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Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

14 March 2022

Outline
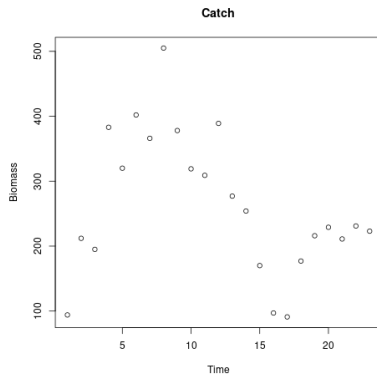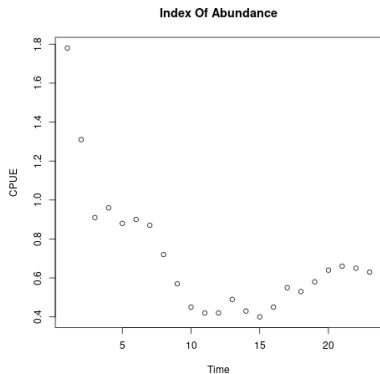
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   ■ Project #3: Catch Interpolation
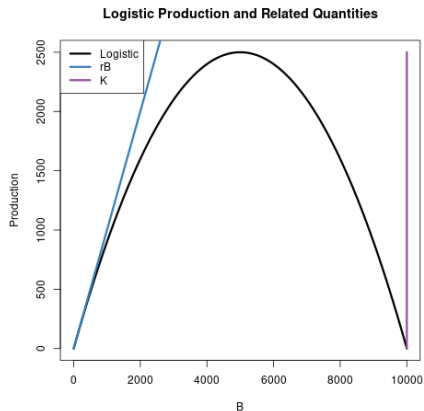
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$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - C(t)$$

Nick Grunloh

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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Schaefer Model

$$P_\theta(B) = rB\left(1 - \frac{B}{K}\right)$$

$$\theta = (r, K)$$

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Schaefer Reference Points

$$F^* = \frac{r}{2}$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

$$MSY = \frac{rK}{4}$$
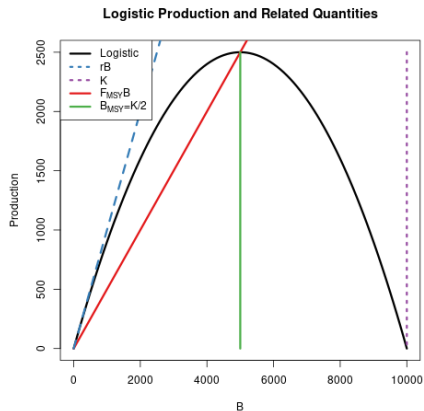
Logistic Production and Related Quantities

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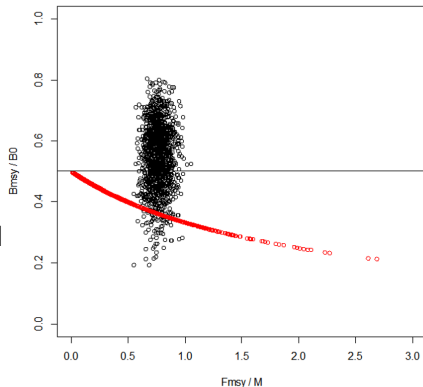Results
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Conceptually:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{F^*/M+2}$
- Similar Constraint for Ricker and other 2 Parameter Curves

Conceptually:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0, 1)$$
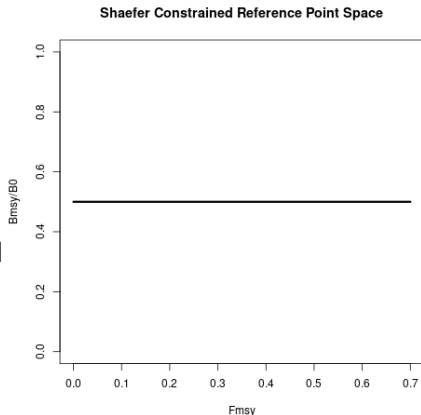
Mangel et al. 2013, CJFAS:

