

Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

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UCSC :: CSTAR :: SWFSC :: NMFS

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Diagnostic

Show Example

Beta-Binomial Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}(\mu_{ijklm\eta}, \sigma_{ijklm\eta}^2)$$

$$\mu_{ijklm\eta} = n \text{ logit}^{-1}(\theta_{ijklm\eta})$$

$$\sigma_{ijklm\eta}^2 = \mu_{ijklm\eta} \left(1 - \frac{\mu_{ijklm\eta}}{n}\right) \left(1 + (n-1) \rho\right)$$

$$\theta_{ijklm\eta} = \beta_0 + \beta_j^{(s)} + \beta_k^{(p)} + \beta_l^{(g)} + \beta_{m\eta}^{(t)} \beta_{m\eta}^{(y:q)}$$

$y_{ijklm\eta}$: i^{th} sample of the j^{th} species' integer weight, in the k^{th} port, caught with the l^{th} gear, in the η^{th} quarter, of year m , for a particular market category.

$j \in \{1, \dots, J\}$ Species

$k \in \{1, \dots, K\}$ Ports

$l \in \{1, \dots, L\}$ Gears

$m \in \{1, \dots, M\}$ Years

$\eta \in \{1, \dots, H\}$ Quarters

Time Models

(M1)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, 32^2)$$

$$\beta_{\eta}^{(q)} \sim N(0, 32^2)$$

(M2)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

(M3)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)} + \beta_{m\eta}^{(y:q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M4)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M5)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_{\eta})$$

(M6)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

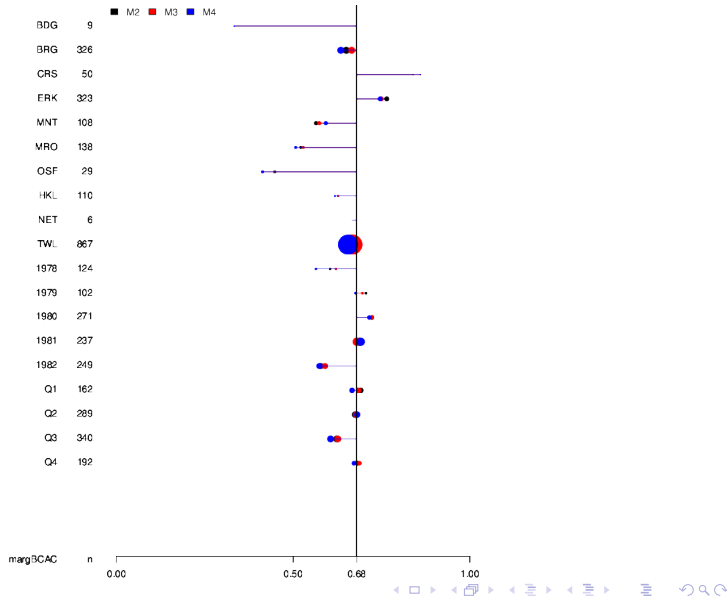
$$\beta_{m\eta}^{(y:q)} \sim N(0, v_m)$$

