Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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Outline

Outline

- MSA?
- RPs
- Steepness paper

Surplus Production Model General Structure

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2). \tag{1}$$

$$\frac{dB}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t). \tag{2}$$

Outline

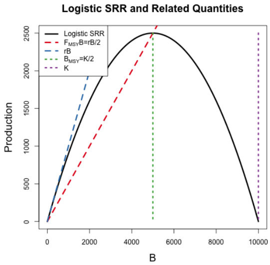
$$P_p(B;[r,K]) = rB\left(1-\left(\frac{B}{K}\right)\right)$$

$$F^* = \frac{r}{2}$$

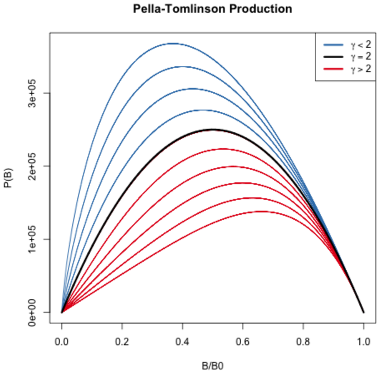
$$B^* = \frac{K}{2}$$

$$B_0 = K$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$



$$P_p(B;[r,K,\gamma]) = \frac{rB}{\gamma-1}\left(1-\left(\frac{B}{K}\right)^{(\gamma-1)}\right). \tag{3}$$



$$\frac{dB}{dt} = \frac{rB}{\gamma - 1}\left(1 - \left(\frac{B}{K}\right)^{\gamma-1}\right) - FB. \tag{4}$$

$$\bar{B}(F) = K\left(1 - \frac{F(\gamma - 1)}{r}\right)^{\frac{1}{(\gamma-1)}}. \tag{5}$$

$$F^* = \frac{r}{\gamma} \qquad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \tag{6}$$

$$F^* = \frac{r}{\gamma} \qquad\qquad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \tag{7}$$

$$r = \gamma F^* \qquad\qquad \gamma = \frac{W\left(\frac{B^*}{\bar{B}(0)}\log\left(\frac{B^*}{\bar{B}(0)}\right)\right)}{\log\left(\frac{B^*}{\bar{B}(0)}\right)} \tag{8}$$

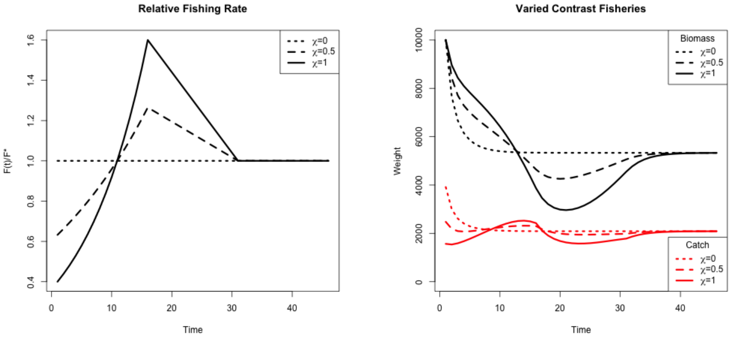
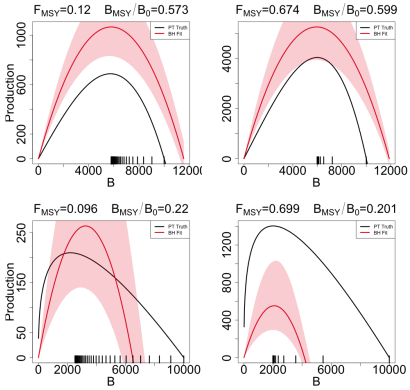


Figure: (*left*) Relative fishing with low, medium, and high contrast. (*right*) Population biomass and catch at each associated level of contrast.