- BH Model:
  $F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{F^*/M+2}$
- Similar Constraint for Ricker and other 2 Parameter Curves

Schaefer Model:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{2}$$

Shaefer Constrained Reference Point Space

Nick Grunloh

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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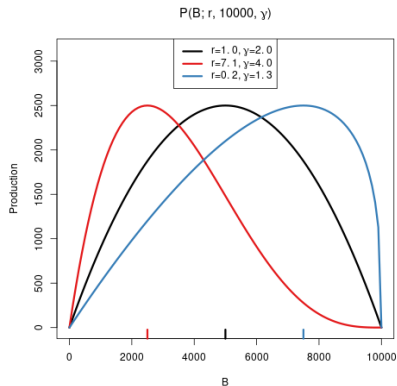- Project #2: Growth & Productivity Extension
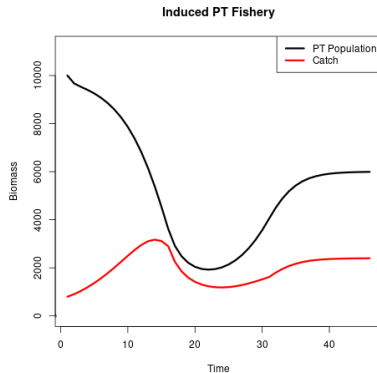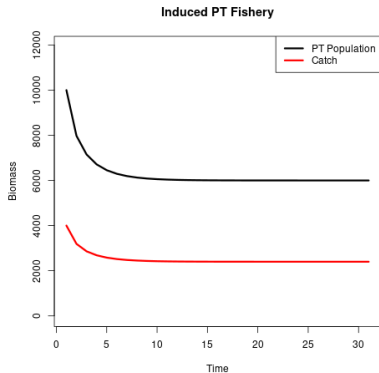
- Project #3: Catch Interpolation

5 End

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Pella-Tomlinson Production Model

$$I(t) \sim LN(qB(t), \sigma^2)$$

$$\frac{dB(t)}{dt} = P_{\boldsymbol{\theta}}(B(t)) - F(t)B(t)$$

$$P_{\boldsymbol{\theta}}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma - 1}$$

$$\boldsymbol{\theta} = (r, K, \gamma)$$

$\gamma = 2 \Rightarrow$ Schaefer Model

P(B; r, 10000, γ)

Legend:
- r=1.0, γ=2.0
- r=7.1, γ=4.0
- r=0.2, γ=1.3

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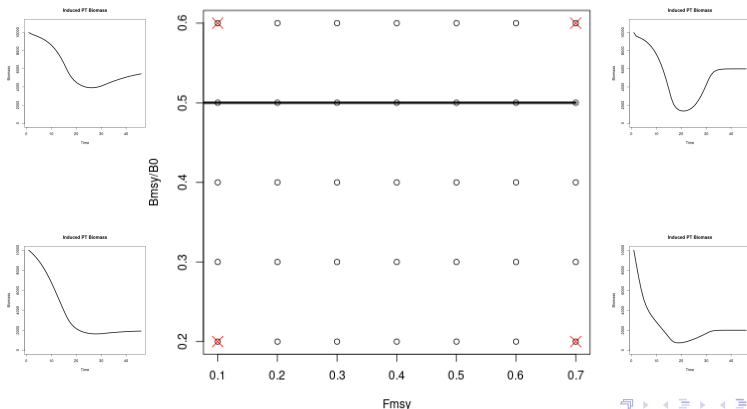○○○○○○○○

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Catch

$$\boldsymbol{\theta} = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)}, \; K = 10000, \; \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$

Reference Point Space

$$\boldsymbol{\theta} = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)}, \ K = 10000, \ \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$

