

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

14 March 2022



Outline

- 1 Introduction
- 2 Simulation
- 3 Results
 - Low Contrast
 - High Contrast
- 4 Proposals
 - Project #2: Growth & Productivity Extension
 - Project #3: Catch Interpolation
- 5 End

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1 Introduction

2 Simulation

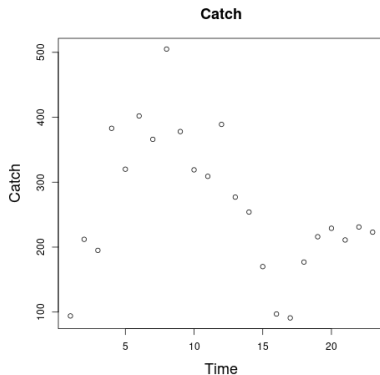
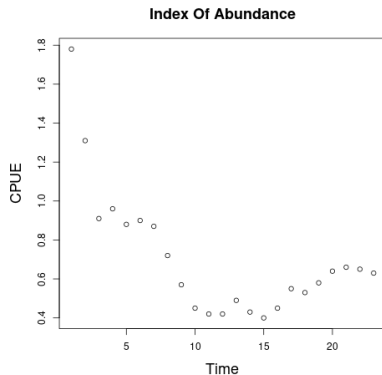
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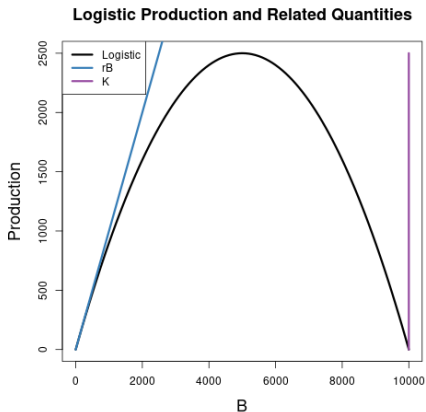
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \theta) - C(t)$$

Schaefer Model

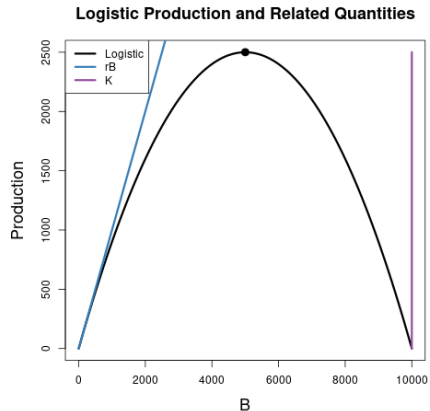
$$P_{\theta}(B) = rB \left(1 - \frac{B}{K}\right)$$

$$\theta = (r, K)$$



Schaefer Reference Points

$$MSY = \frac{rK}{4}$$

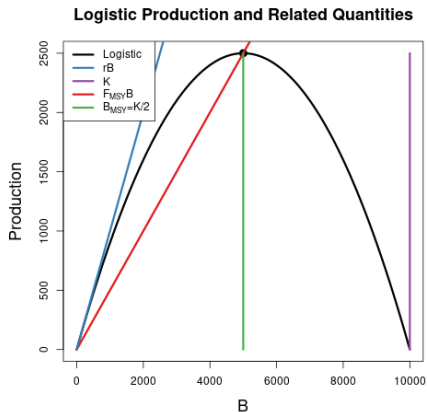


Schaefer Reference Points

$$MSY = \frac{rK}{4}$$

$$F_{MSY} = \frac{r}{2}$$

$$\frac{B_{MSY}}{B_0} = \frac{1}{2}$$



Conceptually:

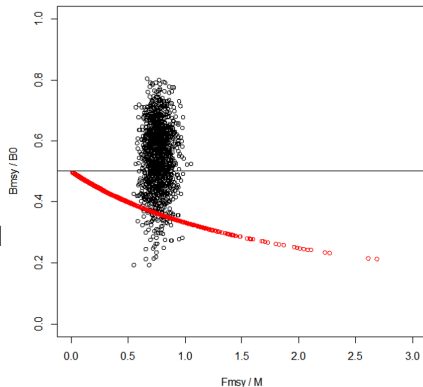
$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

- BH Model:

