Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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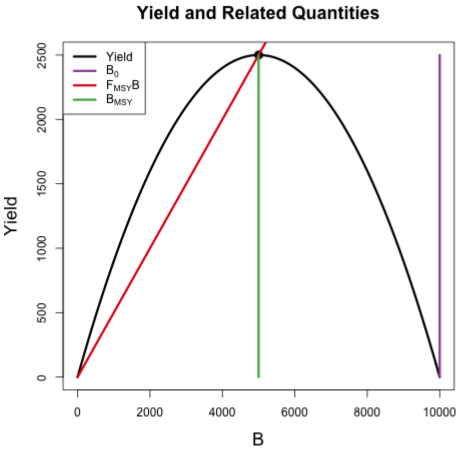
General Modeling Structures

$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

$$\text{Reference Points} : MSY, \frac{F_{MSY}}{M}, \frac{B_{MSY}}{B_0}$$

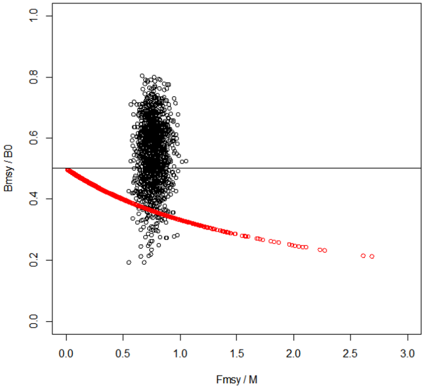


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0,1)$$

Two Parameter:

- Shaefer Model:
  $F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{2}$
- BH Model:
  $\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$



$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

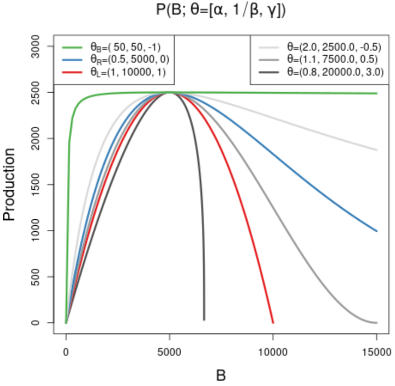
$$\frac{dB}{dt} = P(B; \boldsymbol{\theta}) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma \rightarrow 0 \Rightarrow$ Ricker

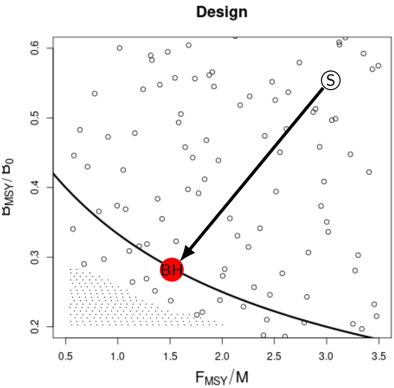
$\gamma = 1 \Rightarrow$ Logistic



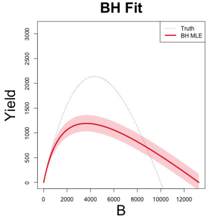
Simulation Design

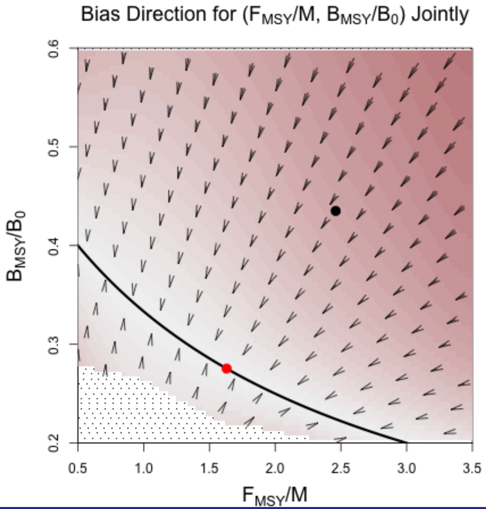
- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS
- GP Metamodeling of RP bias

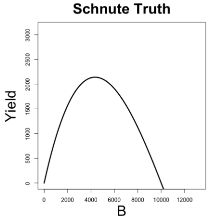
$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{GP}{\mapsto} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{\hat{B}_{MSY}}{\bar{B}(0)}\right)}_{\text{BH Estimate}}$$