Reference Point Space

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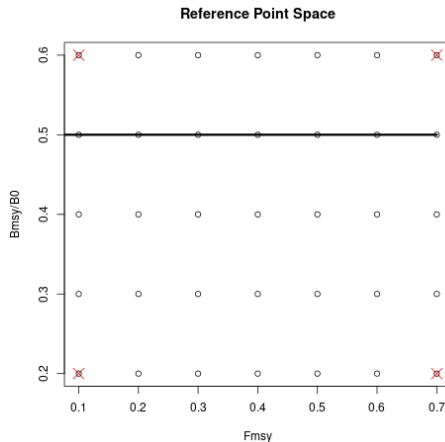
Metamodel

$$\underbrace{\left(F^*, \frac{B^*}{\bar{B}(0)}\right)}_{\text{PT Truth}} \overset{\text{GP}}{\mapsto} \underbrace{\left(\hat{F}^*, \frac{\hat{B}^*}{\bar{B}(0)}\right)}_{\text{Shaefer Estimate}}$$

- GP interpolates over degrees of RP model misspecification.
- Propogation of estimate uncertainty smooths bias estimation.
- Explicitely highlights trade-offs induced in RPs.

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Low Contrast

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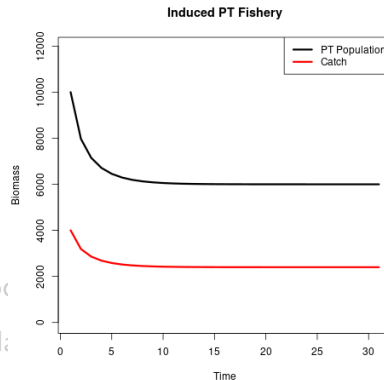
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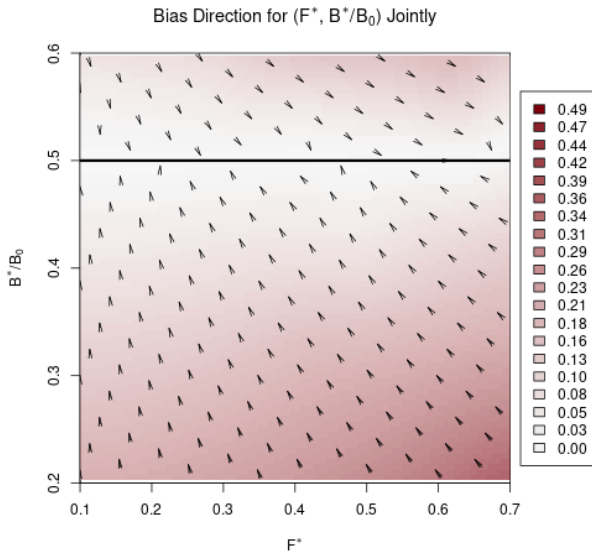
4 Proposals

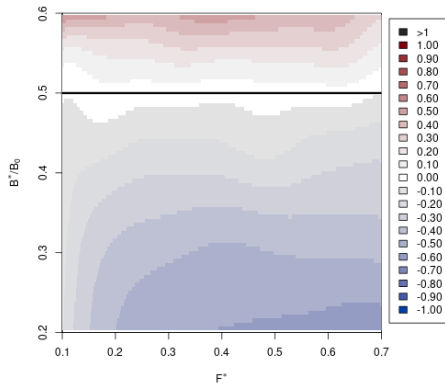   ■ Project #2: Growth & Pro

   ■ Project #3: Catch Interpola

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Induced PT Fishery

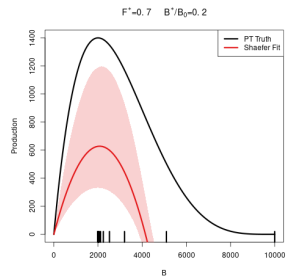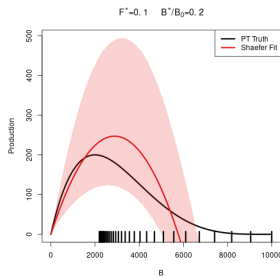| Introduction	Simulation	**Results**	Proposals	End
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Low Contrast

