Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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In collaboration with:
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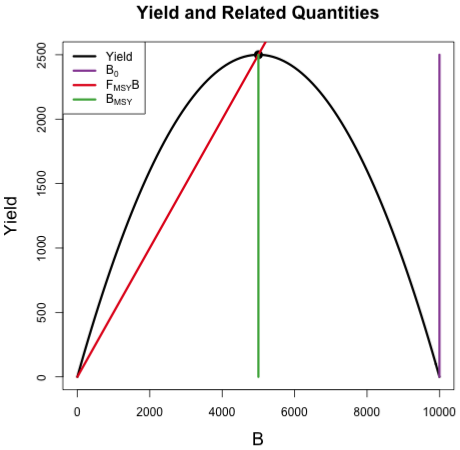
General Modeling Structures

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

$$RP : MSY, \ \frac{F_{MSY}}{M}, \ \frac{B_{MSY}}{B_0}$$

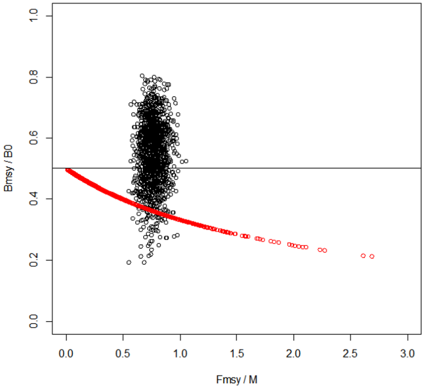


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^{+} \quad \frac{B_{MSY}}{B_0} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $\frac{F_{MSY}}{M} \in \mathbb{R}^{+} \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Curves



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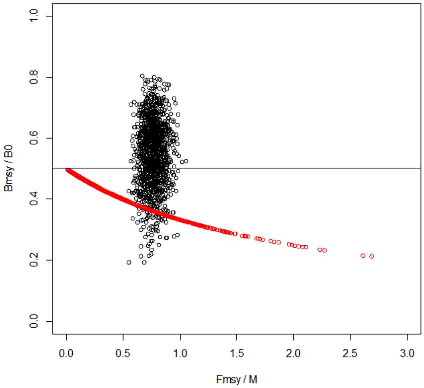
- Similar Constraints for other Two-Parameter Curves
- Three-Parameter Relationships Allow Independent RP Estimation



Schnute 1985, CJFAS

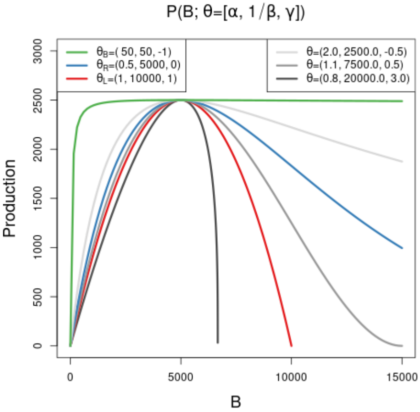
$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma \to 0 \Rightarrow$ Ricker

$\gamma = 1 \Rightarrow$ Logistic



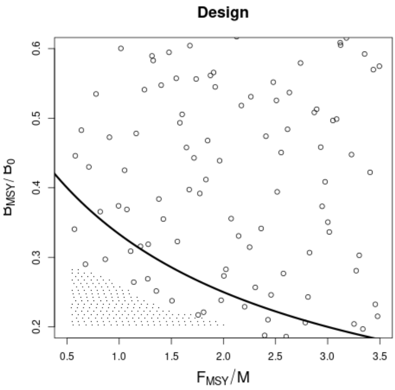
- Isolalting RP Bias is Hard:
  - Chaos in the Dynamical System
  - Time Integrator Inaccuracy
  - Model Identifiability
  - Global Optimization
  - etc...
- Production Models are simplified places to build intuition
- See my analysis of the mechanisms of bias in the Schaefer Model $\Longrightarrow$

