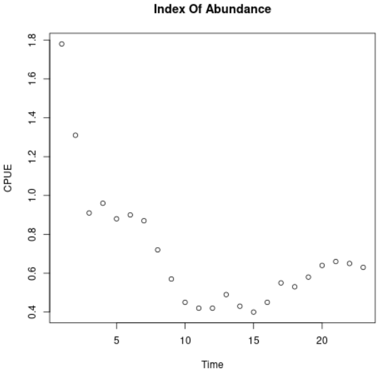
Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

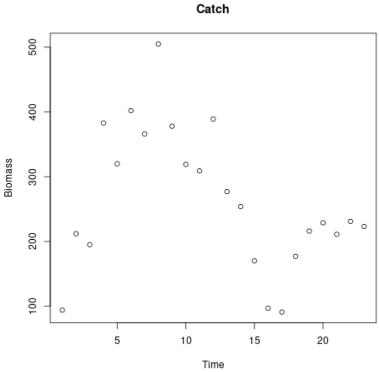
Nick Grunloh

14 March 2022

Outline

Outline





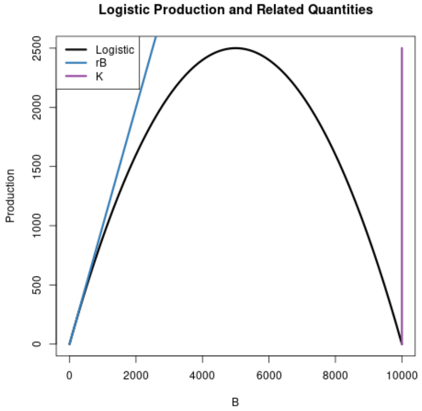
$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - C(t)$$

Schaefer Model

$$P_{\boldsymbol{\theta}}(B) = rB\left(1 - \frac{B}{K}\right)$$

$$\boldsymbol{\theta} = (r, K)$$

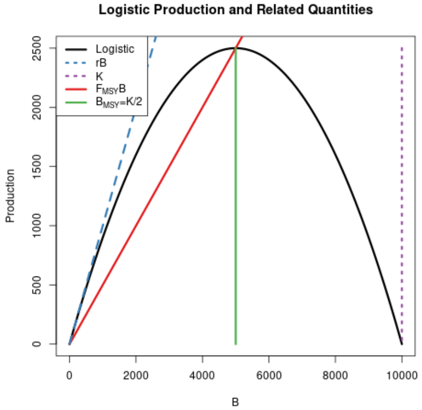


Schaefer Reference Points

$$F^* = \frac{r}{2}$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

$$MSY = \frac{rK}{4}$$

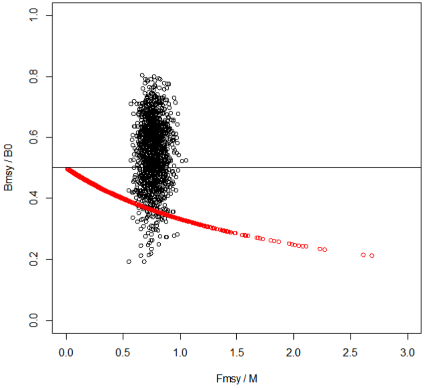


Conceptually:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{F^*/M+2}$
- Similar Constraint for Ricker and other 2 Parameter Curves



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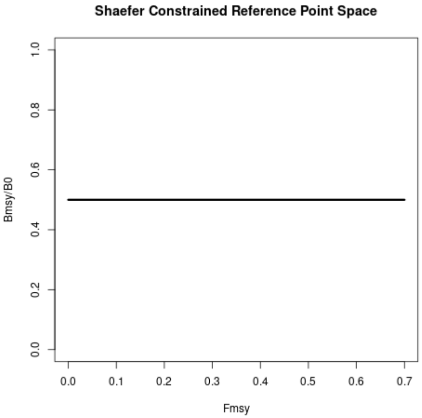
Schaefer Model:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{2}$$