$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$$

- Similar Constraint for Ricker and other 2 Parameter Curves



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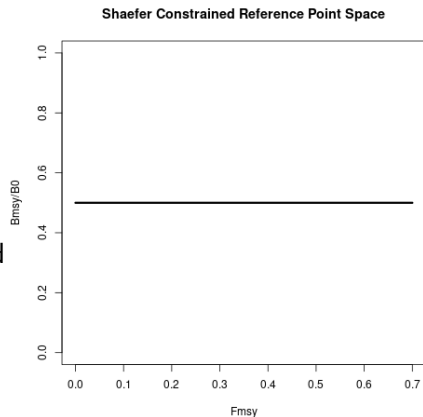
- BH Model:

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Schaefer Model:

$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{2}$$



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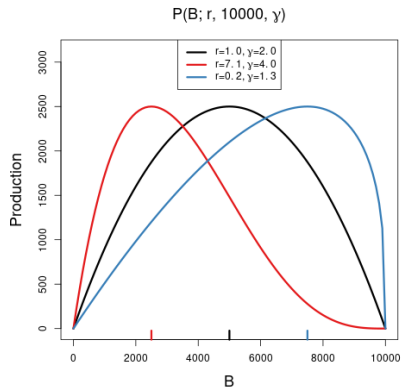
5 End

Pella-Tomlinson Production Model

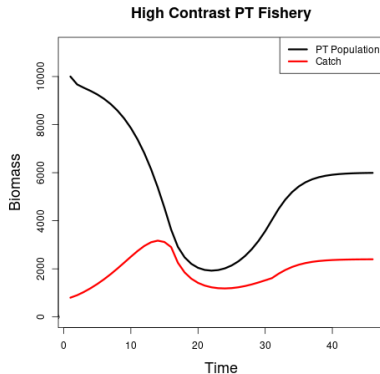
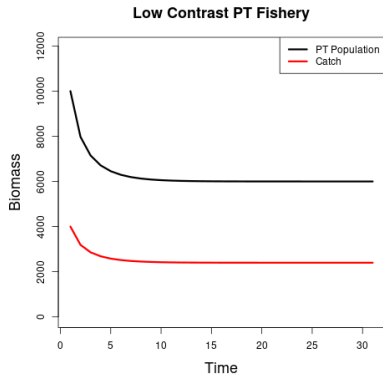
$$I(t) \sim LN(qB(t), \sigma^2)$$
$$\frac{dB(t)}{dt} = P_{\theta}(B(t)) - F(t)B(t)$$

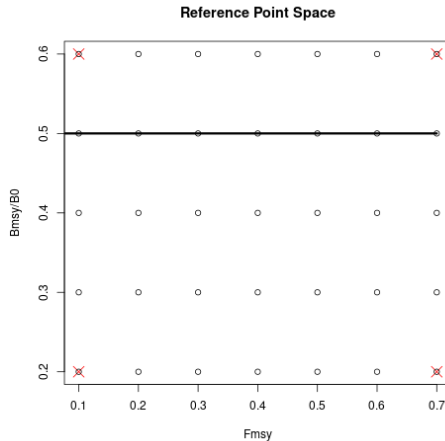
$$P_{\theta}(B) = \frac{rB}{\gamma - 1} \left(1 - \frac{B}{K}\right)^{\gamma-1}$$
$$\theta = (r, K, \gamma)$$