Bias Direction for (F^*, B^*/B_0) Jointly

Components of Bias

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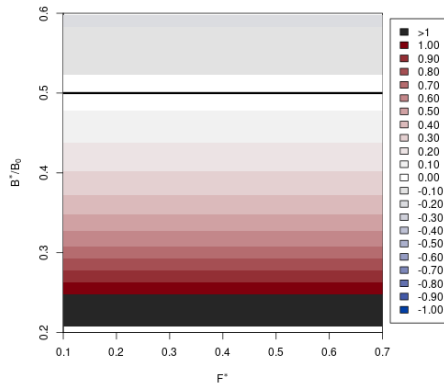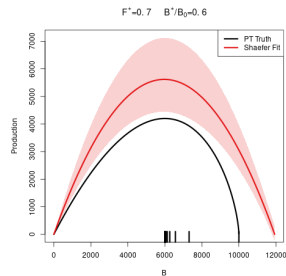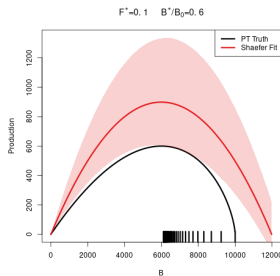
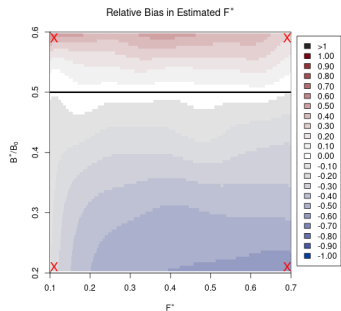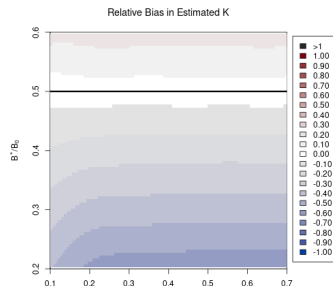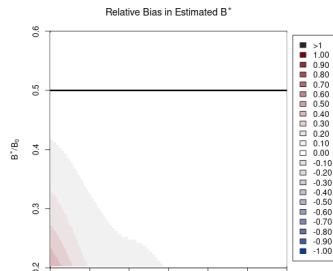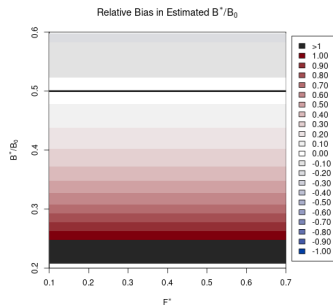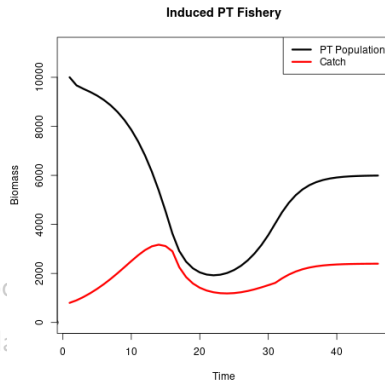Low Contrast
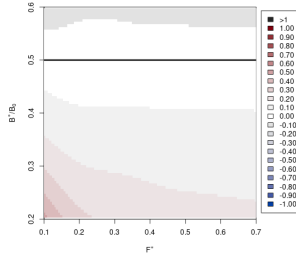
Relative Bias in Estimated B^*

Relative Bias in Estimated B^*/B_0

Relative Bias in Estimated K

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High Contrast

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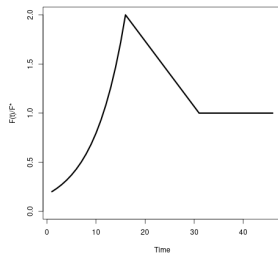   ▪ **High Contrast**

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Induced PT Fishery

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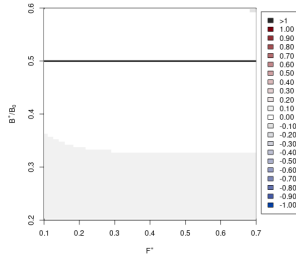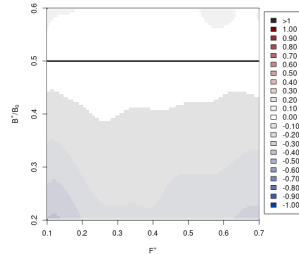
Summary

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
  - ⇒ Productivity Extensions
  - ⇒ Individual growth and maturity dynamics

- In this severly constrained settings we pay for our modeling mistakes primarily in estimate bias.