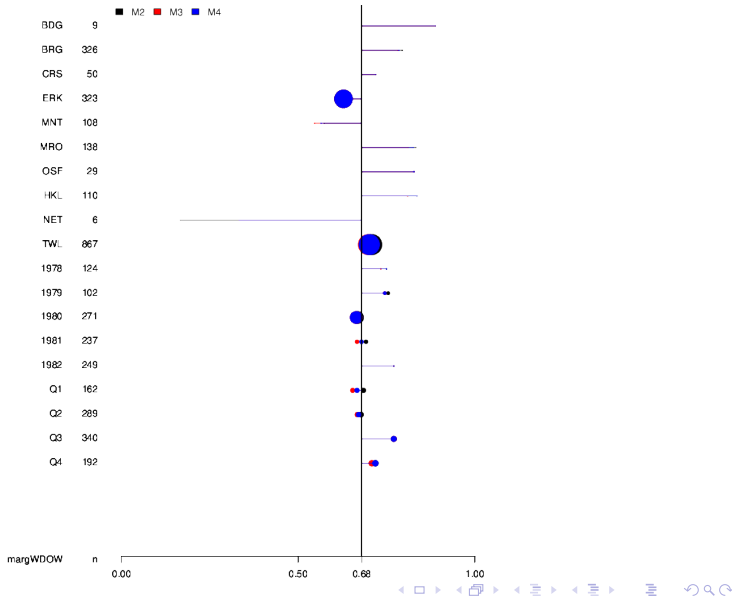
MCAT 250

	M1	M2	M3	M4	M5	M6
Δ DIC	6448.98	0.33	0	4.45	9.3	7.42
Δ WAIC	6421.5	0.37	0	4.52	8.25	6.55
$pr(M y)$	0	0	0	0.03	0	0.97

M4

MAD Ordered by Species



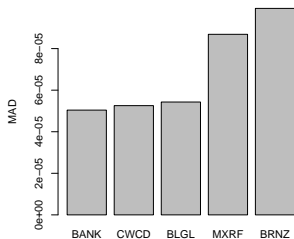


M2

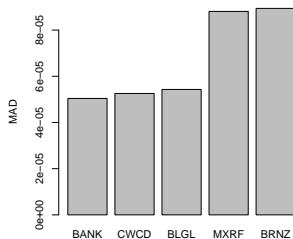
M3

M4

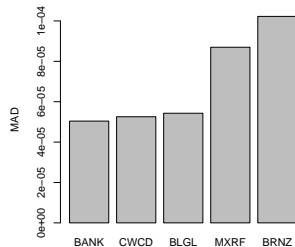
MAD Ordered by Species

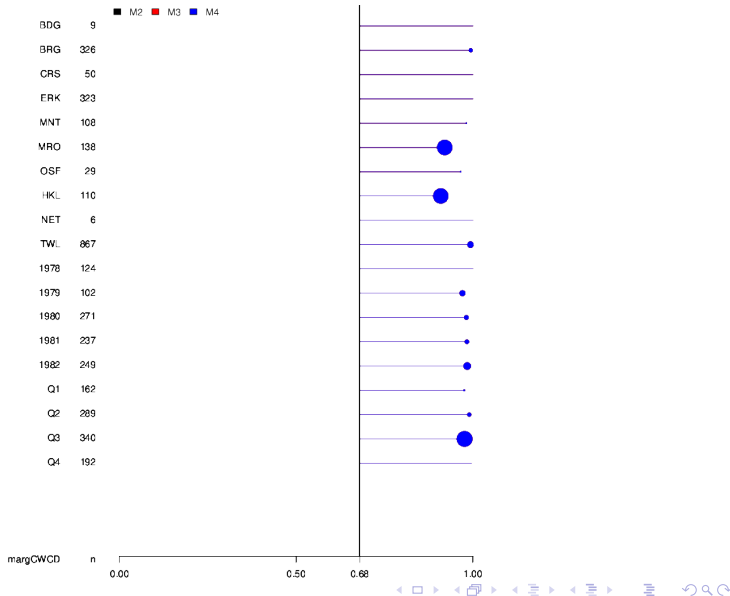


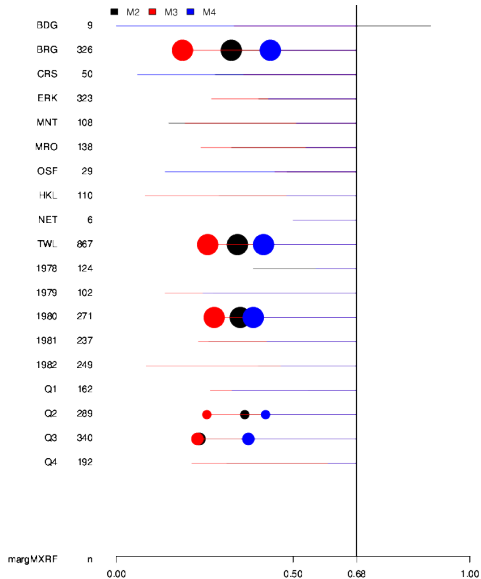
MAD Ordered by Species



MAD Ordered by Species







MCAT 253

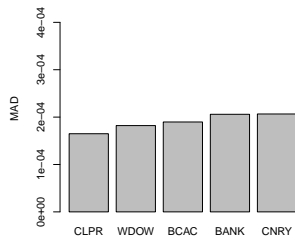
	M1	M2	M3	M4	M5	M6
Δ DIC	1409.81	0.09	0.1	0.07	0.05	0
Δ WAIC	1391.66	0.16	0.18	0	0.13	0.08
$pr(M y)$	0	0	0	1	0	0

M4

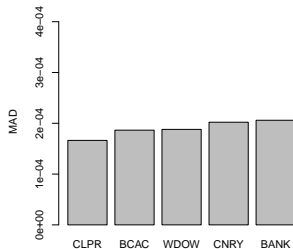
M5

M6

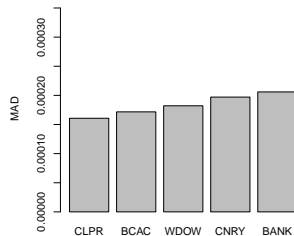
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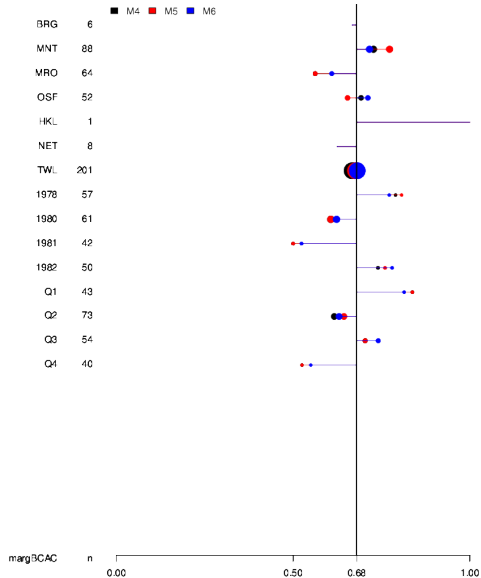


