

# Improving Catch Estimation Methods in Sparsely Sampled, Mixed Stock Fisheries.

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**UCSC :: CSTAR :: SWFSC :: NMFS**

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# Diagnostic

Show Example

# Beta-Binomial Model

$$y_{ijklm\eta} \sim \text{Beta-Binomial}(\mu_{ijklm\eta}, \sigma_{ijklm\eta}^2)$$

$$\mu_{ijklm\eta} = n \text{ logit}^{-1}(\theta_{ijklm\eta})$$

$$\sigma_{ijklm\eta}^2 = \mu_{ijklm\eta} \left(1 - \frac{\mu_{ijklm\eta}}{n}\right) \left(1 + (n-1) \rho\right)$$

$$\theta_{ijklm\eta} = \beta_0 + \beta_j^{(s)} + \beta_k^{(p)} + \beta_l^{(g)} + \beta_{m\eta}^{(t)} \beta_{m\eta}^{(y:q)}$$

$y_{ijklm\eta}$ :  $i^{\text{th}}$  sample of the  $j^{\text{th}}$  species' integer weight, in the  $k^{\text{th}}$  port, caught with the  $l^{\text{th}}$  gear, in the  $\eta^{\text{th}}$  quarter, of year  $m$ , for a particular market category.

$j \in \{1, \dots, J\}$  Species

$k \in \{1, \dots, K\}$  Ports

$l \in \{1, \dots, L\}$  Gears

$m \in \{1, \dots, M\}$  Years

$\eta \in \{1, \dots, H\}$  Quarters

# Time Models

**(M1)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, 32^2)$$

$$\beta_{\eta}^{(q)} \sim N(0, 32^2)$$

**(M2)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

**(M3)**

$$\beta_{m\eta}^{(t)} = \beta_m^{(y)} + \beta_{\eta}^{(q)} + \beta_{m\eta}^{(y:q)}$$

$$\beta_m^{(y)} \sim N(0, v^{(y)})$$

$$\beta_{\eta}^{(q)} \sim N(0, v^{(q)})$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

**(M4)**

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v)$$

**(M5)**

$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_{\eta})$$

**(M6)**

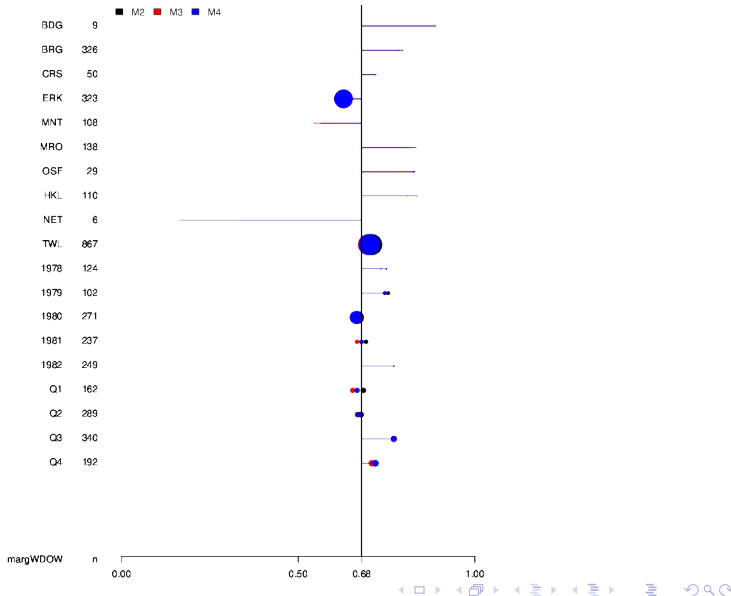
$$\beta_{m\eta}^{(t)} = \beta_{m\eta}^{(y:q)}$$