F_{MSY}=0.12 B_{MSY}/B_0=0.573
F_{MSY}=0.674 B_{MSY}/B_0=0.599
F_{MSY}=0.096 B_{MSY}/B_0=0.22
F_{MSY}=0.699 B_{MSY}/B_0=0.201
PT Truth
BH Fit
Production
B

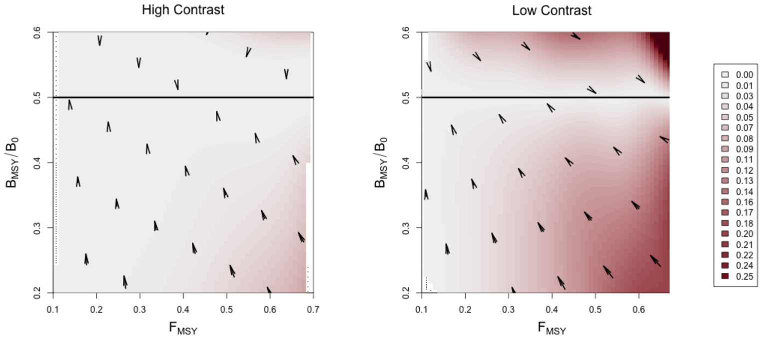
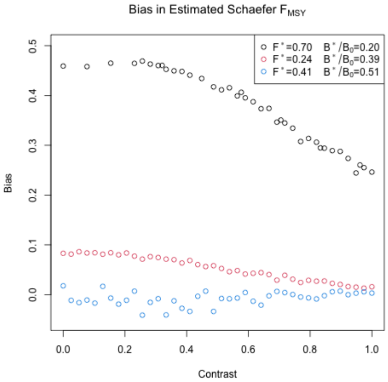


Figure: Joint bias direction for $(F^*, \frac{B^*}{B_0})$ estimates under the misspecified Schaefer Model. The intensity of color represents the excess bias relative to the shortest possible mapping. Results in the low contrast setting are shown *right*, and the high contrast setting is shown *left*.

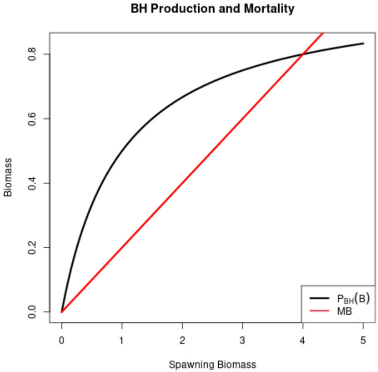


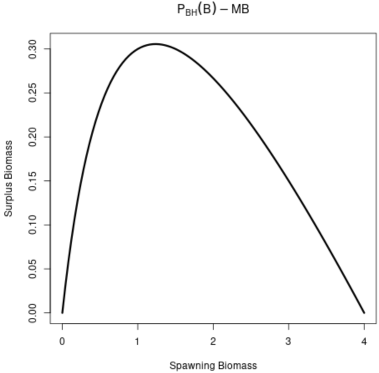
Bias in Estimated Schaefer F_{MSY}
F^*=0.70 B^*/B_0=0.20
F^*=0.24 B^*/B_0=0.39
F^*=0.41 B^*/B_0=0.51
Bias
0.0
0.1
0.2
0.3
0.4
0.5
0.0
0.2
0.4
0.6
0.8
1.0
Contrast

Outline

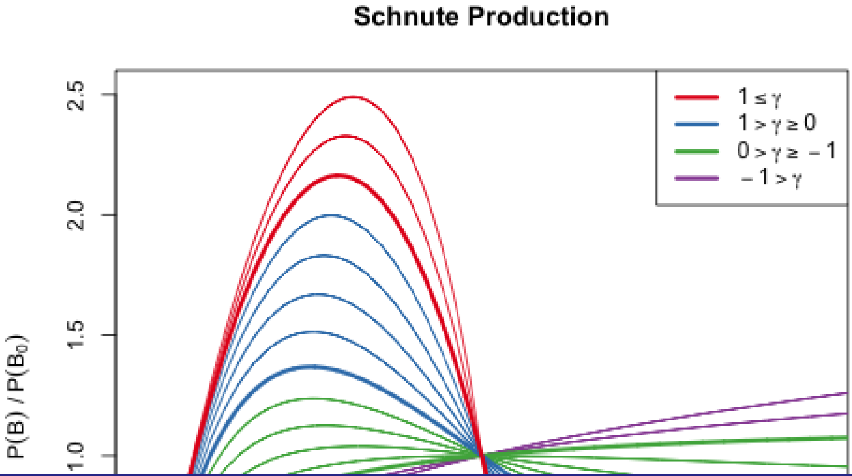
$$P(B) = \frac{\alpha B}{1 + \beta B}. \tag{9}$$

