Beverton-Holt (BH)

$$R(P) = \frac{\alpha P}{1+\beta P}$$

$$\frac{dP}{dt} = \frac{\alpha P}{1+\beta P} - (M+F)P$$

Set 0

$$\cancel{(M+F)\bar{P}} = \frac{\alpha \cancel{\bar{P}}}{1+\beta\bar{P}}$$

$$1+\beta\bar{P} = \frac{\alpha}{M+F}$$

$$\bar{P} = \left(\frac{\alpha}{M+F} - 1\right)\frac{1}{\beta}$$

$$\frac{d\bar{P}}{dF} = \frac{1}{\beta}\frac{d}{dF}\left(\frac{\alpha}{M+F} - 1\right)$$

$$= \frac{\alpha}{\beta}\frac{d}{dF}(M+F)^{-1}$$

$$= -\frac{\alpha}{\beta}(M+F)^{-2}$$

$$F_{MSY} = \underset{F}{\mathrm{Arg\,Max}}\ \underbrace{F\,\bar{P}(F)}_{\bar{Y}}$$

$$\frac{d\bar{Y}}{dF} = \bar{P}(F) + F\frac{d\bar{P}}{dF}$$

$$= \frac{1}{\beta}\left(\frac{\alpha}{M+F} - 1\right) + F\left(\frac{-\alpha}{\beta}\right)(M+F)^{-2}$$

Set 0:

$$0 = \cancel{\frac{1}{\beta}}\left(\frac{\alpha}{M+F^*} - 1\right) + F^*\left(\frac{-\alpha}{\cancel{\beta}}\right)(M+F^*)^{-2}$$

$$= \frac{\alpha}{M+F^*} - 1 - \alpha\left(\frac{F^*}{(M+F^*)^2}\right)$$

$$= \alpha - (M+F^*) - \alpha\left(\frac{F^*}{M+F^*}\right)$$

$$= \alpha(M+F^*) - (M+F^*)^2 - \alpha F^*$$

$$= \alpha M - (M+F^*)^2$$

$$M+F^* = \sqrt{\alpha M}$$

$$F^* = \sqrt{\alpha M} - M$$

$$\frac{F^*}{M} = \sqrt{\frac{\alpha}{M}} - 1$$

$$P^* = \left(\frac{\alpha}{M+(\sqrt{\alpha M} - \cancel{M})} - 1\right)\frac{1}{\beta}$$

$$= \left(\sqrt{\frac{\alpha}{M}} - 1\right)\frac{1}{\beta} = \frac{F^*}{M}\left(\frac{1}{\beta}\right)$$

$$P_0 = \left(\frac{\alpha}{M} - 1\right)\frac{1}{\beta}$$

$$\boxed{\frac{P^*}{P_0} = \frac{\cancel{\frac{1}{\beta}}\left(\sqrt{\frac{\alpha}{M}} - 1\right)}{\cancel{\frac{1}{\beta}}\left(\frac{\alpha}{M} - 1\right)}}$$

$$\frac{\mathcal{E}}{(\mathcal{E}+1)^2 - 1^2}$$

$$(\mathcal{E}+\cancel{1}-\cancel{1})(\mathcal{E}+1+1)$$

$$= \frac{\mathcal{E}}{\mathcal{E}+2}$$

$$= \frac{\frac{F^*}{M}}{\left(\frac{F^*}{M}+1\right)^2 - 1}$$