$$\beta_{m\eta}^{(y:q)} \sim N(0, v_m)$$



M4





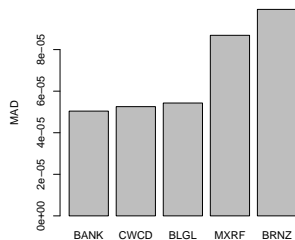


M2

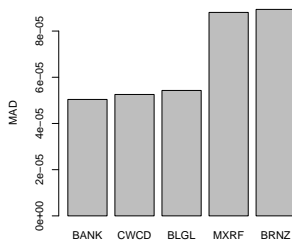
M3

M4

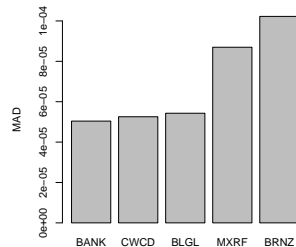
MAD Ordered by Species

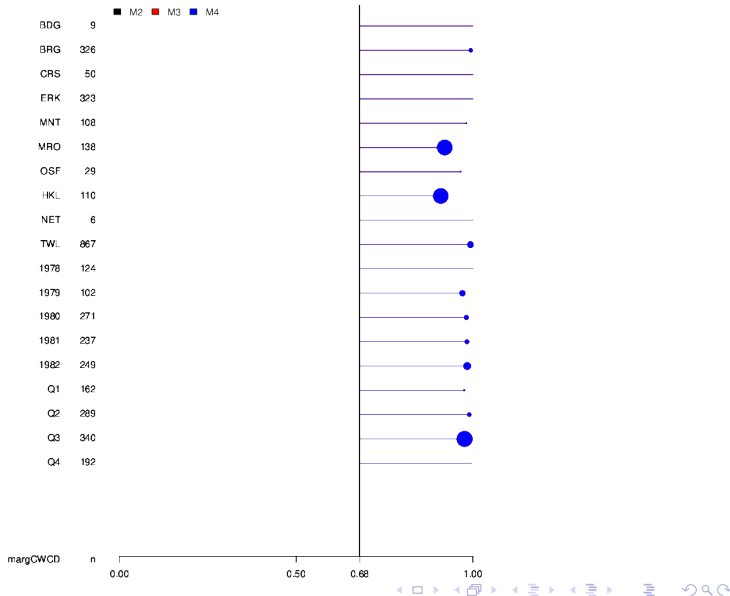


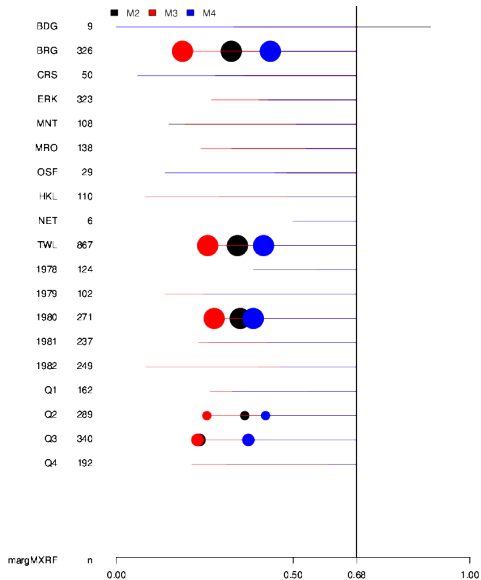
MAD Ordered by Species



MAD Ordered by Species







# MCAT 253

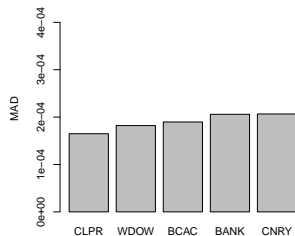
	M1	M2	M3	M4	M5	M6
$\Delta$ DIC	1409.81	0.09	0.1	0.07	0.05	0
$\Delta$ WAIC	1391.66	0.16	0.18	0	0.13	0.08
$pr(M y)$	0	0	0	1	0	0