- In practice the Schaefer model is at best only likely to reasonably estimate one of either B^* or F^*.

- The observed contrast serves to distribute the available information among B^* and F^*.

  - ⇒ Models of catch contextulize interpretation of RP estimation.

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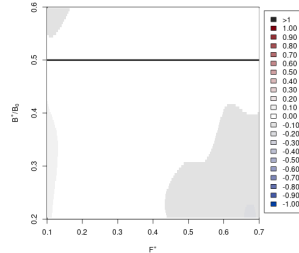
   - Project #3: Catch Interpolation

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Project #2: Growth & Productivity Extension

Productivity Extension

$$\frac{dB}{dt} = P(B; \boldsymbol{\theta}) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B (1 - \beta \gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma = 1 \Rightarrow$ Logistic

$\gamma \to 0 \Rightarrow$ Ricker

$P(B; \theta = [\alpha, 1/\beta, \gamma])$
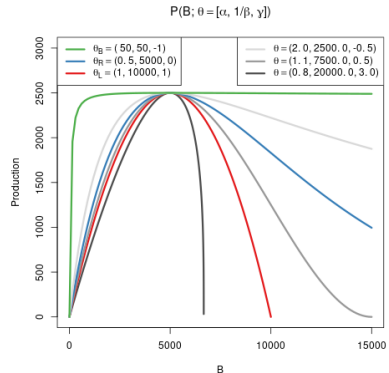
Productivity Extension

$$P_{\text{BH}}(B; [\alpha, \beta, -1]) = \frac{\alpha B}{(1 + \beta B)}$$

$$\frac{B^*}{\bar{B}(0)} = \frac{1}{\frac{F^*}{M} + 2}$$

BH Constrained RP-Space

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Project #2: Growth & Productivity Extension

Growth Extension

$$\frac{dB}{dt} = \overbrace{w(a_0)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$
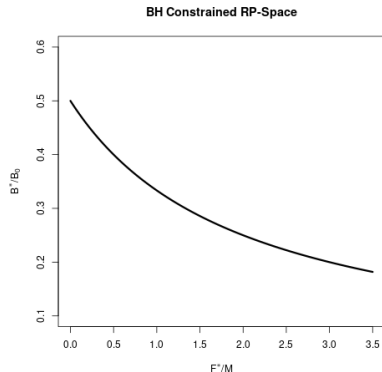
$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

$$R(B;[\alpha,\beta,\gamma]) = \alpha B(t-a_0)(1-\beta\gamma B(t-a_0))^{\frac{1}{\gamma}}$$

$$w(a) = w_\infty(1-e^{-\kappa a})$$

$$\theta' = [\alpha,\beta,\gamma] \qquad \text{Species Properties: } a_0,\ \kappa,\ w_\infty,\ M$$

Outline

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$c(\tau) = \int_\tau^{\tau+1} C(t)dt$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

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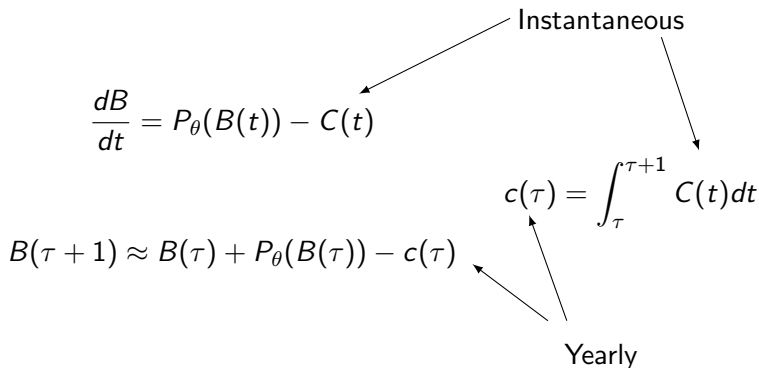
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Project #3: Catch Interpolation

