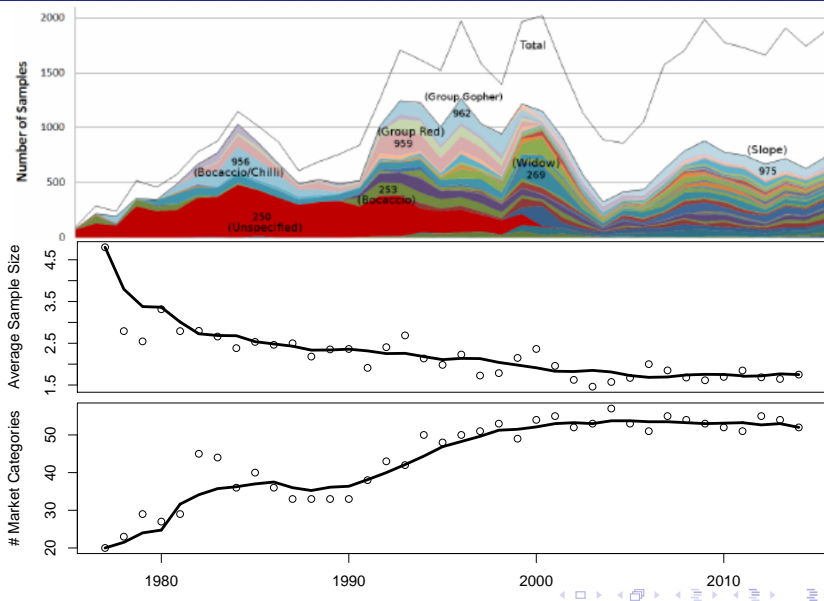


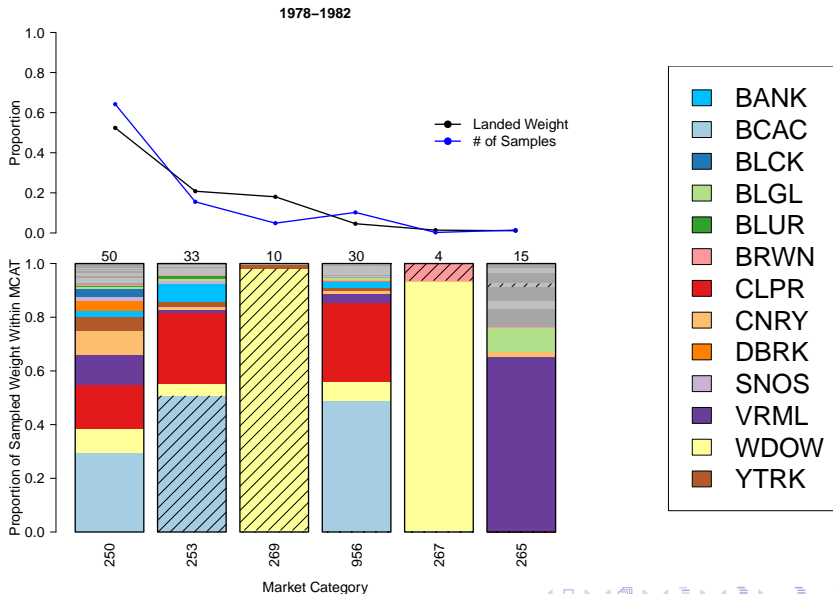
# Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