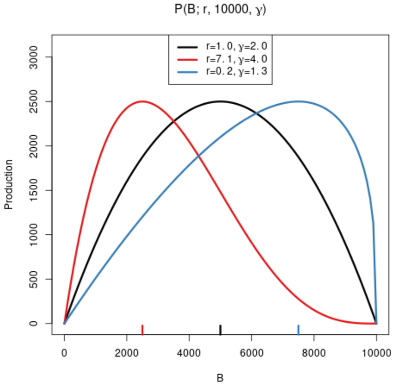
Outline

Pella-Tomlinson Production Model

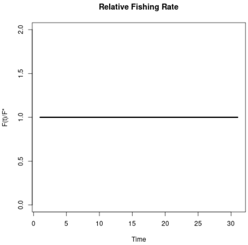
$$I(t) \sim LN(qB(t), \sigma^2)$$
$$\frac{dB(t)}{dt} = P_{\boldsymbol{\theta}}(B(t)) - F(t)B(t)$$

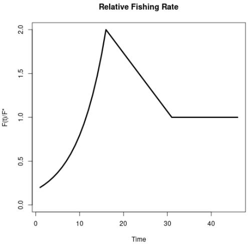
$$P_{\boldsymbol{\theta}}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma - 1}$$
$$\boldsymbol{\theta} = (r, K, \gamma)$$

$$\gamma = 2 \Rightarrow \text{Schaefer Model}$$



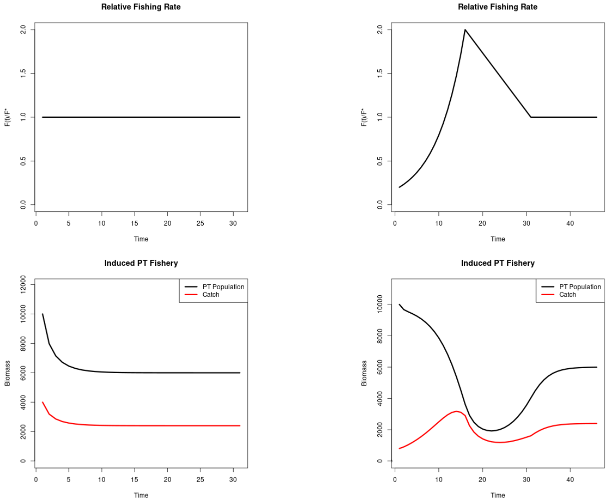
Catch



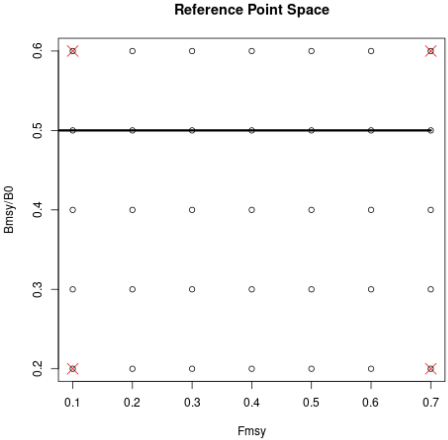


$$
\begin{aligned}
C(t) &= F(t)B(t) \\
&= F^* \left(\frac{F(t)}{F^*}\right) B(t)
\end{aligned}
$$