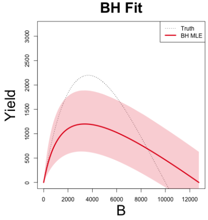
High Contrast

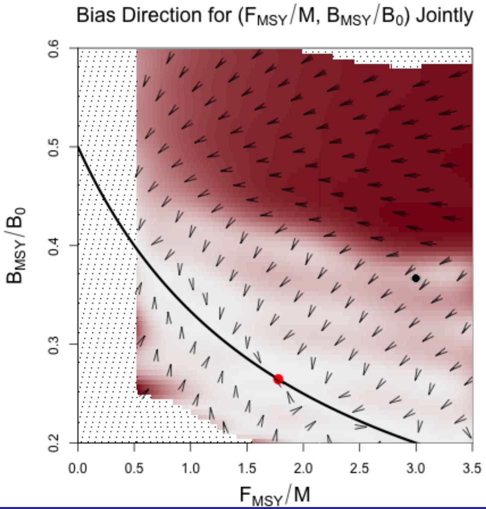


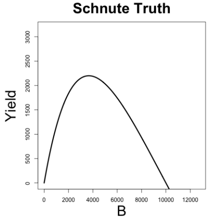




Low Contrast







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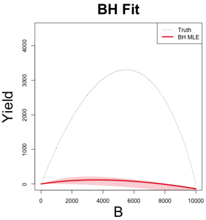
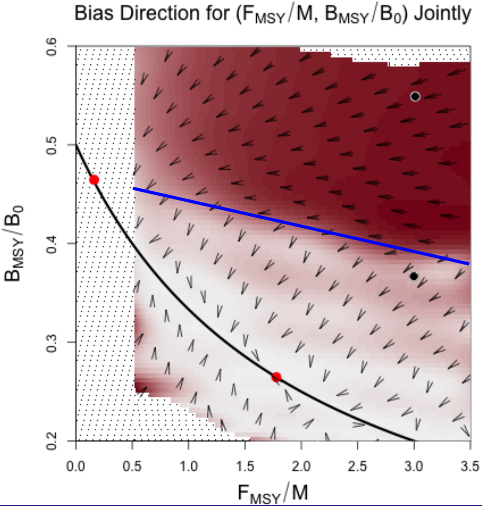
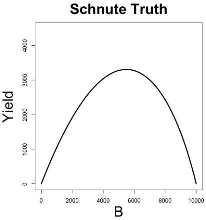
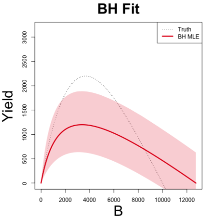
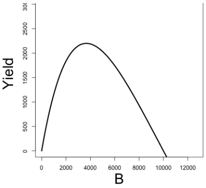










Individual Growth Extension

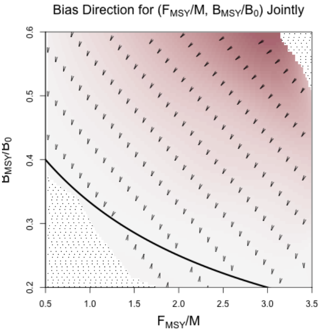
$$\frac{dB}{dt} = \overbrace{w(a_0)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

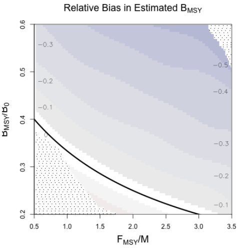
$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

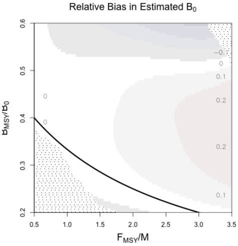
$$R(B;[\alpha,\beta,\gamma]) = \alpha B(t-a_0)(1-\beta\gamma B(t-a_0))^{\frac{1}{\gamma}}$$

