Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

In collaboration with:
Dr. E.J. Dick
Dr. H. K.H. Lee

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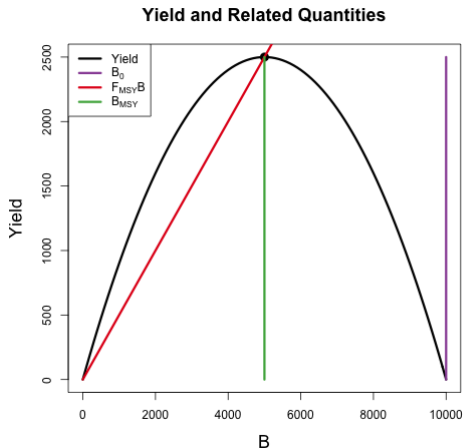
General Modeling Structures

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

$$\text{Reference Points} : MSY, \ \frac{F_{MSY}}{M}, \ \frac{B_{MSY}}{B_0}$$
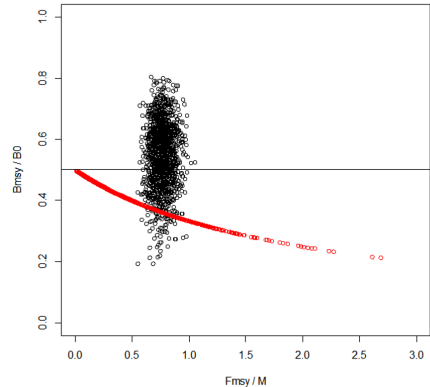
Yield and Related Quantities

Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Curves

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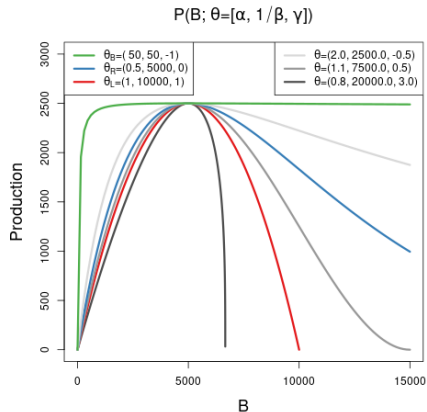- Three-Parameter Relationships Allow Independent RP Estimation
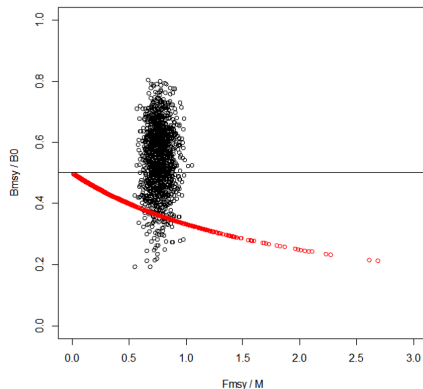
Schnute 1985, CJFAS

$$\frac{dB}{dt} = P(B; \boldsymbol{\theta}) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma \to 0 \Rightarrow$ Ricker

$\gamma = 1 \Rightarrow$ Logistic

$P(B; \theta=[\alpha, 1/\beta, \gamma])$

Legend:
- θ_B=(50, 50, -1)
- θ_R=(0.5, 5000, 0)
- θ_L=(1, 10000, 1)
- θ=(2.0, 2500.0, -0.5)
- θ=(1.1, 7500.0, 0.5)
- θ=(0.8, 20000.0, 3.0)

Production (y-axis), B (x-axis)

- Isolalting RP Bias is Hard:
    - Chaos in the Dynamical System
    - Time Integrator Inaccuracy
    - Model Identifiability
    - Global Optimization
    - etc...

- Production Models are simplified places to build intuition

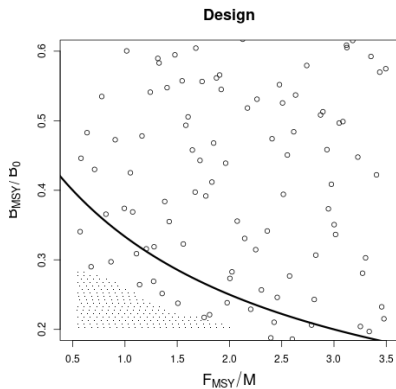- See my analysis of the mechanisms of bias in the Schaefer Model $\implies$
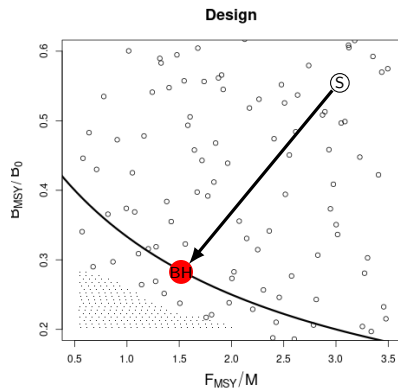
Schaefer RP Analysis

https://ggle.io/5EnI

Simulation Design

- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS

Design

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- GP Metamodeling of RP bias