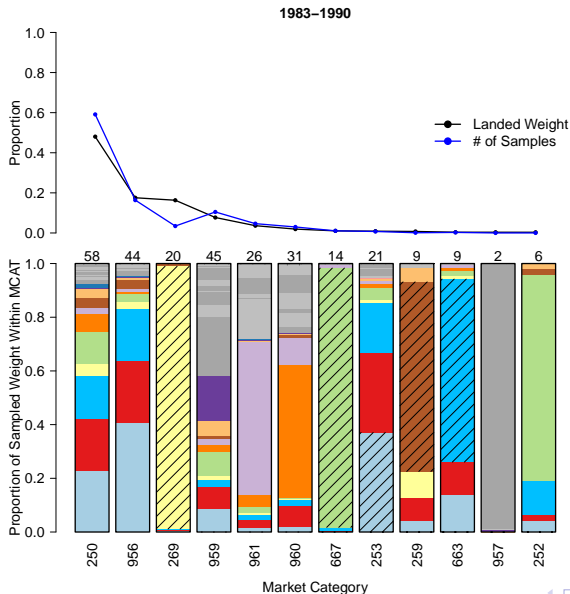
Nick Grunloh

UCSC :: CSTAR :: SWFSC :: NMFS

28 March 2018





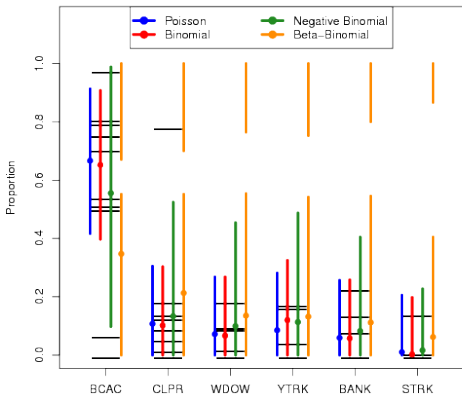


# Likelihood

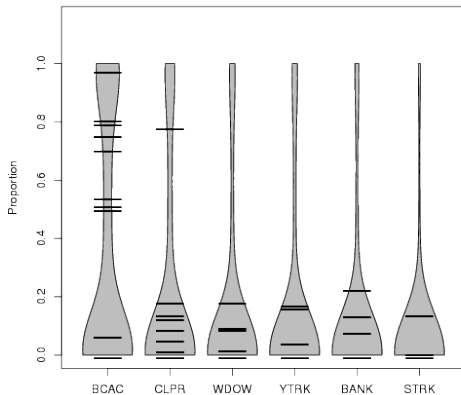
$y_{ij}$ :  $i^{\text{th}}$  sample of the  $j^{\text{th}}$  species' integer weight from market category 250, in the Monterey port complex trawl fishery for the second quarter of 1982.

$$y_{ij} \sim \text{Pois}(\theta_j) \quad y_{ij} \sim \text{Bin}(\theta_j) \quad y_{ij} \sim \text{NB}(\theta_j, \phi) \quad y_{ij} \sim \text{BB}(\theta_j, \phi)$$

95% Predictive HDI Model Comparison



Beta-Binomial Posterior Predictive Species Compositions



	Poisson	Binomial	NB	BB
MSE	0.06412	0.06264	0.05171	0.04479
$\Delta$ DIC	1001.41	1230.60	5.03	0
$\Delta$ WAIC	1079.95	1323.75	3.43	0
$pr(M y)$	$\approx 0$	$\approx 0$	$\approx 10^{-7}$	$\approx 1 - 10^{-7}$

# Beta-Binomial Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}(\mu_{ijklm\eta}, \sigma_{ijklm\eta}^2)$$

$$\mu_{ijklm\eta} = n \operatorname{logit}^{-1}(\theta_{ijklm\eta})$$

$$\sigma_{ijklm\eta}^2 = \mu_{ijklm\eta} \left(1 - \frac{\mu_{ijklm\eta}}{n}\right) \left(1 + (n-1)\rho\right)$$

$$\theta_{ijklm\eta} = \beta_0 + \beta_j^{(s)} + \beta_k^{(p)} + \beta_l^{(g)} + \beta_{m\eta}^{(t)}$$

$y_{ijklm\eta}$ :  $i^{\text{th}}$  sample of the  $j^{\text{th}}$  species',  
integer weight, in the  $k^{\text{th}}$  port, caught  
with the  $l^{\text{th}}$  gear, in the  $\eta^{\text{th}}$  quarter,  
of year  $m$ , for a particular market  
category.

$j \in \{1, \dots, J\}$  Species

$k \in \{1, \dots, K\}$  Ports

$l \in \{1, \dots, L\}$  Gears

$m \in \{1, \dots, M\}$  Years

$\eta \in \{1, \dots, H\}$  Quarters



# Time Model

**(M1)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, 32^2)$$

$$\beta_{\eta}^{(q)} \sim N(0, 32^2)$$

**(M2)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

**(M3)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)} + \beta_{m\eta}^{(y:q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