M4

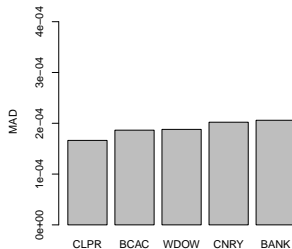
M5

M6

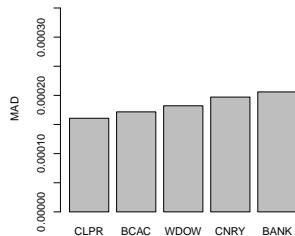
MAD Ordered by Species



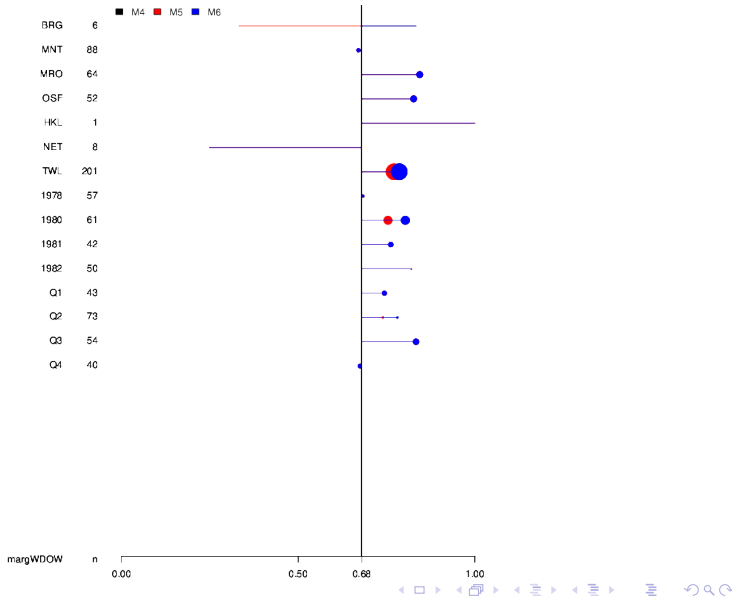
MAD Ordered by Species



MAD Ordered by Species





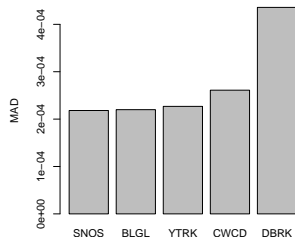


M4

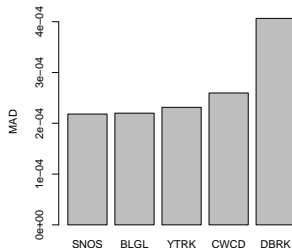
M5

M6

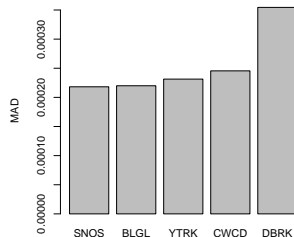
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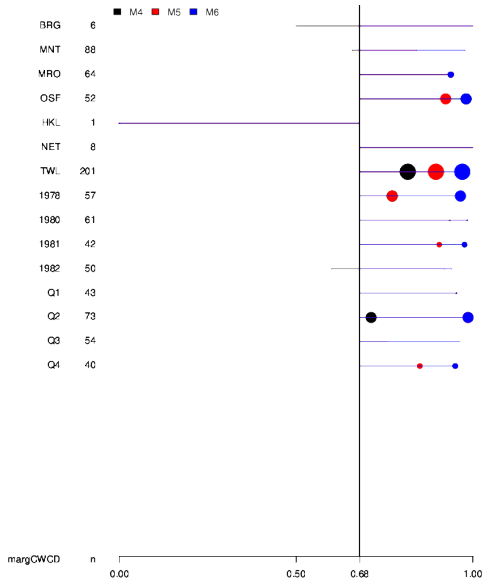
MAD Ordered by Species

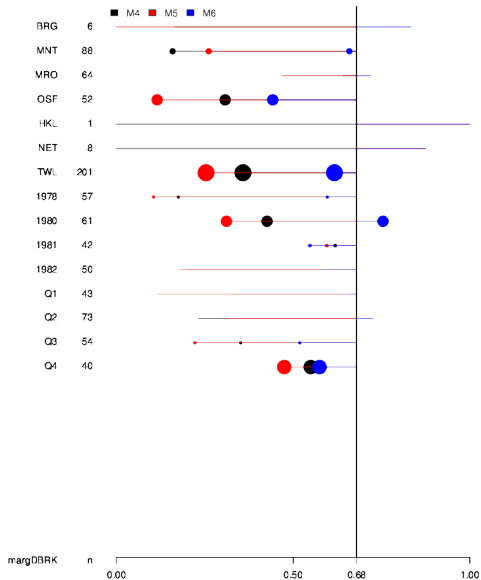


MAD Ordered by Species









## MCAT 269

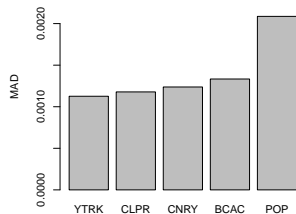
	M1	M2	M3	M4	M5	M6
$\Delta$ DIC	572.51	176.63	599.41	0.57	0	193.35
$\Delta$ WAIC	427.48	69.37	454.41	0.23	0	78.07
$pr(M y)$	0	0	0	0	0	1