Common Discretization

Instantaneous

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$c(\tau) = \int_\tau^{\tau+1} C(t)dt$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Yearly

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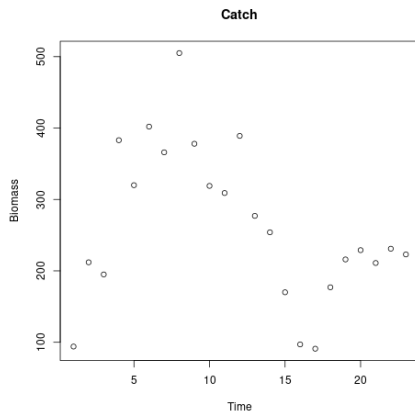○○○○○○●○○

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Project #3: Catch Interpolation

Catch Interpolation

$$t \in \mathbb{R}^+ \qquad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[c(t)] = \int_\tau^t C(t^*)dt^*$$

$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j(t - \tau_j)\mathbb{1}_{t > \tau_j}$$

$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0, 1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$

Observed Catch with Predictive Interpolations

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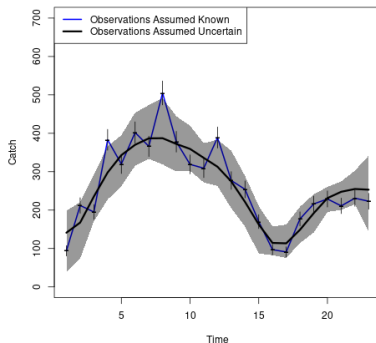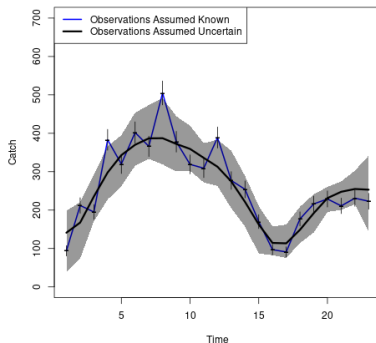○○○○○

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Project #3: Catch Interpolation

$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0, 1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$

Observed Catch with Predictive Interpolations

Interpolated Instantaneous Catch
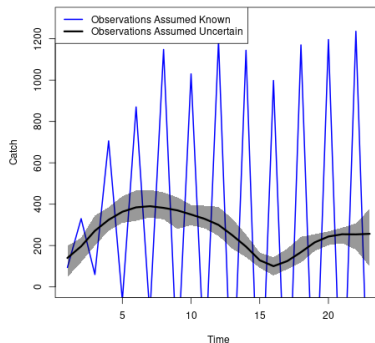
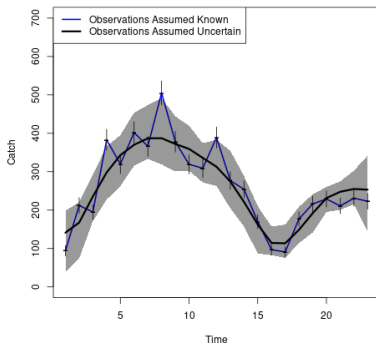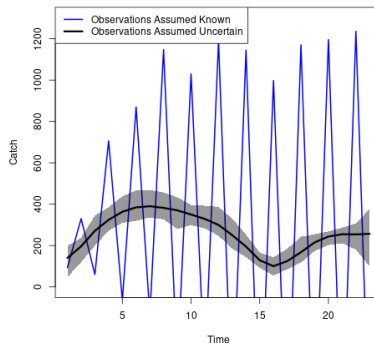| Introduction	Simulation	Results	Proposals	End
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Project #3: Catch Interpolation

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Observed Catch with Predictive Interpolations

