Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

13 August 2024

Outline

1 Introduction

2 The Schaefer Model

3 The Beverton-Holt Model

4 Delay Differential Growth Extension

5 End

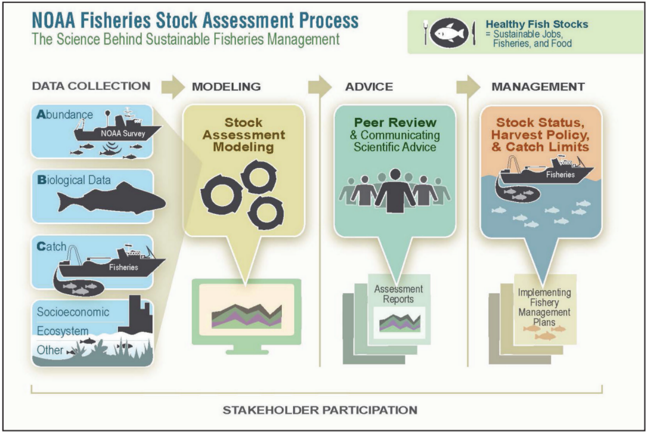


Figure 1: Overview of the stock assessment process from data collection through the provision of scientific advice to fishery managers. Stakeholders and other partners participate in each step of the assessment process. This report captures NOAA Fisheries products associated with the 'Advice' phase of the process.

*

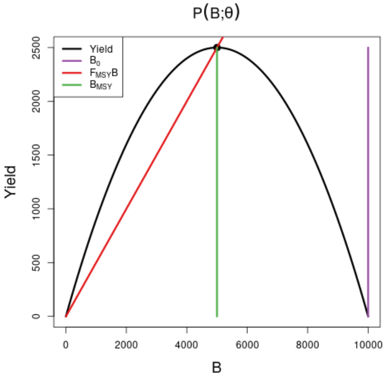
Surplus Production Model General Structure

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2).$$

$$\frac{dB}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t).$$

Reference Points:

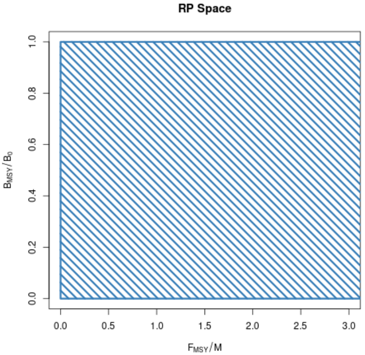
- Maximum sustainable Yield (MSY)
- F_{MSY}[a]: Fishing rate to achieve MSY
- $\frac{B_{MSY}}{B_0}$: Biomass Depletion when at MSY
- Driven by the shape of P as determined by θ.



[a] or $\frac{F_{MSY}}{M}$

Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0,1)$$



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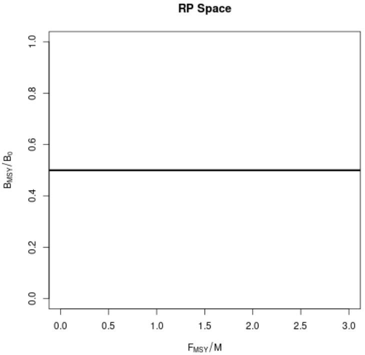
- Schaefer Model:
  $F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{2}$



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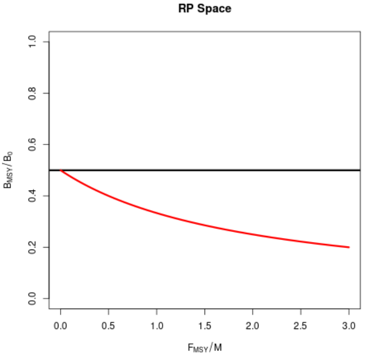
- BH Model:
  $\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$



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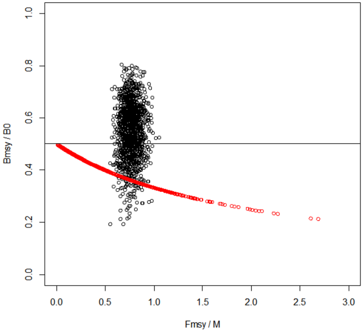
  $\frac{F_{MSY}}{M} \in \mathbb{R}^+ \qquad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Models:
  Fox, Ricker, etc...
- Three-Parameter Models Allow Independent RP Estimation