$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{GP}{\mapsto} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{\hat{B}_{MSY}}{\bar{B}(0)}\right)}_{\text{BH Estimate}}$$
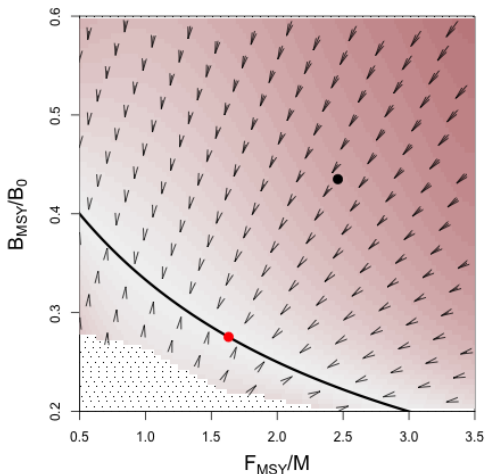

Design
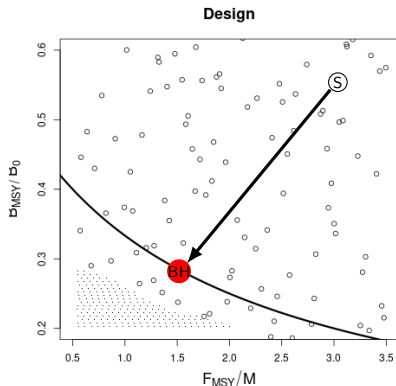
Catch

High Contrast
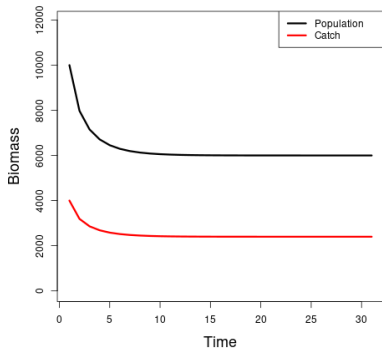
Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
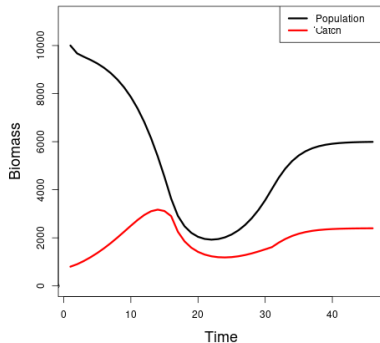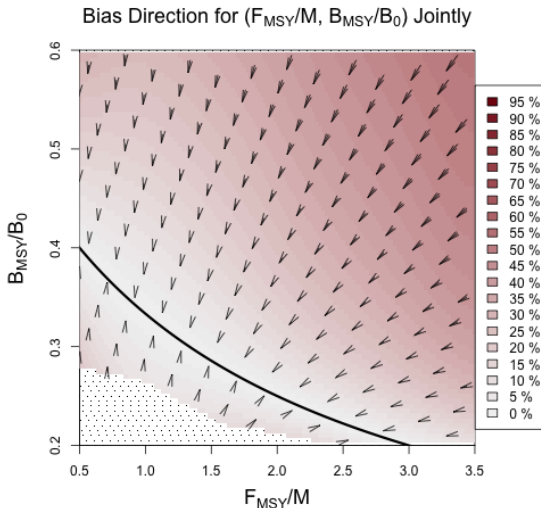
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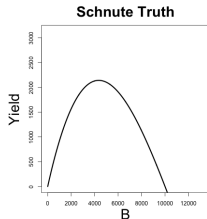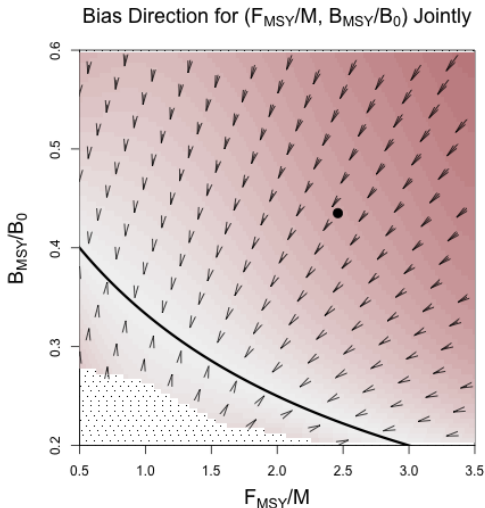
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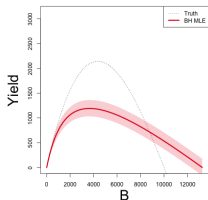
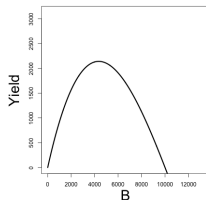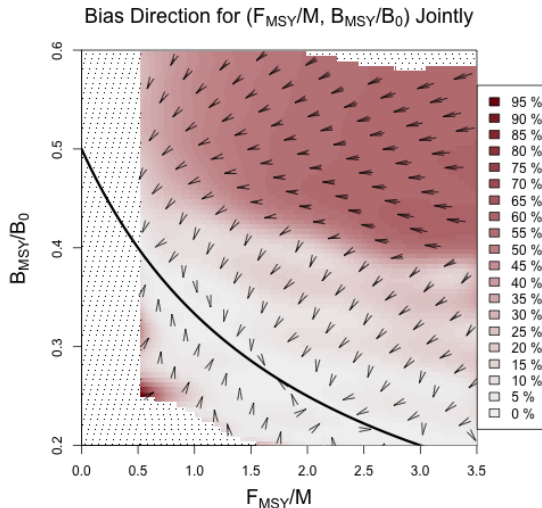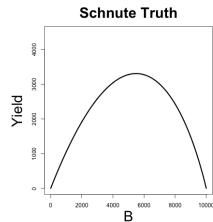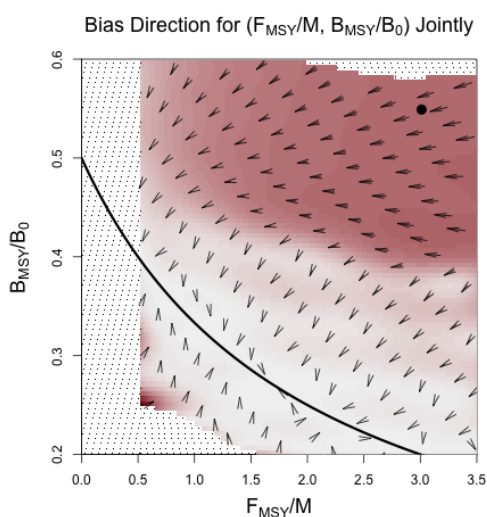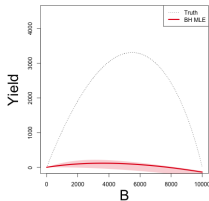Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly

Schnute Truth

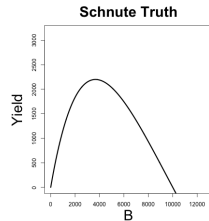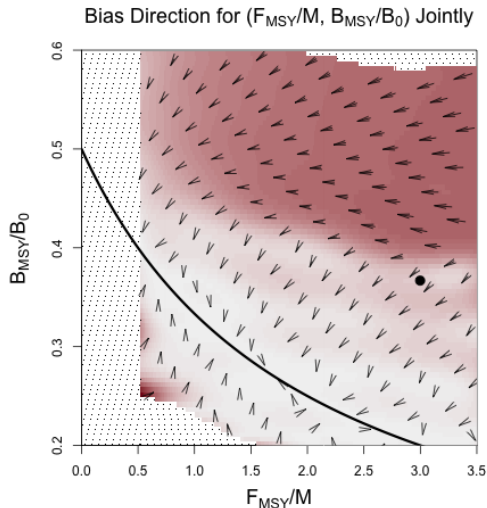## Low Contrast

BH Fit

Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
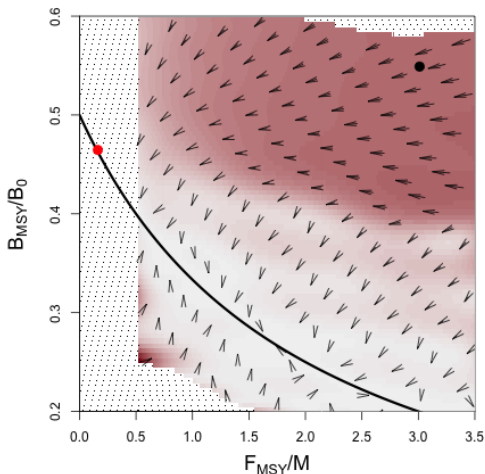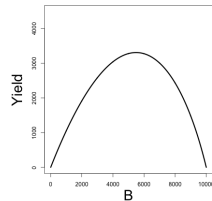
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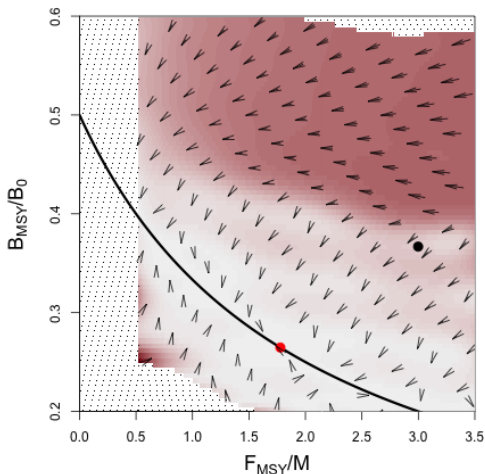
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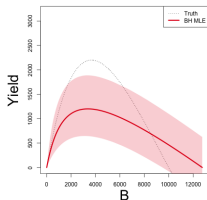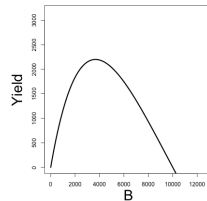
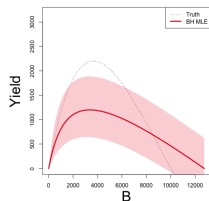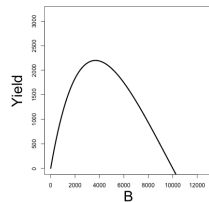## Low Contrast