$$\frac{dB}{dt} = P(B; \theta) - MB - FB \tag{10}$$





$$R(B;\alpha,\beta,\gamma) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$



$$\bar{B}(F) = \frac{1}{\gamma\beta}\left(1 - \left(\frac{M+F}{\alpha}\right)^{\gamma}\right). \tag{11}$$

$$B_0 = \frac{1}{\gamma\beta}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right) \tag{12}$$

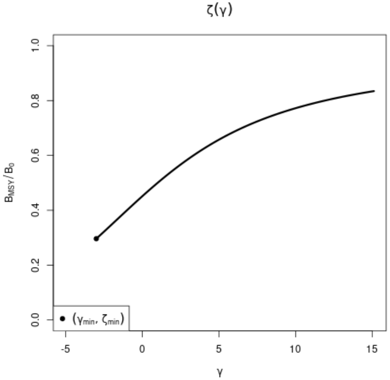
$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}. \tag{13}$$

$$\frac{d\bar{Y}}{dF} = \bar{B}(F) + F\frac{d\bar{B}}{dF} \tag{14}$$

$$\frac{d\bar{B}}{dF} = -\frac{1}{\beta}\left(\frac{\left(\frac{M+F}{\alpha}\right)^{\gamma}}{F+M}\right). \tag{15}$$

$$\alpha = (M + F^*)\left(1 + \frac{\gamma F^*}{M + F^*}\right)^{1/\gamma}$$
$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right) \tag{17}$$
$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}.$$

$$\gamma' \sim \zeta_{min}\delta(\gamma_{min}) + t(\mu,\sigma,\nu)\mathbf{1}_{\gamma>\gamma_{min}} \tag{18}$$



Schnute Recruitment

$$R(B;\alpha,\beta,\gamma) = \alpha B_{t-a_s}(1-\beta\gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic

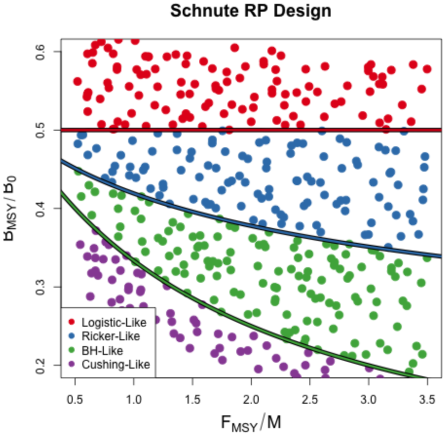
$\gamma = 1$

Ricker

$\gamma \rightarrow 0$

Beverton-Holt

$\gamma = -1$



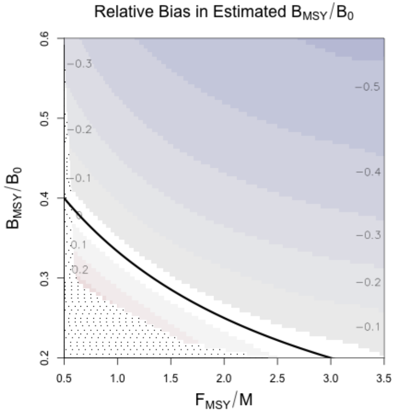
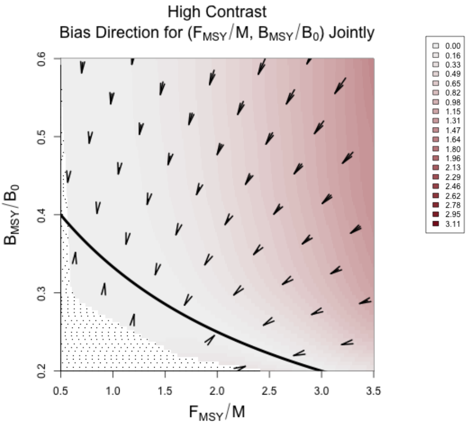
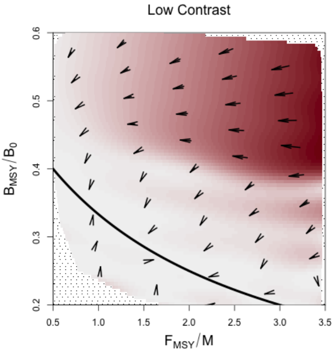