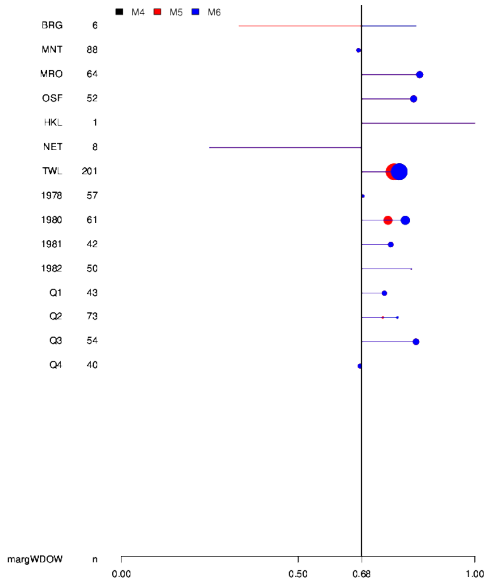
MAD Ordered by Species



MAD Ordered by Species





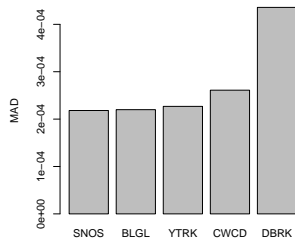


M4

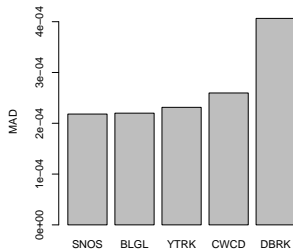
M5

M6

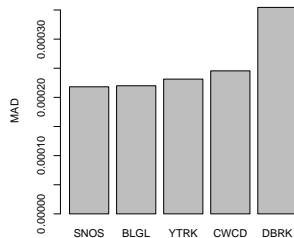
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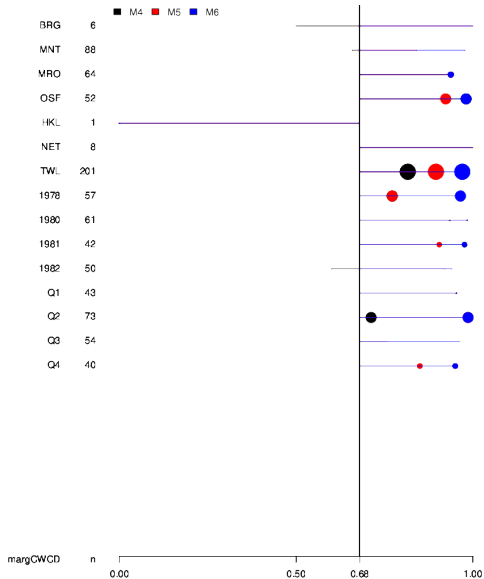


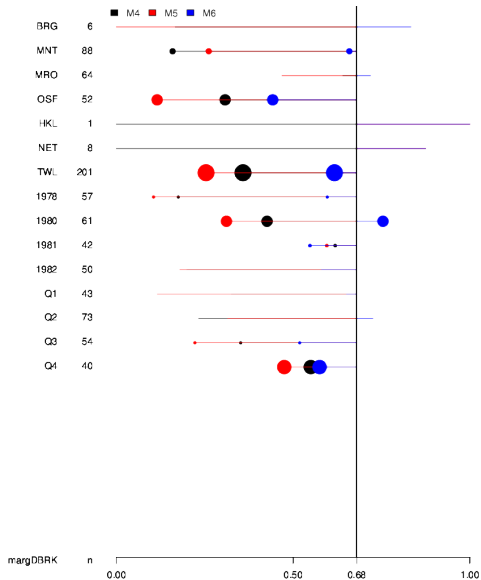
MAD Ordered by Species



MAD Ordered by Species







MCAT 269

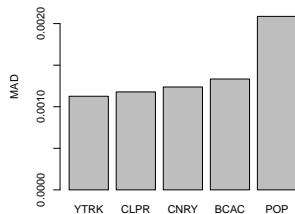
	M1	M2	M3	M4	M5	M6
Δ DIC	572.51	176.63	599.41	0.57	0	193.35
Δ WAIC	427.48	69.37	454.41	0.23	0	78.07
$pr(M y)$	0	0	0	0	0	1

M4

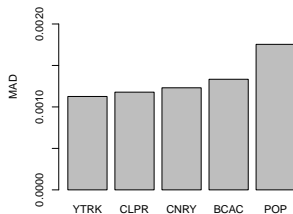
M5

M6

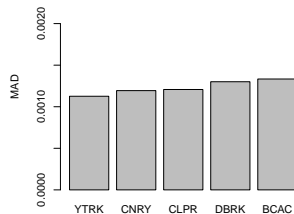
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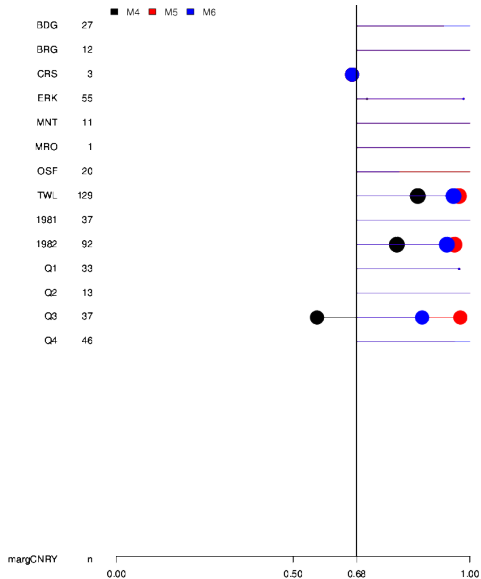