[a]

[a] Mangel et al. 2013, CJFAS

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- P_ℓ is logistic production
- Logistic map in discrete time
- Implicit Natural Mortality
- Explicit Fishing Mortality

$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_\ell(B(t); \boldsymbol{\theta}) - F(t)B(t)$$

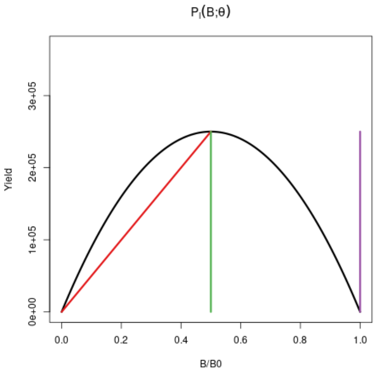
$$P_\ell(B;[r,K]) = rB\left(1-\left(\frac{B}{K}\right)\right)$$

Reference Points:

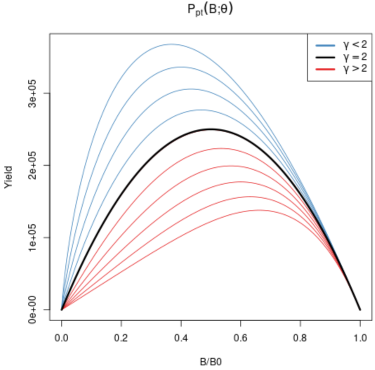
$$F^* = \frac{r}{2}$$

$$B^* = \frac{K}{2} \qquad B_0 = K$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$



$$P_{pt}(B;[r,K,\gamma]) = \frac{rB}{\gamma-1}\left(1-\left(\frac{B}{K}\right)^{(\gamma-1)}\right)$$



Reference Points:

$$F^* = \frac{r}{\gamma}$$

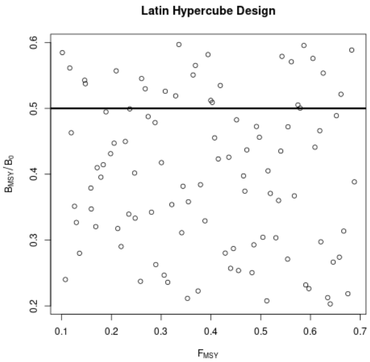
$$B^* = K\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \quad B_0 = K$$

$$\frac{B^*}{B_0} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$F^* = \frac{r}{\gamma} \qquad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

Closed-Form Inversion

$$r = \gamma F^* \quad \gamma = \frac{W\left(\frac{B^*}{\bar{B}(0)} \log\left(\frac{B^*}{\bar{B}(0)}\right)\right)}{\log\left(\frac{B^*}{\bar{B}(0)}\right)}$$



* Lambert W function inverts xe^x $s.t.$ $W(xe^x) = x$

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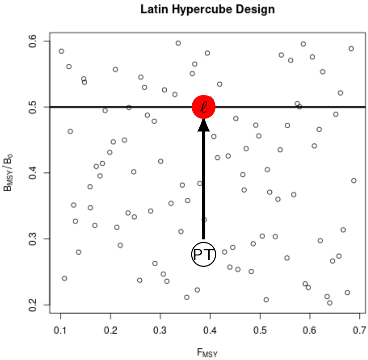
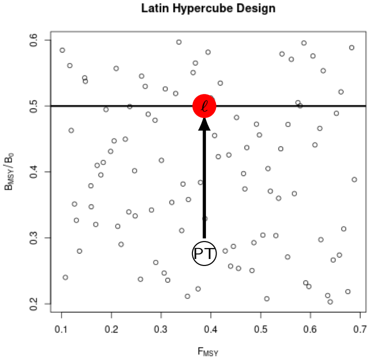
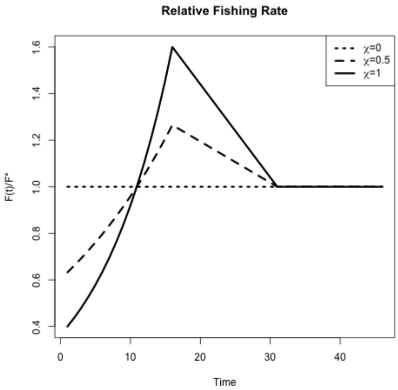