Schaefer RP Analysis

https://ggle.io/5EnI

Simulation Design

- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS



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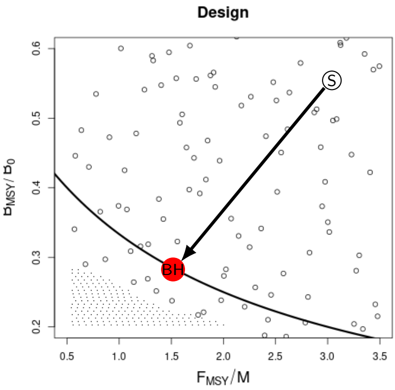
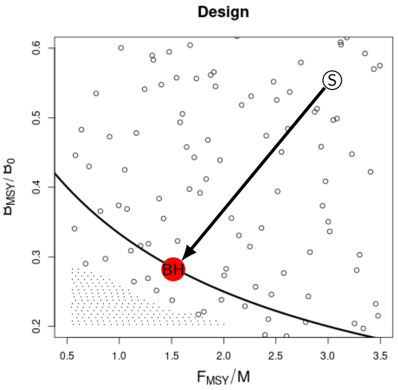


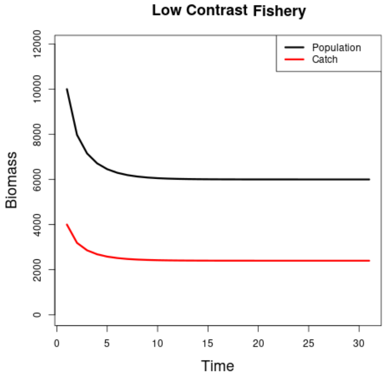
Simulation Design

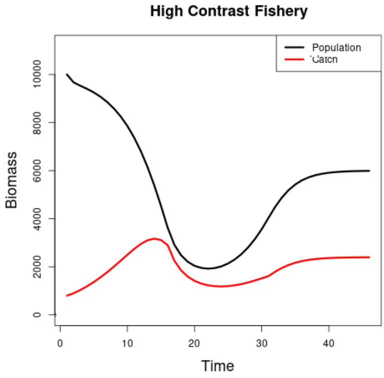
- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS
- GP Metamodeling of RP bias

$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{GP}{\mapsto} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{\hat{B}_{MSY}}{\bar{B}(0)}\right)}_{\text{BH Estimate}}$$

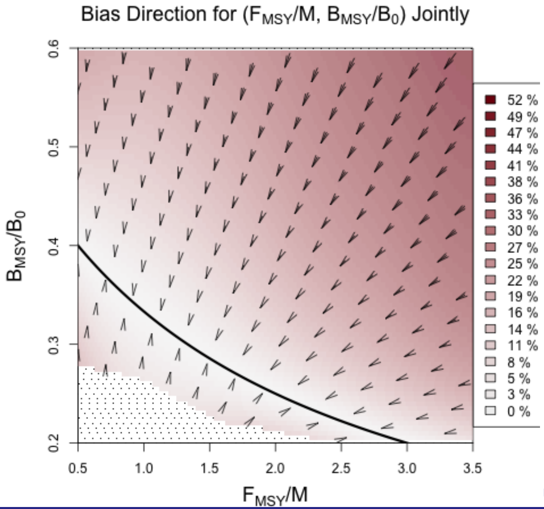


Catch

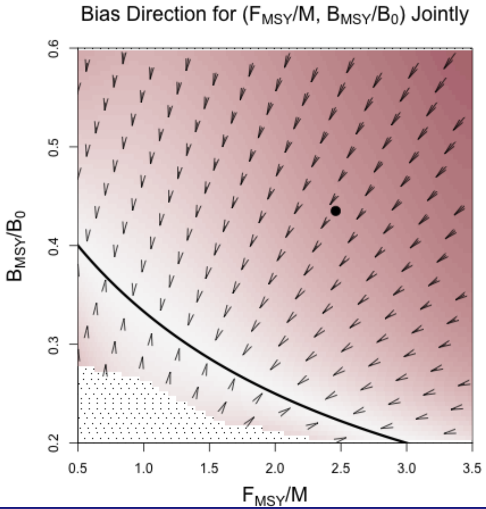

Low Contrast Fishery
Population
Catch
Biomass
0
2000
4000
6000
8000
10000
12000
0
5
10
15
20
25
30
Time