MAD Ordered by Species



MAD Ordered by Species



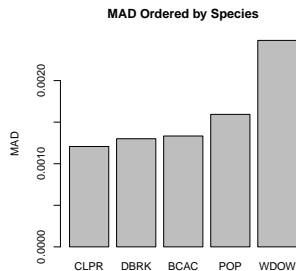
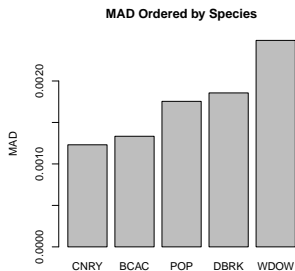
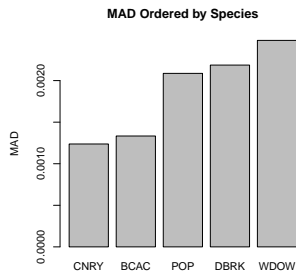


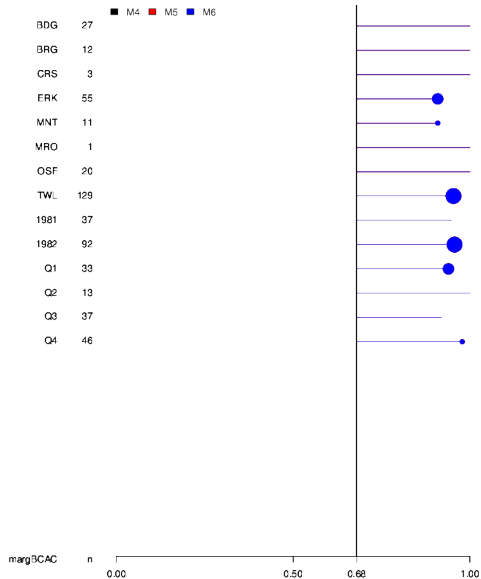


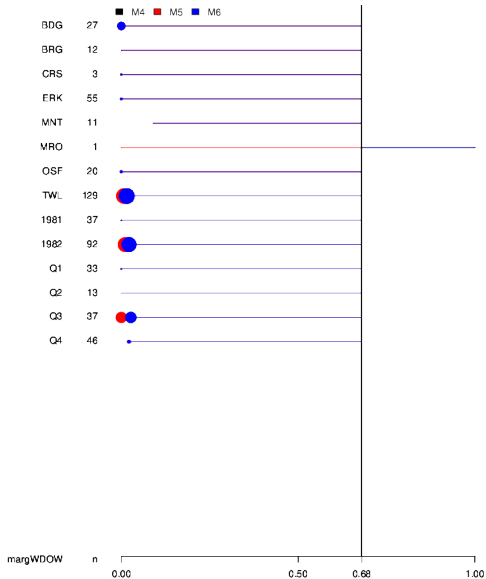
M4

M5

M6







Priors

$$\beta_0 \propto 1$$

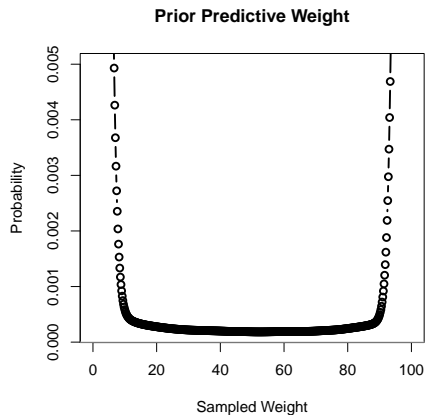
$$\beta_i^{(s)} \sim N(0, 32^2)$$

$$\beta_k^{(p)} \sim N(0, 32^2)$$

$$\beta_l^{(g)} \sim N(0, 32^2)$$

$$\text{logit}(\rho) \sim N(0, 2^2)$$

$$v \sim IG(1, 2 \times 10^3) \quad \forall \quad v$$



MCAT 250

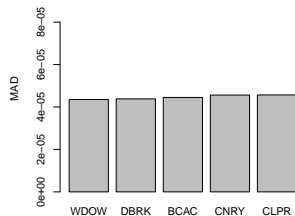
	M4	M4HC1	M4HC3	M4U4
Δ DIC	3.87	0.02	0.1	0
Δ WAIC	3.78	0.03	0.11	0
$pr(M y)$	0	0.21	0.37	0.42