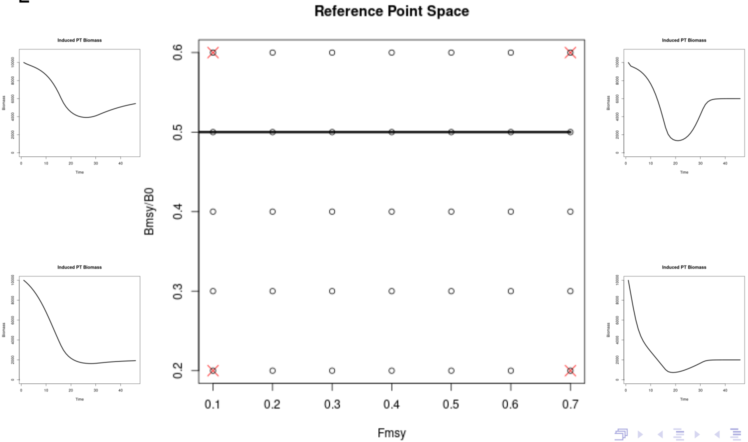
Catch


Relative Fishing Rate
F(t)/F*
Time
Relative Fishing Rate
F(t)/F*
Time
Induced PT Fishery
PT Population
Catch
Biomass
Time
Induced PT Fishery
PT Population
Catch
Biomass
Time

$$\boldsymbol{\theta} = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)},\ K = 10000,\ \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$



$$\boldsymbol{\theta} = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)},\ K = 10000,\ \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$

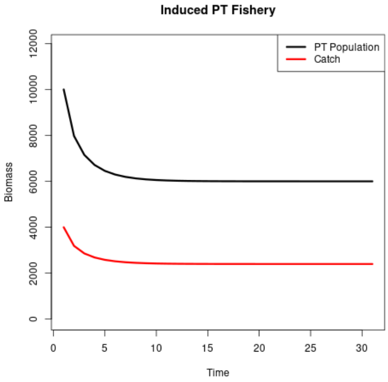


Metamodel

$$\underbrace{\left(F^*, \frac{B^*}{\bar{B}(0)}\right)}_{\text{PT Truth}} \overset{\text{GP}}{\mapsto} \underbrace{\left(\hat{F}^*, \frac{\hat{B}^*}{\bar{B}(0)}\right)}_{\text{Shaefer Estimate}}$$

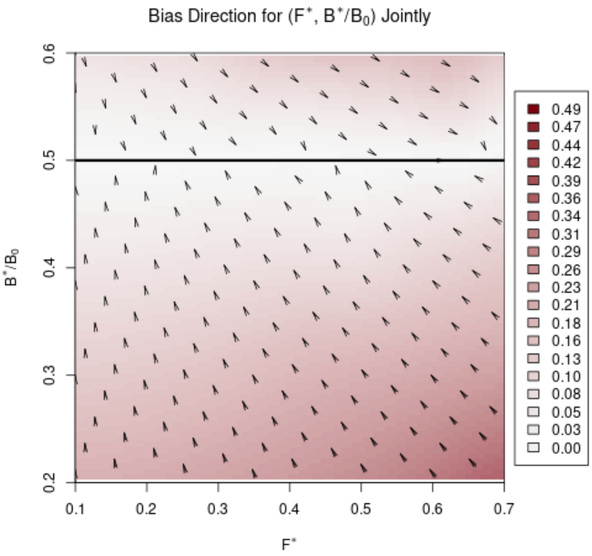
- GP interpolates over degrees of RP model misspecification.
- Propogation of estimate uncertainty smooths bias estimation.
- Explicitely highlights trade-offs induced in RPs.

Outline

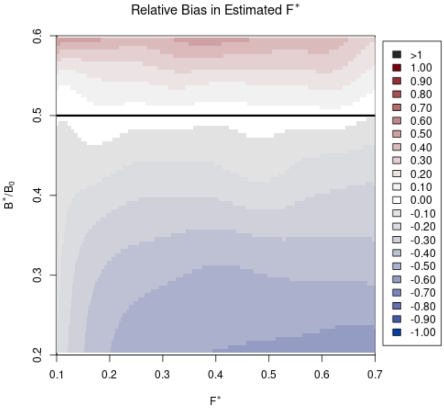


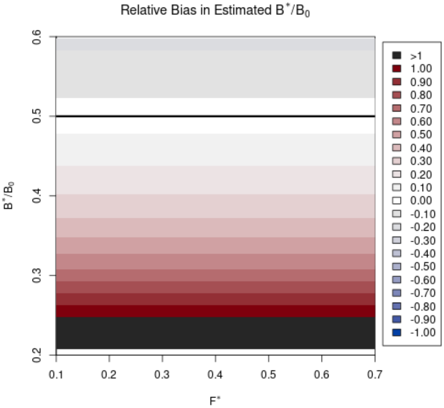
Induced PT Fishery
PT Population
Catch
Biomass
0
2000
4000
6000
8000
10000
12000
0
5
10
15
20
25
30
Time