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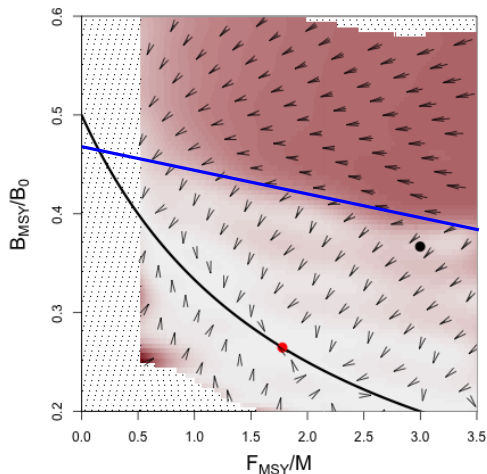
Conclusions

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding more complex models.

  $\Rightarrow$ Individual growth and maturity dynamics

- RPs are not directly observable quantities, but rather model dependent latent quantities.
  - $\Rightarrow$ Subject to Model Misspecification, Uncertainty, & Bias
  - $\Rightarrow$ In severely constrained settings we pay for our modeling mistakes primarily in estimate bias.

- The observed contrast serves to increase the range of potentially "allowable" model misspecification.

Many Thanks:

- UCSC Advisors
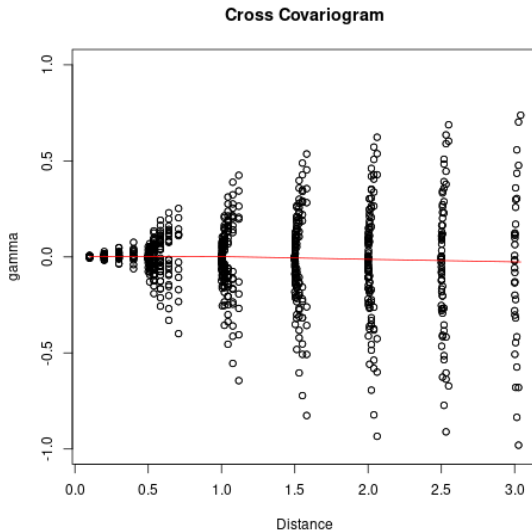- SWFSC Groundfish
- NMFS Sea Grant

Metamodel Details

$$\boldsymbol{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\hat{\mu} = \beta_0 + \boldsymbol{\beta}' \boldsymbol{x} + f(\boldsymbol{x}) + \epsilon$$
$$f(\boldsymbol{x}) \sim \mathsf{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}'))$$
$$\epsilon_i \sim \mathsf{N}(0, \hat{\omega}_i).$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp \left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$

Cross Covariogram

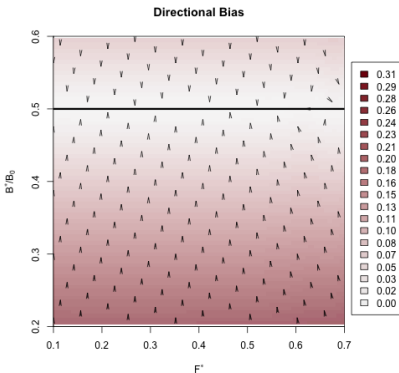## PT Data Fit with the Schaefer Model
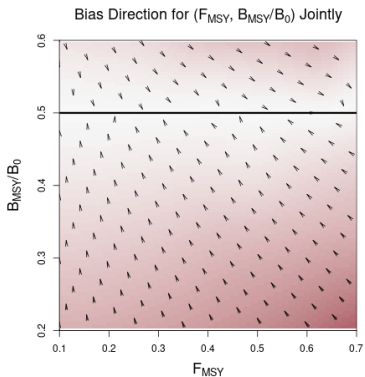
Schnute RP-Parameter System of Equations

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M + F_{MSY}}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}$$

$$\alpha = (M + F_{MSY})\left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$