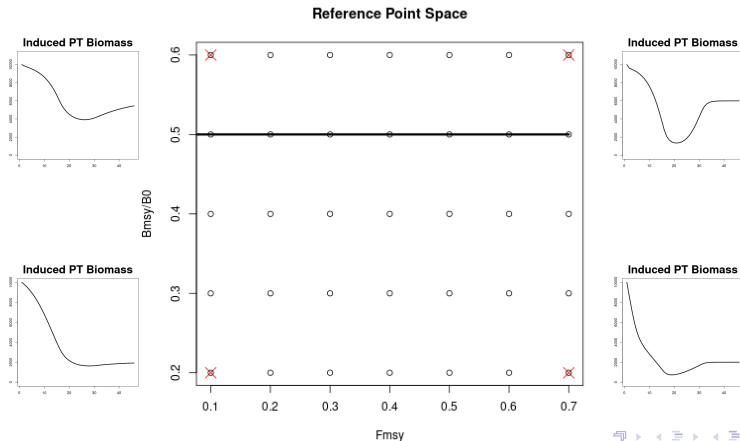
$\gamma = 2 \Rightarrow$ Schaefer Model



Catch



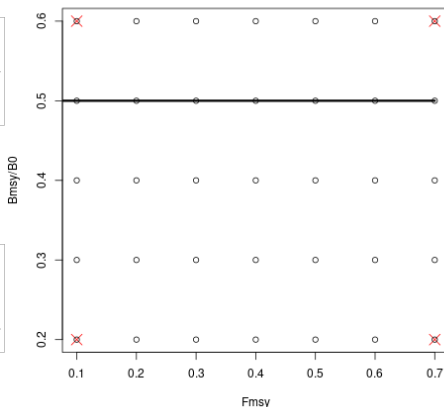
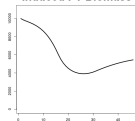




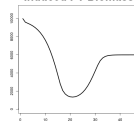
$$\theta = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)}, K = 10000, \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$

Reference Point Space

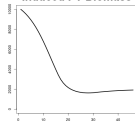
Induced PT Biomass



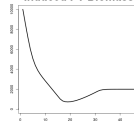
Induced PT Biomass



Induced PT Biomass



Induced PT Biomass



Metamodel

$$\underbrace{\left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{PT Truth}} \xrightarrow{\text{GP}} \underbrace{\left(\hat{F}_{MSY}, \frac{\hat{B}_{MSY}}{\bar{B}(0)} \right)}_{\text{Shaefer Estimate}}$$

- GP approximates constrained RP inference.
- Propagation of estimator uncertainty smooths bias estimation.
- Explicitly highlights trade-offs induced in inferred RPs.

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■ Low Contrast

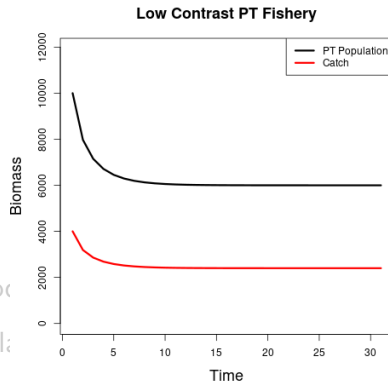
■ High Contrast

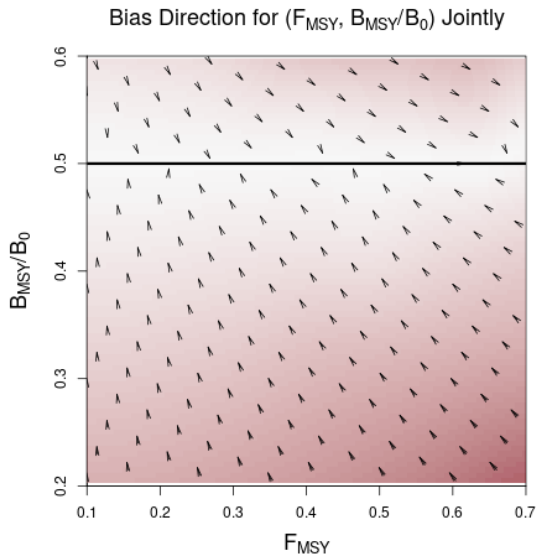
4 Proposals

■ Project #2: Growth & Pro

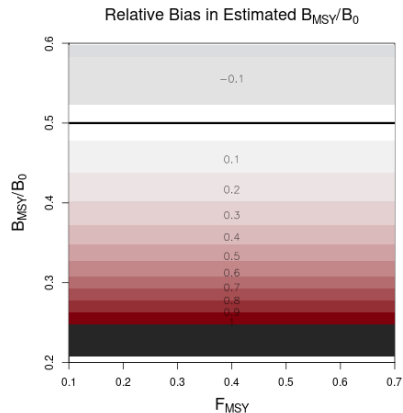
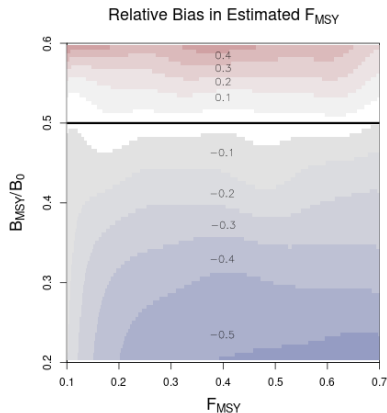
■ Project #3: Catch Interpol