Interpolated Instantaneous Catch

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Timeline

	2022				2023			
	F	W	S	S	F	W	S	S

Submit Project 1

Productivity & **Growth**

Catch Interpolation

Write Thesis

TODAY
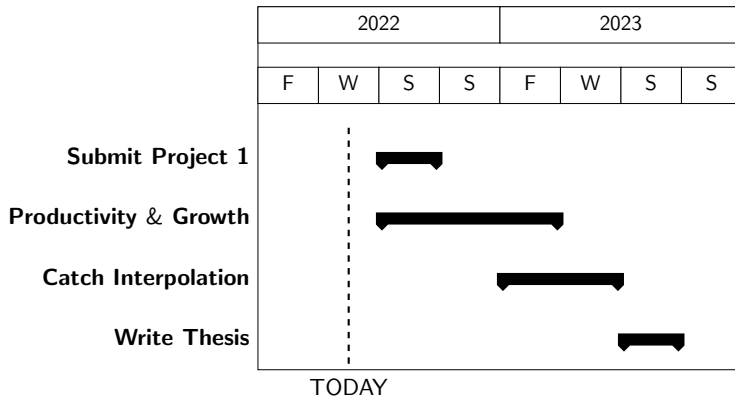
Many Thanks:

- Dr. Marc Mangel
- Collaborators at NOAA
- NMFS Sea Grant

Metamodel Details

$$\hat{\mu} = \widehat{\log(r)} \quad - or - \quad \hat{\mu} = \widehat{\log(K)}$$

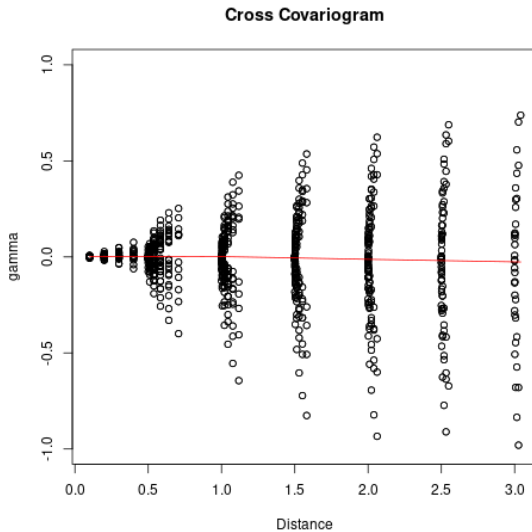$$\boldsymbol{x} = \left(F^*, \frac{B^*}{\bar{B}(0)} \right)$$

$$\hat{\mu} = \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon$$
$$f(\boldsymbol{x}) \sim \mathsf{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}'))$$
$$\epsilon_i \sim \mathsf{N}(0, \hat{\omega}_i).$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$

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Cross Covariogram

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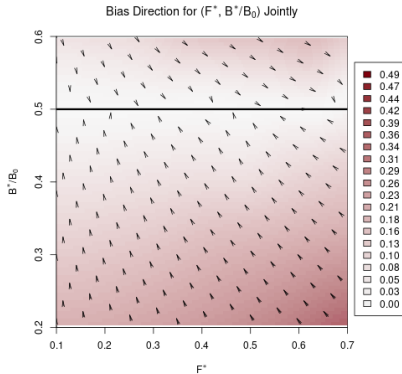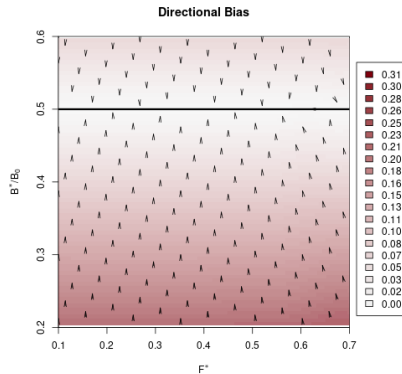
Deriso RP-Parameter System

$$\frac{B^*}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F^*}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$

$$\alpha = (M + F^*)\left[1 - \frac{1}{\gamma}\left(\frac{F^*}{M+F^*}\right)\right]^{-\gamma}$$

$$\beta = \frac{1}{\gamma\bar{B}(0)}\left(1 - \left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}}\right)$$

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Nick Grunloh

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs