Schnute & Richards (1985)

SRR:

$$R_t = \alpha S_{t-1}\left(1 - \beta\gamma S_{t-1}\right)^{\frac{1}{\gamma}}$$

Production Model:

$$P_t = R_t + (1-\delta) S_{t-1}$$

$$S_t = P_t - C_t$$

$$\delta = 1 - e^{-M} \leftarrow \text{constant}$$

$$f = 1 - e^{-F}$$

Equilibrium Eq:

$$\sigma(f) = (1-\delta)(1-f)$$

$$\rho(f) = \frac{1-\sigma(f)}{1-f}$$

$$R(f) = \frac{\rho(f)}{\beta\gamma}\left[1 - \left(\frac{\rho(f)}{\alpha}\right)^{\gamma}\right]$$

$$P(f) = \frac{R(f)}{1-\sigma(f)}$$

@ $f=0$

$$\sigma_0 = 1-\delta$$

$$\rho_0 = \delta$$

$$R_0 = \frac{\delta}{\beta\gamma}\left[1 - \left(\frac{\delta}{\alpha}\right)^{\gamma}\right]$$

$$P_0 = \frac{R_0}{\delta}$$

De-Discretize:

Recall: $P_t = R_t + (1-\delta) S_{t-1}$

Subtract P_{t-1} from both sides aiming for Euler's Method

$$P_t - P_{t-1} = R_t + (1-\delta) S_{t-1} - P_{t-1}$$

Note: $P_t - P_{t-1} = \frac{P_t - P_{t-1}}{1}$ * Finite Difference $\frac{dP}{dt}$ w/ step-size 1.

$$= R_t + (1-\delta)(P_{t-1} - C_{t-1}) - P_{t-1}$$

$$= R_t + (1-\delta)P_{t-1} - P_{t-1} - (1-\delta)C_{t-1}$$

$$= R_t - \delta P_{t-1} + \delta C_{t-1} - C_{t-1}$$

$$= R_t - \delta(P_{t-1} - C_{t-1}) - C_{t-1}$$

$$\frac{dP}{dt} = R_t - \delta S_{t-1} - C_{t-1}$$

Reparameterize: $(f^*, C^*) \mapsto (\alpha, \beta)$

$$\sigma^* = (1-\delta)(1-f^*)$$

$$\alpha = \frac{1-\sigma^*}{1-f^*}\left[1 + \frac{\gamma f^*}{1-\sigma^*}\right]^{\frac{1}{\gamma}}$$

$$\beta = \frac{f^{*2}}{(1-f^*)(1-\sigma^* + \gamma f^*)C^*}$$

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$$P_0 = \frac{-1}{\beta}\left[1 - \left(\frac{\delta}{\alpha}\right)^{-1}\right]$$

$$= \frac{1}{\beta}\left[\frac{\alpha}{\delta} - 1\right]$$

Mangel et. al. (2012)

Under B-H:

? SRR in terms of $S = P - C$ or P?

? $f P_{t-1} = C_{t-1}$ v. observed catch. ?

$$\frac{dP}{dt} = \frac{\alpha P}{1 + \beta P} - (\delta + f) P$$

? Steepness:

$$h = \frac{\frac{\alpha}{\delta}}{4 + \frac{\alpha}{\delta}}$$

? Optimal Fishing Rate RP:

$$\frac{f^*}{\delta} = \sqrt{\frac{4h}{1-h}} - 1$$

Optimal Biomass RP:

$$\frac{P^*}{P_0} = \frac{\sqrt{\frac{4h}{1-h}} - 1}{\frac{4h}{1-h} - 1}$$

RP Relationships Under BH

ξ : Optimal Fishing Rate RP

ζ : Optimal Biomass RP

$$\xi = \sqrt{\frac{4h}{1-h}} - 1$$

$$(\xi + 1)^2 = \frac{4h}{1-h}$$

$$\zeta = \frac{\sqrt{\frac{4h}{1-h}} - 1}{\frac{4h}{1-h} - 1} \overset{\xi \neq 0}{=} \frac{\xi}{(\xi+1)^2 - 1}$$

see as Diff of squares

$$(\xi+1)^2 - (1)^2 = \left((\xi + \cancel{1}) - \cancel{1}\right)\left((\xi+1)+1\right)$$
$$= \xi(\xi+2)$$

$$= \frac{\xi}{\xi(\xi+2)} = \frac{1}{\xi+2}$$

RP Distributional Results Under BH

Assume $\xi \sim LN(\mu, \sigma)$ $\quad \xi \in (0, \infty)$

$$\zeta = \frac{1}{\xi + 2} \qquad \zeta \in (0, \tfrac{1}{2})$$

$$\text{logit}(2\zeta) = \log\left(\frac{\frac{2}{\xi+2}}{1 - \frac{2}{\xi+2}}\right) = \log\left(\frac{\frac{2}{\cancel{\xi+2}}}{\frac{\xi}{\cancel{\xi+2}}}\right) = \log\left(\frac{2}{\xi}\right) = \log(2) - \log(\xi)$$

*Shifted Normal

$$\text{logit}(2\zeta) \sim N(\log(2) - \mu, \sigma)$$

$$\Rightarrow 2\zeta \sim \text{Logit-Normal}(\log(2) - \mu, \sigma)$$

Confusion over RP Notation :

$$h = \frac{\frac{\alpha}{M}}{4 + \frac{\alpha}{M}} \qquad \xi = \frac{F^*}{M} = \sqrt{\frac{4h}{1-h}} - 1 \qquad \zeta = \frac{B^*}{B_0} = \frac{\sqrt{\frac{4h}{1-h}} - 1}{\frac{4h}{1-h} - 1}$$

I have

$$M = 0.2 \Rightarrow \delta = 1 - e^{-0.2} \doteq 0.18 \Rightarrow F^* = \xi M$$

$$B_0 = 3000 \Rightarrow B^* = \zeta B_0$$

$$f^* = 1 - e^{-F^*}$$

$$\operatorname{cloglog}(f^*) = \log(F^*)$$

Fit MLE for

$$\operatorname{cloglog}(f^*) \overset{CLT}{\sim} N(\mu, \sigma) \Leftrightarrow F^* \sim LN(\mu, \sigma)$$

$$\xi = \frac{F^*}{M} \quad \left(\frac{1}{M}\right) \text{ Scales } F^* \cancel{\Rightarrow \xi \sim LN\left(\mu, \frac{\sigma}{M}\right)}$$