M4HC1

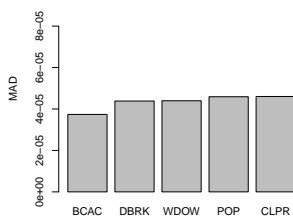
M4HC3

M4U4

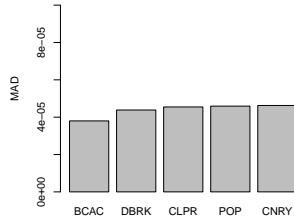
MAD Ordered by Species

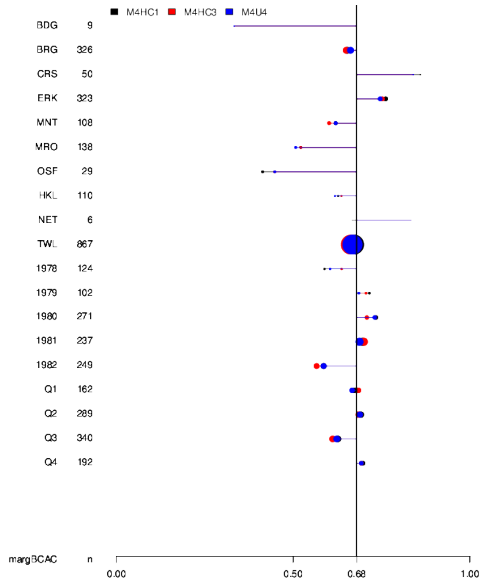


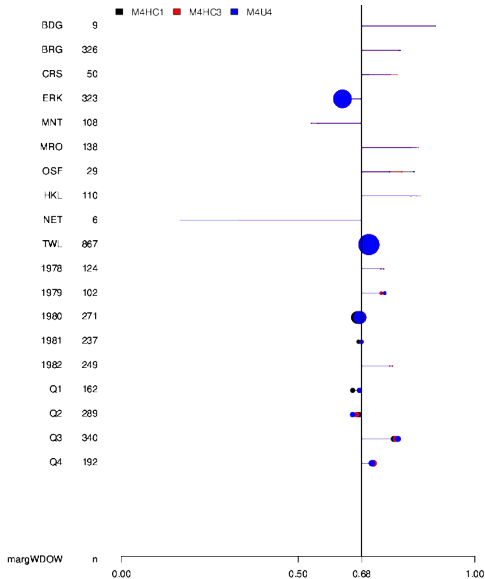
MAD Ordered by Species



MAD Ordered by Species





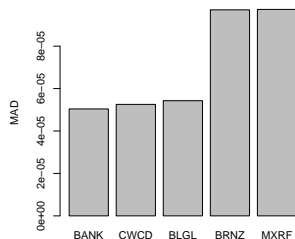


M4HC1

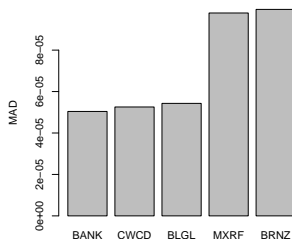
M4HC3

M4U4

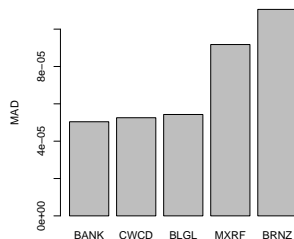
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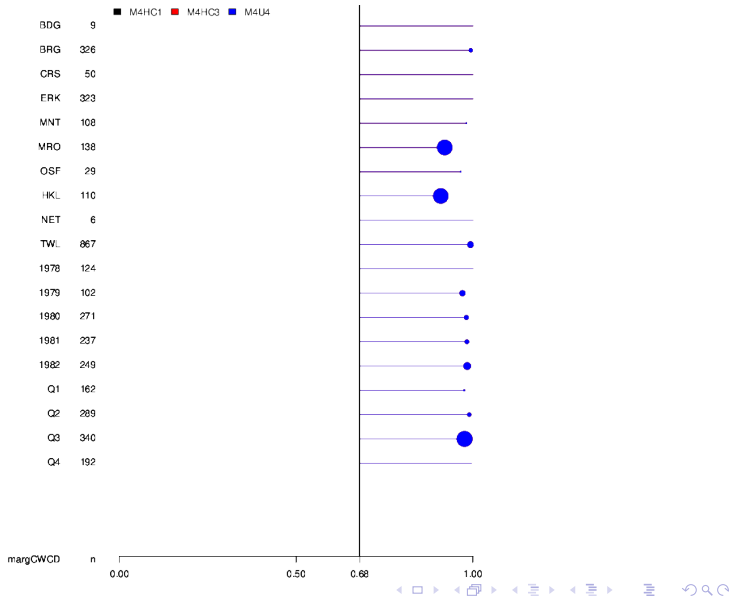