Figure: Heatplots showing the bias in RP estimation induced by model misspecification of the BH model in the high contrast simulation setting. In all cases





Add Yeild curves from AFS presentation.

Outline

General Modeling Structure

$$\frac{dB}{dt} = \overbrace{w(a_s)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

$R(B_{t-a_s};\alpha,\beta,\gamma)$: Schnute Three Parameter Recruitment

$w(a)$: Von Bertalanffy Individual Growth

Knife-Edge Selectivity at age a_s

Schnute Recruitment

$$R(B;\alpha,\beta,\gamma) = \alpha B_{t-a_s}(1-\beta\gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic

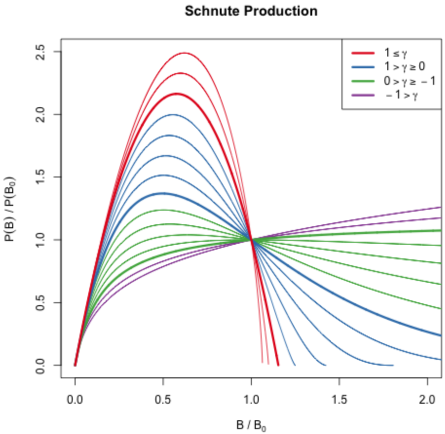
$\gamma = 1$

Ricker

$\gamma \to 0$

Beverton-Holt

$\gamma = -1$



Schnute Recruitment

$$R(B;\alpha,\beta,\gamma) = \alpha B_{t-a_s}(1-\beta\gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic

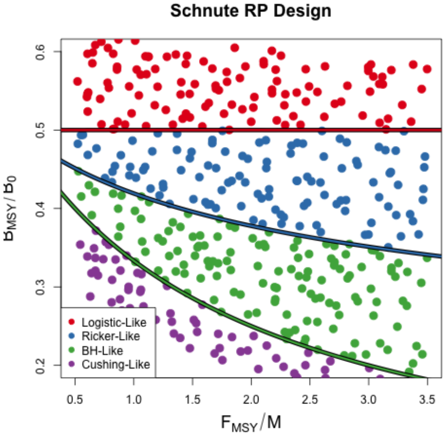
$\gamma = 1$

Ricker

$\gamma \to 0$

Beverton-Holt

$\gamma = -1$

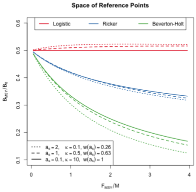


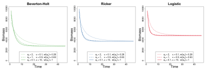
Von Bertalanffy Individual Growth

$$w(a) = w_\infty(1 - e^{-\kappa(a-a_0)})$$

- Instant Growth:
  (Production Model)
  $a_s \to 0 \quad \kappa \to \infty$

- Dynamic Growth:
  a_s : Not Instant
  κ : Peakness of Dynamics





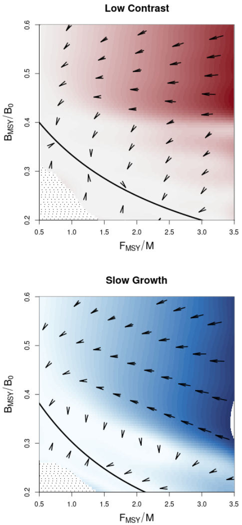
$$\bar{B}(F) = \frac{1}{\beta\gamma}\left(1 - \Big(\frac{(F+M)(F+M+\kappa)}{\alpha w(a_s)(F+M+\frac{\kappa w_\infty}{w(a_s)})}\Big)^{\gamma}\right) \tag{19}$$