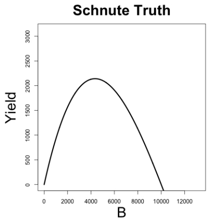

High Contrast Fishery
Population
Catch
Biomass
0
2000
4000
6000
8000
10000
0
10
20
30
40
Time

High Contrast



High Contrast





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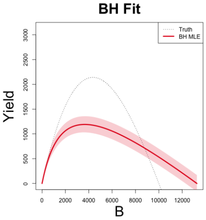
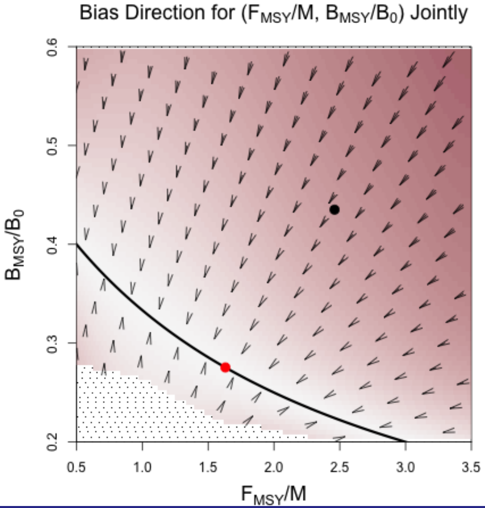
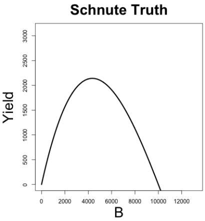
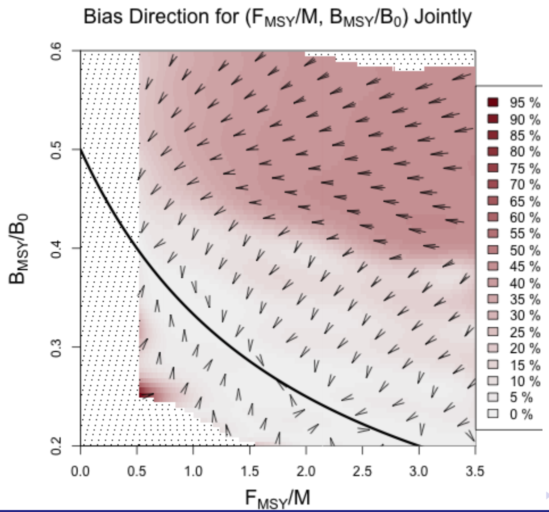


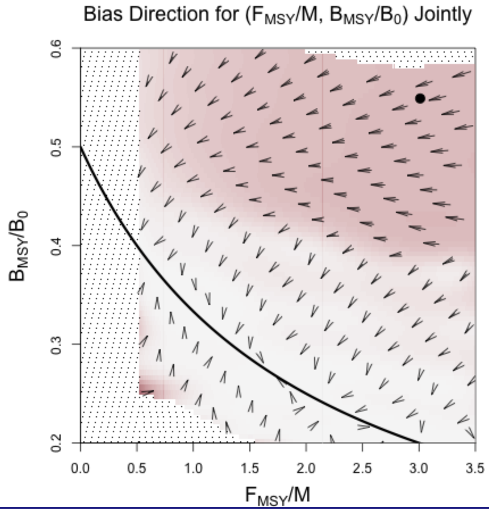


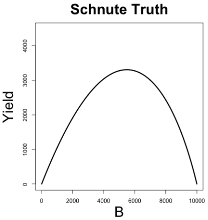


Low Contrast



Low Contrast





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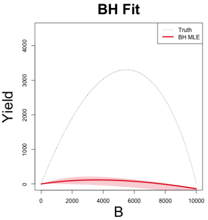

