

# Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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In collaboration with:

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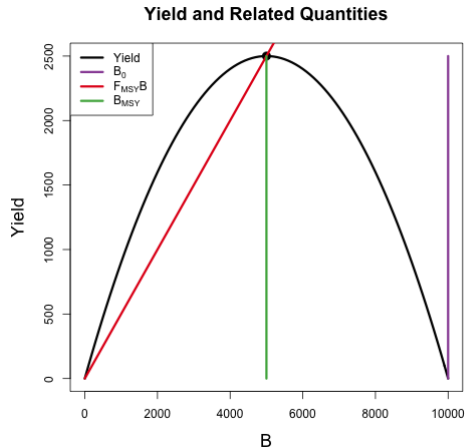


# General Modeling Structures

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \theta) - Z(t)B(t)$$

$$\text{Reference Points : } MSY, \frac{F_{MSY}}{M}, \frac{B_{MSY}}{B_0}$$



Conceptually:

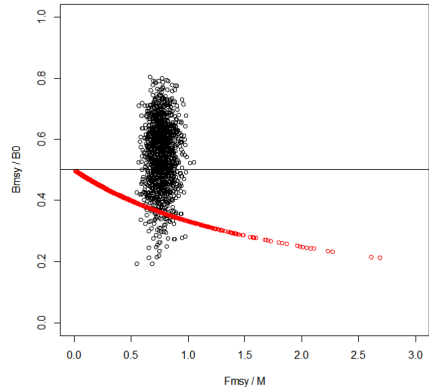
$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

- BH Model:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M+2}$$

- Similar Constraints for other Two-Parameter Curves



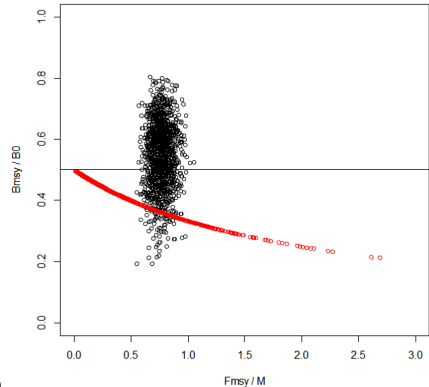
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- Similar Constraints for other Two-Parameter Curves
- Three-Parameter Relationships Allow Independent RP Estimation



# Schnute 1985, CJFAS

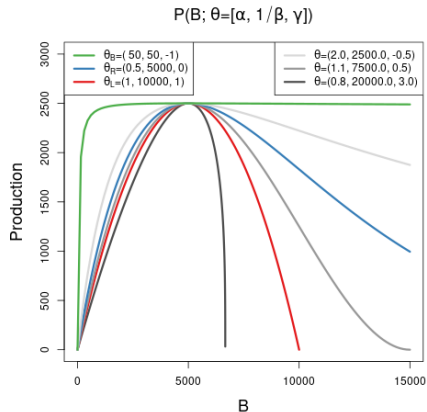
$$\frac{dB}{dt} = P(B; \theta) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$  Beverton-Holt

$\gamma \rightarrow 0 \Rightarrow$  Ricker

$\gamma = 1 \Rightarrow$  Logistic



- Isolalting RP Bias is Hard:
  - Chaos in the Dynamical System
  - Time Integrator Inaccuracy
  - Model Identifiability
  - Global Optimization
  - etc...
- Production Models are simplified places to build intuition
- See my analysis of the mechanisms of bias in the Schaefer Model  $\Rightarrow$

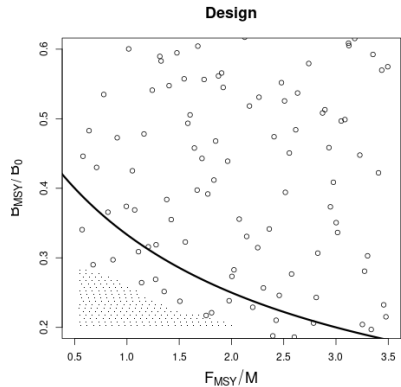
## Schaefer RP Analysis



<https://ggle.io/5EnI>

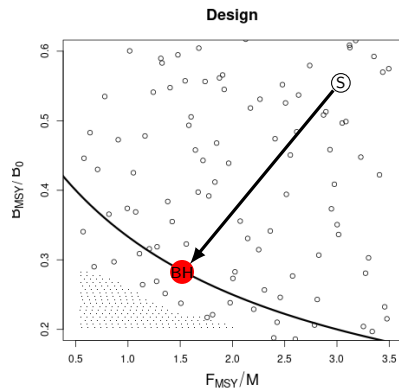
# Simulation Design

- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS



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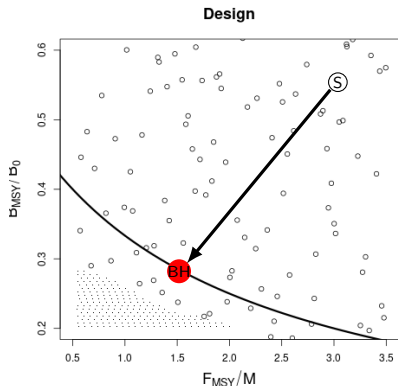




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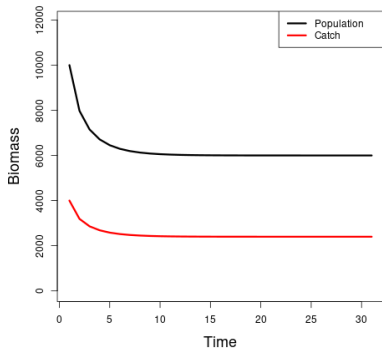
- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS
- GP Metamodeling of RP bias

$$\underbrace{\left( \frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{Schnute Truth}} \xrightarrow{\text{GP}} \underbrace{\left( \frac{\hat{F}_{MSY}}{M}, \frac{\hat{B}_{MSY}}{\bar{B}(0)} \right)}_{\text{BH Estimate}}$$

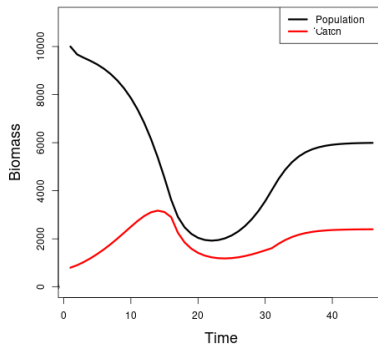


# Catch

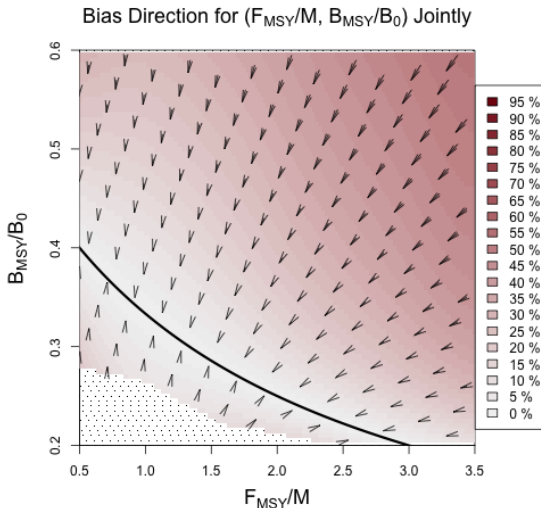
### Low Contrast Fishery



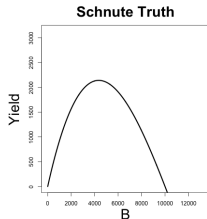
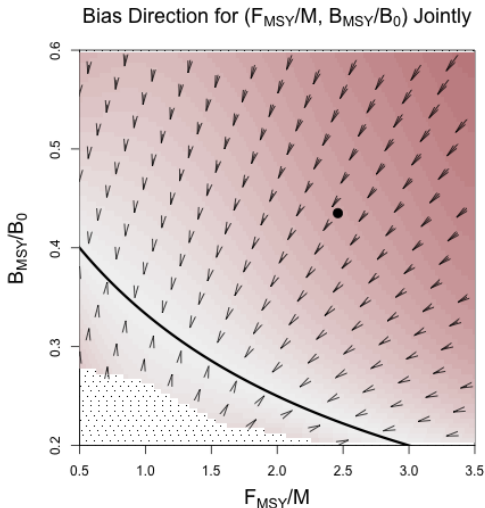
### High Contrast Fishery



# High Contrast

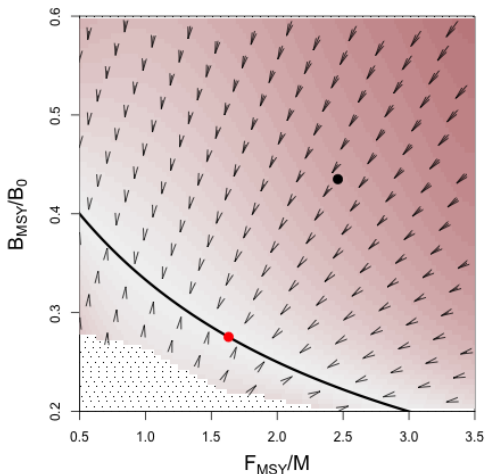


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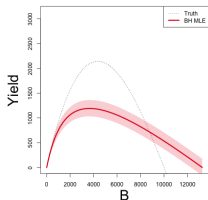


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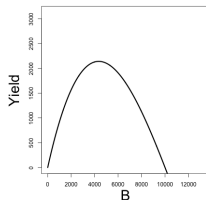
Bias Direction for  $(F_{MSY}/M, B_{MSY}/B_0)$  Jointly



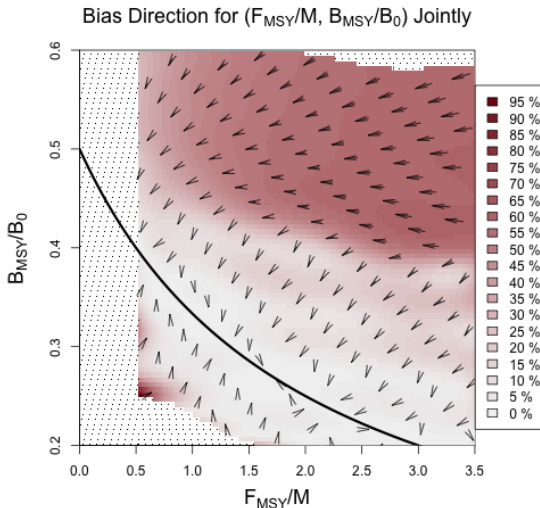
BH Fit



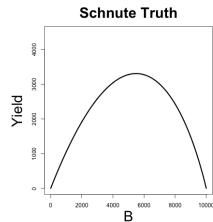
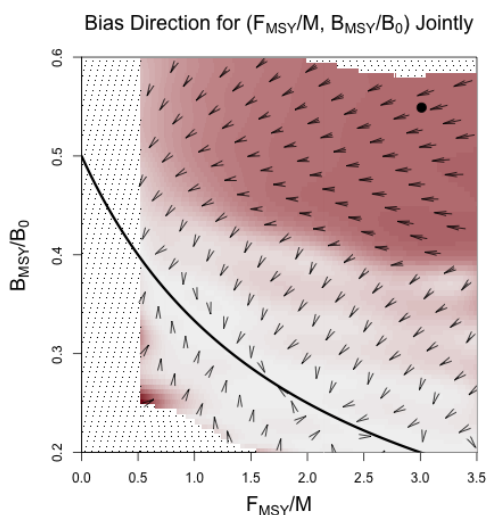
Schnute Truth



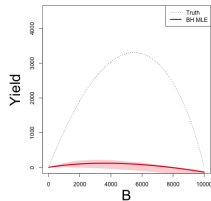
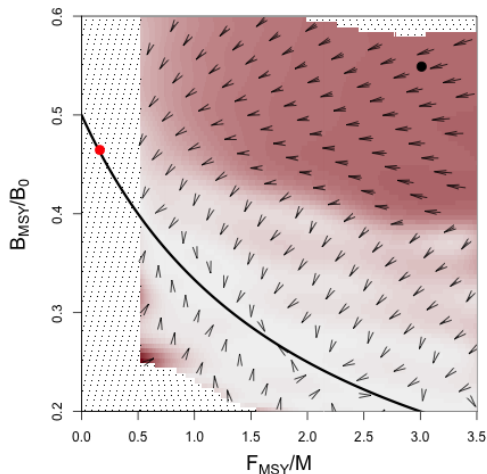
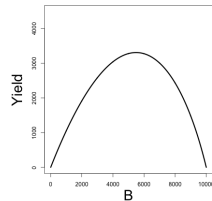
# Low Contrast



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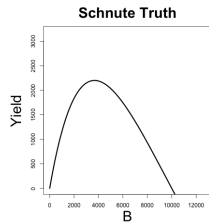
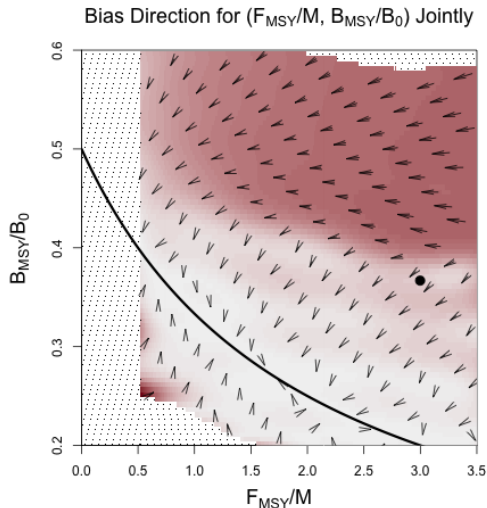


# Low Contrast

**BH Fit****Bias Direction for ( $F_{MSY}/M$ ,  $B_{MSY}/B_0$ ) Jointly****Schnute Truth**

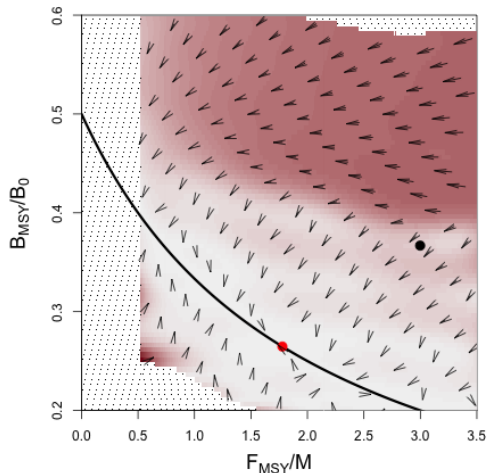


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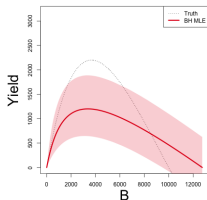


# Low Contrast

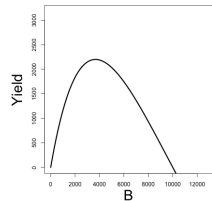
Bias Direction for  $(F_{MSY}/M, B_{MSY}/B_0)$  Jointly



BH Fit

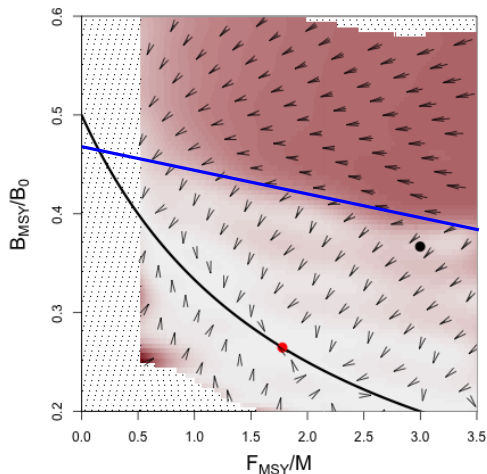


Schnute Truth

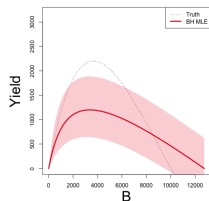


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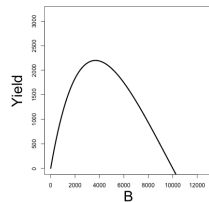
Bias Direction for  $(F_{MSY}/M, B_{MSY}/B_0)$  Jointly



BH Fit



Schnute Truth



# Conclusions

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding more complex models.
  - ⇒ Individual growth and maturity dynamics
- RPs are not directly observable quantities, but rather model dependent latent quantities.
  - ⇒ Subject to Model Misspecification, Uncertainty, & Bias
  - ⇒ In severely constrained settings we pay for our modeling mistakes primarily in estimate bias.
- The observed contrast serves to increase the range of potentially “allowable” model misspecification.

Many Thanks:

- UCSC Advisors
- SWFSC Groundfish
- NMFS Sea Grant



# Metamodel Details

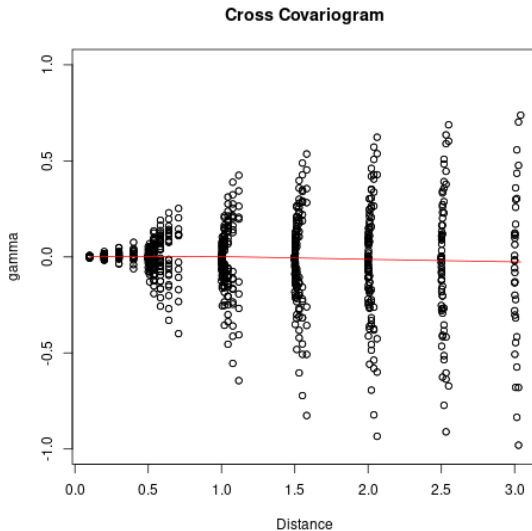
$$\mathbf{x} = \left( F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\hat{\mu} = \beta_0 + \beta' \mathbf{x} + f(\mathbf{x}) + \epsilon$$

$$f(\mathbf{x}) \sim \text{GP}(0, \tau^2 R(\mathbf{x}, \mathbf{x}'))$$

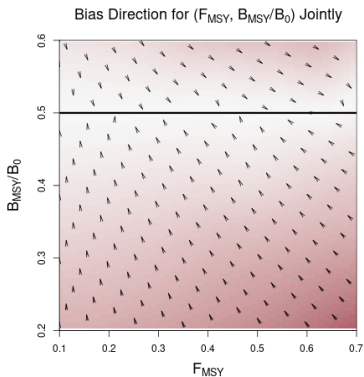
$$\epsilon_i \sim \text{N}(0, \hat{\omega}_i).$$

$$R(\mathbf{x}, \mathbf{x}') = \exp \left( \sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2} \right)$$

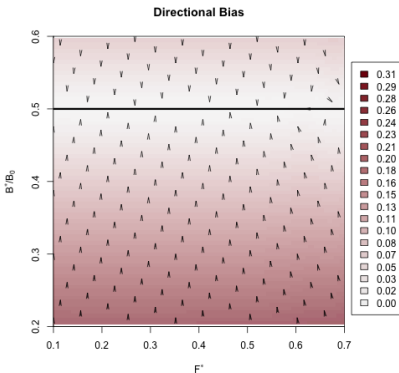


# PT Data Fit with the Schaefer Model

## Low Contrast



## High Contrast





# Schnute RP-Parameter System of Equations

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M + F_{MSY}}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}$$
$$\alpha = (M + F_{MSY}) \left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$
$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^\gamma\right)$$

# Common Discretization

$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - c(\tau)$$

