Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

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Beta-Binomial Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}\left(\mu_{jklm\eta},\ \sigma^2_{jklm\eta}\right)$$

$$\mu_{jklm\eta} = n\ \text{logit}^{-1}(\theta_{jklm\eta})$$

$$\sigma^2_{jklm\eta} = \mu_{jklm\eta}\left(1 - \frac{\mu_{jklm\eta}}{n}\right)\left(1 + (n-1)\ \rho\right)$$

$$\theta_{jklm\eta} = \beta_0 + \beta_j^{(s)} + \beta_k^{(p)} + \beta_l^{(g)} + \beta_{m\eta}^{(t)}$$

$y_{ijklm\eta}$: i^{th} sample of the jth species' integer weight, in the kth port, caught with the lth gear, in the ηth quarter, of year m, for a particular market category.

$$\begin{aligned} j &\in \{1, ..., J\}\ \text{Species} \\ k &\in \{1, ..., K\}\ \text{Ports} \\ l &\in \{1, ..., L\}\ \text{Gears} \\ m &\in \{1, ..., M\}\ \text{Years} \\ \eta &\in \{1, ..., H\}\ \text{Quarters} \end{aligned}$$

Request: As a diagnostic template, for each sampled stratum compare the posterior predictive distributions at the 68th, 95th, and 99th percentiles with the current observed species proportions (create fully stratified versions of tables 2 and 3 in the Grunloh et al. methods documentation). With each row, include sample sizes and associated landing weights with a graphical display to highlight problems and outliers (circle size proportional to landing weights).

Rationale: The Team provided broad-scale summary metrics (e.g., MSE and DIC) for evaluating the goodness-of-fit of the different model forms and structures. Fine-scale diagnostics are needed to help identify aspects of the data that are not adequately addressed by the different models. The diagnostic template will provide a mechanism for fine-scale exploration of goodness-of-fit.

Response: The following slides show the requested diagnostic plot

Likelihood

Information Criterion

MAD Diagnostic

MAD

Marginal Plot

Stratum Plot

Request: Provide a summary table of species sample sizes in each market category by time block.

Rationale: The requested information will assist in understanding where there are gaps in the available data that the model is filling in by means of its pooling structure.

Response: The following slides show tables of total sample sizes and the frequency of occurrence by species within each sample (across all years, gears, ports and quarters within a time period) for the three rockfish market categories that we focus on in the presentation, as well as elasmobranchs and flatfish. Supplementary excel files include data summaries and pivot tables that can be used to review the same information for all other market categories, as well as these and other market categories at higher levels of stratification (year, port complex, gear, quarter) to

Request: Redo the modeling of the early time block without southern CA ports. Explore spatially and temporally (i.e., alternative time blocks).

Rationale: The available dataset does not have any sample data in the early time block from the southern CA ports. It was unclear how this lack of data influenced the model results. The requested analysis will clarify the situation.

Response: All of the model runs and plots from this point forward only model ports north of point conception.

Request: The diagnostic template should be developed for each of the sensitivity runs (vary across a range of plausible time models and priors and limit to the top 2-3 market categories).

Rationale: Application of the diagnostics across a wide range of models will form a test of how well the diagnostics illustrate whether the models capture important structural features that are thought to be embedded in the data.

Response: The following slides show the requested diagnostic plots as applied to models across a range of time models and prior choices.

Time Models

(M1)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)}$$
$$\beta_m^{(y)} \sim N(0, 32^2)$$
$$\beta_\eta^{(q)} \sim N(0, 32^2)$$

(M2)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)}$$
$$\beta_m^{(y)} \sim N(0, v^{(y)})$$
$$\beta_\eta^{(q)} \sim N(0, v^{(q)})$$

(M3)

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_\eta^{(q)} + \beta_{m\eta}^{(y:q)}$$
$$\beta_m^{(y)} \sim N(0, v^{(y)})$$
$$\beta_\eta^{(q)} \sim N(0, v^{(q)})$$
$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M4)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$
$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

(M5)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$
$$\beta_{m\eta}^{(y:q)} \sim N(0, v_\eta)$$

(M6)

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$
$$\beta_{m\eta}^{(y:q)} \sim N(0, v_m)$$

