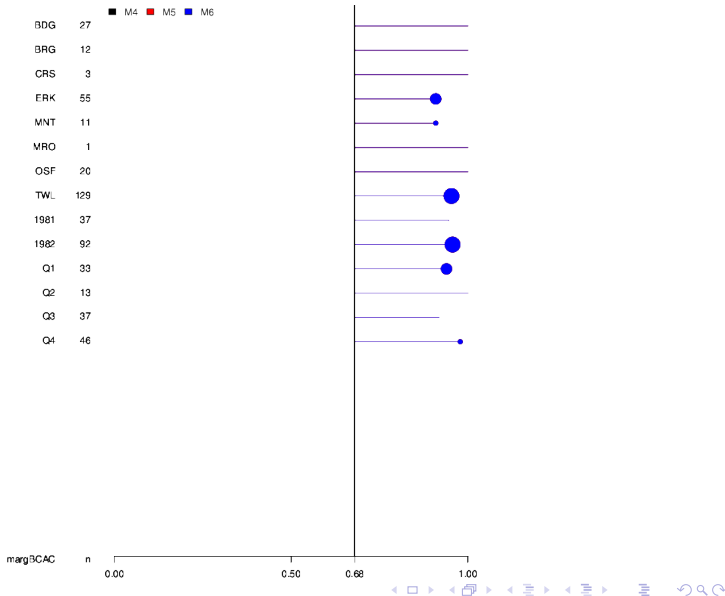
MAD Ordered by Species

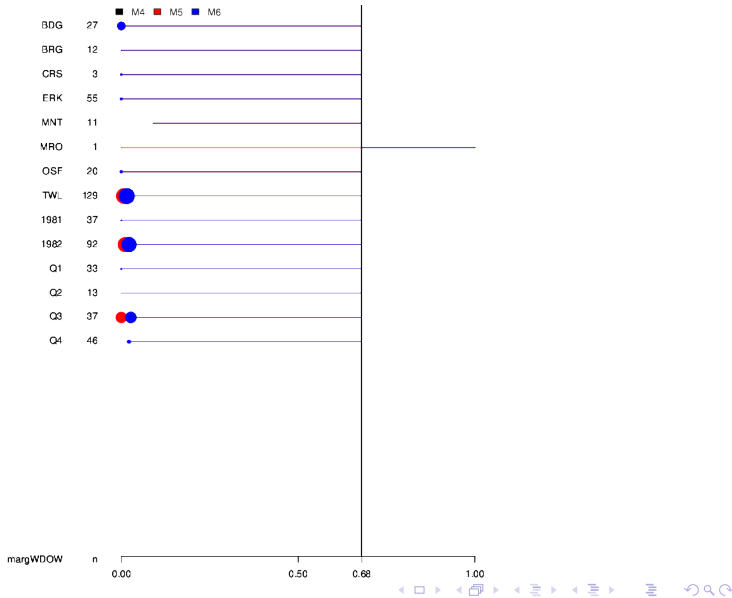


MAD Ordered by Species



Combined





Priors

$$\beta_0 \propto 1$$

$$\beta_j^{(s)} \sim N(0, 32^2)$$

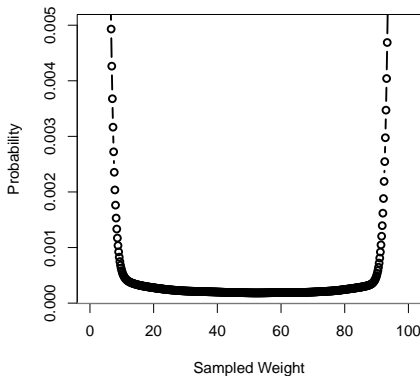
$$\beta_k^{(p)} \sim N(0, 32^2)$$

$$\beta_l^{(g)} \sim N(0, 32^2)$$

$$\text{logit}(\rho) \sim N(0, 2^2)$$

$$v \sim IG(1, 2 \times 10^3) \quad \forall \quad v$$

Prior Predictive Weight

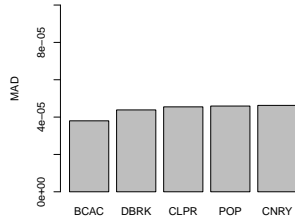


MCAT 250

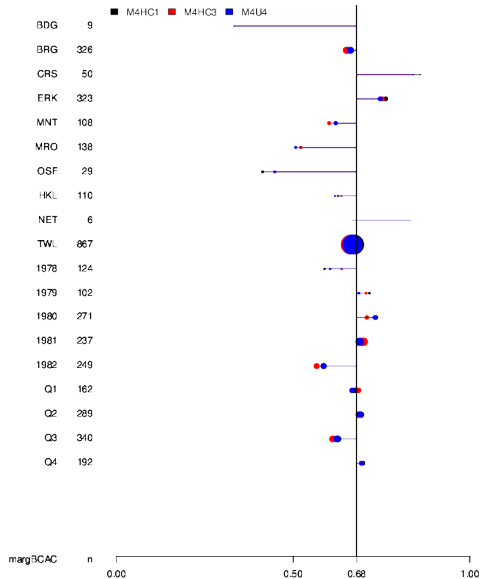
	M4	M4HC1	M4HC3	M4U4
Δ DIC	3.87	0.02	0.1	0
Δ WAIC	3.78	0.03	0.11	0
$pr(M y)$	0	0.21	0.37	0.42