**(M4)**

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

**(M5)**

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_{\eta})$$

**(M6)**

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_m)$$

# Priors

$$\beta_0 \propto 1$$

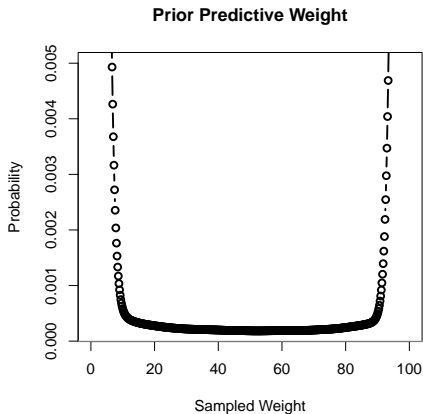
$$\beta_j^{(s)} \sim N(0, 32^2)$$

$$\beta_k^{(p)} \sim N(0, 32^2)$$

$$\beta_l^{(g)} \sim N(0, 32^2)$$

$$\text{logit}(\rho) \sim N(0, 2^2)$$

$$\nu \sim IG(1, 2 \times 10^3) \quad \forall \quad \nu$$



**1978-1982**

	M1	M2	M3	M4	M5	M6
MSE	0.12725	0.12704	0.12680	0.12237	0.12724	0.12657
$\Delta$ DIC	2558.56	2259.94	2013.21	0	2175.32	2174.71
$\Delta$ WAIC	2562.65	2263.58	2009.32	0	2171.18	2170.56
$pr(M y)$	$\approx 0$	$\approx 0$	$\approx 0$	$\approx 1$	$\approx 0$	$\approx 0$

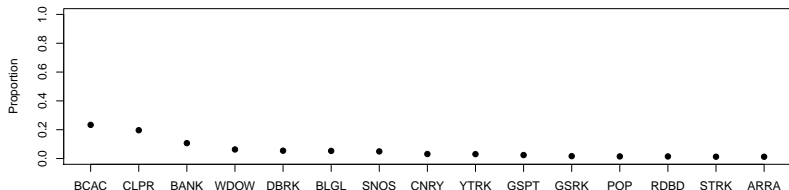
**1983-1990\***

	M1	M2	M3	M4	M5	M6
MSE	0.12968	?	0.12604	0.12604	0.12594	?
$\Delta$ DIC	68.85	?	0	33.45	35.55	?
$\Delta$ WAIC	2381.16	?	2316.41	0	1775.02	?
$pr(M y)$	$\approx 0$	?	$\approx 0$	$\approx 1$	$\approx 0$	?

# Posterior Predictive Weight

$$p(y_{jklm\eta}^* | \mathbf{y}) = \iint \text{BB}(y_{jklm\eta}^* | \mu_{jklm\eta}, \sigma_{jklm\eta}^2) P(\mu_{jklm\eta}, \sigma_{jklm\eta}^2 | \mathbf{y}) d\mu_{jklm\eta} d\sigma_{jklm\eta}^2$$

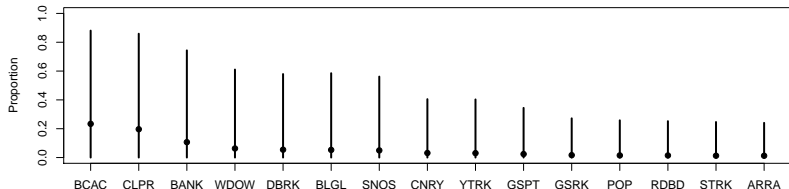
$$\pi_{jklm\eta}^* = \frac{y_{jklm\eta}^*}{\sum_j y_{jklm\eta}^*} \quad \mathbf{y}_{klm\eta}^* \neq \mathbf{0}$$