M4

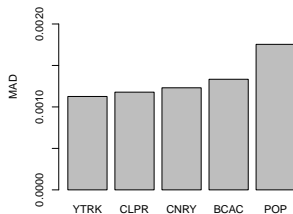
M5

M6

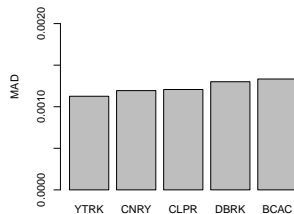
MAD Ordered by Species



MAD Ordered by Species



MAD Ordered by Species



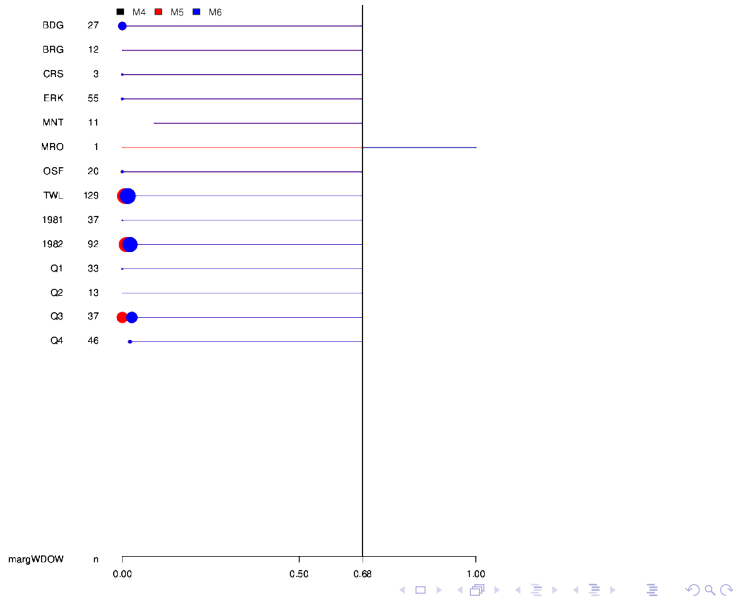




## M6







# Priors

$$\beta_0 \propto 1$$

$$\beta_j^{(s)} \sim N(0, 32^2)$$

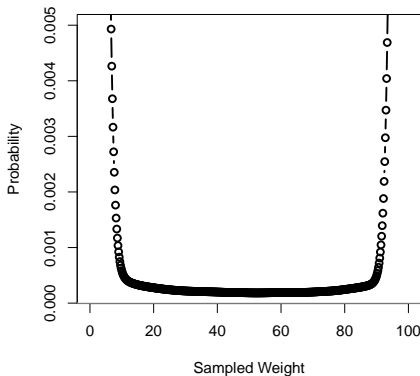
$$\beta_k^{(p)} \sim N(0, 32^2)$$

$$\beta_l^{(g)} \sim N(0, 32^2)$$

$$\text{logit}(\rho) \sim N(0, 2^2)$$

$$v \sim IG(1, 2 \times 10^3) \quad \forall \quad v$$

Prior Predictive Weight



# MCAT 250

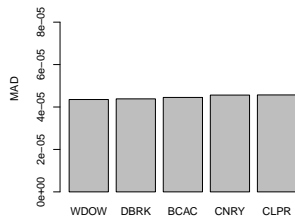
	M4	M4HC1	M4HC3	M4U4
$\Delta$ DIC	3.87	0.02	0.1	0
$\Delta$ WAIC	3.78	0.03	0.11	0
$pr(M y)$	0	0.21	0.37	0.42