	M1	M2	M3	M4	M5	M6
Δ DIC	6448.98	0.33	0	4.45	9.3	7.42
Δ WAIC	6421.5	0.37	0	4.52	8.25	6.55

M2

M3

M4

Combined

MCAT 250

M2
M3
M4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margBCAC n
0.00
0.50
0.68
1.00

MCAT 250

M2
M3
M4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margWDOW n
0.00
0.50
0.68
1.00

M2 M3 M4

Combined

MCAT 250

M2
M3
M4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margCWCD n
0.00
0.50
0.68
1.00

MCAT 250

M2
M3
M4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margMXRF n
0.00
0.50
0.68
1.00

	M1	M2	M3	M4	M5	M6
Δ DIC	1409.81	0.09	0.1	0.07	0.05	0
Δ WAIC	1391.66	0.16	0.18	0	0.13	0.08

M4 M5 M6

Combined

MCAT 253

M4
M5
M6
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margBCAC n
0.00
0.50
0.68
1.00

MCAT 253

M4
M5
M6
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margWDOW n
0.00
0.50
0.68
1.00

M4 M5 M6

Combined

MCAT 253

M4
M5
M6
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
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1982 50
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Q2 73
Q3 54
Q4 40
margCWCD n
0.00
0.50
0.68
1.00

MCAT 253

M4
M5
M6
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margDBRK n
0.00
0.50
0.68
1.00

	M1	M2	M3	M4	M5	M6
Δ DIC	572.51	176.63	599.41	0.57	0	193.35
Δ WAIC	427.48	69.37	454.41	0.23	0	78.07

M4 M5 M6

Combined

MCAT 269

M4
M5
M6
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margCNRY n
0.00
0.50
0.68
1.00

MCAT 269

M4
M5
M6
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margYTRK n
0.00
0.50
0.68
1.00

M4 M5 M6

Combined

MCAT 269

M4
M5
M6
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margBCAC n
0.00
0.50
0.68
1.00

MCAT 269

M4
M5
M6
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margWDOW n
0.00
0.50
0.68
1.00

Priors

$$\beta_0 \propto 1$$
$$\beta_j^{(s)} \sim N(0, 32^2)$$
$$\beta_k^{(p)} \sim N(0, 32^2)$$
$$\beta_l^{(g)} \sim N(0, 32^2)$$
$$\text{logit}(\rho) \sim N(0, 2^2)$$

$$\begin{aligned} \text{IG}: \quad & v \sim \text{Inv-Gamma}(1,\ 2x10^3) \quad \forall \quad v \\ \text{HC1}: \quad & \sqrt{v} \sim \text{Half-Cauchy}(10^1) \quad \forall \quad v \\ \text{HC3}: \quad & \sqrt{v} \sim \text{Half-Cauchy}(10^3) \quad \forall \quad v \\ \text{U4}: \quad & \sqrt{v} \sim \text{Uniform}(0, 10^4) \quad \forall \quad v \end{aligned}$$

	M4IG	M4HC1	M4HC3	M4U4
Δ DIC	3.87	0.02	0.1	0
Δ WAIC	3.78	0.03	0.11	0

M4HC1 M4HC3 M4U4

Combined

MCAT 250

M4HC1
M4HC3
M4U4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margBCAC n
0.00
0.50
0.68
1.00

MCAT 250

M4HC1
M4HC3
M4U4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margWDOW n
0.00
0.50
0.68
1.00

M4HC1 M4HC3 M4U4

Combined

MCAT 250

M4HC1
M4HC3
M4U4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margCWCD n
0.00
0.50
0.68
1.00

MCAT 250

M4HC1
M4HC3
M4U4
BDG 9
BRG 326
CRS 50
ERK 323
MNT 108
MRO 138
OSF 29
HKL 110
NET 6
TWL 867
1978 124
1979 102
1980 271
1981 237
1982 249
Q1 162
Q2 289
Q3 340
Q4 192
margMXRF n
0.00
0.50
0.68
1.00

	M4IG	M4HC1	M4HC3	M4U4
Δ DIC	0.88	0.8	0.8	0
Δ WAIC	0.76	0.83	0.83	0

M4HC1 M4HC3 M4U4

Combined

MCAT 253

M4HC1
M4HC3
M4U4
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margBCAC n
0.00
0.50
0.68
1.00