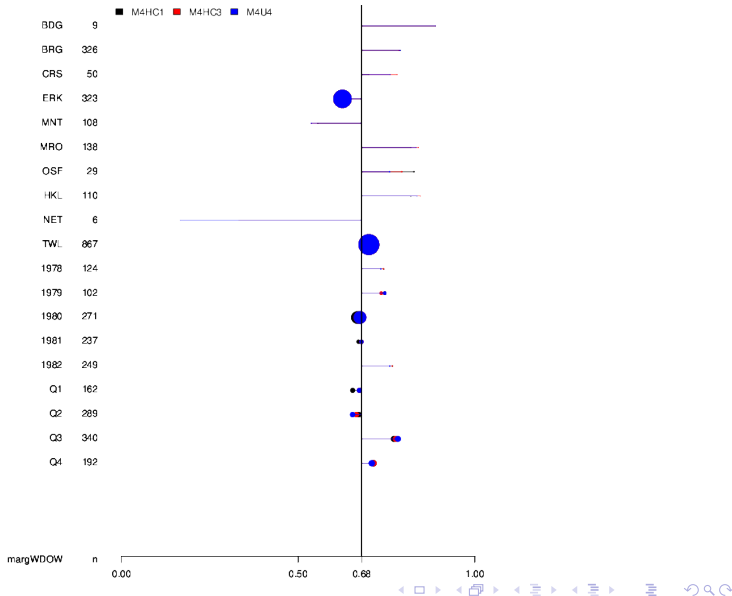
M4U4

MAD Ordered by Species



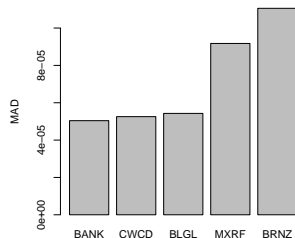
Combined



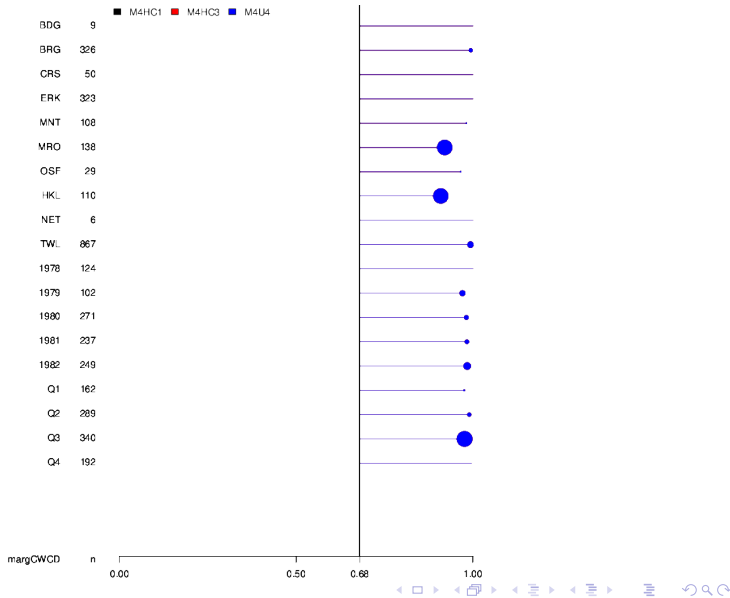


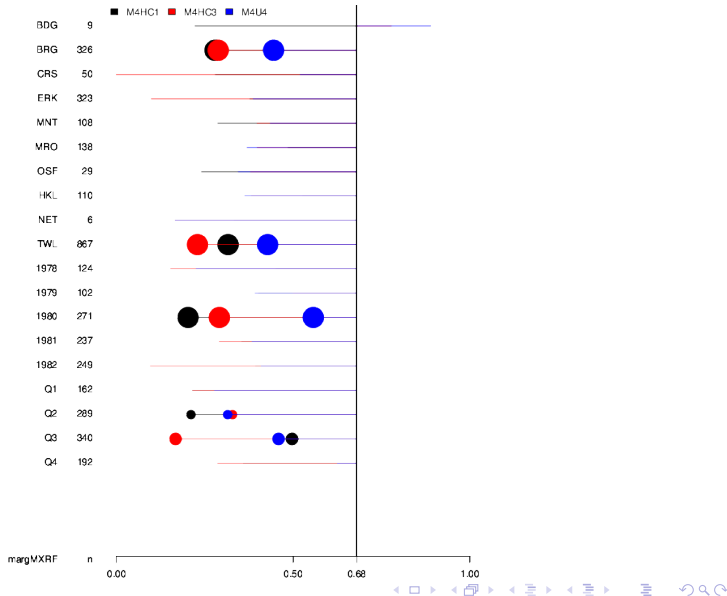
M4U4

MAD Ordered by Species



Combined



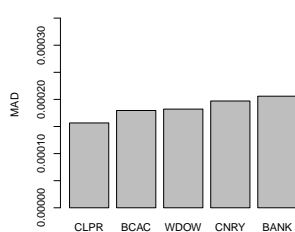


MCAT 253

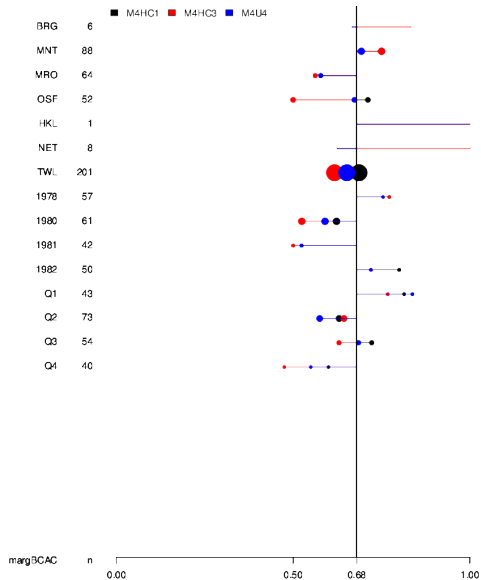
	M4	M4HC1	M4HC3	M4U4
Δ DIC	0.88	0.8	0.8	0
Δ WAIC	0.76	0.83	0.83	0
$pr(M y)$	0.01	0.99	0	0