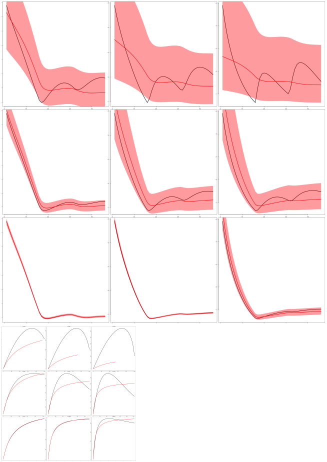
$$w(a) = w_\infty(1-e^{-\kappa a})$$

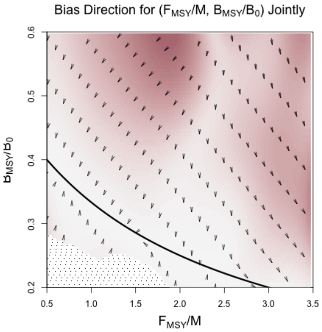
$\theta' = [\alpha,\beta,\gamma]$ Species Properties: $a_0,\ \kappa,\ w_\infty,\ M$

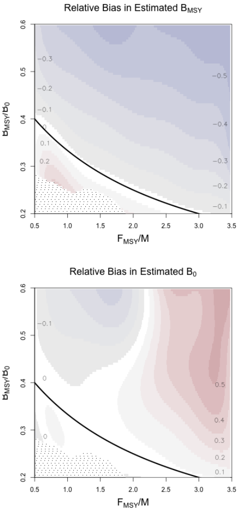

Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.2
0.3
0.4
0.5
0.6

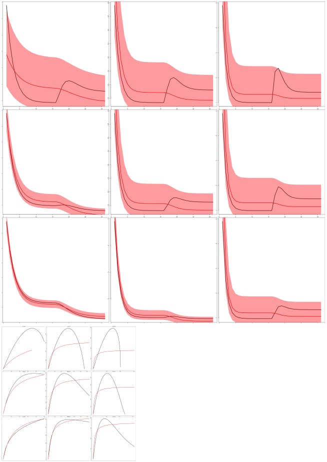

Relative Bias in Estimated B_{MSY}
B_{MSY}/B_0
F_{MSY}/M
−0.3
−0.2
−0.1
−0.5
−0.4
−0.3
−0.2
−0.1


Relative Bias in Estimated B_0
B_{MSY}/B_0
F_{MSY}/M
−0.1
0
0.1
0.2
0.2
0.1




Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M

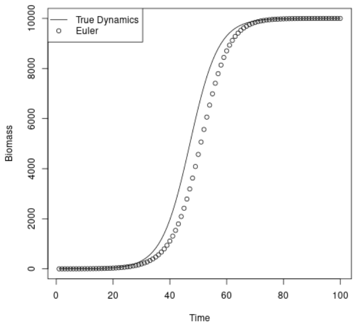

Relative Bias in Estimated B_{MSY}
B_{MSY}/B_0
F_{MSY}/M
Relative Bias in Estimated B_0
B_{MSY}/B_0
F_{MSY}/M



Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$



Common Discretization

Instantaneous

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$c(\tau) = \int_{\tau}^{\tau+1} C(t)dt$$

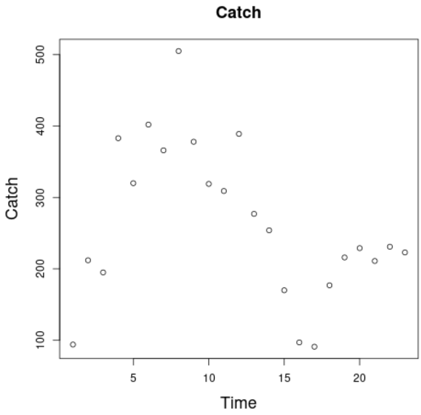
$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Yearly

$$t \in \mathbb{R}^+ \qquad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[c(t)] = \int_{\tau}^{t} C(t^*)dt^*$$