MCAT 253

M4HC1
M4HC3
M4U4
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margCLPR n
0.00
0.50
0.68
1.00

M4HC1 M4HC3 M4U4

Combined

MCAT 253

M4HC1
M4HC3
M4U4
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margCWCD n
0.00
0.50
0.68
1.00

MCAT 253

M4HC1
M4HC3
M4U4
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margDBRK n
0.00
0.50
0.68
1.00

	M4IG	M4HC1	M4HC3	M4U4
Δ DIC	0.18	176.33	0.2	0
Δ WAIC	0.08	69.19	0.08	0

M4HC1

M4HC3

M4U4

Combined

MCAT 269

M4HC1
M4HC3
M4U4
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margCLPR n
0.00
0.50
0.68
1.00

MCAT 269

M4HC1
M4HC3
M4U4
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margYTRK n
0.00
0.50
0.68
1.00

M4HC1 M4HC3 M4U4

Combined

MCAT 269

M4HC1
M4HC3
M4U4
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margBCAC n
0.00
0.50
0.68
1.00

MCAT 269

M4HC1
M4HC3
M4U4
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margWDOW n
0.00
0.50
0.68
1.00

Request: Explore various two-way interactions (beyond the current explorations; e.g., Species : Port and Species : Gear).

Rationale: The Team did not have time to search across the multitude of possible interaction terms that they could have included in the model. From various anecdotal comments made during the review it seemed likely that the model would benefit from the inclusion of other interaction terms. Explorations with the diagnostic template may suggest potentially beneficial terms.

Response: The following slides show the diagnostic plots as applied to models exploring the inclusion of species:port and species:gear interaction terms.

Port and Gear Interactions

Model Description

	M4	M4IGSG	M4IGSP
Δ DIC	58229.78	21403.07	0
Δ WAIC	24203.2	10186.57	0

	M4	M4SG	M4SP
Δ DIC	403.42	308.65	0
Δ WAIC	872.35	778.17	0

M4 M4SG M4SP

Combined

MCAT 253

M4
M4IGSG
M4IGSP
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margBCAC n
0.00
0.50
0.68
1.00

MCAT 253

M4
M4IGSG
M4IGSP
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margCLPR n
0.00
0.50
0.68
1.00

M4 M4SG M4SP

Combined

MCAT 253

M4
M4IGSG
M4IGSP
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margCWCD n
0.00
0.50
0.68
1.00

MCAT 253

M4
M4IGSG
M4IGSP
BRG 6
MNT 88
MRO 64
OSF 52
HKL 1
NET 8
TWL 201
1978 57
1980 61
1981 42
1982 50
Q1 43
Q2 73
Q3 54
Q4 40
margDBRK n
0.00
0.50
0.68
1.00

	M4	M4SG	M4SP
Δ DIC	0	0.31	182.15
Δ WAIC	0	0.13	68.48

M4 M4SG M4SP

Combined

MCAT 269

M4
M4IGSG
M4IGSP
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margCNRY n
0.00
0.50
0.68
1.00

MCAT 269

M4
M4IGSG
M4IGSP
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margYTRK n
0.00
0.50
0.68
1.00

M4 M4SG M4SP

Combined

MCAT 269

M4
M4IGSG
M4IGSP
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margBCAC n
0.00
0.50
0.68
1.00

MCAT 269

M4
M4IGSG
M4IGSP
BDG 27
BRG 12
CRS 3
ERK 55
MNT 11
MRO 1
OSF 20
TWL 129
1981 37
1982 92
Q1 33
Q2 13
Q3 37
Q4 46
margWDOW n
0.00
0.50
0.68
1.00

Request: Explore an alternative time block: an extension of 1983 and 1984 to the first time block.

Rationale: The panel expressed concerns about how the model would perform when applied to shorter time periods, as will occur when the model is used with data more recent than 1990. Results from the above recommendation could be compared to the results from the current two time blocks (1978-1982; 1983-1990) to explore how fits to data from the late period degrade when the model for the late period is based on fewer years of data. Also, comparisons of the two forms of blocking serve as a sensitivity evaluation of the selection of the block boundary, which was chosen on a fairly arbitrary basis.