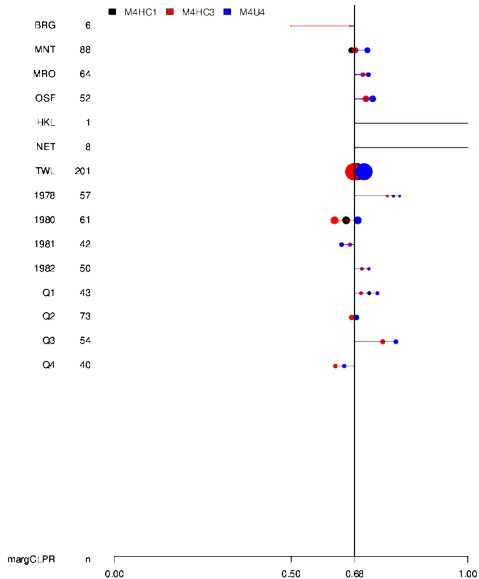
M4U4

MAD Ordered by Species



Combined



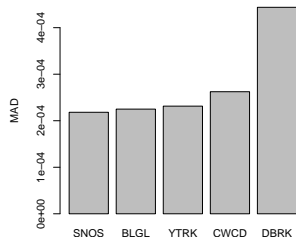


M4HC1

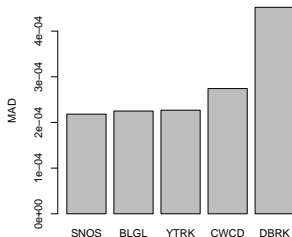
M4HC3

M4U4

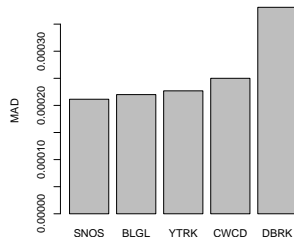
MAD Ordered by Species



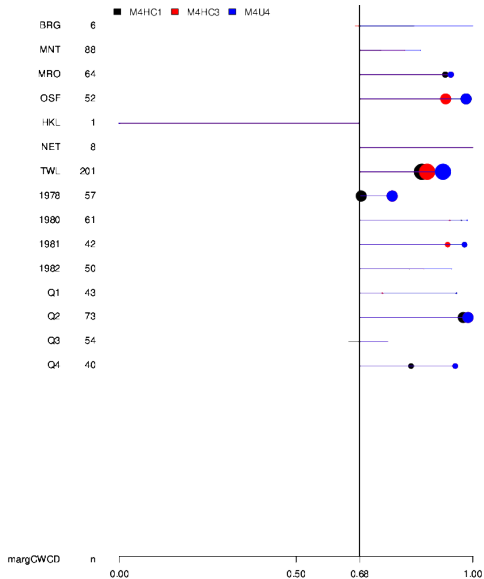
MAD Ordered by Species

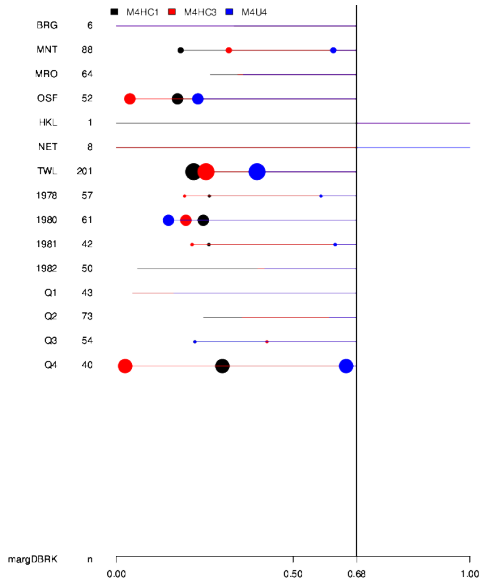


MAD Ordered by Species



Combined





MCAT 269

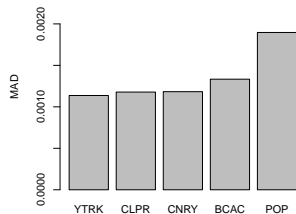
	M4	M4HC1	M4HC3	M4U4
Δ DIC	0.18	176.33	0.2	0
Δ WAIC	0.08	69.19	0.08	0
$pr(M y)$	0	1	0	0