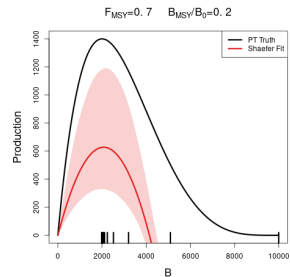
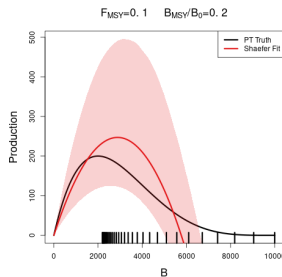
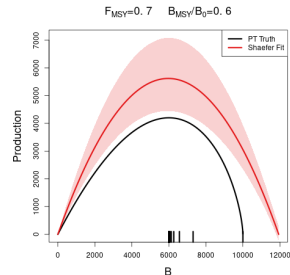
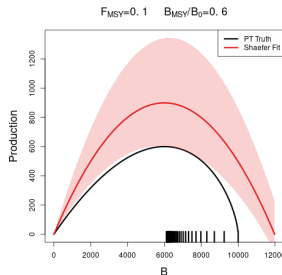
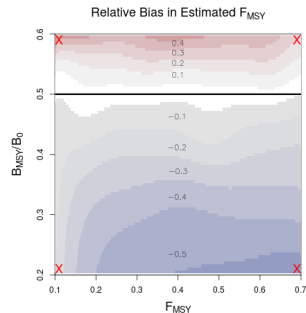
5 End

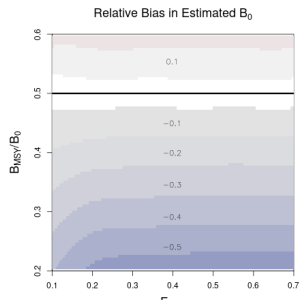
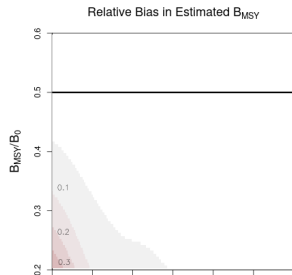
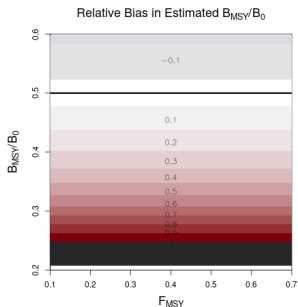




Components of Bias







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■ High Contrast

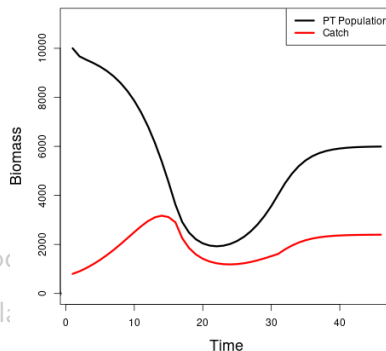
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■ Project #2: Growth & Pro

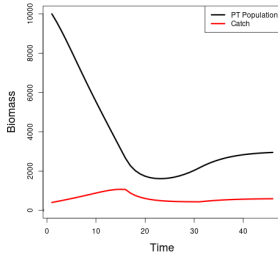
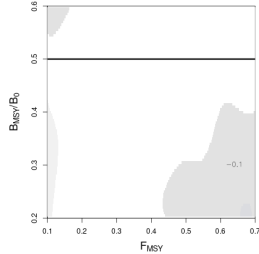
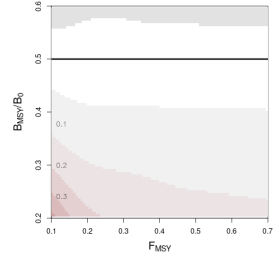
■ Project #3: Catch Interpol