$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



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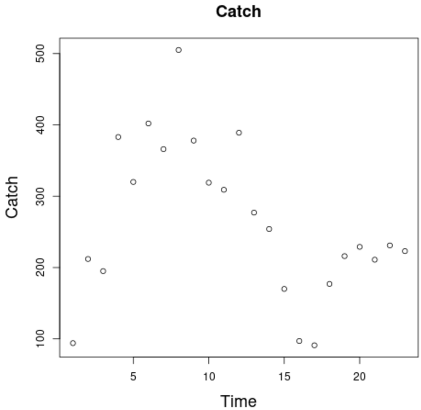
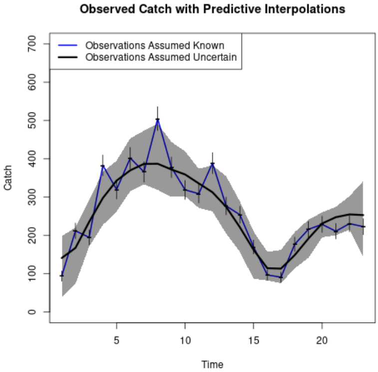
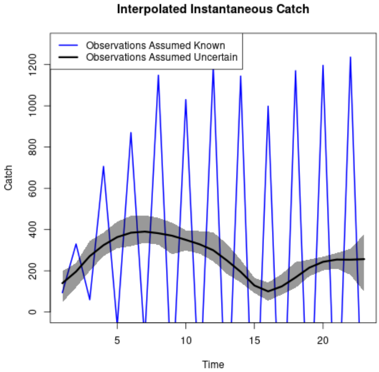


$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0,1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$
$$B(\tau+1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$





Summary

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
  - ⇒ Productivity Extensions
  - ⇒ Individual growth and maturity dynamics
- In this severly constrained settings we pay for our modeling mistakes primarily in estimate bias.
- In practice the Schaefer model is at best only likely to reasonably estimate one of either B_{MSY} or F_{MSY}.
- The observed contrast serves to distribute the available information among B_{MSY} and F_{MSY}.
  - ⇒ Models of catch contextulize interpretation of RP estimation.

Conclusions

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding more complex models.
  - $\Rightarrow$ Individual growth and maturity dynamics
- RPs are not directly observable quantities, but rather model dependent latent quantities.
  - $\Rightarrow$ Subject to Model Misspecification, Uncertainty, & Bias
  - $\Rightarrow$ In severly constrained settings we pay for our modeling mistakes primarily in estimate bias.
- The observed contrast serves to increase the range of potentially "allowable" model misspecification.

Many Thanks:

- UCSC Advisors
- SWFSC Groundfish
- NMFS Sea Grant

Metamodel Details

$$\boldsymbol{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\begin{aligned} \hat{\mu} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon \\ f(\boldsymbol{x}) &\sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}')) \\ \epsilon_i &\sim \mathrm{N}(0, \hat{\omega}_i). \end{aligned}$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$

Schnute RP-Parameter System of Equations

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M+F_{MSY}}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}$$

$$\alpha = (M + F_{MSY})\left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

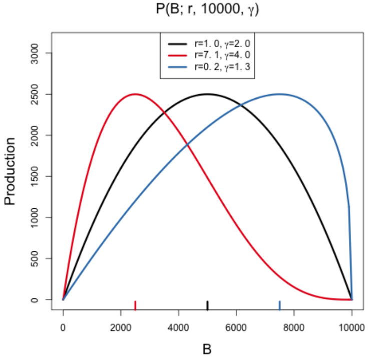
Pella-Tomlinson Production Model

$$I(t) \sim LN(qB(t), \sigma^2)$$

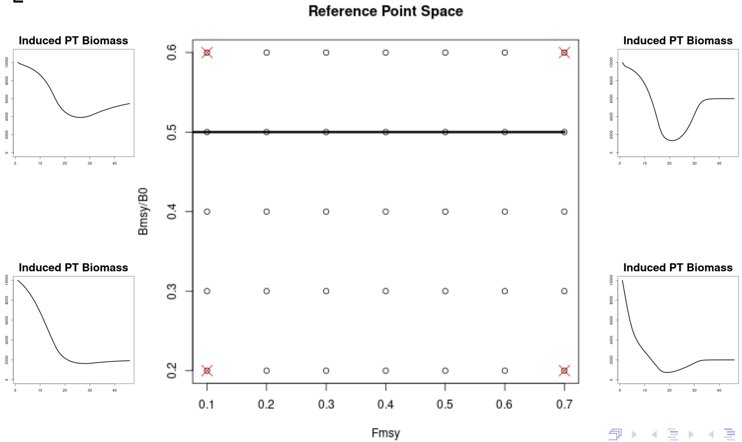
$$\frac{dB(t)}{dt} = P_{\theta}(B(t)) - F(t)B(t)$$