M4HC1

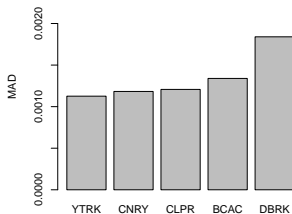
M4HC3

M4U4

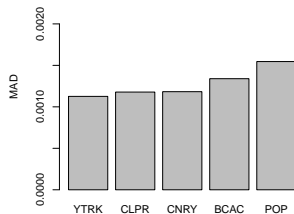
MAD Ordered by Species



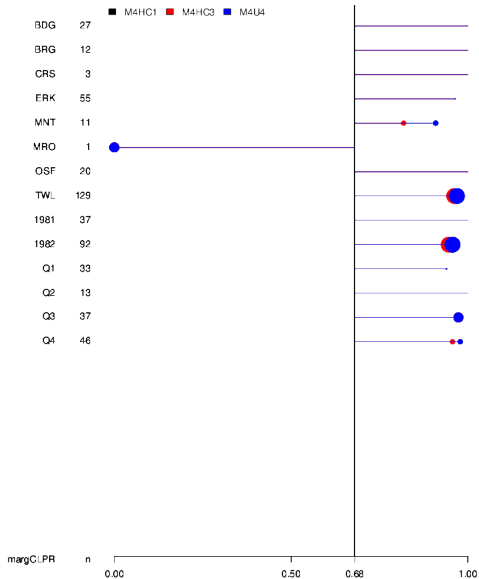
MAD Ordered by Species

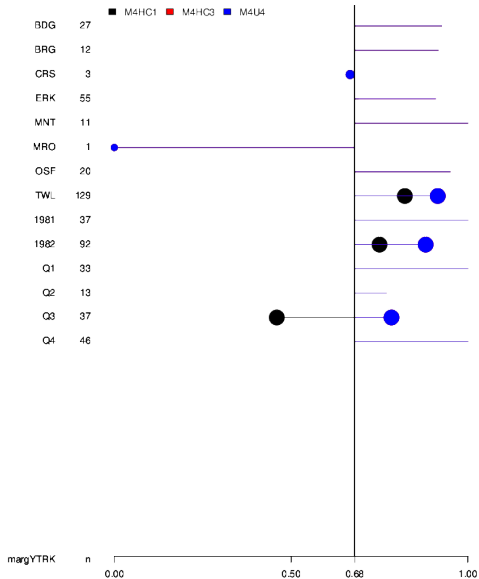


MAD Ordered by Species



Combined



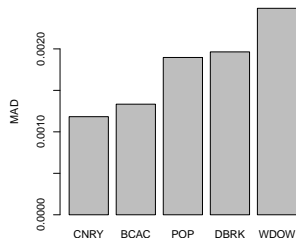


M4HC1

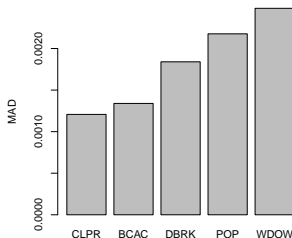
M4HC3

M4U4

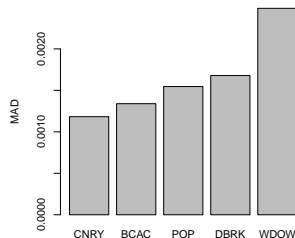
MAD Ordered by Species



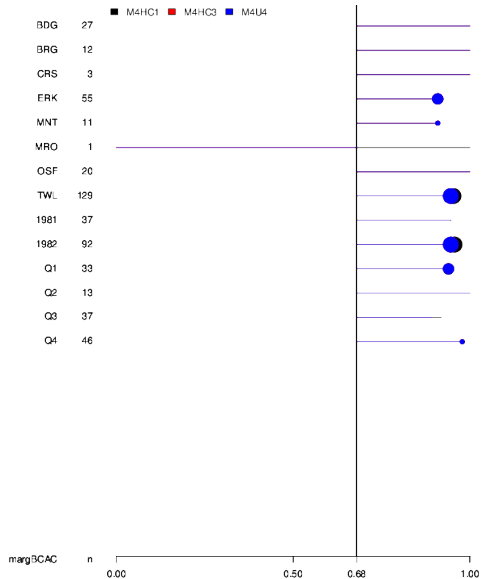
MAD Ordered by Species

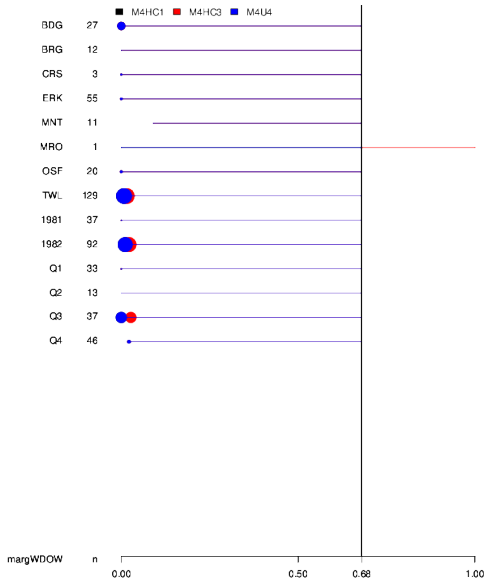


MAD Ordered by Species



Combined





Port and Gear Interactions

Model Description

MCAT 250

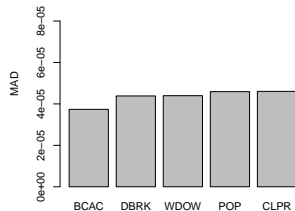
	M4HC3	M4HC3SG	M4HC3SP
Δ DIC	537.64	0	536.22
Δ WAIC	541.26	0	538.48
$pr(M y)$	0	1	0

M4HC3

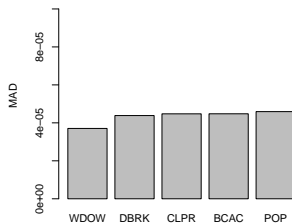
M4HC3SG

M4HC3SP

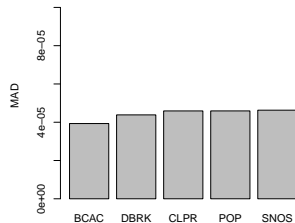
MAD Ordered by Species



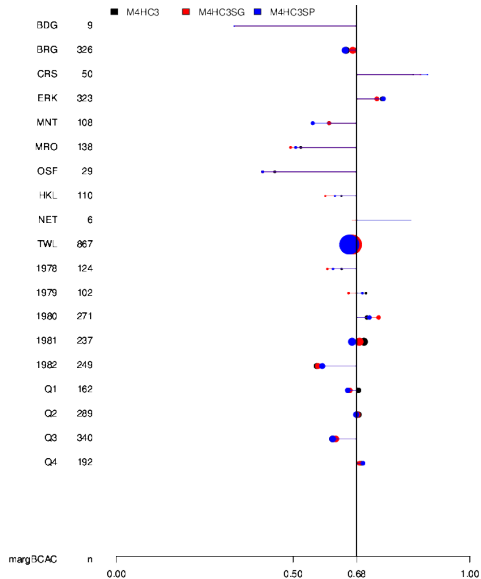
MAD Ordered by Species

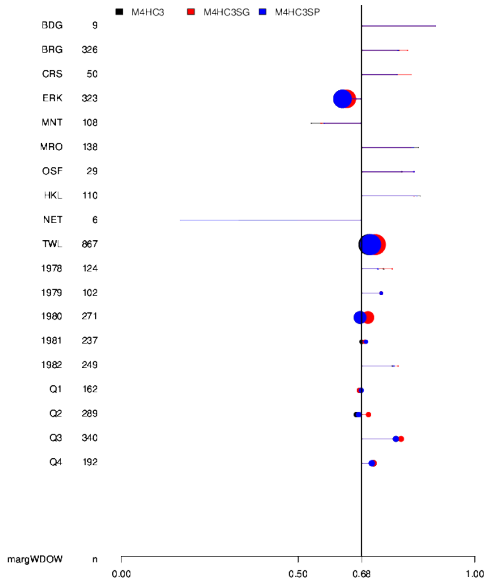


MAD Ordered by Species



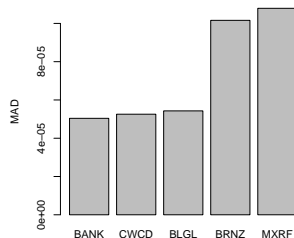
Combined



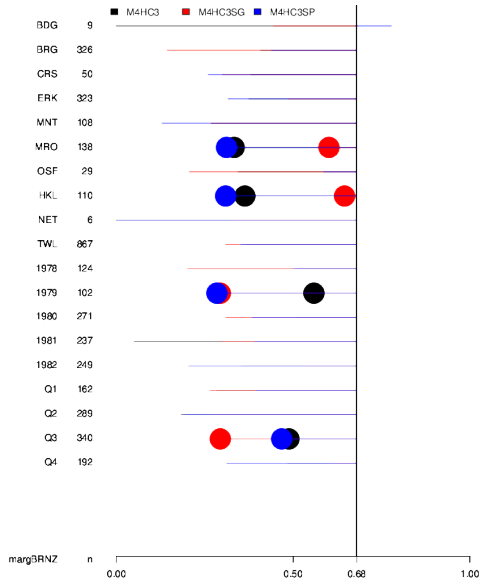


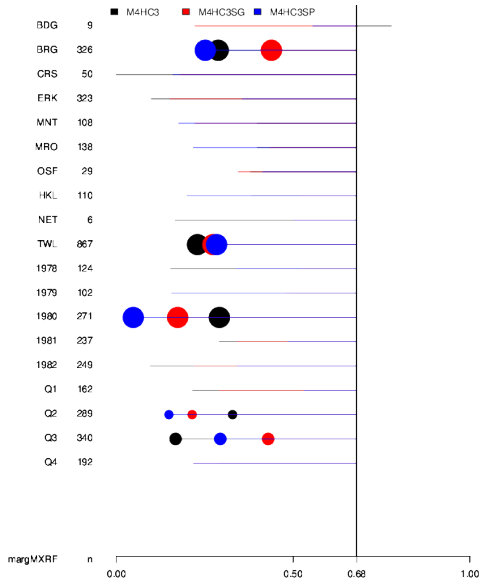
M4HC3SP

MAD Ordered by Species



Combined





MCAT 253

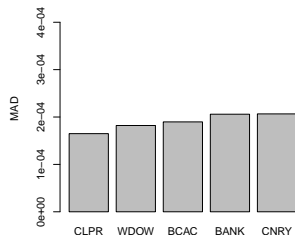
	M4	M4SG	M4SP
Δ DIC	403.42	308.65	0
Δ WAIC	872.35	778.17	0
$pr(M y)$	0	1	0