## M4HC1

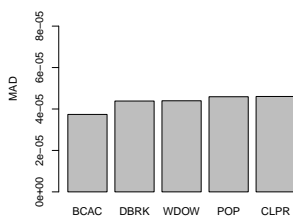
## M4HC3

## M4U4

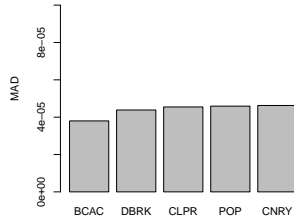
MAD Ordered by Species

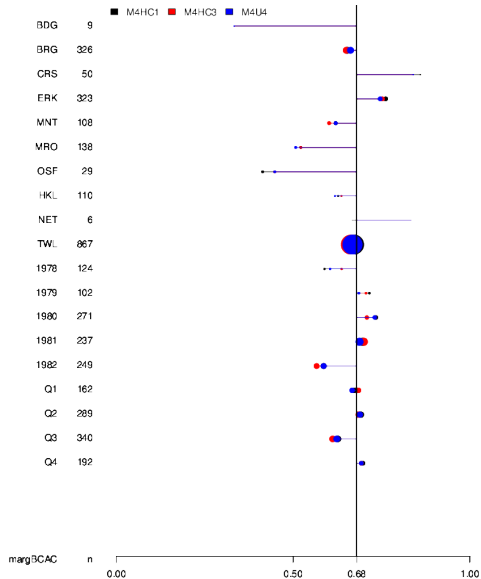


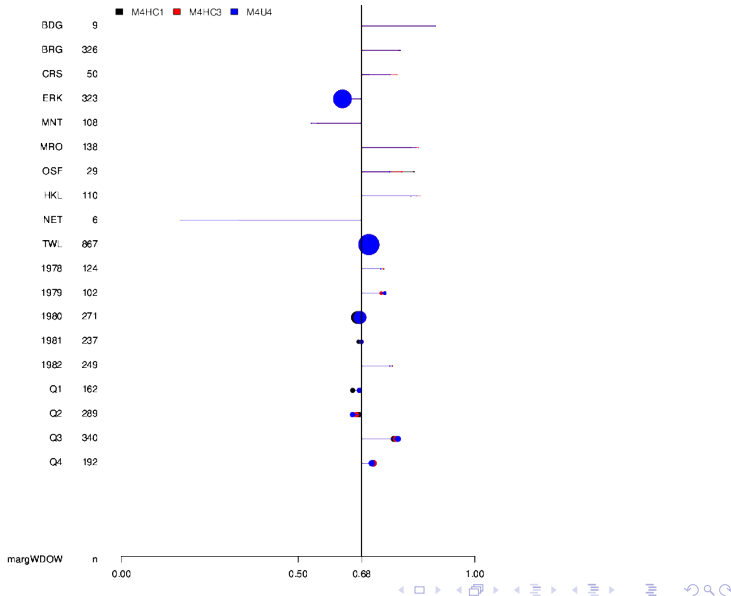
MAD Ordered by Species



MAD Ordered by Species





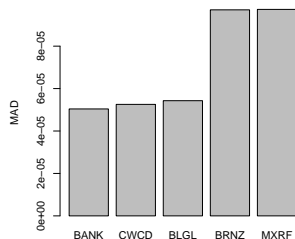


M4HC1

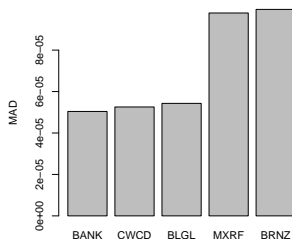
M4HC3

M4U4

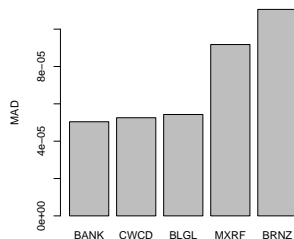
MAD Ordered by Species

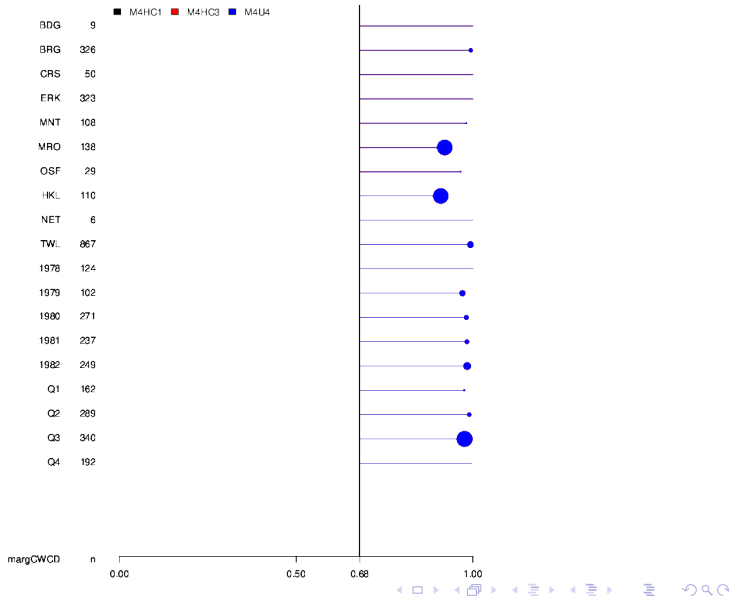


MAD Ordered by Species

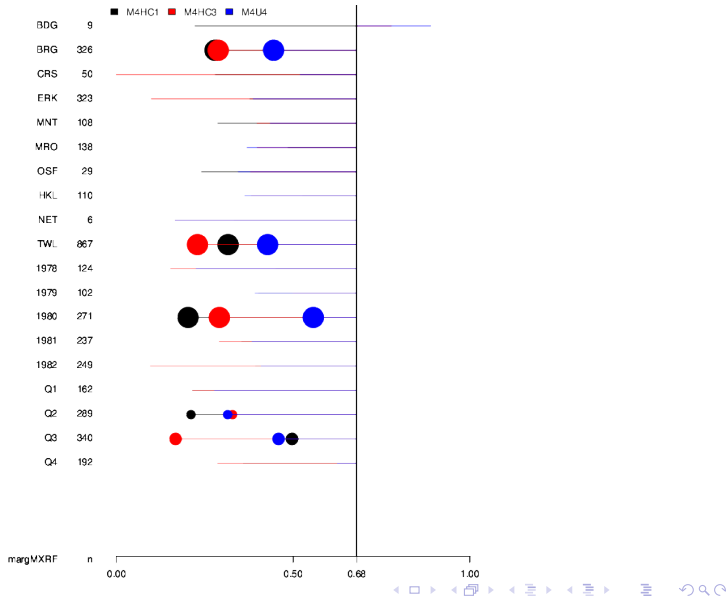


MAD Ordered by Species









# MCAT 253

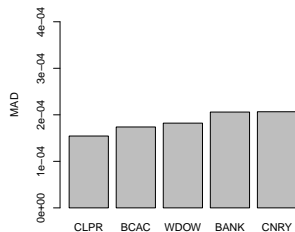
	M4	M4HC1	M4HC3	M4U4
$\Delta$ DIC	0.88	0.8	0.8	0
$\Delta$ WAIC	0.76	0.83	0.83	0
$pr(M y)$	0.01	0.99	0	0