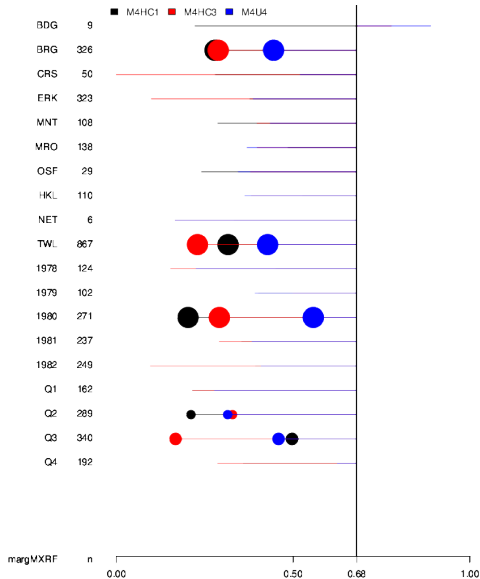
MAD Ordered by Species



MAD Ordered by Species







MCAT 253

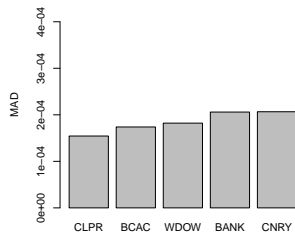
	M4	M4HC1	M4HC3	M4U4
Δ DIC	0.88	0.8	0.8	0
Δ WAIC	0.76	0.83	0.83	0
$pr(M y)$	0.01	0.99	0	0

M4HC1

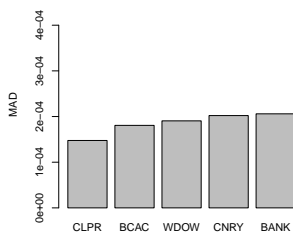
M4HC3

M4U4

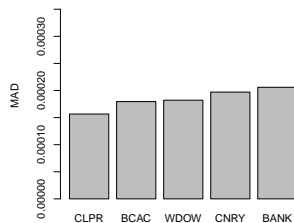
MAD Ordered by Species

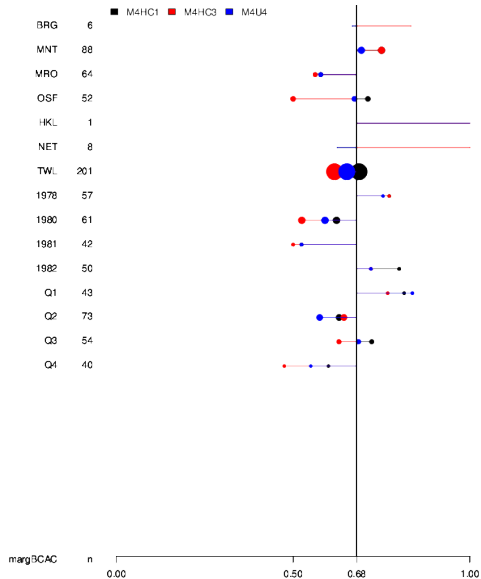


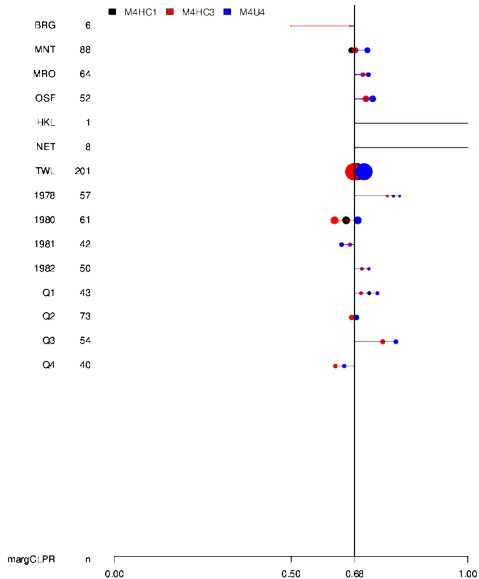
MAD Ordered by Species



MAD Ordered by Species





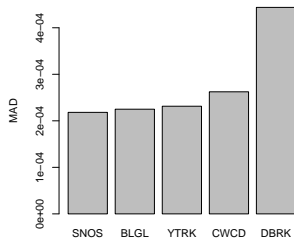


M4HC1

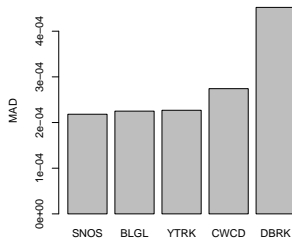
M4HC3

M4U4

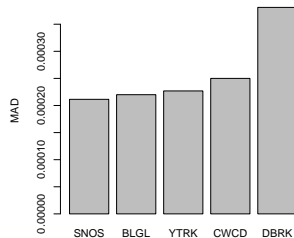
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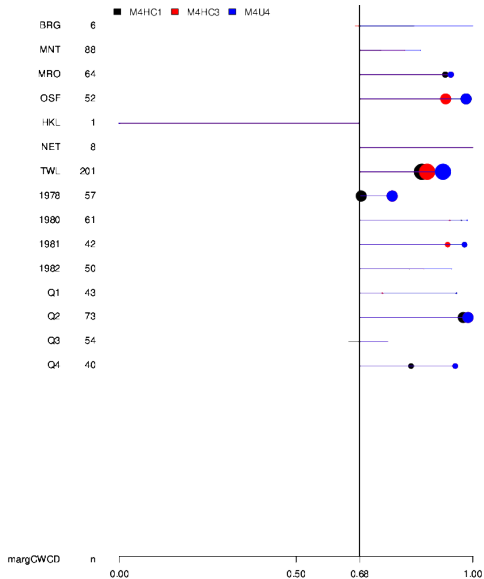