M4

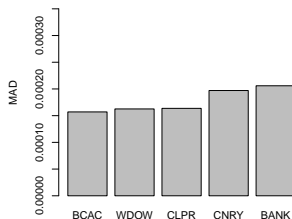
M4SG

M4SP

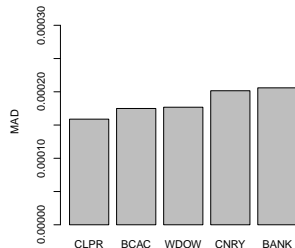
MAD Ordered by Species



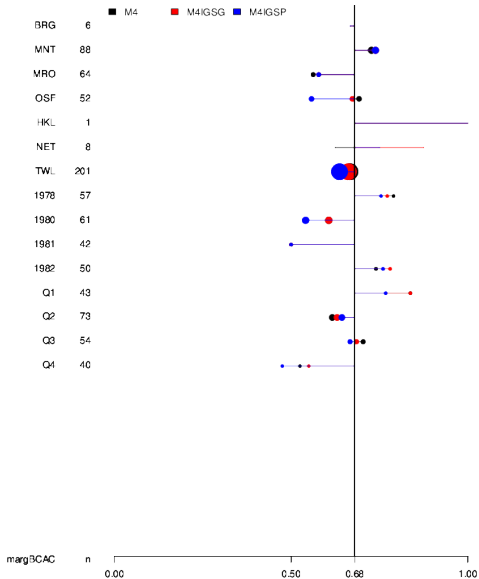
MAD Ordered by Species

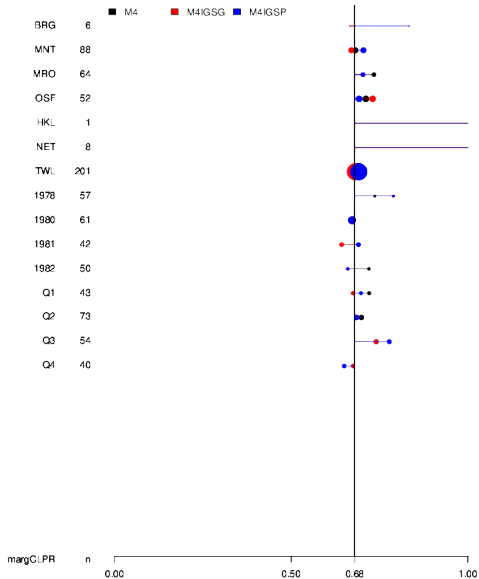


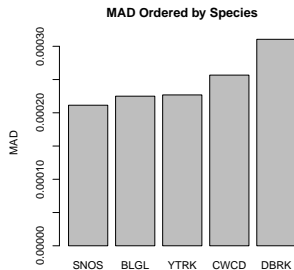
MAD Ordered by Species



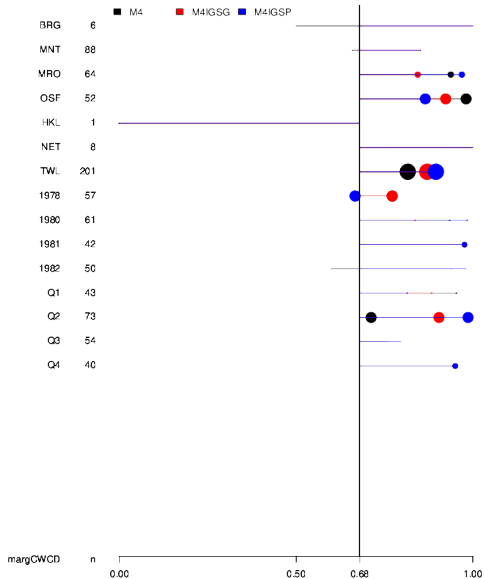
Combined

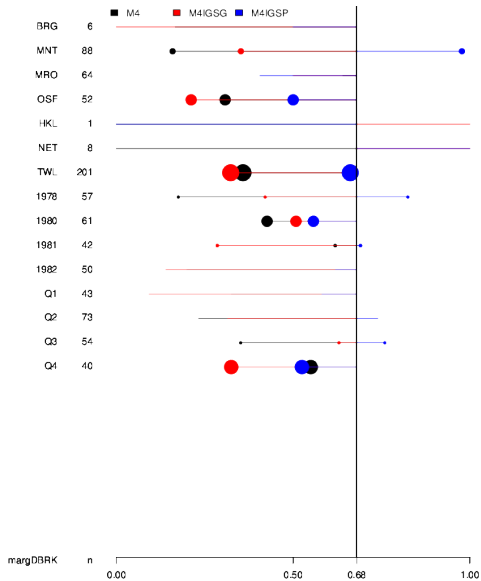






Combined





MCAT 269

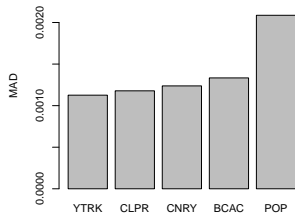
	M4	M4SG	M4SP
Δ DIC	0	0.31	182.15
Δ WAIC	0	0.13	68.48
$pr(M y)$	0	0	1

