Improving Catch Estimation Methods in Sparsely Sampled Mixed-Stock Fisheries.

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Abstract

Introduction

Context

Data

- Collection issues
    - funding => nature of sparcity
- Lay down goal modeling goal
    - mean
    - uncertainty

Methods

Data Generating Model

Something something heirarchical poisson model. Something something (Shelton, 2012).

For the purposes of accuratly modeling not only species composition means, but also higher moments of the data, such as species composition variances, it is neccisary to recognize model limitations with respect to over-dispersed data. Amoung the simplest models for count data are the poisson and binomial models. Both models are typically specificed with a single degree of freedom for modeling all of the moments of the data, and thus they rely heavily on their respective data generating processes to accuratly represent higher moments in the data. McCullagh and Nelder (1989, pg. 124) commiserate about the prevalence of over-dispersed data in cluster sampling, and explain the numerious ways in which cluster sampling may result in over-dispersion.

Extending the Poisson and binomial models to deal with over-dispersion, typically involves adding additional parameters for the purpose of modeling higher moments of the data. The negative binomial (NB) distribution is often used as an over-dispersed extension of the poisson model, since it can be expressly written as an infite mixture of poisson distributions. While the beta-binomial model is typically used to as an over-dispersed extension of the binomial model.

An Example

To discern between these models we consider a small scale example of the Poisson, binomial, negative binomial, and beta-binomial models fit to the port sampling integer weight data from market category 250, in the Montery port complex trawl fishery in 1990. $(anywillwork)$ This stratum was visited 38 times by port samplers, collecting a total of 67 cluster samples, resulting in 344 model observations across 21 $(atleast; URCK)$ unique species. Each of the above models are fit to these data. The predictive species composition distributions from each model are visualized in Figure (1) as 95% Highest Density Intervals (HDI) $(citations)$, plotted on top of the predictive means for each model and the observed species compositions from the data in Figure(1). For brevity we only consider the most prevalent six species in this example (CLPR, BCAC, WDOW, BLGL, ARRA, BANK). Additionally, the MSE, DIC, WAIC, and Bayesian marginal likelyhood model probabilities are computed for each model as measures of model fit as seen in Table(1).

	Poisson	Binomial	NB	BB
MSE	0.05286	0.05683	0.05188	0.05170
DIC	5675.25	6759.86	1301.51	1261.00
WAIC	5840.56	6939.74	1302.19	1261.30
$pr(M\|y)$	≈ 0	≈ 0	$< 10^{-10}$	≈ 1

The large spread of the observed species compositions seen in Figure(1) visually demonstrate the degree of overdispersion present in port sampling data. The Poisson and binomial models disregaurd this overdispersion to prioritize fitting the data mean. In contrast, the negative binomial and beta-binomial models estimate an additional parameter which is intended to disentangle the mean and residual variance estimates. Thus the negative binomial and beta-binomial models are able to produce more accurate estimates of both the mean and residual variance.

All of the measures in Table(1) consistently agree that the negative binomial and beta-binomial models out perform the overdispersed Poisson and binomial