M4HC1

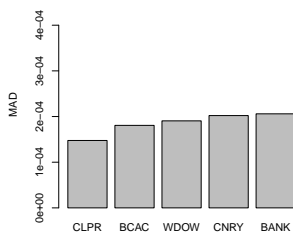
M4HC3

M4U4

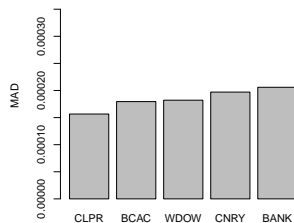
MAD Ordered by Species



MAD Ordered by Species



MAD Ordered by Species





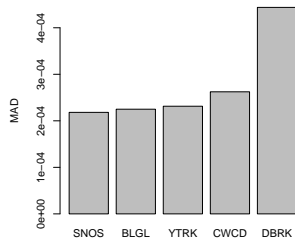


M4HC1

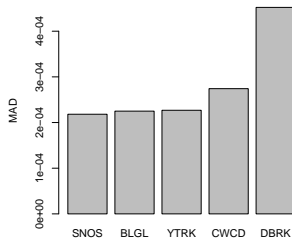
M4HC3

M4U4

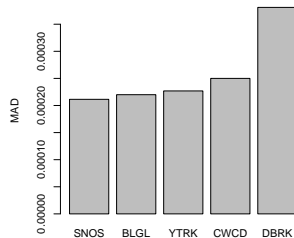
MAD Ordered by Species

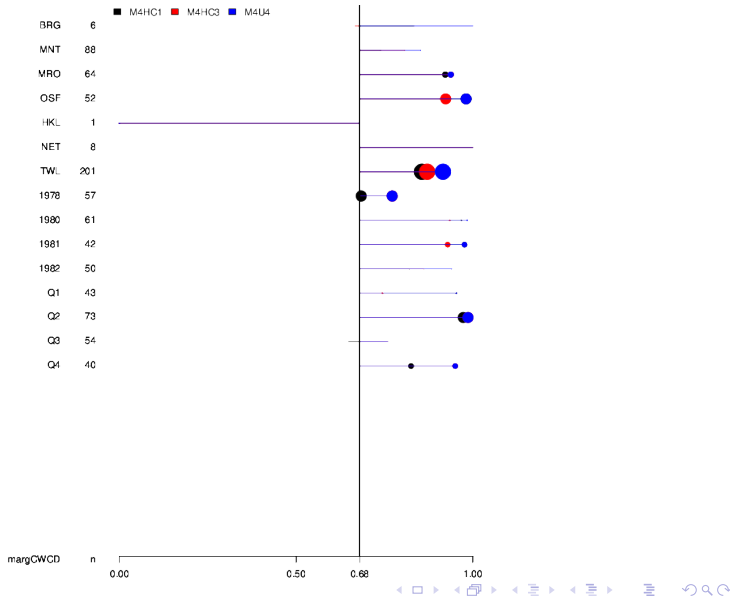


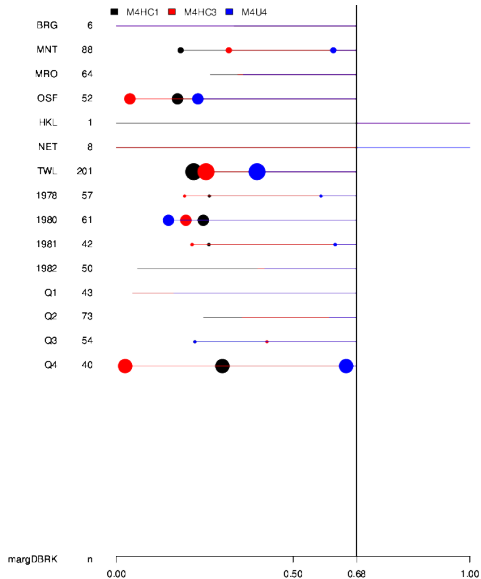
MAD Ordered by Species



MAD Ordered by Species









# MCAT 269

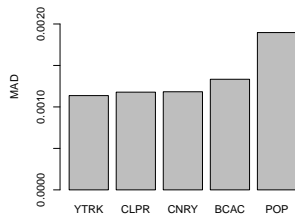
	M4	M4HC1	M4HC3	M4U4
$\Delta$ DIC	0.18	176.33	0.2	0
$\Delta$ WAIC	0.08	69.19	0.08	0
$pr(M y)$	0	1	0	0

