Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

14 March 2022

Outline

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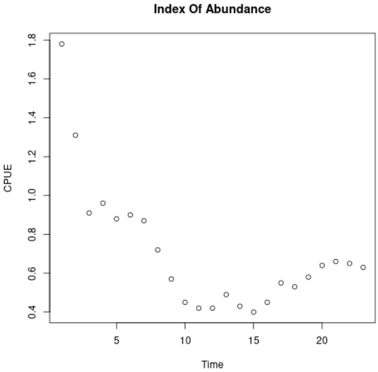
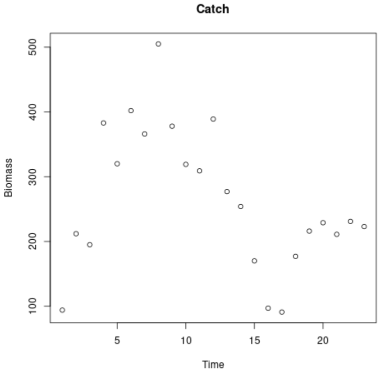




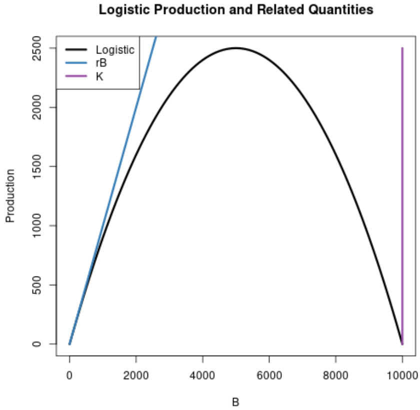
$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - C(t)$$

Schaefer Model

$$P_{\boldsymbol{\theta}}(B) = rB\left(1 - \frac{B}{K}\right)$$

$$\boldsymbol{\theta} = (r, K)$$

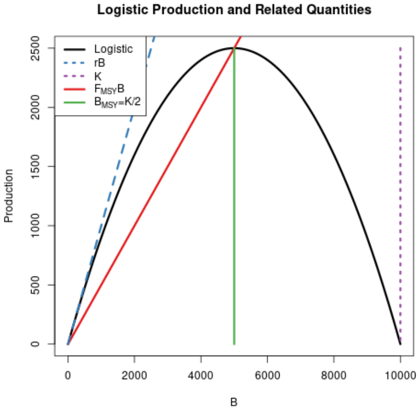


Schaefer Reference Points

$$F^* = \frac{r}{2}$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

$$MSY = \frac{rK}{4}$$

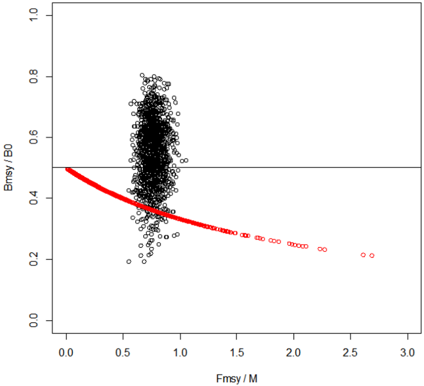


Conceptually:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{F^*/M+2}$
- Similar Constraint for Ricker and other 2 Parameter Curves



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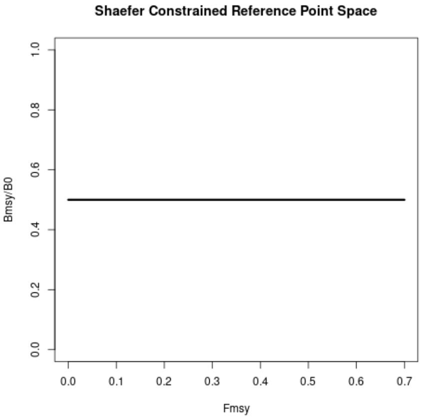
Schaefer Model:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{2}$$



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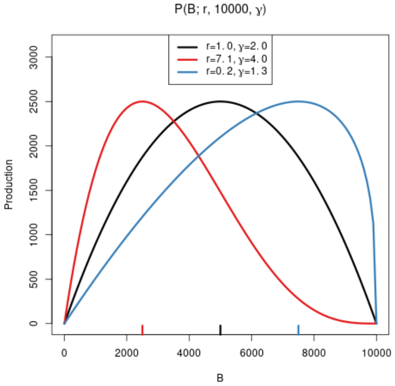
Pella-Tomlinson Production Model