$$P(B;[r,K,\gamma]) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma-1}$$

$\gamma = 2 \Rightarrow$ Schaefer Model

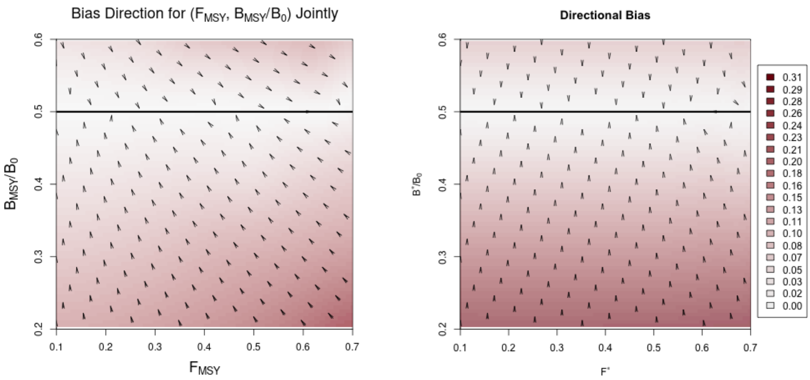


$$\theta = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)},\ K = 10000,\ \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$



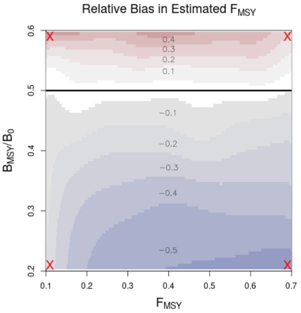
Low Contrast

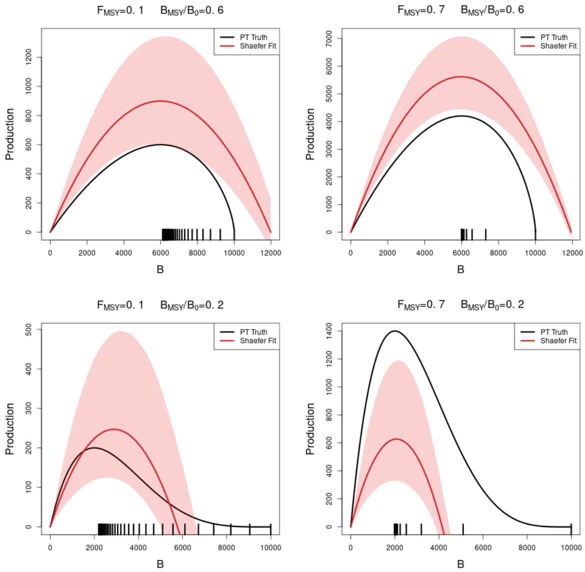
High Contrast



Low Contrast

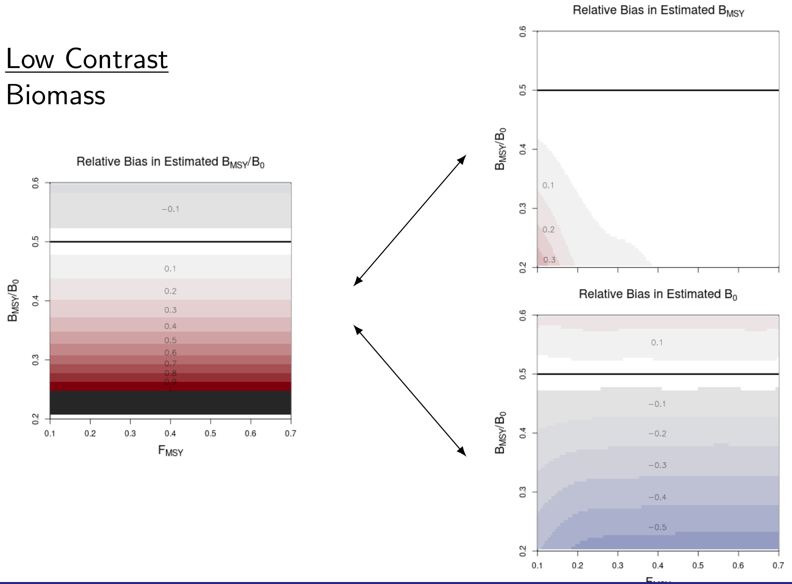
F_{MSY}





Low Contrast

Biomass



High Contrast