* Lambert W function inverts xe^x $s.t.$ $W(xe^x) = x$

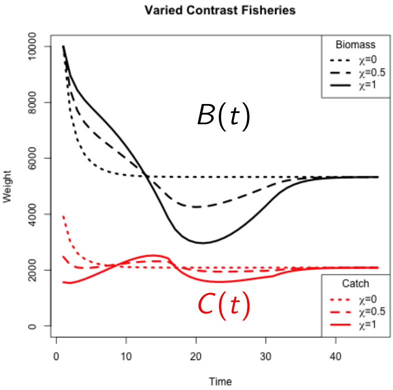
$$\underbrace{\left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{PT Truth}} \overset{GP}{\mapsto} \underbrace{\left(\hat{F}_{MSY}, \frac{1}{2}\right)}_{\text{Shaefer Estimate}}$$

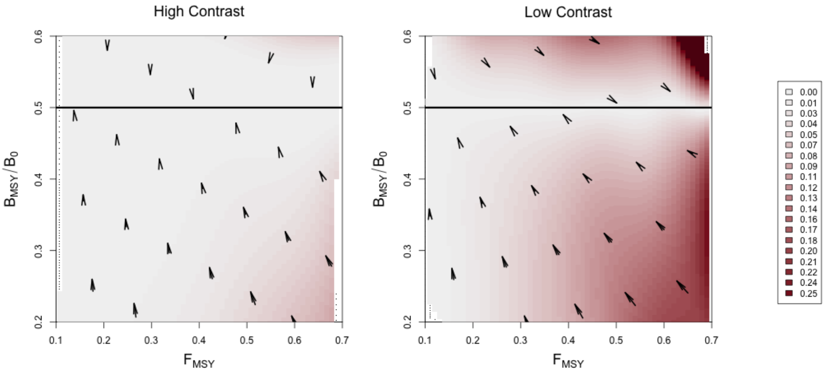


$$\frac{dB}{dt} = P(B(t);\theta) - F(t)B(t)$$
$$= P(B(t);\theta) - F^*_\theta \underbrace{\frac{F(t)}{F^*_\theta}}_{\substack{\text{Relative}\\ \text{Fishing}}} B(t)$$



$$\frac{dB}{dt} = P(B(t);\theta) - F(t)B(t)$$
$$= P(B(t);\theta) - \underbrace{F^*_\theta \frac{F(t)}{F^*_\theta} B(t)}_{C(t)}$$





High Contrast
Low Contrast
B_{MSY}/B_0
F_{MSY}
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.00
0.01
0.03
0.04
0.05
0.07
0.08
0.09
0.11
0.12
0.13
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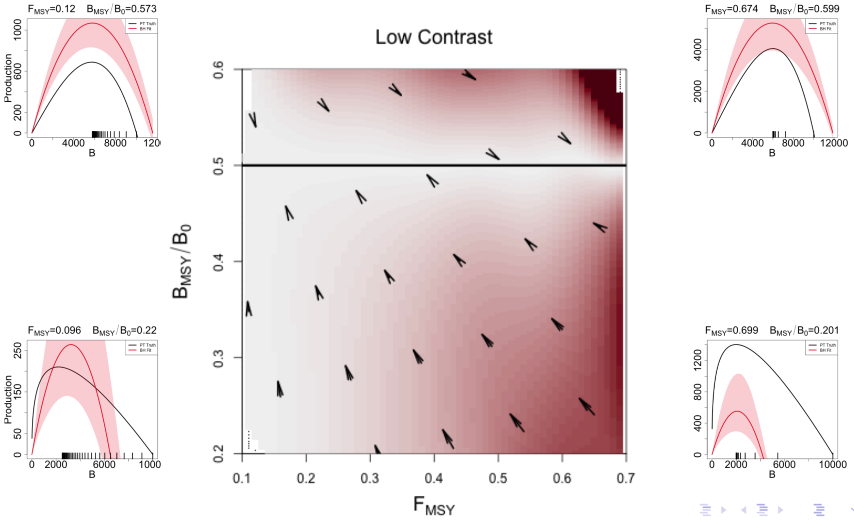
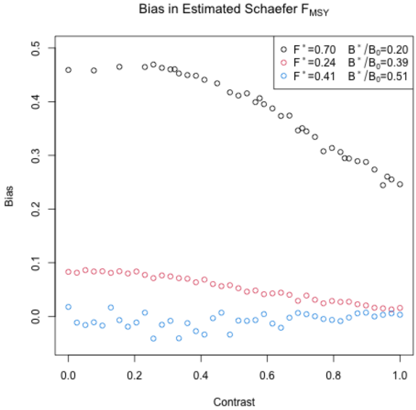
0.21
0.22
0.24
0.25