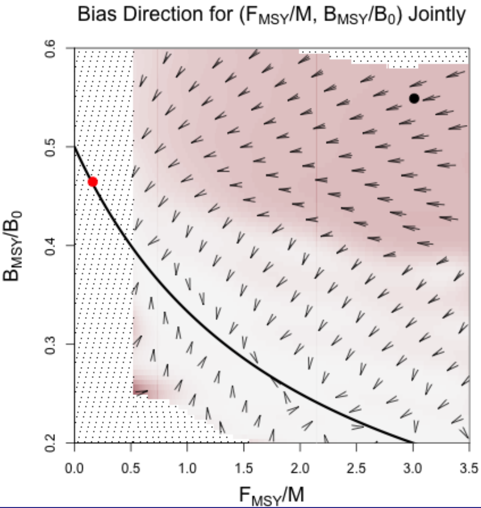

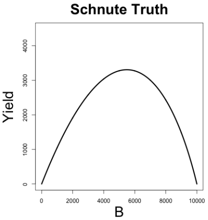

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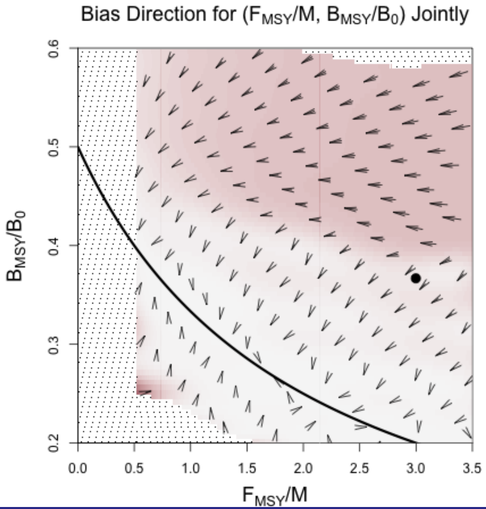

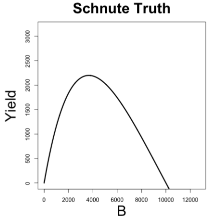

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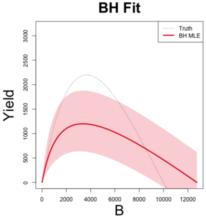