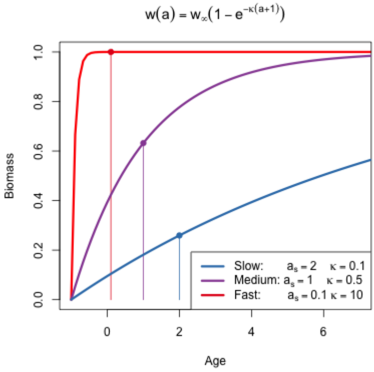
$$\bar{N}(F) = \frac{\alpha\bar{B}(F)(1-\beta\gamma\bar{B}(F))^{1/\gamma}}{F+M} \tag{20}$$

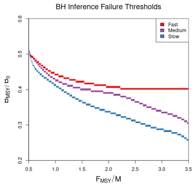
$$\alpha = \left[\left(\frac{Z^*(Z^*+\kappa)}{w(a_s)(Z^*+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma} + \left(\frac{\gamma F^*}{w(a_s)}\right)\left(\frac{Z^*(Z^*+\kappa)}{w(a_s)(Z^*+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma-1}\left(1+\frac{\left(\frac{\kappa w_\infty}{w(a_s)}\right)\left(\kappa-\frac{\kappa w_\infty}{w(a_s)}\right)}{(Z^*+\frac{\kappa w_\infty}{w(a_s)})^2}\right)\right]^{\frac{1}{\gamma}} \tag{21}$$

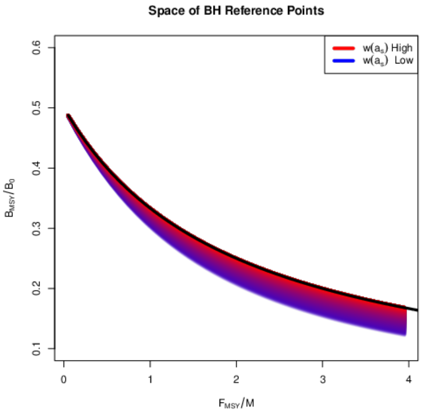
$$\beta = \frac{1}{\gamma B_0}\left(1 - \Big(\frac{M(M+\kappa)}{\alpha w(a_s)(M+\frac{\kappa w_\infty}{w(a_s)})}\Big)^{\gamma}\right) \tag{22}$$

$$\frac{B^*}{B_0} = \frac{1-\left(\frac{(F^*+M)(F^*+M+\kappa)}{\alpha w(a_s)(F^*+M+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma}}{1-\left(\frac{M(M+\kappa)}{\alpha w(a_s)(M+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma}} \tag{23}$$


Low Contrast
Medium Growth
Slow Growth
B_{MSY}/B_0
F_{MSY}/M
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5


w(a) = w∞(1 − e^{−κ(a+1)})
Biomass
Age
Slow: a_s = 2 κ = 0.1
Medium: a_s = 1 κ = 0.5
Fast: a_s = 0.1 κ = 10


BH Inference Failure Thresholds
Fast
Medium
Slow
B_MSY / B_0
F_MSY / M


Space of BH Reference Points
$w(a_s)$ High
$w(a_s)$ Low
B_{MSY}/B_0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
F_{MSY}/M

Many Thanks:

- Dr. Marc Mangel
- Collaborators at NOAA
- NMFS Sea Grant

Metamodel Details

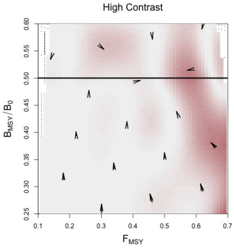
$$\hat{\mu} = \widehat{log(r)} \quad - or - \quad \hat{\mu} = \widehat{log(K)}$$

$$\boldsymbol{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

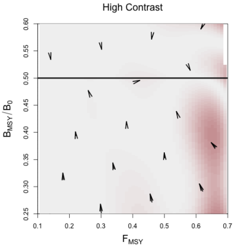
$$\begin{aligned} \hat{\mu} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon \\ f(\boldsymbol{x}) &\sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}')) \\ \epsilon_i &\sim \mathrm{N}(0, \hat{\omega}_i). \end{aligned}$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$