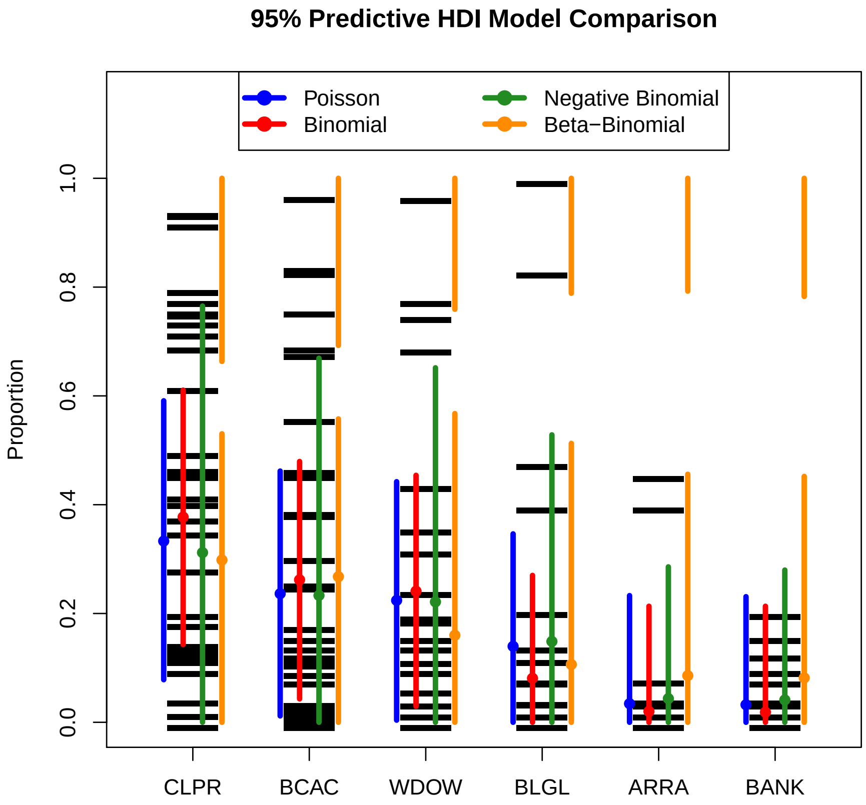

Figure 1: Interval Plot

models. Furthermore, all of the metrics in Table(1) indicate that the beta-binomial model outperforms the negative binomial model. Depending on the users value system toward model selection (e.g. predictive or inferential), the support for the beta-binomial model over the negative binomial model may vary, but it is worth noting that the more robust model selction tools show stronger support for the beta-binomial model, with Bayesian model probabilities indicating practically conclusive support for the beta-binomial model.

The split beta-binomial intervals seen in Figure(1) are the consequence of confining a large amount of residual variability to the unit interval. The beta-binomial is the only model considered here, which estimates such a large degree of variablility and thus it is the only model that produces predictive species composition distributions of the sort. Figure(2) shows the beta-binomial predictive distributions as a violin plot demonstrating how the beta-binomial model arranges predictive density over the unit interval. The predictive intervals in Figure(1) are the smallest possible regions on each density so that the intervals contain 95% of the predictive density (i.e. these regions represent the densest packing of 95% probbaility under each predictive distribution). For the cases of Aurora and Bank rockfish, the empty upper regions seen in Figure(1) are understandable in terms of the relatively low density region of the posterior they represent, as seen in Figure(2).

Operationalized Model

For a particular market category, $y_{ijklm\eta}$ is the i^{th} sample of the j^{th} species' weight, in the k^{th} port, caught with the l^{th} gear, in the η^{th} quarter, of year m. The $y_{ijklm\eta}$ are modeled as observations from a beta-binomial distribution conditional on parameters $\mu_{jklm\eta}$ and $\sigma^2_{jklm\eta}$,

$$y_{ijklm\eta} \sim BB(\mu_{jklm\eta},\ \sigma^2_{jklm\eta}).$$

Where $\mu_{jklm\eta}$ is the stratum level beta-binomial mean weight and $\sigma^2_{jklm\eta}$ is the stratum level residual variance. $\mu_{jklm\eta}$ is related to a linear predictor, $\theta_{jklm\eta}$, via the mean function,

$$\mu_{jklm\eta} = n_{ijklm\eta}\frac{\exp(\theta_{jklm\eta})}{1+\exp(\theta_{jklm\eta})} = n\ \text{expit}(\theta_{jklm\eta}) = n\ \text{logit}^{-1}(\theta_{jklm\eta}).$$

Here $n_{ijklm\eta}$ is the known cluster size for each sample. Additionally, $\sigma^2_{jklm\eta}$ is related to $\mu_{jklm\eta}$ and the overdispersion parameter, ρ, via the following equation,

$$\sigma^2_{jklm\eta} = \mu_{jklm\eta}\Big(1-\frac{\mu_{jklm\eta}}{n_{ijklm\eta}}\Big)\Big(1+(n_{ijklm\eta}-1)\ \rho\Big).$$