Low Contrast



Components of Bias





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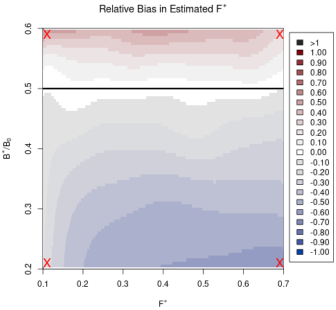

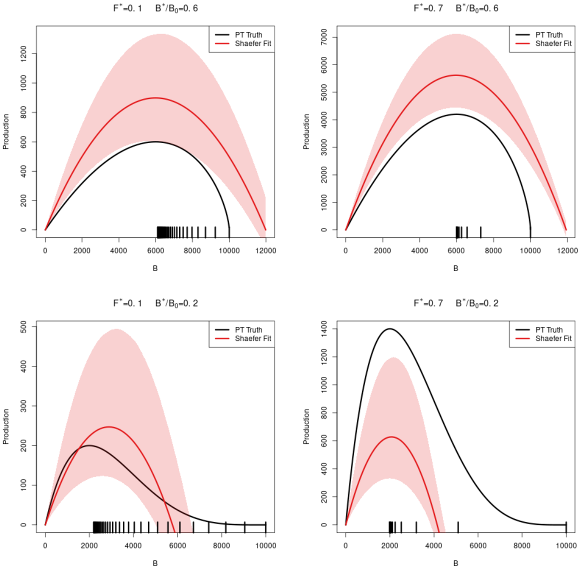

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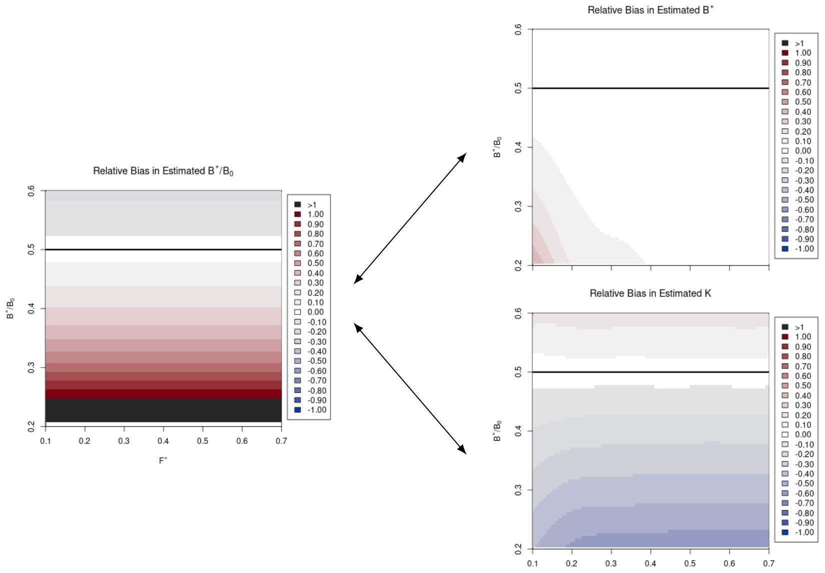
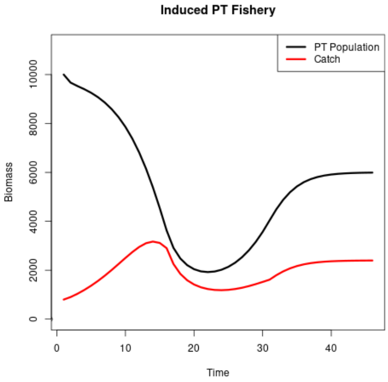
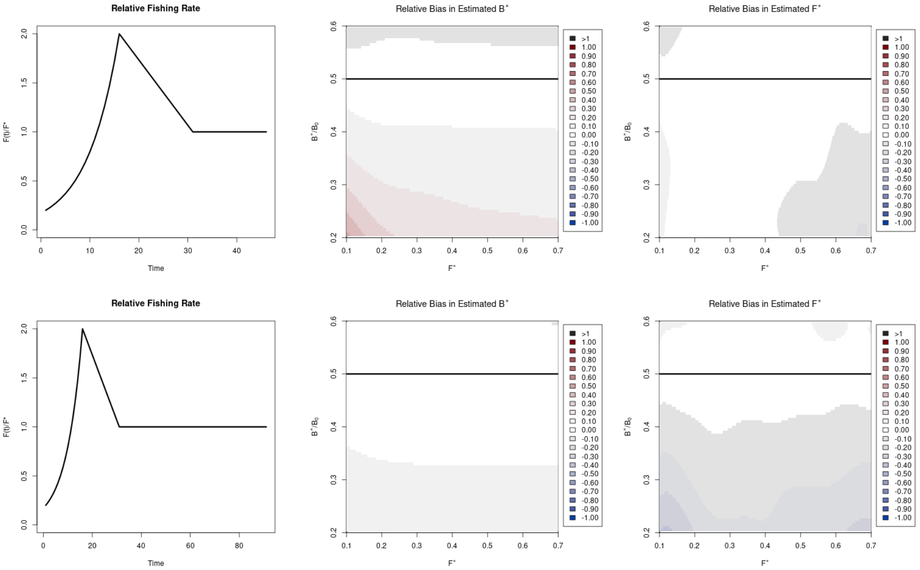


