Shepherd

4/17/21

$$R(P) = \frac{\alpha P}{1+\beta P^{1/\gamma}}$$

$$\frac{dP}{dt} = \frac{\alpha P}{1+\beta P^{1/\gamma}} - (M+F)P$$

set 0

$$\cancel{(M+F)\bar{P}} = \frac{\alpha \cancel{\bar{P}}}{1+\beta \bar{P}^{1/\gamma}}$$

$$1+\beta\bar{P}^{1/\gamma} = \frac{\alpha}{M+F}$$

$$\bar{P}^{1/\gamma} = \left(\frac{\alpha}{M+F} - 1\right)\frac{1}{\beta}$$

$$\bar{P} = \left(\frac{\alpha}{M+F} - 1\right)^{\gamma} \beta^{-\gamma}$$

$$\frac{d\bar{P}}{dF} = \frac{1}{\beta^{\gamma}} \frac{d}{dF}\left(\frac{\alpha}{M+F} - 1\right)^{\gamma}$$

$$= \beta^{-\gamma}\gamma\left(\frac{\alpha}{M+F} - 1\right)^{\gamma-1}\left[\frac{d}{dF}\alpha(M+F)^{-1}\right]$$

$$= -\alpha\gamma\beta^{-\gamma}\left(\frac{\alpha}{M+F} - 1\right)^{\gamma-1}(M+F)^{-2}$$

$$F_{MSY} = \underset{F}{\text{ArgMax}}\ \bar{Y} = \underset{F}{\text{ArgMax}}\ F\bar{P}(F)$$

$$\frac{d\bar{Y}}{dF} = \bar{P}(F) + F\frac{d\bar{P}}{dF}$$

$$= \cancel{\left(\frac{\alpha}{M+F} - 1\right)^{\gamma}\beta^{-\gamma}} - \alpha\gamma F\cancel{\beta^{-\gamma}}\frac{\cancel{\left(\frac{\alpha}{M+F} - 1\right)^{\gamma}}}{\left(\frac{\alpha}{M+F} - 1\right)}(M+F)^{-2}$$

set

$$0 = 1 - \frac{\alpha\gamma F}{\left(\frac{\alpha}{M+F} - 1\right)(M+F)^2} \Rightarrow F^{*}$$

* A quadratic Solution Exists, but... Just use a Rootfinging Numerical Alg.

$$P^{*} = \bar{P}(F^{*})\ ;\ P_0 = \bar{P}(0)$$

* Note F^{*} Does Not Depend on β.

↳ For Known Fixed P_0 ; $\beta|\alpha,\gamma$ is Fully Determined.