M4

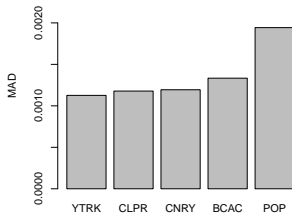
M4SG

M4SP

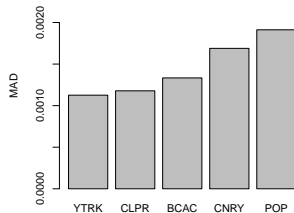
MAD Ordered by Species



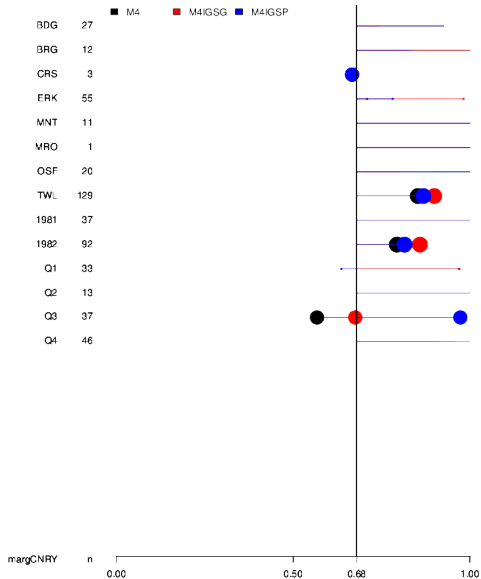
MAD Ordered by Species

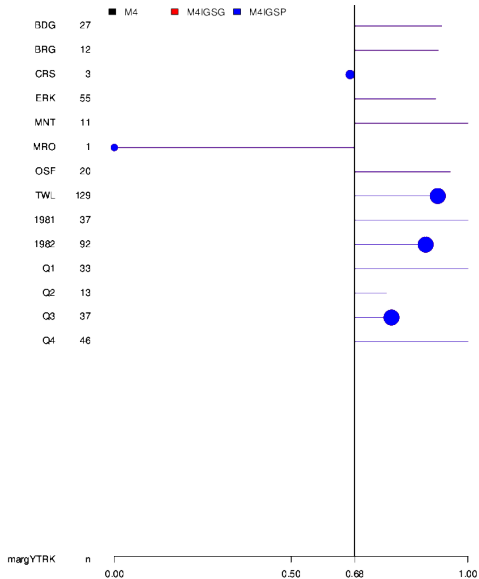


MAD Ordered by Species



Combined



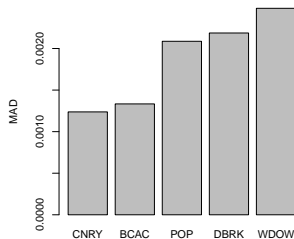


M4

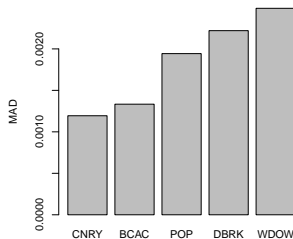
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M4SP

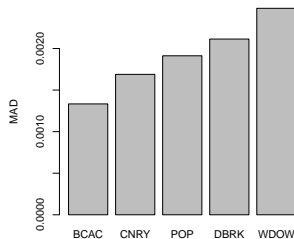
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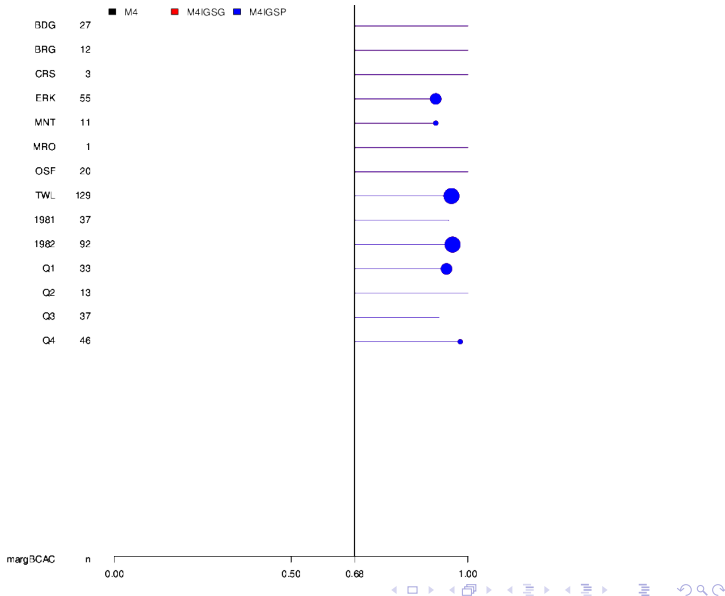
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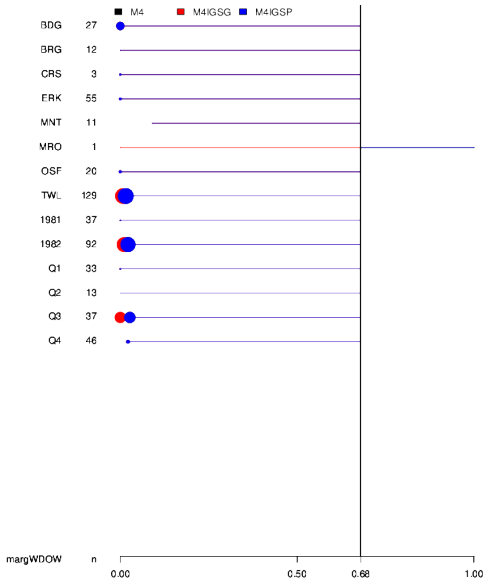