Outline


Induced PT Fishery
PT Population
Catch
Biomass
0
2000
4000
6000
8000
10000
0
10
20
30
40
Time

High Contrast



Summary

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
  - ⇒ Productivity Extensions
  - ⇒ Individual growth and maturity dynamics
- In this severly constrained settings we pay for our modeling mistakes primarily in estimate bias.
- In practice the Schaefer model is at best only likely to reasonably estimate one of either B^* or F^*.
- The observed contrast serves to distribute the available information among B^* and F^*.
  - ⇒ Models of catch contextulize interpretation of RP estimation.

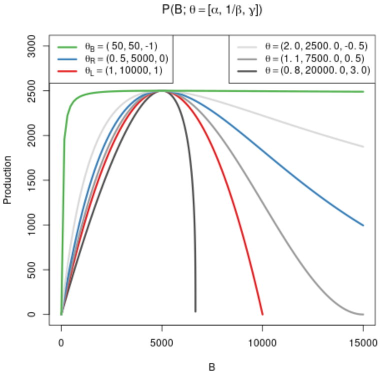
Outline

Productivity Extension

$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

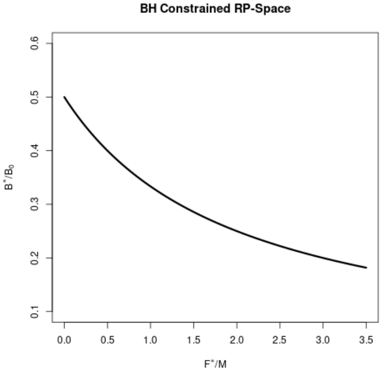
$$\gamma = -1 \Rightarrow \text{Beverton-Holt}$$
$$\gamma = 1 \Rightarrow \text{Logistic}$$
$$\gamma \rightarrow 0 \Rightarrow \text{Ricker}$$