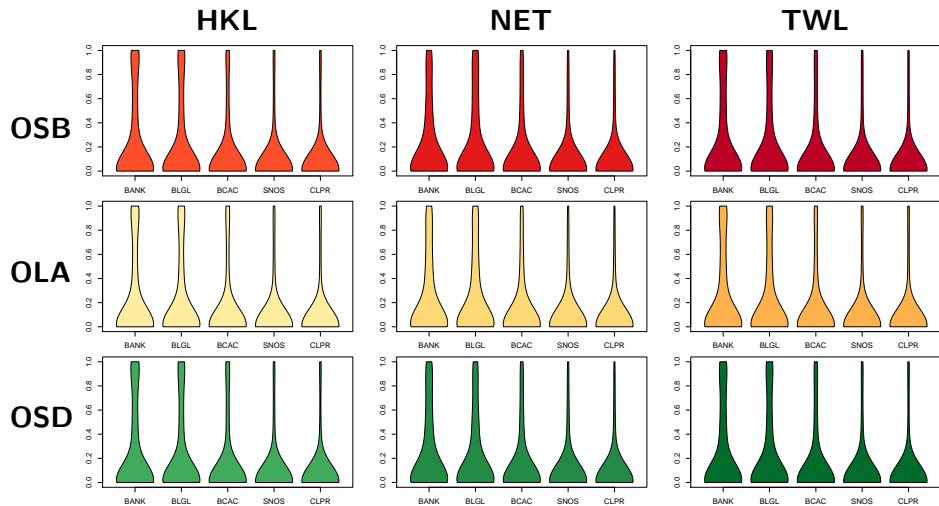
# Posterior Predictive Weight

$$p(y_{jklm\eta}^* | \mathbf{y}) = \iint \text{BB}(y_{jklm\eta}^* | \mu_{jklm\eta}, \sigma_{jklm\eta}^2) P(\mu_{jklm\eta}, \sigma_{jklm\eta}^2 | \mathbf{y}) d\mu_{jklm\eta} d\sigma_{jklm\eta}^2$$

$$\pi_{jklm\eta}^* = \frac{y_{jklm\eta}^*}{\sum_j y_{jklm\eta}^*} \quad \mathbf{y}_{klm\eta}^* \neq \mathbf{0}$$



# Single Quarter Hindcast (MCAT 250)



# Predictive Accuracy

	68%	95%	99%
250	67.1%	96.1%	98.7%
253	67.3%	96.3%	98.9%
262	67.4%	93.8%	95.3%
265	69.6%	96.0%	97.8%
269	68.2%	88.8%	90.2%
270	68.6%	93.6%	96.7%
956	68.3%	96.7%	99.2%
959	68.5%	96.3%	98.1%
961	69.3%	93.2%	95.3%
AVG	68.3%	94.5%	96.7%

**1978-1982**

	68%	95%	99%
245	60.8%	94.9%	97.7%
250	68.1%	96.0%	99.0%
253	69.3%	97.1%	98.9%
259	83.8%	91.9%	92.9%
262	68.5%	95.1%	95.9%
269	68.6%	94.2%	94.7%
270	67.9%	94.2%	96.7%
663	68.1%	94.1%	96.3%
667	69.4%	92.5%	93.5%
956	67.5%	96.2%	99.0%
959	67.4%	96.4%	99.0%
960	68.0%	96.1%	98.6%
961	68.6%	94.6%	97.8%
AVG	68.9%	94.9%	96.9%

**1983-1990**

# Speciated Landings

If  $\lambda_{\cdot klm\eta}$  is the observed landings of **all species** in the  $k^{th}$  port, caught with the  $l^{th}$  gear, in the  $\eta^{th}$  quarter, of year  $m$ , in particular market category. Then,

$$\lambda_{jklm\eta}^* = \lambda_{\cdot klm\eta} \pi_{jklm\eta}^*$$

$$\lambda_{jklm\cdot}^* = \sum_{\eta} \lambda_{jklm\eta}^*$$

$$\lambda_{j\cdot lm\cdot}^* = \sum_k \sum_{\eta} \lambda_{jklm\eta}^*$$

$$\lambda_{j\cdot\cdot m\cdot}^* = \sum_l \sum_k \sum_{\eta} \lambda_{jklm\eta}^*$$



$$\text{MSE}(\hat{\theta}) = \mathbb{E} \left[ (\hat{\theta} - \theta)^2 \right] = \mathbb{E} \left[ \overbrace{\left( \hat{\theta} - \mathbb{E}(\hat{\theta}) \right)^2}^{\text{Var}(\hat{\theta})} \right] + \overbrace{\left( \mathbb{E}(\hat{\theta}) - \theta \right)^2}^{\text{Bias}(\hat{\theta}, \theta)^2}$$

OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



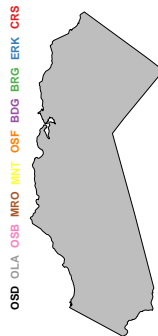
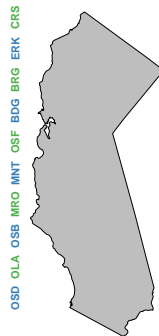
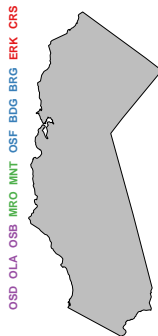
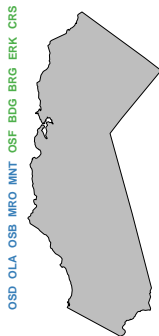
OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



$$B_K = \sum_{\kappa=0}^K \frac{1}{\kappa!} \left( \sum_{j=0}^{\kappa} (-1)^{\kappa-j} \binom{\kappa}{j} j^K \right)$$



$$B_{10} = 115975$$

OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



$$\bar{B}_{10} = 61136$$



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



$$\hat{B}_{10} = 512$$

OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



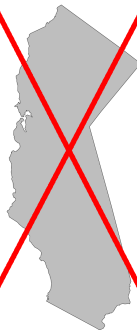
OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