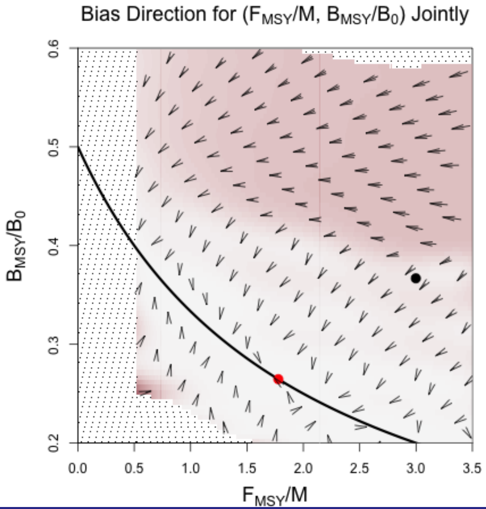

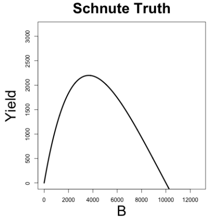

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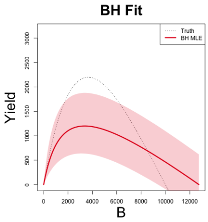
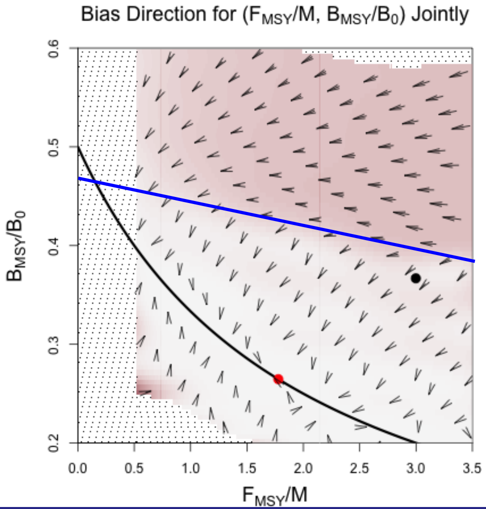
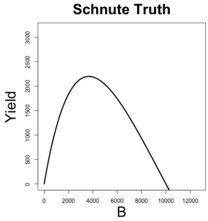






Conclusions

- A rich simulation-based method for describing global RP bias and a stepping stone for understanding more complex models.
  - $\Rightarrow$ Individual growth and maturity dynamics
- RPs are not directly observable quantities, but rather model dependent latent quantities.
  - $\Rightarrow$ Subject to Model Misspecification, Uncertainty, & Bias
  - $\Rightarrow$ In severly constrained settings we pay for our modeling mistakes primarily in estimate bias.
- The observed contrast serves to increase the range of potentially "allowable" model misspecification.

Many Thanks:

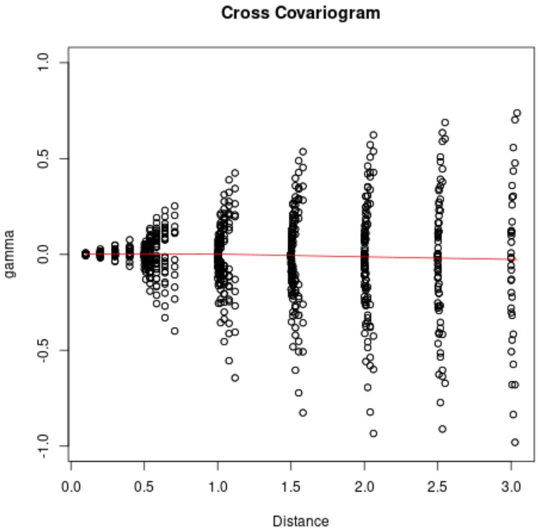
- UCSC Advisors
- SWFSC Groundfish
- NMFS Sea Grant

Metamodel Details

$$\boldsymbol{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

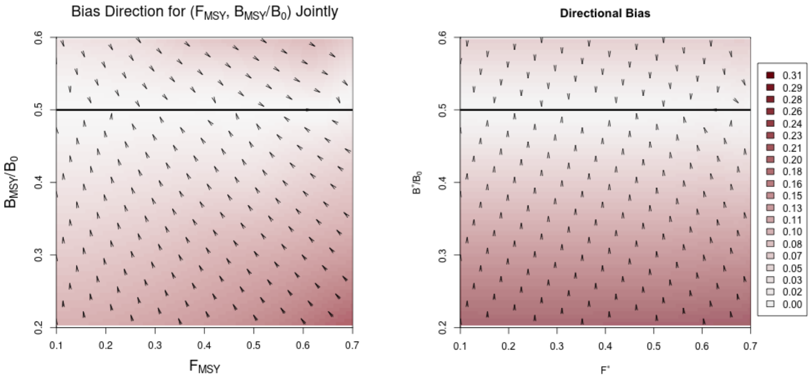
$$\begin{aligned}
\hat{\mu} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon \\
f(\boldsymbol{x}) &\sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}')) \\
\epsilon_i &\sim \mathrm{N}(0, \hat{\omega}_i).
\end{aligned}$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$


Cross Covariogram
gamma
1.0
0.5
0.0
-0.5
-1.0
Distance
0.0
0.5
1.0
1.5
2.0
2.5
3.0

PT Data Fit with the Schaefer Model

Low Contrast High Contrast



Schnute RP-Parameter System of Equations

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M+F_{MSY}}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}$$

$$\alpha = (M + F_{MSY}) \left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau+1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$