MAD Ordered by Species



Combined





Time Blocks

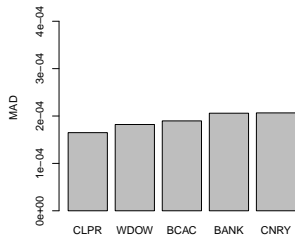
MCAT 250

78-82

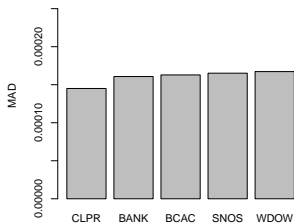
78-83

78-84

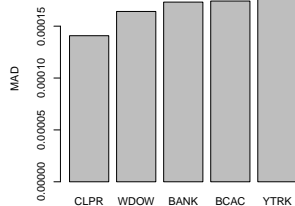
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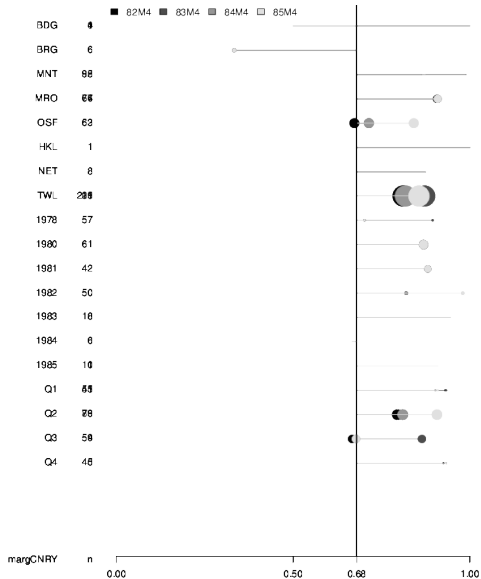
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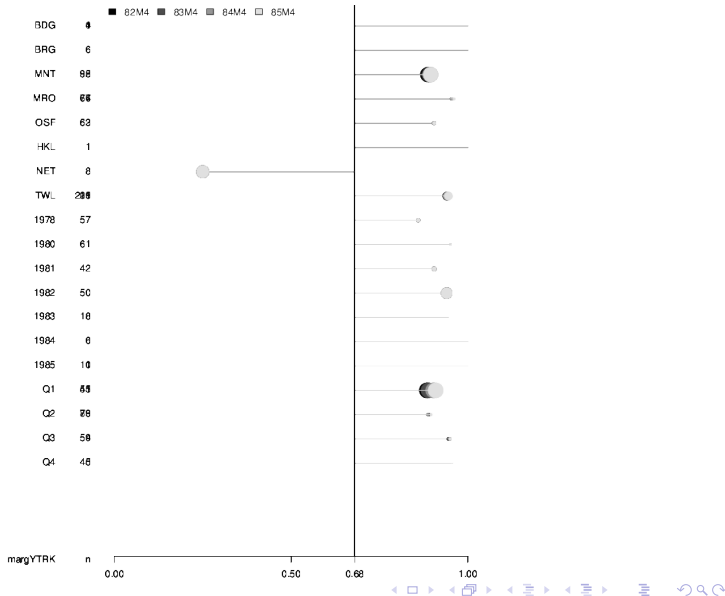


MAD Ordered by Species



Combined



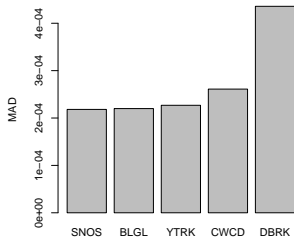


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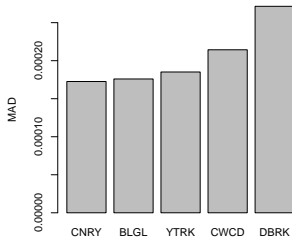
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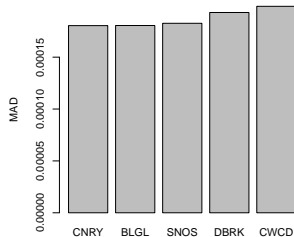
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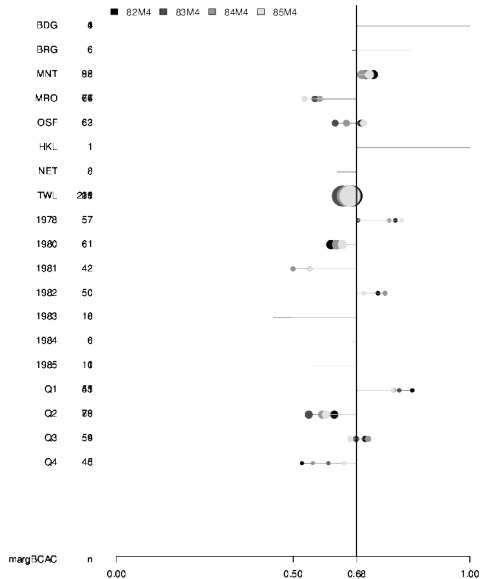
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MAD Ordered by Species



Combined

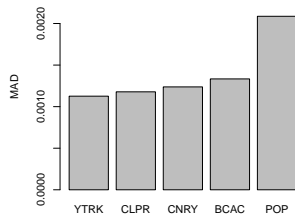


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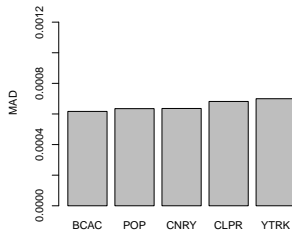
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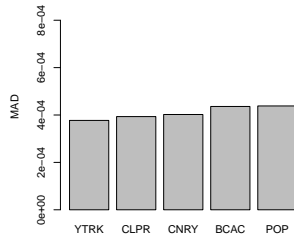
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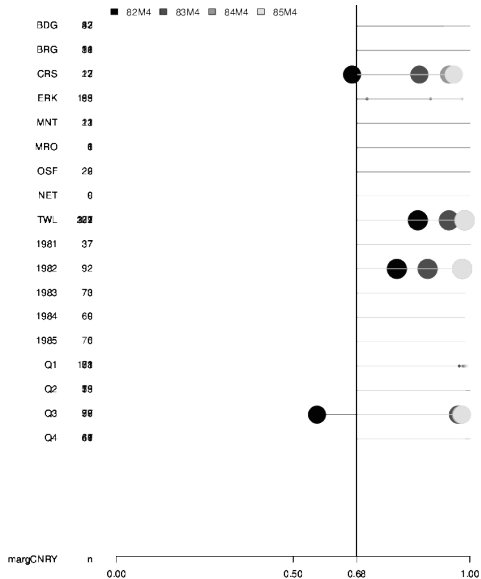
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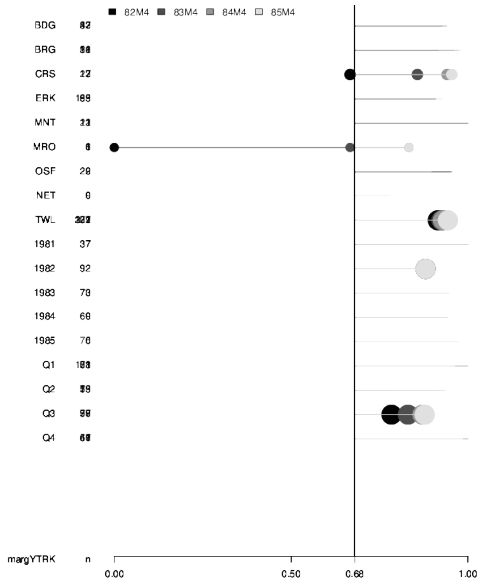