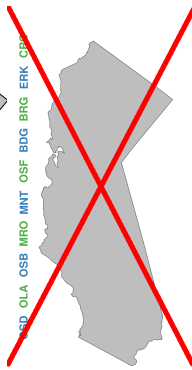
OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



$$\hat{B}_{10} = 274$$



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS



OSD OLA OSB MRO MNT OSF BDG BRG ERK CRS

# Bayesian Model Averaging (BMA)

Consider a set of Models ( $M$ ) indexed by  $\iota$ :

$$\omega_{\iota} = Pr(M_{\iota}|y) = \frac{p(y|M_{\iota})p(M_{\iota})}{\sum_{\iota} p(y|M_{\iota})p(M_{\iota})}$$

$$\bar{p}(\theta|\mathbf{y}) = \sum_{\iota} \omega_{\iota} p(\theta|\mathbf{y}, M_{\iota})$$

if  $f$  only depends on  $M$  through  $\theta$ , then

$$\bar{p}(y^*|\mathbf{y}) = \int f(y^*|\theta) \bar{p}(\theta|\mathbf{y}) d\theta$$

\* Hoeting, J. A., Madigan, D., Raftery, A. E., and Volinsky, C. T. (1999). Bayesian model averaging: a tutorial.

*Statistical science*, 382-401.

## 1978-1982

MCAT 250										
$\omega$	0.32	0.14	0.13	0.12	0.02	0.02	0.02	0.02	0.02	0.02
CRS										
ERK										
BRG										
BDG										
OSF										
MNT										
MRO										
OSB										
OLA										
OSD										



## 1978-1982

MCAT 253					
$\omega$	0.14	0.14	0.14	0.10	0.06
CRS					
ERK					
BRG					
BDG					
OSF					
MNT					
MRO					
OSB					
OLA					
OSD					

MCAT 269					
$\omega$	0.19	0.14	0.14	0.13	0.07
CRS					
ERK					
BRG					
BDG					
OSF					
MNT					
MRO					
OSB					
OLA					
OSD					

## 1983-1990

MCAT 250							
$\omega$	0.73	0.25	0.00	0.00	0.00	0.00	0.00
CRS							
ERK							
BRG							
BDG							
OSF							
MNT							
MRO							
OSB							
OLA							
OSD							

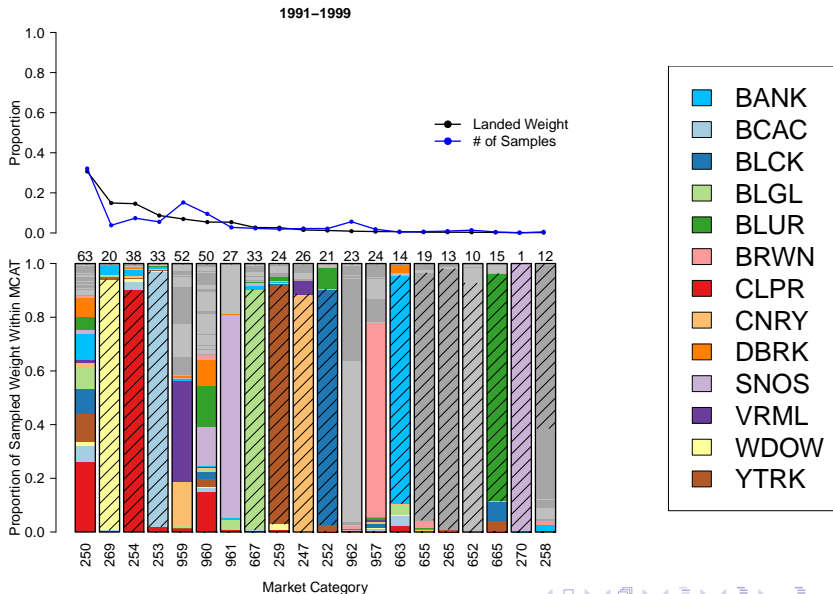
## 1983-1990

MCAT 956					
$\omega$	0.26	0.21	0.19	0.11	0.10
CRS					
ERK					
BRG					
BDG					
OSF					
MNT					
MRO					
OSB					
OLA					
OSD					