Productivity Extension

$$P_{\mathrm{BH}}(B;[\alpha,\beta,-1]) = \frac{\alpha B}{(1+\beta B)}$$

$$\frac{B^*}{\bar{B}(0)} = \frac{1}{\frac{F^*}{M}+2}$$



Growth Extension

$$\frac{dB}{dt} = \overbrace{w(a_0)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

$$R(B;[\alpha,\beta,\gamma]) = \alpha B(t-a_0)(1-\beta\gamma B(t-a_0))^{\frac{1}{\gamma}}$$

$$w(a) = w_\infty(1-e^{-\kappa a})$$

$\theta' = [\alpha,\beta,\gamma]$ Species Properties: $a_0,\ \kappa,\ w_\infty,\ M$

Outline

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$c(\tau) = \int_{\tau}^{\tau+1} C(t)dt$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Common Discretization

Instantaneous

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$c(\tau) = \int_{\tau}^{\tau+1} C(t)dt$$

$$B(\tau+1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

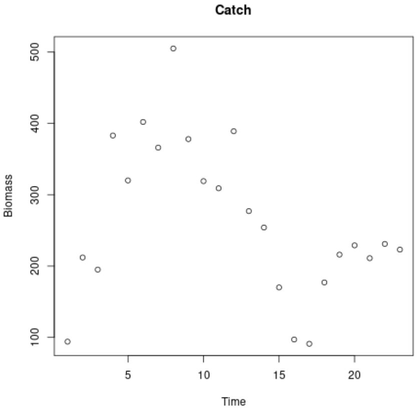
Yearly

Catch Interpolation

$$t \in \mathbb{R}^+ \qquad \tau = \lceil t \rceil - 1$$

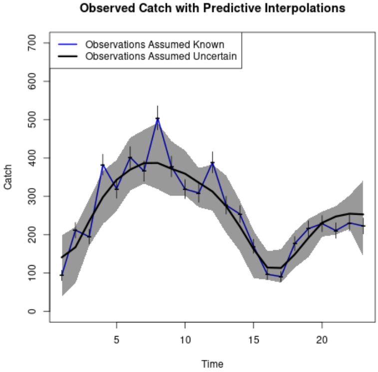
$$\mathbb{E}[c(t)] = \int_{\tau}^{t} C(t^*)dt^*$$

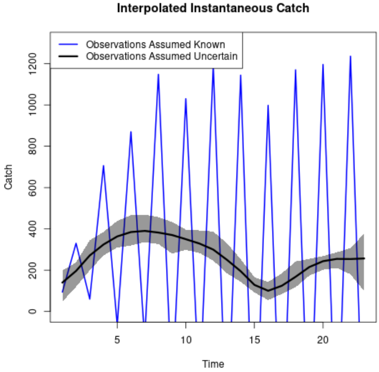
$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



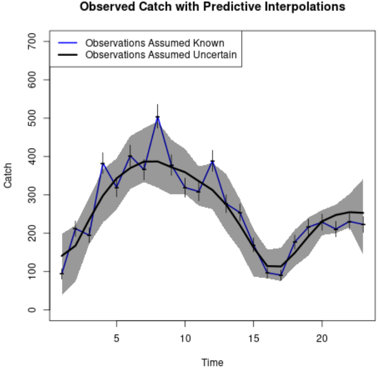
$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

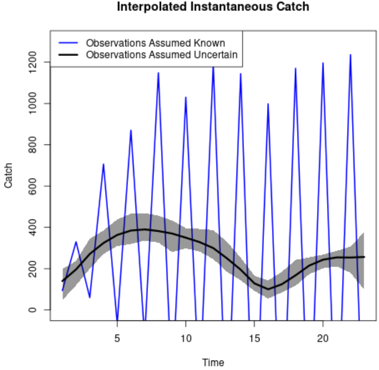
$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0,1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$