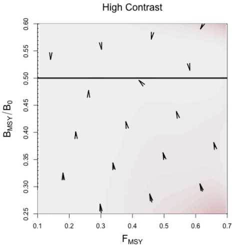
High Contrast PT $\sigma = 0.12$ Data



1x Samples

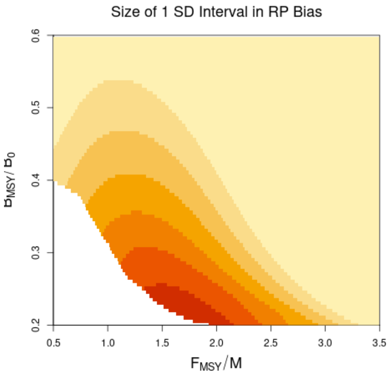


2x Samples

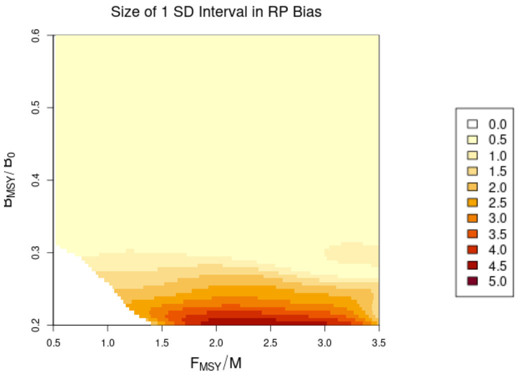


4x Samples

Contrast

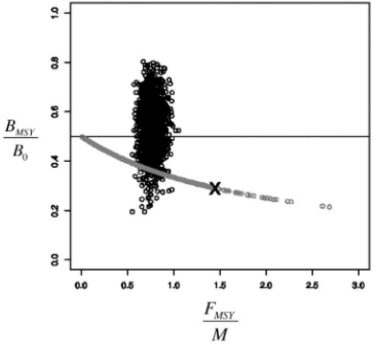


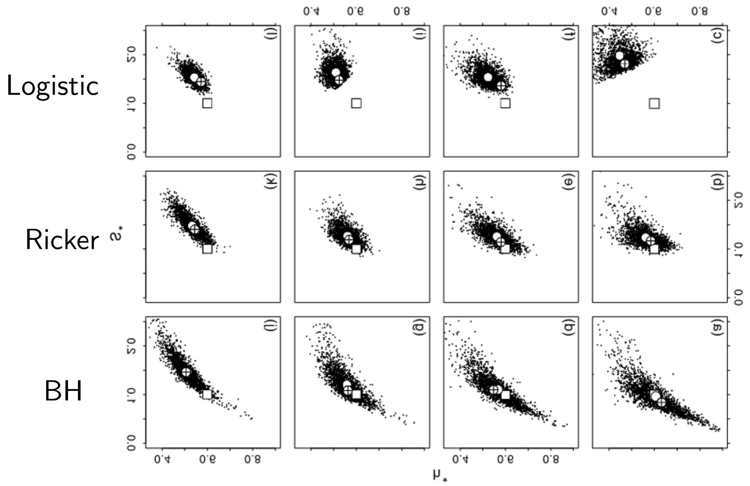
No Contrast



Mangel et al.

Fig. 4. DeYoreo et al. (2012) used both a BH-SRR and three-parameter SRR, similar to the S-SRR in a stock assessment of cowcod (*Sebastes levis*). We show samples from posterior distributions arising from different values of steepness. Unlike most stock assessments, we plot B_{MSY}/B_0 versus F_{MSY}/M. The grey circles show the results for the BH-SRR. This curve is another way of representing the constraint placed on a stock assessment by using a BH-SRR and specifying steepness — results must lie along this curve. The black circles represent the outcome of the three-parameter SRR. The black X represents the result when steepness is asserted to be 0.6.





Schnute, J. T., & Kronlund, A. R. (2002). Estimating salmon stock recruitment relationships from catch and escapement data. Canadian Journal of Fisheries and Aquatic Sciences, 59(3), 433–449.