## M4HC1

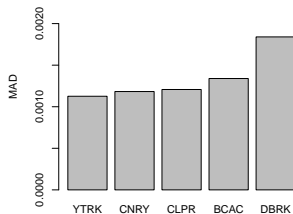
## M4HC3

## M4U4

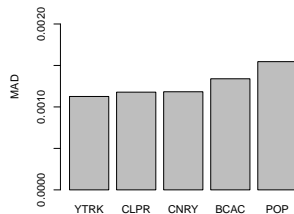
MAD Ordered by Species

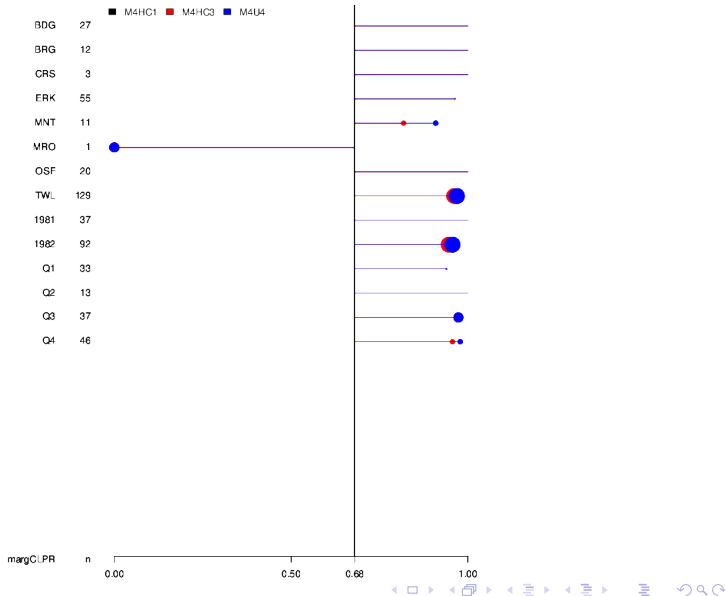


MAD Ordered by Species



MAD Ordered by Species





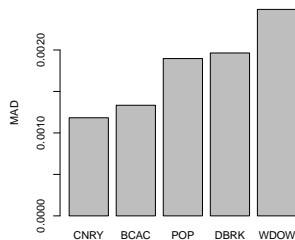


## M4HC1

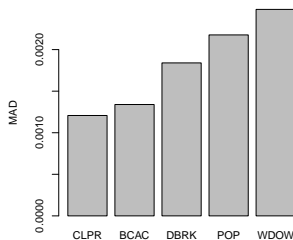
## M4HC3

## M4U4

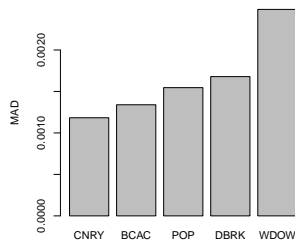
MAD Ordered by Species

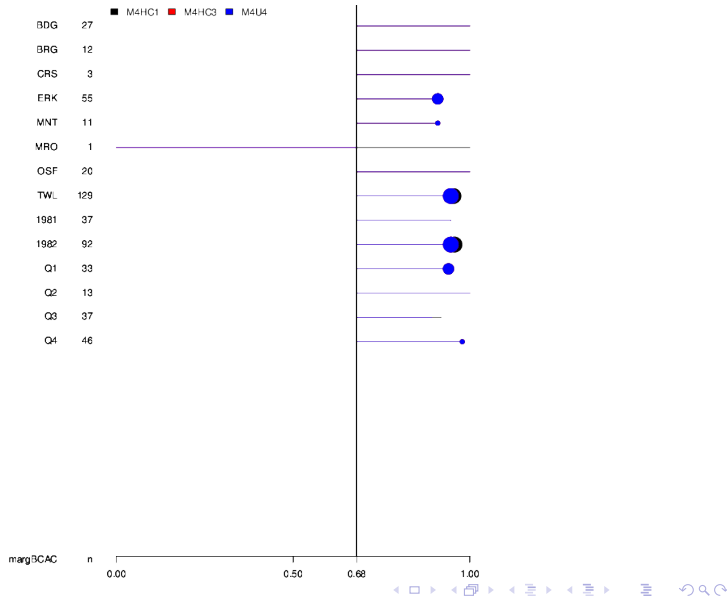


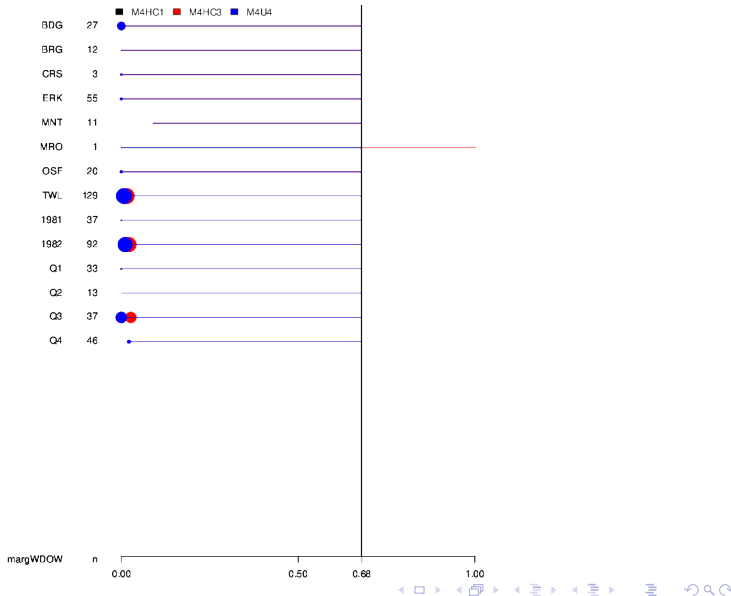
MAD Ordered by Species



MAD Ordered by Species







# Port and Gear Interactions



# Time Blocks

# Proof: Species Comps Sum to One... as do Their Means.

If  $y_{jk}$  is the  $k^{\text{th}}$  draw,  $k \in \{1, \dots, K\}$ , of the posterior predictive weight of species  $j$  in a particular stratum. Then,

$$\pi_{jk} = \frac{y_{jk}}{\sum_j y_{jk}} \quad \mathbf{y}_k \neq \mathbf{0}. \quad (1)$$

The predictive mean for species  $j$  is,

$$\hat{\pi}_j = \frac{\sum_k^K \pi_{jk}}{K}. \quad (2)$$

Summing  $\hat{\pi}_j$  across species, it follows from (1) and (2) that,

$$\sum_j \hat{\pi}_j \stackrel{(2)}{=} \sum_j \frac{\sum_k^K \pi_{jk}}{K} = \frac{\sum_k^K \sum_j \pi_{jk}}{K} \stackrel{(1)}{=} \frac{\sum_k^K \sum_j \frac{y_{jk}}{\sum_j y_{jk}}}{K} = \frac{\sum_k^K 1}{K} = \frac{K}{K} = 1. \quad \blacksquare$$

# Proof: Species Comps are Negatively Correlated.

Here we seek to show for any two species  $l \neq m$ ,  $\text{Corr}(\pi_l, \pi_m) < 0$ .