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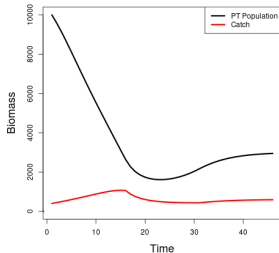
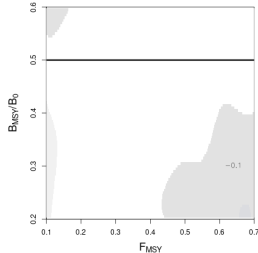
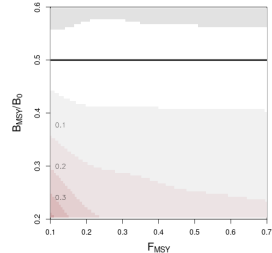
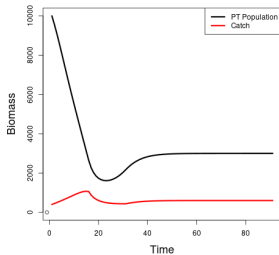
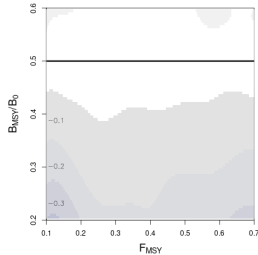
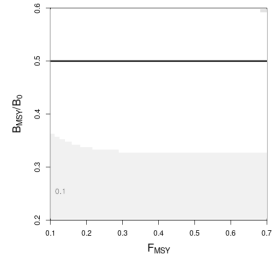
High Contrast PT Fishery



High Contrast

 $F_{MSY}=0.2$ $B_{MSY}/B_0=0.3$ Relative Bias in Estimated F_{MSY} Relative Bias in Estimated B_{MSY} 

High Contrast

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Summary

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
 - ⇒ Productivity Extensions
 - ⇒ Individual growth and maturity dynamics
- In this severely constrained settings we pay for our modeling mistakes primarily in estimate bias.
- In practice the Schaefer model is at best only likely to reasonably estimate one of either B_{MSY} or F_{MSY} .
- The observed contrast serves to distribute the available information among B_{MSY} and F_{MSY} .
 - ⇒ Models of catch contextualize interpretation of RP estimation.

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Productivity Extension

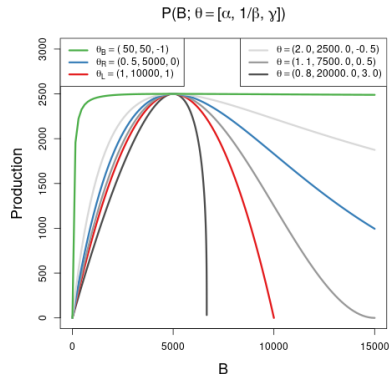
$$\frac{dB}{dt} = P(B; \theta) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ **Beverton-Holt**

$\gamma \rightarrow 0 \Rightarrow$ **Ricker**

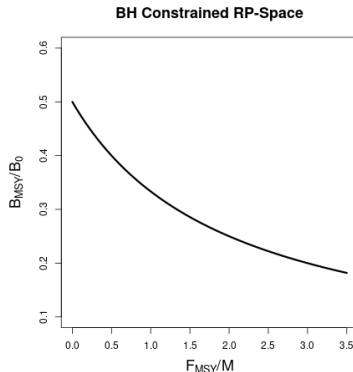
$\gamma = 1 \Rightarrow$ **Logistic**



Productivity Extension

$$P_{BH}(B; [\alpha, \beta, -1]) = \frac{\alpha B}{(1 + \beta B)}$$

$$\frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{\frac{F_{MSY}}{M} + 2}$$



Growth Extension

$$\begin{aligned}\frac{dB}{dt} &= \overbrace{w(a_0)R(B; \theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa [w_\infty N - B]}^{\text{Net Growth}} - \overbrace{(M + F)B}^{\text{Mortality}} \\ \frac{dN}{dt} &= R(B; \theta) - (M + F)N\end{aligned}$$

$$\begin{aligned}R(B; [\alpha, \beta, \gamma]) &= \alpha B(t - a_0)(1 - \beta \gamma B(t - a_0))^{\frac{1}{\gamma}} \\ w(a) &= w_\infty(1 - e^{-\kappa a})\end{aligned}$$

$$\theta' = [\alpha, \beta, \gamma] \quad \text{Species Properties: } a_0, \kappa, w_\infty, M$$