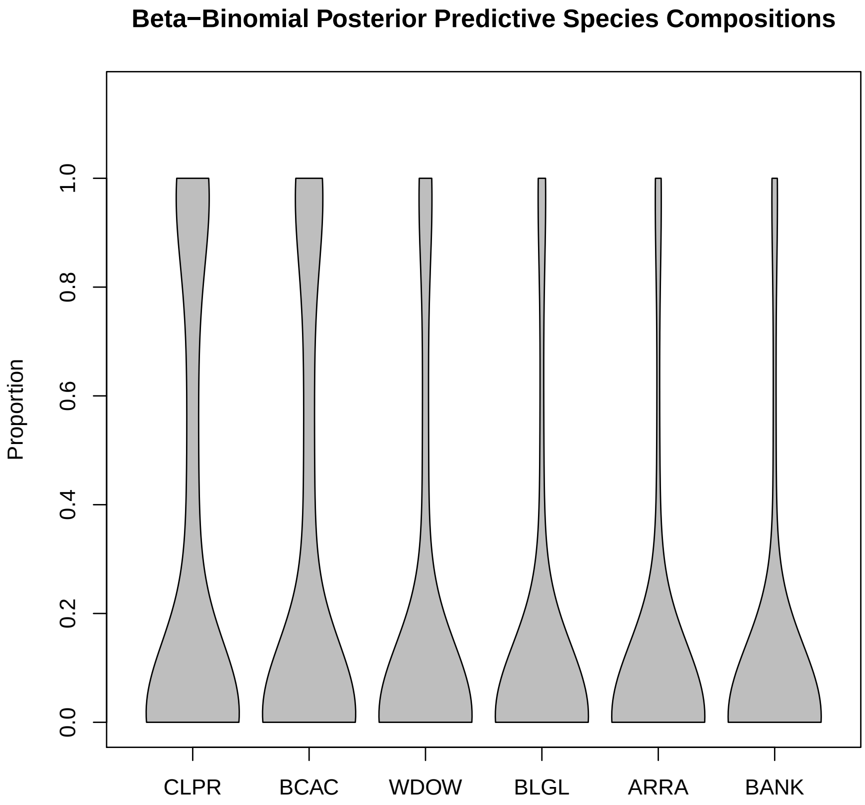

Figure 2: Violin Plot

ρ is the within cluster correlation. The situation where $\rho \to 1$ represents identical information content amoung replicates within a cluster, with maximal overdispersion relative to the binomial distribution. The situation where $\rho \to 0$ represents totally independent information content amoug replicates within a cluster, and the beta-binomial model approaches the binomial model. ρ explicitly models average overdispersion across all stratum, while $\mu_{jklm\eta}$ gives the model flexiblity at the stratum level through through it's linear predictor,

$$\theta_{jklm\eta} = \beta_0 + \beta^{(s)}_j + \beta^{(p)}_k + \beta^{(g)}_l + \beta^{(y:q)}_{m\eta}.$$

Firstly, θ includes an intercept (β_0) shared among all strata. Secondly, θ is factored among the many strata by simple additive offsets for each of the species ($\beta^{(s)}_j$), port-complex ($\beta^{(p)}_k$), and gear-group ($\beta^{(g)}_l$) categories. Finally, a year-quarter interaction ($\beta^{(y:q)}_{m\eta}$) is included to give this model the flexibility to model differing seasonality from year to year. In addition to offering flexibility in modeling seasonalities, the year-quarter interaction provides an ideal structure for partially pooling data through time via a heirarchical prior discussed later in $Section(XX)/thefollowingsection$.