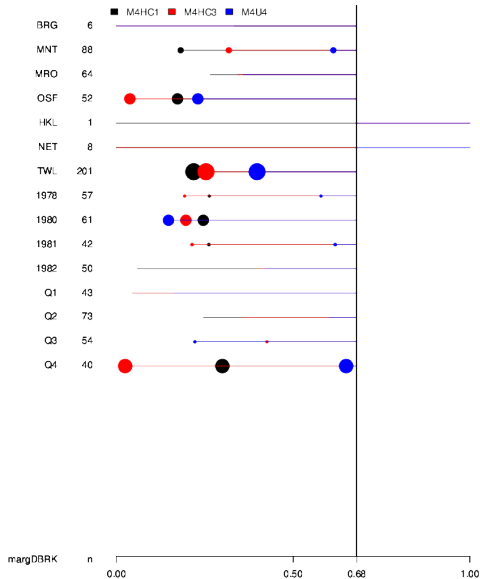
MAD Ordered by Species



MAD Ordered by Species







MCAT 269

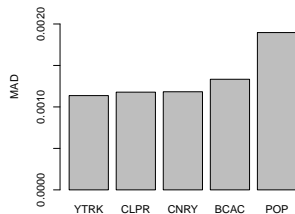
	M4	M4HC1	M4HC3	M4U4
Δ DIC	0.18	176.33	0.2	0
Δ WAIC	0.08	69.19	0.08	0
$pr(M y)$	0	1	0	0

M4HC1

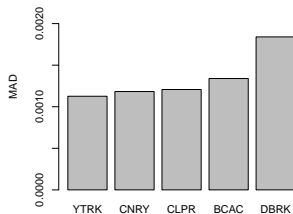
M4HC3

M4U4

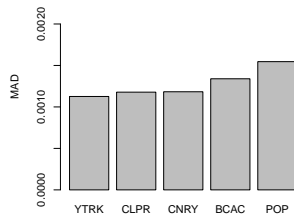
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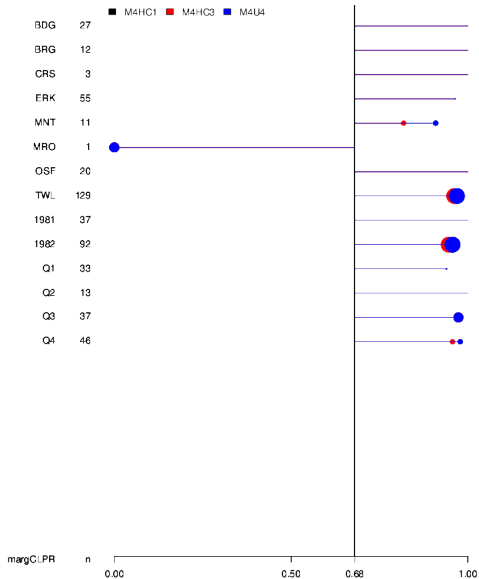


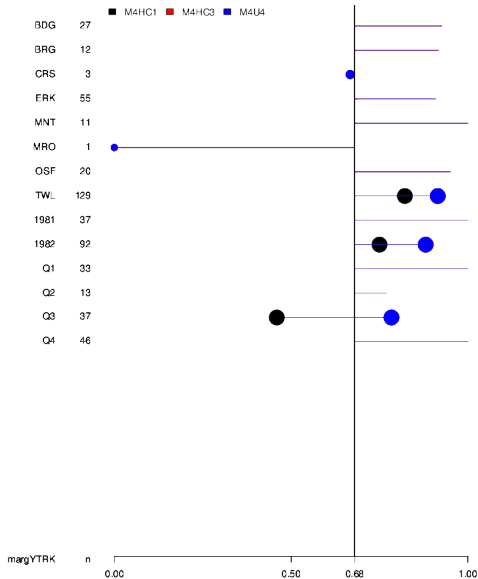
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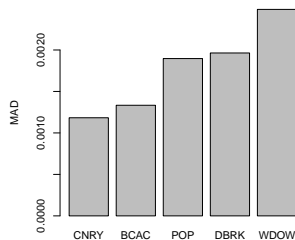


M4HC1

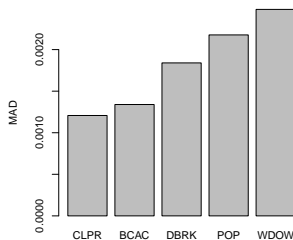
M4HC3

M4U4

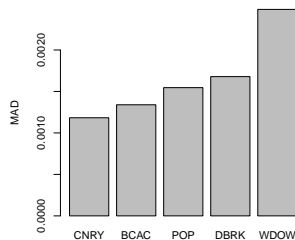
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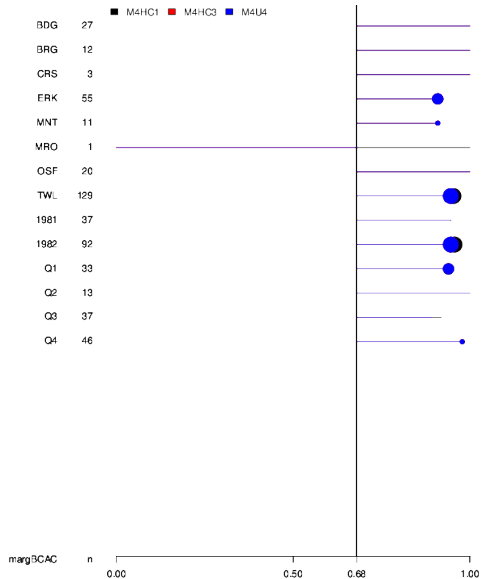