Response: The following slides show the diagnostic plots as applied to model M4 when fit to data from 78-82 as well as data

Time Blocks

78-82

78-83

78-84

Combined

MCAT 250

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
HKL
NET
TWL
1978 124
1979 102
1980 271
1981 237
1982 249
1983 312
1984 270
1985 330
Q1
Q2
Q3
Q4
margBCAC n
0.00
0.50
0.68
1.00

MCAT 250

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
HKL
NET
TWL
1978 124
1979 102
1980 271
1981 237
1982 249
1983 312
1984 270
1985 330
Q1
Q2
Q3
Q4
margWDOW n
0.00
0.50
0.68
1.00

78-82 78-83 78-84

Combined

MCAT 250

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
HKL
NET
TWL
1978 124
1979 102
1980 271
1981 237
1982 249
1983 312
1984 270
1985 330
Q1
Q2
Q3
Q4
margCWCD n
0.00
0.50
0.68
1.00

MCAT 250

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
HKL
NET
TWL
1978 124
1979 102
1980 271
1981 237
1982 249
1983 312
1984 270
1985 330
Q1
Q2
Q3
Q4
margMXRF n
0.00
0.50
0.68
1.00

78-82	78-83	78-84

Combined

MCAT 253

82M4
83M4
84M4
85M4
BDG
BRG
MNT
MRO
OSF
HKL
NET
TWL
1978
1980
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margCNRY
n
0.00
0.50
0.68
1.00

MCAT 253

82M4
83M4
84M4
85M4
BDG
BRG
MNT
MRO
OSF
HKL
NET
TWL
1978
1980
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margYTRK
n
0.00
0.50
0.68
1.00

78-82

78-83

78-84

Combined

MCAT 253

82M4
83M4
84M4
85M4
BDG
BRG
MNT
MRO
OSF
HKL
NET
TWL
1978
1980
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margBCAC
n
0.00
0.50
0.68
1.00

MCAT 253

82M4
83M4
84M4
85M4
BDG
BRG
MNT
MRO
OSF
HKL
NET
TWL
1978
1980
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margWDOW
n
0.00
0.50
0.68
1.00

78-82 78-83 78-84

Combined

MCAT 269

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
NET
TWL
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margCNRY
n
0.00
0.50
0.68
1.00

MCAT 269

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
NET
TWL
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margYTRK
n
0.00
0.50
0.68
1.00

78-82 78-83 78-84

Combined

MCAT 269

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
NET
TWL
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margBCAC
n
0.00
0.50
0.68
1.00

MCAT 269

82M4
83M4
84M4
85M4
BDG
BRG
CRS
ERK
MNT
MRO
OSF
NET
TWL
1981
1982
1983
1984
1985
Q1
Q2
Q3
Q4
margWDOW
n
0.00
0.50
0.68
1.00

Request: Compare alternative ComX outputs and the current time series of estimated catches.

Rationale: It would be informative to see the landings estimates corresponding to the additional models developed in response to the above requests. The landings estimates can be generated for a small set of illustrative species and do not need to be comprehensive.

Response: The following slides show explanded landings (by year and by year:gear) from the early time period, summing across market categories 250, 253, and 269 associated with time model, prior, intereaction model, and time block sensativity runs.

All Species

Time Model

WDOW
WDOW:NET
WDOW:TWL
WDOW:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Time Model

BCAC
BCAC:NET
BCAC:TWL
BCAC:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Time Model

CLPR
CLPR:NET
CLPR:TWL
CLPR:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Time Model

DBRK
DBRK:NET
DBRK:TWL
DBRK:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Time Model

CWCD
CWCD:NET
CWCD:TWL
CWCD:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Time Model

MXRF
MXRF:NET
MXRF:TWL
MXRF:HKL
M2/
M4/
M5/
M6/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

All Species

Prior Model

WDOW
WDOW:NET
WDOW:TWL
WDOW:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Prior Model

BCAC
BCAC:NET
BCAC:TWL
BCAC:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Prior Model

CLPR
CLPR:NET
CLPR:TWL
CLPR:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Prior Model

