24 Apl 2020

Steepness Redux for N in G.

$$N(t+1) = e^{-\mu} N(t) + \frac{\alpha N(t)}{1+\beta N(t)}$$

Steady state

$$\bar{N} = e^{-\mu} \bar{N} + \frac{\alpha \bar{N}}{1+\beta \bar{N}}$$

$$\left[\mu \text{ small} : e^{-\mu} \approx 1-\mu \right]$$

$$1 - e^{-\mu} = \frac{\alpha}{1+\beta\bar{N}}$$

$$1+\beta\bar{N} = \frac{\alpha}{1-e^{-\mu}}$$

$$\bar{N} = \frac{1}{\beta}\left[\frac{\alpha}{1-e^{-\mu}} - 1\right] \simeq \frac{1}{\beta}\left[\frac{\alpha}{\mu} - 1\right]$$

$$R(\bar{N}) = \frac{\alpha\bar{N}}{1+\beta\bar{N}} = \cancel{\frac{\frac{\alpha}{\beta}\left[\frac{\alpha}{\mu}-1\right]}{1+\frac{\alpha}{1-}}}$$

24.1

$$R(\bar{N}) = \frac{\alpha \bar{N}}{1+\beta \bar{N}} = \frac{\frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-m}} - 1\right]}{1 + \left[\frac{\alpha}{1-e^{-m}} - 1\right]}$$

$$= \frac{\frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-m}} - 1\right]}{\left[\frac{\alpha}{1-e^{-m}}\right]}$$

$$R(0.2\bar{N}) = \frac{\alpha\, 0.2\bar{N}}{1+0.2\beta \bar{N}} = \frac{0.2\, \frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-m}} - 1\right]}{1 + 0.2\left[\frac{\alpha}{1-e^{-m}} - 1\right]}$$

$$= \frac{0.2\, \frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-m}} - 1\right]}{0.8 + 0.2\left[\frac{\alpha}{1-e^{-m}}\right]}$$

$$ch = \frac{R(0.2\tilde{N})}{R(\tilde{N})} = \frac{0.2\,\frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-M}}-1\right]}{0.8 + 0.2\left[\frac{\alpha}{1-e^{-M}}\right]} \cdot \left[\frac{\frac{\alpha}{1-e^{-M}}}{\frac{\alpha}{\beta}\left[\frac{\alpha}{1-e^{-M}}-1\right]}\right] \qquad (24.2)$$

$$= \frac{\frac{\alpha}{1-e^{-M}}}{4 + \frac{\alpha}{1-e^{-M}}}$$

M small

$$\approx \frac{\frac{\alpha}{M}}{4 + \frac{\alpha}{M}} = \frac{\alpha}{4M+\alpha}$$