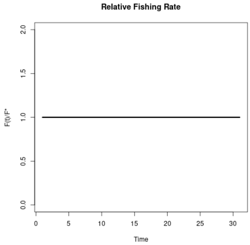
$$I(t) \sim LN(qB(t), \sigma^2)$$
$$\frac{dB(t)}{dt} = P_{\boldsymbol{\theta}}(B(t)) - F(t)B(t)$$

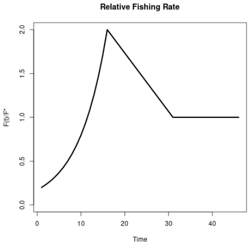
$$P_{\boldsymbol{\theta}}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma-1}$$
$$\boldsymbol{\theta} = (r, K, \gamma)$$

$\gamma = 2 \Rightarrow$ Schaefer Model



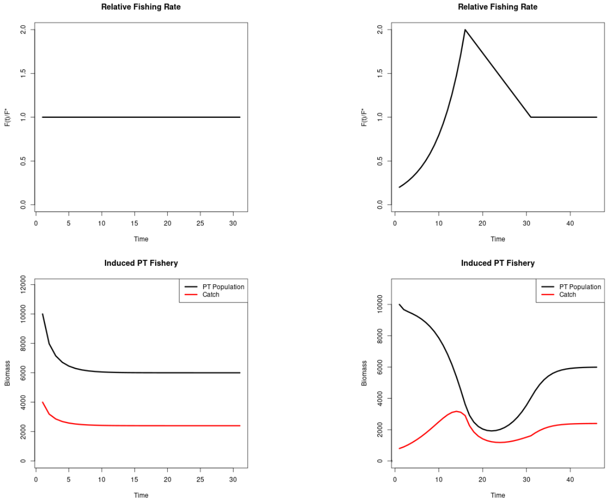
Catch



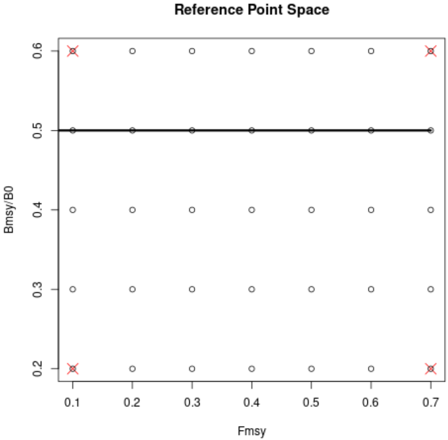


$$
\begin{aligned}
C(t) &= F(t)B(t) \\
&= F^* \left(\frac{F(t)}{F^*} \right) B(t)
\end{aligned}
$$

Catch


Relative Fishing Rate
F(t)/F*
Time
Relative Fishing Rate
F(t)/F*
Time
Induced PT Fishery
Biomass
Time
PT Population
Catch
Induced PT Fishery
Biomass
Time
PT Population
Catch

$$\theta = \left[r = F^*\left(\frac{1-\frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}}\right)\left(1-\frac{B^*}{\bar{B}(0)}\right)^{\left(\frac{\frac{B^*}{\bar{B}(0)}-1}{\frac{B^*}{\bar{B}(0)}}\right)},\ K = 10000,\ \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$



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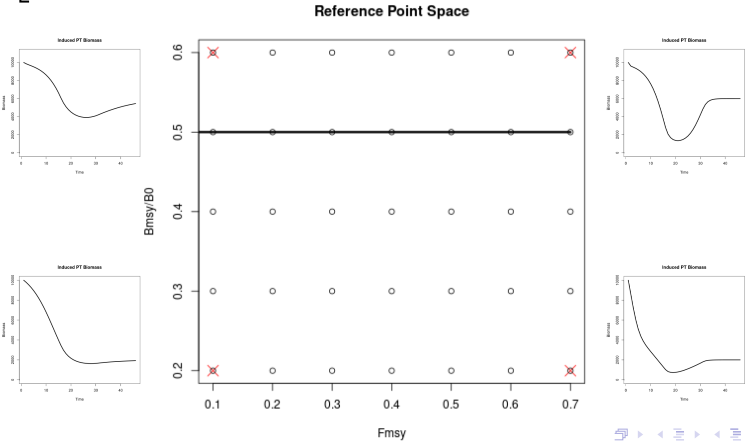
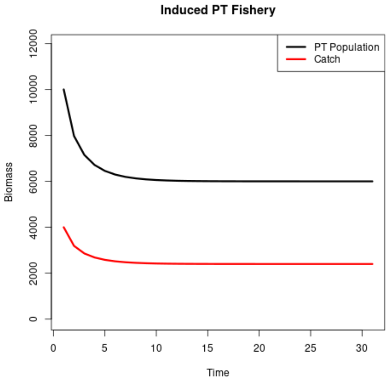


Metamodel

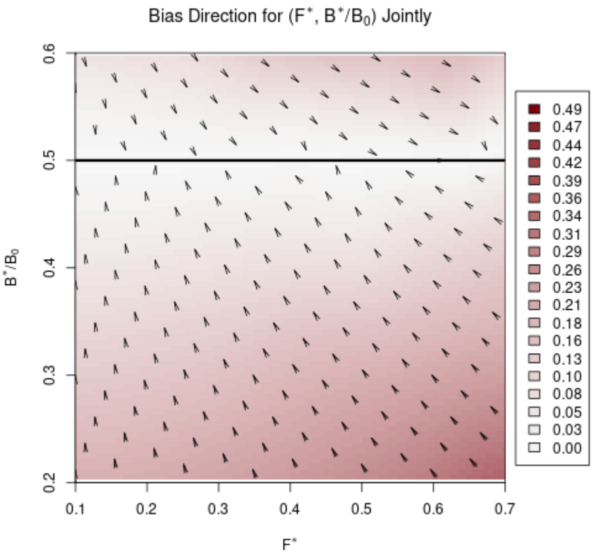
$$\underbrace{\left(F^*, \frac{B^*}{\bar{B}(0)}\right)}_{\text{PT Truth}} \overset{\text{GP}}{\mapsto} \underbrace{\left(\hat{F}^*, \frac{\hat{B}^*}{\bar{B}(0)}\right)}_{\text{Shaefer Estimate}}$$

- GP interpolates over degrees of RP model misspecification.
- Propogation of estimate uncertainty smooths bias estimation.
- Explicitely highlights trade-offs induced in RPs.

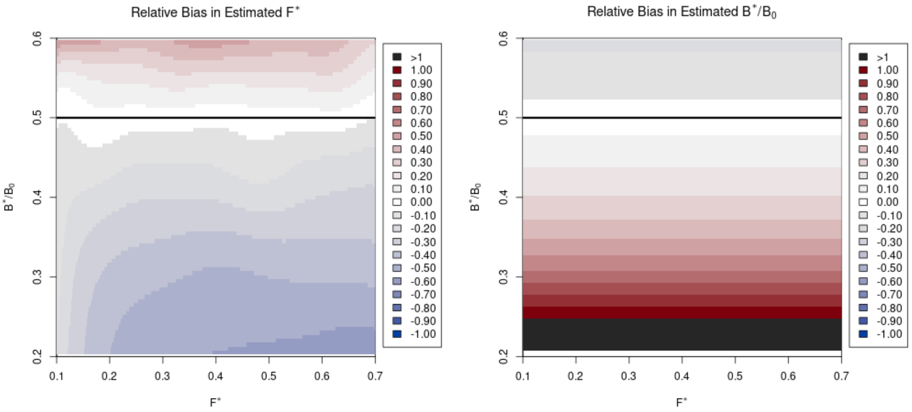
Outline


Induced PT Fishery
PT Population
Catch
Biomass
0
2000
4000
6000
8000
10000
12000
Time
0
5
10
15
20
25
30

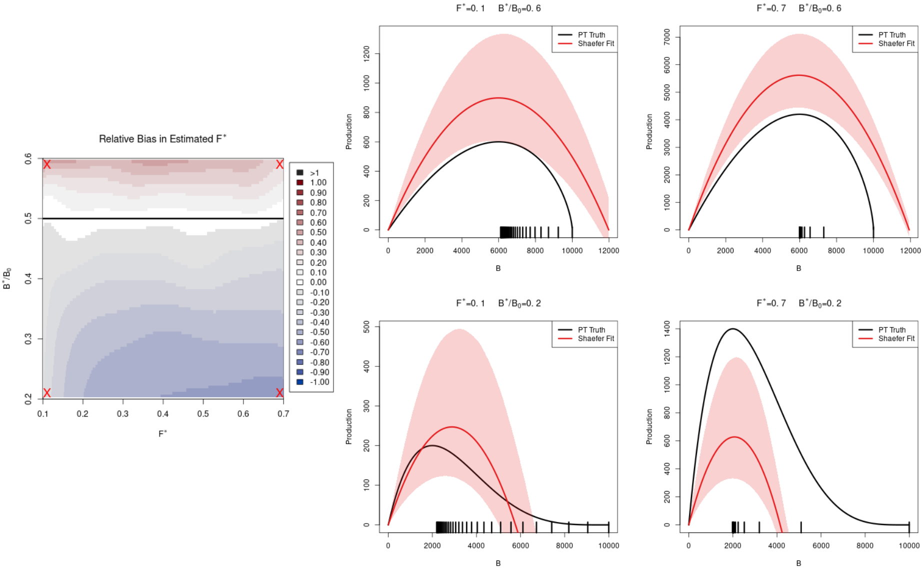
Low Contrast



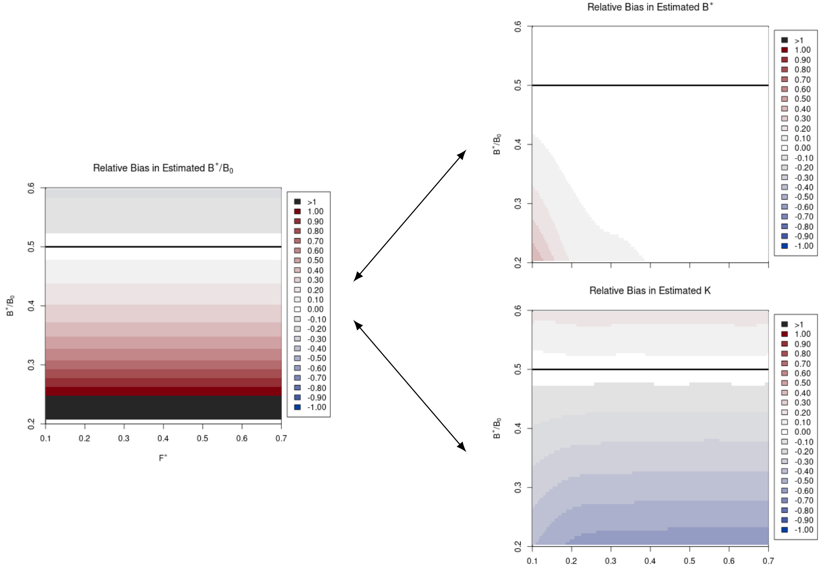
Components of Bias



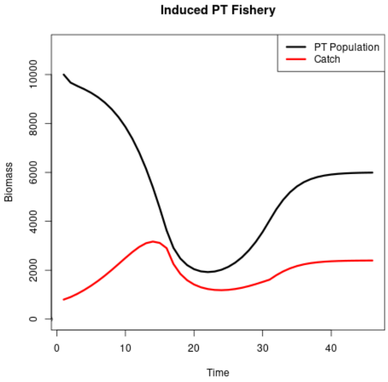
Low Contrast



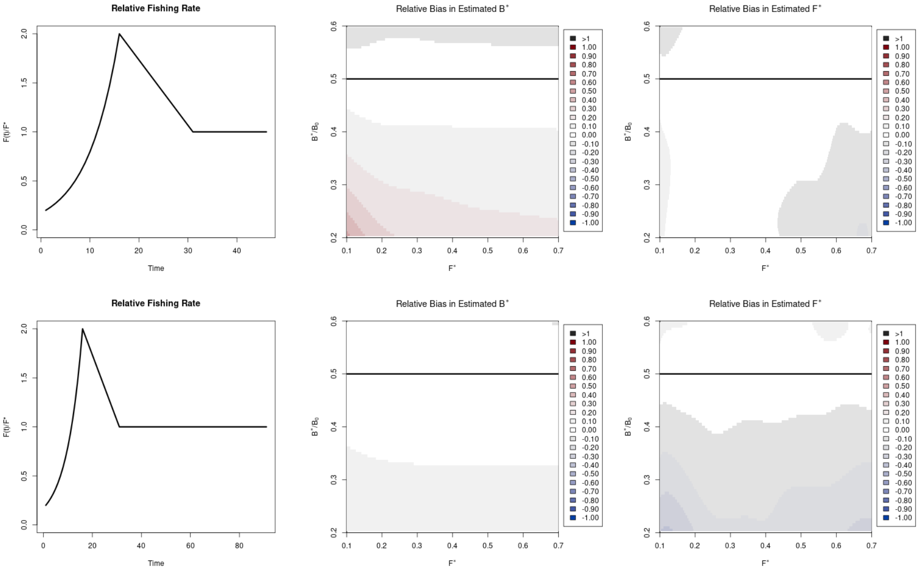
Low Contrast



Outline


Induced PT Fishery
PT Population
Catch
Biomass
0
2000
4000
6000
8000
10000
0
10
20
30
40
Time

High Contrast



Summary

- Given unbias estimation (i.e. MLE), model misspecification is a necessary but not sufficient condition for inducing bias.
- Different data informs different parts of the production function differently
- In the overconstrained setting we pay for our modeling mistakes in bias In practice, when the true model is not known and the Schaefer model is unlikely to be correctly specified, one should at best expect to only estimate either B^* or F^* correctly depending on the particular degree of model misspecification. The observed contrast then serves to distribute the available information among B^* and F^*.
- F^* bias is strongly contrast dependent
  - $\Rightarrow$ Bias depends on how similar the modeled and true production functions can be at the observed biomasses
- A rich simulation-based method for describing global RP bias

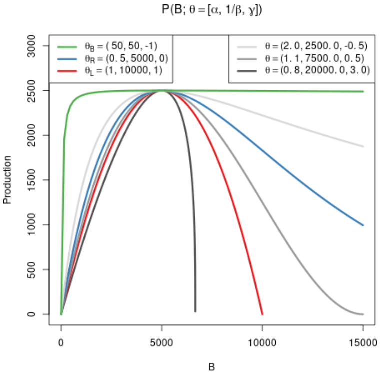
Outline

Productivity Extension

$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

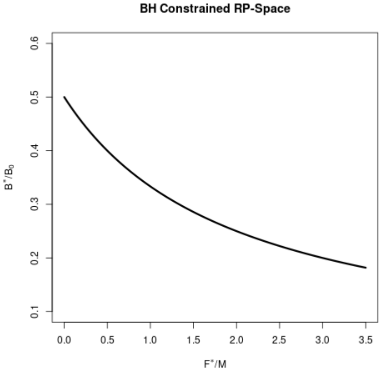
$$\gamma = -1 \Rightarrow \text{Beverton-Holt}$$
$$\gamma = 1 \Rightarrow \text{Logistic}$$
$$\gamma \to 0 \Rightarrow \text{Ricker}$$



Productivity Extension

$$P_{\mathrm{BH}}(B;[\alpha,\beta,-1]) = \frac{\alpha B}{(1+\beta B)}$$

$$\frac{B^*}{\bar{B}(0)} = \frac{1}{\frac{F^*}{M}+2}$$



Growth Extension

$$\frac{dB}{dt} = \overbrace{w(a_0)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

$$R(B;[\alpha,\beta,\gamma]) = \alpha B(t-a_0)(1-\beta\gamma B(t-a_0))^{\frac{1}{\gamma}}$$

$$w(a) = w_\infty(1-e^{-\kappa a})$$

bullets of primary points of individual growth and maturity

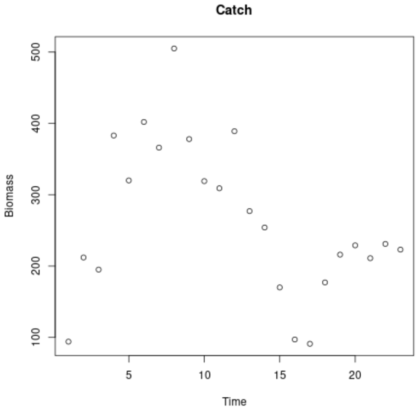
Outline

Catch Interpolation

$$t \in \mathbb{R}^+ \qquad \tau = \lceil t \rceil - 1$$

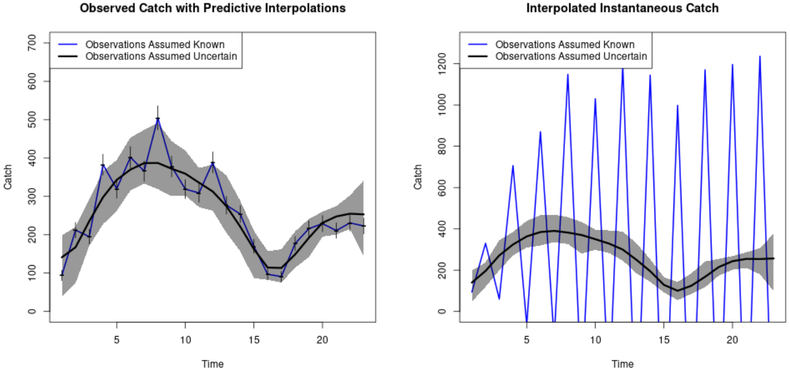
$$\mathbb{E}[y(t)] = \int_{\tau}^{t} x(t^*)dt^*$$