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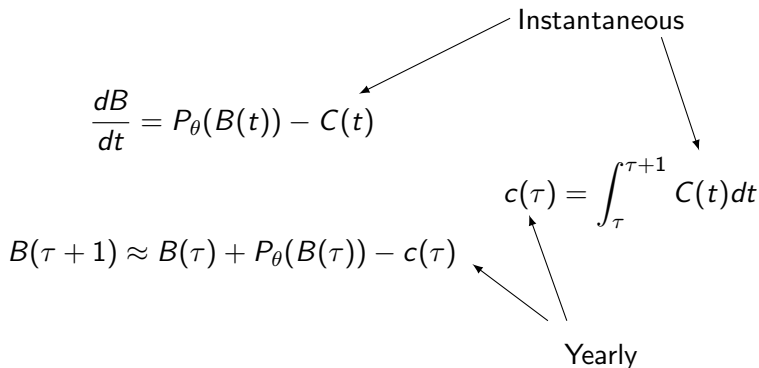
Common Discretization

$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$c(\tau) = \int_{\tau}^{\tau+1} C(t)dt$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - c(\tau)$$

Common Discretization

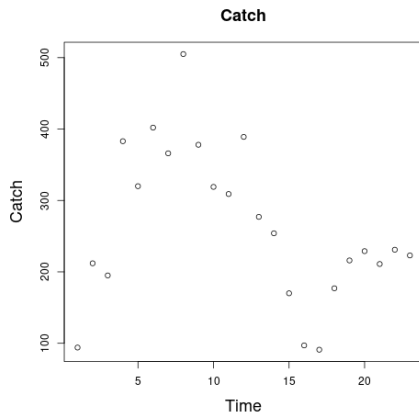


Catch Interpolation

$$t \in \mathbb{R}^+ \quad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[c(t)] = \int_{\tau}^t C(t^*) dt^*$$

$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$

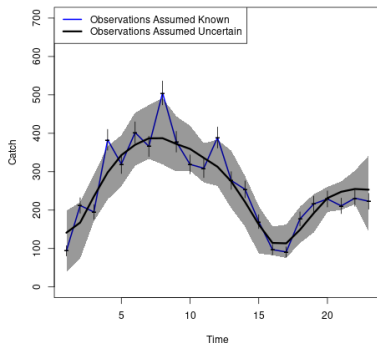


Project #3: Catch Interpolation

$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \quad \phi \sim \text{Half-Cauchy}(0, 1) \quad \epsilon_i \sim N(0, \sigma_i^2)$$

Observed Catch with Predictive Interpolations

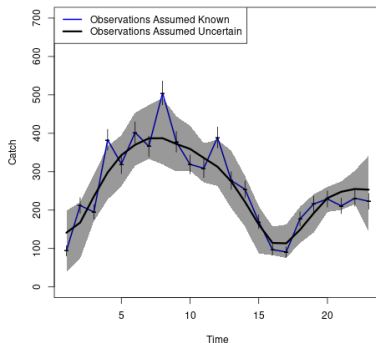


Project #3: Catch Interpolation

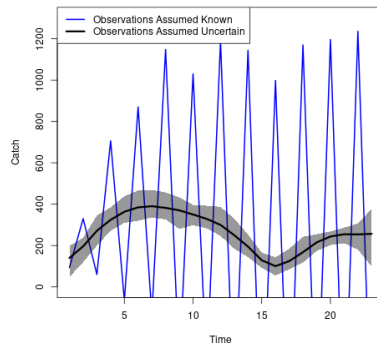
$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

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Observed Catch with Predictive Interpolations



Interpolated Instantaneous Catch

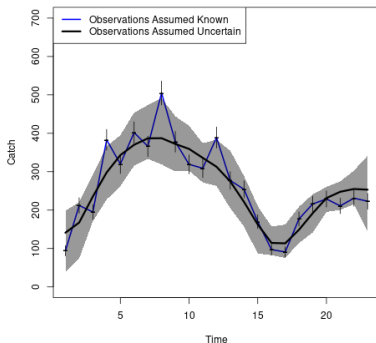


Project #3: Catch Interpolation

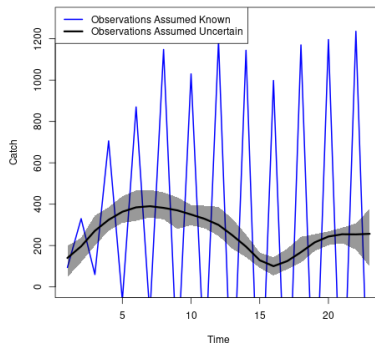
$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - c(\tau)$$