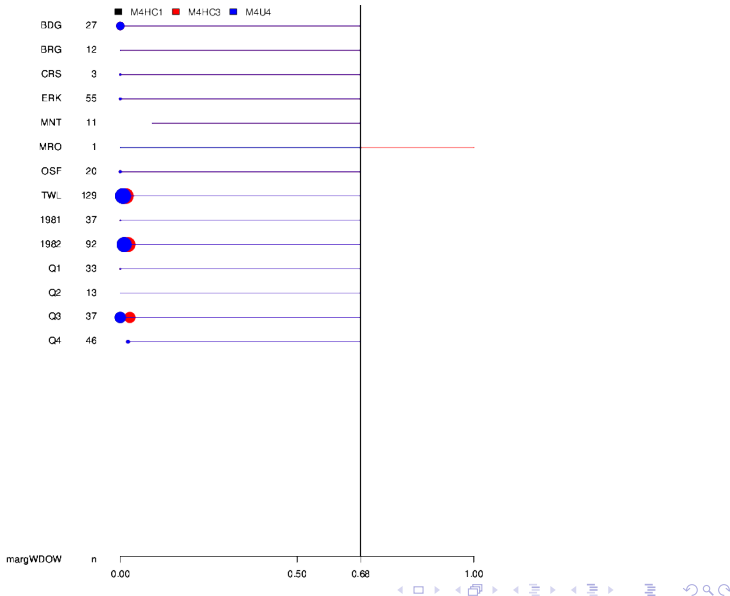
MAD Ordered by Species



MAD Ordered by Species







Time Blocks

Proof: Species Comps Sum to One... as do Their Means.

If y_{jk} is the k^{th} draw, $k \in \{1, \dots, K\}$, of the posterior predictive weight of species j in a particular stratum. Then,

$$\pi_{jk} = \frac{y_{jk}}{\sum_j y_{jk}} \quad \mathbf{y}_k \neq \mathbf{0}. \quad (1)$$

The predictive mean for species j is,

$$\hat{\pi}_j = \frac{\sum_k^K \pi_{jk}}{K}. \quad (2)$$

Summing $\hat{\pi}_j$ across species, it follows from (1) and (2) that,

$$\sum_j \hat{\pi}_j \stackrel{(2)}{=} \sum_j \frac{\sum_k^K \pi_{jk}}{K} = \frac{\sum_k^K \sum_j \pi_{jk}}{K} \stackrel{(1)}{=} \frac{\sum_k^K \sum_j \frac{y_{jk}}{\sum_j y_{jk}}}{K} = \frac{\sum_k^K 1}{K} = \frac{K}{K} = 1. \quad \blacksquare$$

Proof: Species Comps are Negatively Correlated.

Here we seek to show for any two species $l \neq m$, $\text{Corr}(\pi_l, \pi_m) < 0$.

Recall:

$$\text{Corr}(\pi_l, \pi_m) = \frac{\text{Cov}(\pi_l, \pi_m)}{\sigma_{\pi_l} \sigma_{\pi_m}} \quad \sigma_{\pi_l} \geq 0, \sigma_{\pi_m} \geq 0$$

$$\text{Corr}(\pi_l, \pi_m) \leq 0 \iff \text{Cov}(\pi_l, \pi_m) \leq 0$$

$$\begin{aligned} \text{Cov}(\pi_l, \pi_m) &= \mathbb{E}[(\pi_l - \mathbb{E}[\pi_l])(\pi_m - \mathbb{E}[\pi_m])] \\ &= \mathbb{E}[\pi_l \pi_m] - \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \end{aligned}$$

$$\text{Cov}(\pi_l, \pi_m) \leq 0 \iff \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m]$$

Proof: Species Comps are Negatively Correlated Cont.

Consider the strictly concave function: $f(\mathbf{x}) = \prod_i x_i : \sum_i x_i \leq 1, x_i \geq 0$.
Jensen's Inequality for f is,

$$f(\mathbb{E}[\mathbf{x}]) \geq \mathbb{E}[f(\mathbf{x})]. \quad (3)$$

From the previous proof: $\sum_j \pi_j \leq 1, \pi_j \geq 0$ and $\sum_j \hat{\pi}_j \leq 1, \hat{\pi}_j \geq 0$.
Thus applying (3) to π gives

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l\pi_m] \quad (4)$$

with equality only if π is a constant. Since π is never a constant,

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] > \mathbb{E}[\pi_l\pi_m]$$

$$\text{Cov}(\pi_l, \pi_m) < 0$$

$$\text{Corr}(\pi_l, \pi_m) < 0. \quad \blacksquare$$