$$x(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



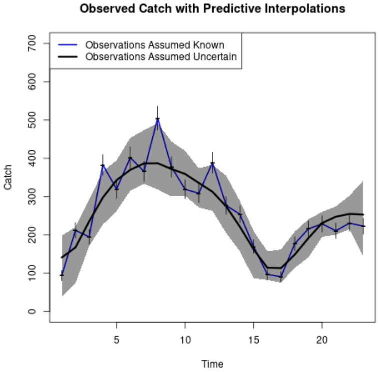
$$y(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

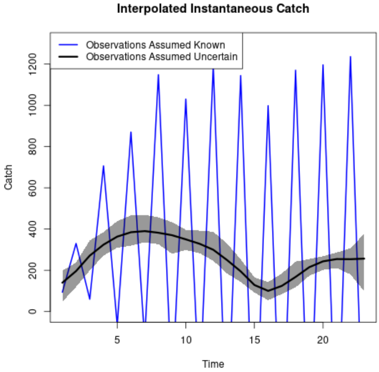
$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0, 1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$



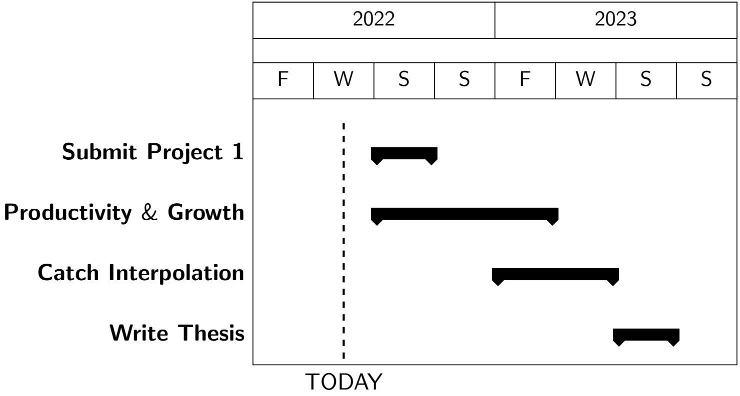
$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - C(\tau)$$





Timeline


2022
2023
F
W
S
S
F
W
S
S
Submit Project 1
Productivity & Growth
Catch Interpolation
Write Thesis
TODAY

Thanks and Acknowldgements NOAA, Sea Grant Ecetra

$$\hat{\mu} = \widehat{log(r)} \quad - or - \quad \hat{\mu} = \widehat{log(K)}$$

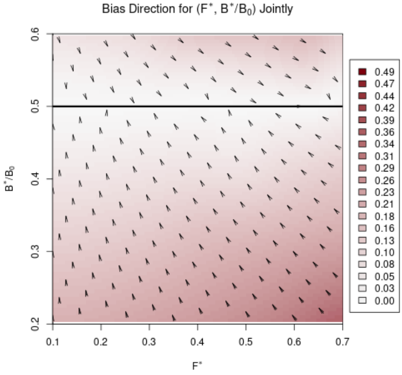
$$\boldsymbol{x} = \left(F^*, \frac{B^*}{\bar{B}(0)}\right)$$

$$\begin{aligned} \hat{\mu} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon \\ f(\boldsymbol{x}) &\sim \text{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}')) \\ \epsilon_i &\sim \text{N}(0, \hat{\omega}_i). \end{aligned}$$

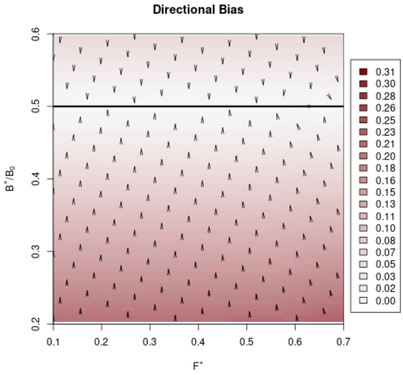
$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2}\right)$$

Cross-Covariogram

Low Contrast



High Contrast



$$\frac{B^*}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F^*}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$

$$\alpha = (M + F^*)\left[1 - \frac{1}{\gamma}\left(\frac{F^*}{M + F^*}\right)\right]^{-\gamma}$$

$$\beta = \frac{1}{\gamma\bar{B}(0)}\left(1 - \left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}}\right)$$

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - C(\tau)$$