DBRK
DBRK:NET
DBRK:TWL
DBRK:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Prior Model

CWCD
CWCD:NET
CWCD:TWL
CWCD:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Prior Model

MXRF
MXRF:NET
MXRF:TWL
MXRF:HKL
M4/
M4HC1/
M4HC3/
M4U4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982

Interaction Model

All Species

Time Block

WDOW
WDOW:NET
WDOW:TWL
WDOW:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Time Block

BCAC
BCAC:NET
BCAC:TWL
BCAC:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Time Block

CLPR
CLPR:NET
CLPR:TWL
CLPR:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Time Block

DBRK
DBRK:NET
DBRK:TWL
DBRK:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Time Block

CWCD
CWCD:NET
CWCD:TWL
CWCD:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Time Block

MXRF
MXRF:NET
MXRF:TWL
MXRF:HKL
82M4/
83M4/
84M4/
85M4/
CALCOM
Landings (mt)
Year
1978
1979
1980
1981
1982
1983
1984
1985

Proof: Species Comps Sum to One... as do Their Means.

If y_{jk} is the kth draw, $k \in \{1, ..., K\}$, of the posterior predictive weight of species j in a particular stratum. Then,

$$\pi_{jk} = \frac{y_{jk}}{\sum_j y_{jk}} \quad \boldsymbol{y}_k \neq \mathbf{0}. \tag{1}$$

The predictive mean for species j is,

$$\hat{\pi}_j = \frac{\sum_k^K \pi_{jk}}{K}. \tag{2}$$

Summing $\hat{\pi}_j$ across species, it follows from (1) and (2) that,

$$\sum_j \hat{\pi}_j \overset{(2)}{=} \sum_j \frac{\sum_k^K \pi_{jk}}{K} = \frac{\sum_k^K \sum_j \pi_{jk}}{K} \overset{(1)}{=} \frac{\sum_k^K \sum_j \frac{y_{jk}}{\sum_j y_{jk}}}{K} = \frac{\sum_k^K 1}{K} = \frac{K}{K} = 1. \quad \blacksquare$$

Proof: Species Comps are Negatively Correlated.

Here we seek to show for any two species $l \neq m$, $Corr(\pi_l, \pi_m) < 0$. Recall:

$$Corr(\pi_l, \pi_m) = \frac{Cov(\pi_l, \pi_m)}{\sigma_{\pi_l}\sigma_{\pi_m}} \quad \sigma_{\pi_l} \geq 0, \ \sigma_{\pi_m} \geq 0$$

$$Corr(\pi_l, \pi_m) \leq 0 \iff Cov(\pi_l, \pi_m) \leq 0$$

$$\begin{aligned} Cov(\pi_l, \pi_m) &= \mathbb{E}[(\pi_l - \mathbb{E}[\pi_l])(\pi_m - \mathbb{E}[\pi_m])] \\ &= \mathbb{E}[\pi_l \pi_m] - \mathbb{E}[\pi_l]\mathbb{E}[\pi_m] \end{aligned}$$

$$Cov(\pi_l, \pi_m) \leq 0 \iff \mathbb{E}[\pi_l]\mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m]$$

Proof: Species Comps are Negatively Correlated Cont.

Consider the strictly concave function:

$$f(\boldsymbol{x}) = \prod_i x_i : \boldsymbol{x} \subset \left\{ \boldsymbol{y} \mid \sum_i y_i = 1,\ y_i \geq 0 \right\}$$

Jensen's Inequality for f is,

$$f(\mathbb{E}[\boldsymbol{x}]) \geq \mathbb{E}[f(\boldsymbol{x})]. \tag{3}$$

From the previous proof: $\sum_j \pi_j = 1,\ \pi_j \geq 0$ and $\sum_j \hat{\pi}_j = 1,\ \hat{\pi}_j \geq 0$. Thus applying (3) to $\boldsymbol{\pi}$ gives

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m] \tag{4}$$

with equality only if π is a constant. Since π is never a constant,

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] > \mathbb{E}[\pi_l \pi_m] \implies Cov(\pi_l, \pi_m) < 0 \implies Corr(\pi_l, \pi_m) < 0. \quad \blacksquare$$