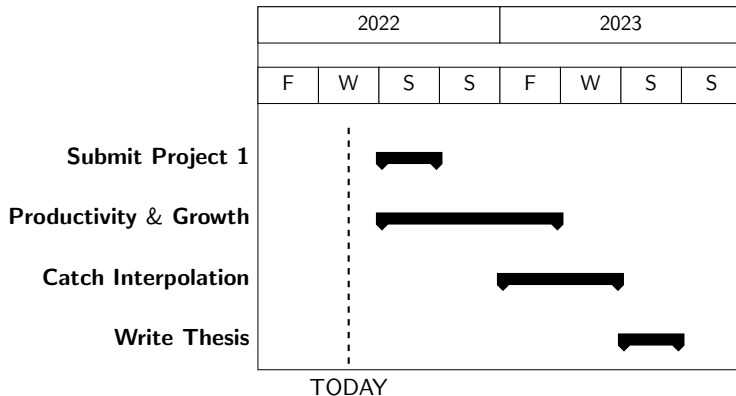
Observed Catch with Predictive Interpolations



Interpolated Instantaneous Catch



Timeline



Many Thanks:

- Dr. Marc Mangel
- Collaborators at NOAA
- NMFS Sea Grant



Metamodel Details

$$\hat{\mu} = \widehat{\log(r)} \quad - \text{ or } - \quad \hat{\mu} = \widehat{\log(K)}$$

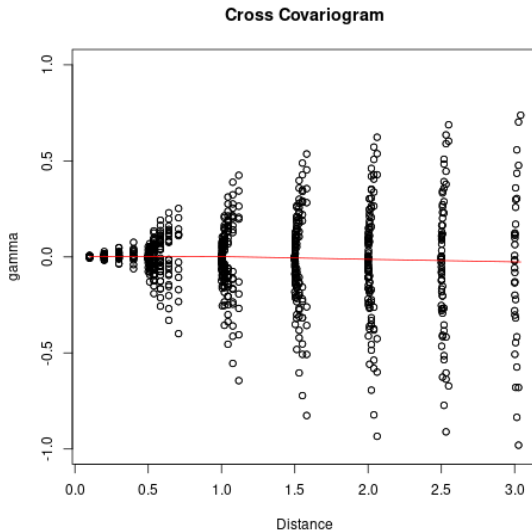
$$\mathbf{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\hat{\mu} = \beta_0 + \beta' \mathbf{x} + f(\mathbf{x}) + \epsilon$$

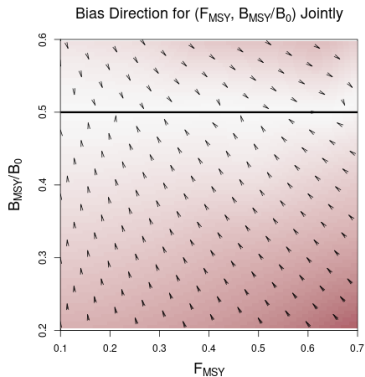
$$f(\mathbf{x}) \sim \text{GP}(0, \tau^2 R(\mathbf{x}, \mathbf{x}'))$$

$$\epsilon_i \sim \text{N}(0, \hat{\omega}_i).$$

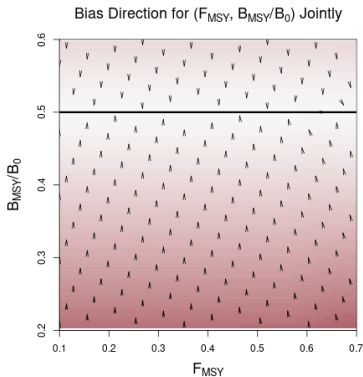
$$R(\mathbf{x}, \mathbf{x}') = \exp \left(\sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2} \right)$$



Low Contrast



High Contrast



Deriso RP-Parameter System

$$\frac{B_{MSY}}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F_{MSY}}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$
$$\alpha = (M + F_{MSY}) \left[1 - \frac{1}{\gamma} \left(\frac{F_{MSY}}{M + F_{MSY}} \right) \right]^{-\gamma}$$
$$\beta = \frac{1}{\gamma \bar{B}(0)} \left(1 - \left(\frac{\alpha}{M} \right)^{\frac{1}{\gamma}} \right)$$

Common Discretization

$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - c(\tau)$$