Recall:

$$\text{Corr}(\pi_l, \pi_m) = \frac{\text{Cov}(\pi_l, \pi_m)}{\sigma_{\pi_l} \sigma_{\pi_m}} \quad \sigma_{\pi_l} \geq 0, \sigma_{\pi_m} \geq 0$$

$$\text{Corr}(\pi_l, \pi_m) \leq 0 \iff \text{Cov}(\pi_l, \pi_m) \leq 0$$

$$\begin{aligned} \text{Cov}(\pi_l, \pi_m) &= \mathbb{E}[(\pi_l - \mathbb{E}[\pi_l])(\pi_m - \mathbb{E}[\pi_m])] \\ &= \mathbb{E}[\pi_l \pi_m] - \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \end{aligned}$$

$$\text{Cov}(\pi_l, \pi_m) \leq 0 \iff \mathbb{E}[\pi_l] \mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l \pi_m]$$

# Proof: Species Comps are Negatively Correlated Cont.

Consider the strictly concave function:  $f(\mathbf{x}) = \prod_i x_i : \sum_i x_i \leq 1, x_i \geq 0$ .  
Jensen's Inequality for  $f$  is,

$$f(\mathbb{E}[\mathbf{x}]) \geq \mathbb{E}[f(\mathbf{x})]. \quad (3)$$

From the previous proof:  $\sum_j \pi_j \leq 1, \pi_j \geq 0$  and  $\sum_j \hat{\pi}_j \leq 1, \hat{\pi}_j \geq 0$ .  
Thus applying (3) to  $\pi$  gives

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] \geq \mathbb{E}[\pi_l\pi_m] \quad (4)$$

with equality only if  $\pi$  is a constant. Since  $\pi$  is never a constant,

$$\mathbb{E}[\pi_l]\mathbb{E}[\pi_m] > \mathbb{E}[\pi_l\pi_m]$$

$$\text{Cov}(\pi_l, \pi_m) < 0$$

$$\text{Corr}(\pi_l, \pi_m) < 0. \quad \blacksquare$$