- justify linear predictor/transistion to priors

A Heirarchical Prior

$$\text{logit}(\rho) \sim N(0, 2^2)$$
$$\left\{\beta^{(s)}_j, \beta^{(p)}_k, \beta^{(g)}_l\right\} \sim N(0, 32^2)$$
$$\beta_0 \propto 1$$

$$\beta^{(y:q)}_{m\eta} \sim N(0, v_m)$$

-or-

$$\beta^{(y:q)}_{m\eta} \sim N(0, v_\eta)$$

-or-

$$\beta^{(y:q)}_{m\eta} \sim N(0, v)$$

$$v \sim IG(1,\ 2\times 10^3) \quad \forall \quad v$$

	v_m	v_η	v
MSE	NA	NA	NA
DIC	NA	NA	NA
WAIC	NA	NA	NA
$pr(M\|y)$	NA	NA	NA

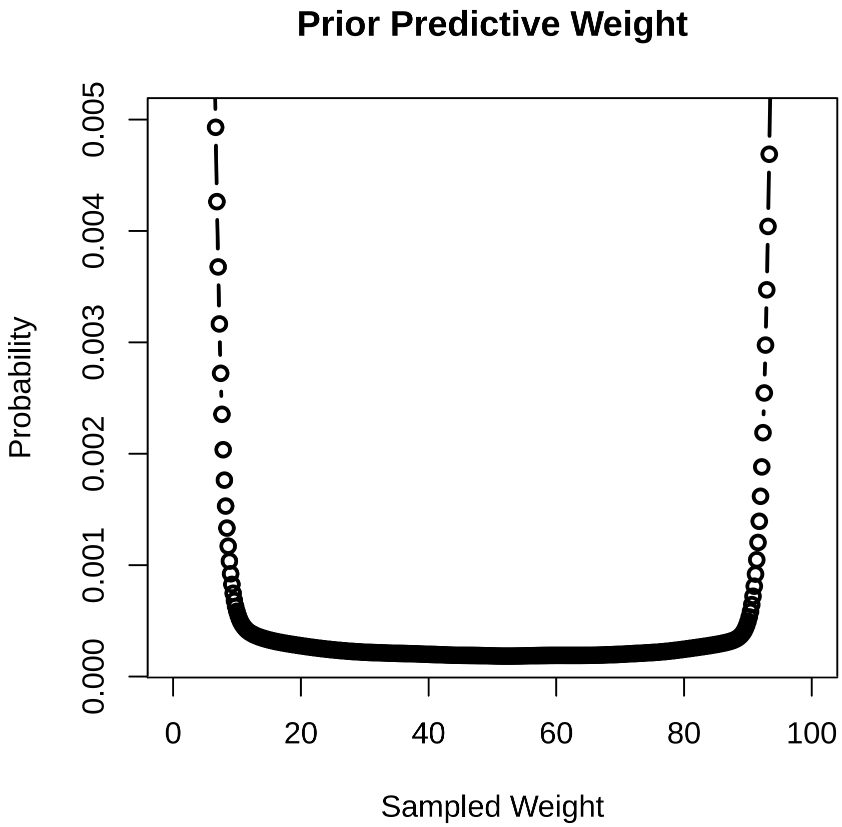

Figure 3: Prior Prediction

Prediction

$$p(y^*_{jklm\eta}|y) = \iint \mathrm{BB}\Big(y^*_{jklm\eta}|\mu_{jklm\eta}, \sigma^2_{jklm\eta}\Big) P\Big(\mu_{jklm\eta}, \sigma^2_{jklm\eta}|y\Big) d\mu_{jklm\eta} d\sigma^2_{jklm\eta}$$

$$\pi^*_{jklm\eta} = \frac{y^*_{jklm\eta}}{\sum_j y^*_{jklm\eta}} \quad y^*_{klm\eta} \neq 0$$

Model Exploration & Averaging

Results

- General Products
- Degree of smoothing (heirarchical parameters)
- Posterior v. Current
  - Report degree of similarity
- Prediction v. Data
  - Report predictive accuracy

Conclusions

- General Math/Science
- Database Stuff
- Looking Forward
  - forcasting/hindcasting
    - simple
    - timeseries models
  - more computation faster
    - broader model exploration
    - broader spatial expansion

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