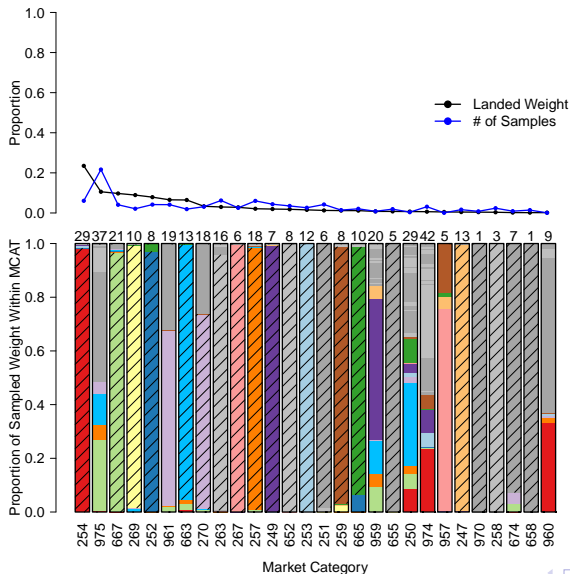
MCAT 269					
$\omega$	0.64	0.12	0.07	0.06	0.04
CRS					
ERK					
BRG					
BDG					
OSF					
MNT					
MRO					
OSB					
OLA					
OSD					

# Conclusion

- Red stuff
- Species Composition Proof



2000–2015



$\rho$  Posterior

MCAT	Mean	Median	SD
250	0.55	0.55	0.004
253	0.39	0.39	0.001
262	0.35	0.35	0.008
265	0.64	0.64	0.002
269	0.52	0.52	0.019
270	0.53	0.54	0.020
956	0.35	0.35	0.007
959	0.47	0.47	0.070
961	0.55	0.55	0.004

**1978-1982**

MCAT	Mean	Median	SD
245	0.65	0.65	0.014
250	0.51	0.51	0.002
253	0.47	0.47	0.010
259	0.75	0.75	0.009
262	0.41	0.41	0.001
269	0.57	0.57	0.046
270	0.74	0.75	0.027
663	0.51	0.51	0.001
667	0.49	0.49	0.022
956	0.43	0.43	0.003
959	0.55	0.55	0.004
960	0.45	0.45	0.004
961	0.59	0.59	0.001

**1983-1990**

# v Posterior

MCAT	Mean	Median	SD
250	12915.85	18523.12	8699.87
253	22747.87	23063.76	1535.53
262	20254.41	20506.36	2581.87
265	15846.22	16694.98	7601.15
269	20135.05	19975.15	4667.11
270	19931.96	19955.13	6033.35
956	19659.11	19795.60	1227.99
959	19159.69	13375.80	19256.94
961	18631.44	19498.31	7970.44

**1978-1982**

MCAT	Mean	Median	SD
245	20211.82	20204.95	1276.83
250	236.03	192.53	134.67
253	20455.18	20140.50	1521.72
259	20246.14	20186.61	898.99
262	20445.49	20348.56	343.70
269	34386.49	25951.03	24030.32
270	20253.34	19908.07	9269.02
663	19563.87	19624.09	331.04
667	20089.55	20078.27	2723.34
956	20581.67	20664.71	913.92
959	19242.41	18707.09	5076.03
960	20059.66	20012.80	1703.89
961	20127.69	20141.04	580.80

**1983-1990**



# Proof: Species Comps Sum to One... as do Their Means.

If  $y_{jk}$  is the  $k^{\text{th}}$  draw,  $k \in \{1, \dots, K\}$ , of the posterior predictive weight of species  $j$  in a particular stratum. Then,

$$\pi_{jk} = \frac{y_{jk}}{\sum_j y_{jk}} \quad \mathbf{y}_k \neq \mathbf{0}. \quad (1)$$

The predictive mean for species  $j$  is,

$$\hat{\pi}_j = \frac{\sum_k^K \pi_{jk}}{K}. \quad (2)$$

Summing  $\hat{\pi}_j$  across species, it follows from (1) and (2) that,

$$\sum_j \hat{\pi}_j \stackrel{(2)}{=} \sum_j \frac{\sum_k^K \pi_{jk}}{K} = \frac{\sum_k^K \sum_j \pi_{jk}}{K} \stackrel{(1)}{=} \frac{\sum_k^K \sum_j \frac{y_{jk}}{\sum_j y_{jk}}}{K} = \frac{\sum_k^K 1}{K} = \frac{K}{K} = 1. \quad \blacksquare$$