MAD Ordered by Species



Combined



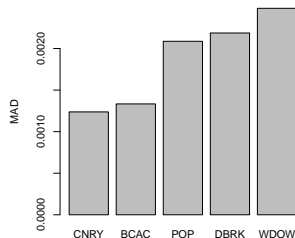


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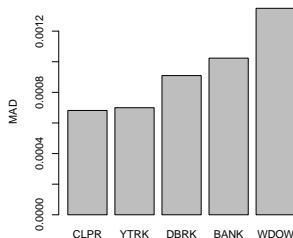
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78-84

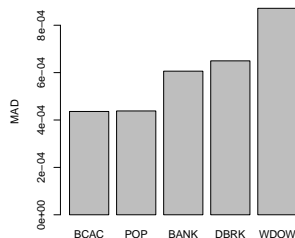
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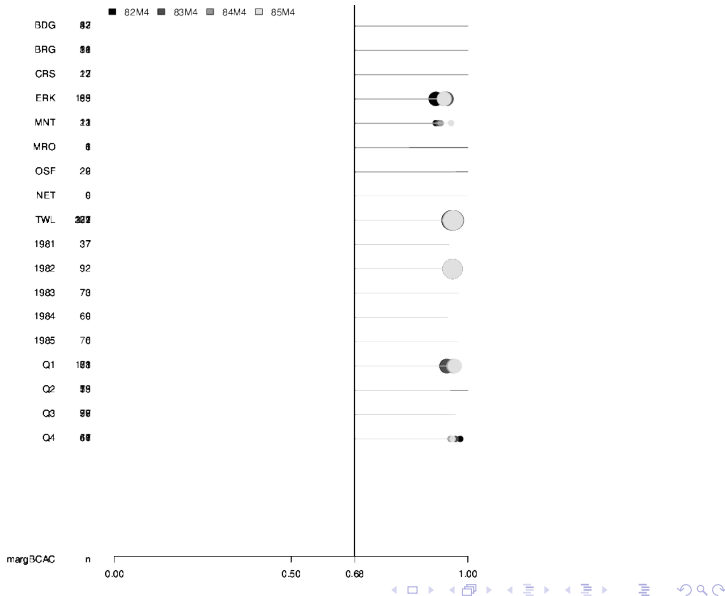
MAD Ordered by Species

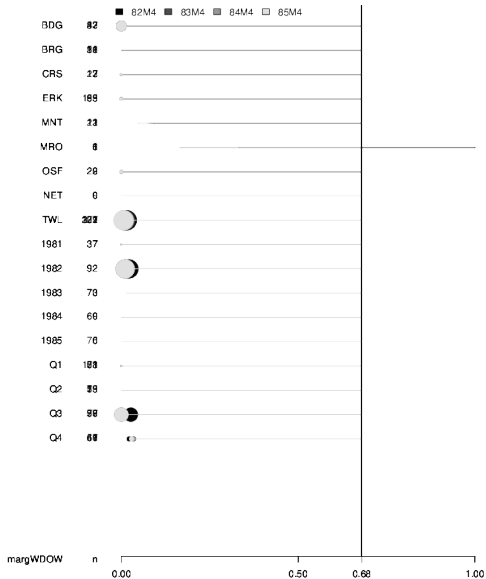


MAD Ordered by Species



Combined





Proof: Species Comps Sum to One... as do Their Means.

If y_{jk} is the k^{th} draw, $k \in \{1, \dots, K\}$, of the posterior predictive weight of species j in a particular stratum. Then,

$$\pi_{jk} = \frac{y_{jk}}{\sum_j y_{jk}} \quad \mathbf{y}_k \neq \mathbf{0}. \quad (1)$$

The predictive mean for species j is,

$$\hat{\pi}_j = \frac{\sum_k^K \pi_{jk}}{K}. \quad (2)$$

Summing $\hat{\pi}_j$ across species, it follows from (1) and (2) that,

$$\sum_j \hat{\pi}_j \stackrel{(2)}{=} \sum_j \frac{\sum_k^K \pi_{jk}}{K} = \frac{\sum_k^K \sum_j \pi_{jk}}{K} \stackrel{(1)}{=} \frac{\sum_k^K \sum_j \frac{y_{jk}}{\sum_j y_{jk}}}{K} = \frac{\sum_k^K 1}{K} = \frac{K}{K} = 1. \quad \blacksquare$$

Proof: Species Comps are Negatively Correlated.

Here we seek to show for any two species $l \neq m$, $\text{Corr}(\pi_l, \pi_m) < 0$.

Recall:

$$\text{Corr}(\pi_l, \pi_m) = \frac{\text{Cov}(\pi_l, \pi_m)}{\sigma_{\pi_l} \sigma_{\pi_m}} \quad \sigma_{\pi_l} \geq 0, \sigma_{\pi_m} \geq 0$$

$$\text{Corr}(\pi_l, \pi_m) \leq 0 \iff \text{Cov}(\pi_l, \pi_m) \leq 0$$

$$\begin{aligned} \text{Cov}(\pi_l, \pi_m) &= \mathbb{E}[(\pi_l - \mathbb{E}[\pi_l])(\pi_m - \mathbb{E}[\pi_m])] \\ &= \mathbb{E}[\pi_l \pi_m] - \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \end{aligned}$$

$$\text{Cov}(\pi_l, \pi_m) \leq 0 \iff \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m]$$

Proof: Species Comps are Negatively Correlated Cont.

Consider the strictly concave function:

$$f(\mathbf{x}) = \prod_i x_i : \mathbf{x} \in \left\{ \mathbf{y} \mid \sum_i y_i = 1, y_i \geq 0 \right\}$$

Jensen's Inequality for f is,

$$f(\mathbb{E}[\mathbf{x}]) \geq \mathbb{E}[f(\mathbf{x})]. \quad (3)$$

From the previous proof: $\sum_j \pi_j = 1$, $\pi_j \geq 0$ and $\sum_j \hat{\pi}_j = 1$, $\hat{\pi}_j \geq 0$.

Thus applying (3) to π gives

$$\mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m] \quad (4)$$

with equality only if π is a constant. Since π is never a constant,

$$\mathbb{E}[\pi_l] \mathbb{E}[\pi_m] > \mathbb{E}[\pi_l \pi_m]$$

$$\text{Cov}(\pi_l, \pi_m) < 0$$

$$\text{Corr}(\pi_l, \pi_m) < 0. \quad \blacksquare$$