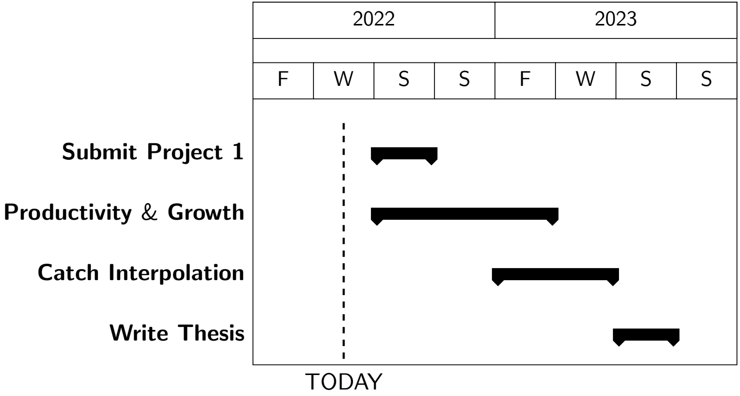


$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$
$$B(\tau+1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$





Timeline


2022
2023
F
W
S
S
F
W
S
S
Submit Project 1
Productivity & Growth
Catch Interpolation
Write Thesis
TODAY

Many Thanks:

- Dr. Marc Mangel
- Collaborators at NOAA
- NMFS Sea Grant
- Members of my Committee

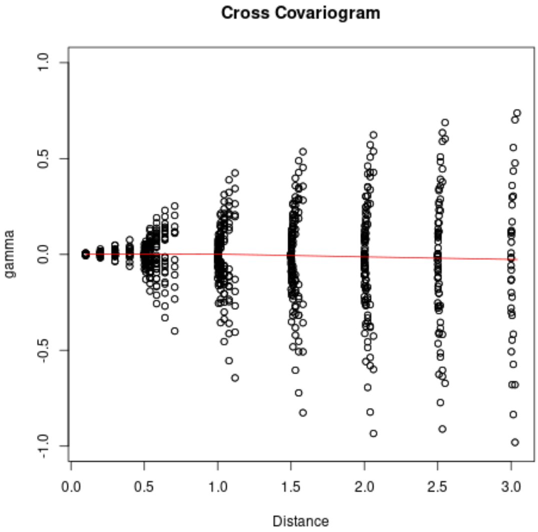
Metamodel Details

$$\hat{\mu} = \widehat{log(r)} \quad - or - \quad \hat{\mu} = \widehat{log(K)}$$

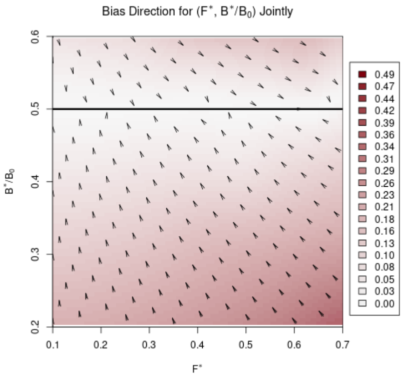
$$\boldsymbol{x} = \left(F^*, \frac{B^*}{\bar{B}(0)}\right)$$

$$\begin{aligned} \hat{\mu} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon \\ f(\boldsymbol{x}) &\sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}')) \\ \epsilon_i &\sim \mathrm{N}(0, \hat{\omega}_i). \end{aligned}$$

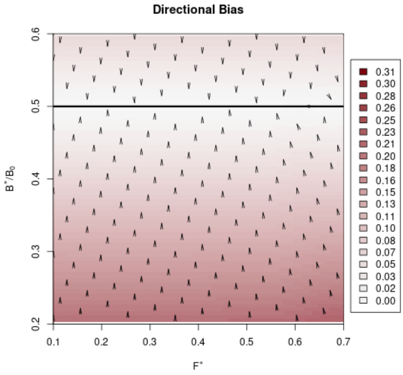
$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2}\right)$$


Cross Covariogram
gamma
Distance
1.0
0.5
0.0
-0.5
-1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Low Contrast



High Contrast



Deriso RP-Parameter System

$$\frac{B^*}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F^*}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$

$$\alpha = (M + F^*)\left[1 - \frac{1}{\gamma}\left(\frac{F^*}{M + F^*}\right)\right]^{-\gamma}$$

$$\beta = \frac{1}{\gamma \bar{B}(0)}\left(1 - \left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}}\right)$$

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

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