Mechanism for Bias in F_{MSY} via Contrast



Mechanism for Bias in F_{MSY} via Contrast

- Only observe upper half
- learn about slope at origin from upper biomass range
- as contrast increases biases perspective deminishes



Summary

- A quick summary
- just a few points will suffice
- the quicker and more concise the better
- transition to BH

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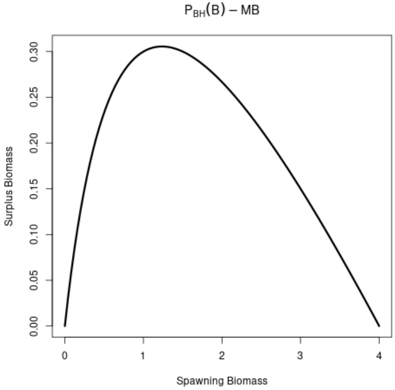
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$
$$\frac{dB}{dt} = P_{BH}(B; [\alpha, \beta]) - (M + F)B$$

$$P_{BH}(B; [\alpha, \beta]) = \frac{\alpha B}{1 + \beta B}$$

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \underbrace{P_{BH}(B;[\alpha,\beta]) - MB}_{\text{Surplus Production}} - FB$$

$$P_{BH}(B;[\alpha,\beta]) = \frac{\alpha B}{1+\beta B}$$



$$P_s(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

Logistic

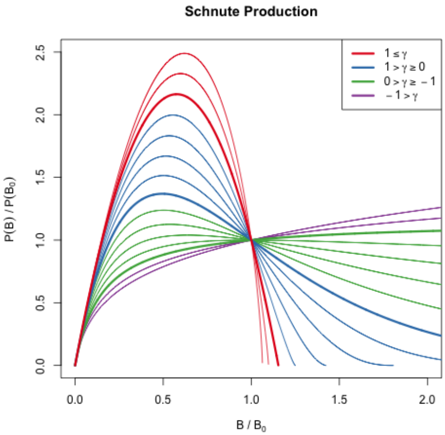
$\gamma = 1$

Ricker

$\gamma \to 0$

Beverton-Holt

$\gamma = -1$



$$\text{RPs} \mapsto \theta$$

Incomplete
Inversion

$$\theta \mapsto \text{RPs}$$

$$\alpha = (M + F^*)\left(1 + \frac{\gamma F^*}{M + F^*}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}.$$

Schnute & Richards (1998). CJFAS.

$$\text{RPs} \mapsto \theta$$

Incomplete Inversion

$$\theta \mapsto \text{RPs}$$

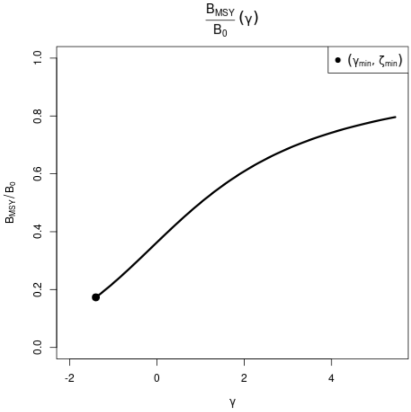
$$\alpha(\gamma)$$

$$\beta(\alpha(\gamma), \gamma)$$

$$\frac{B^*}{B_0}(\alpha(\gamma), \gamma)$$

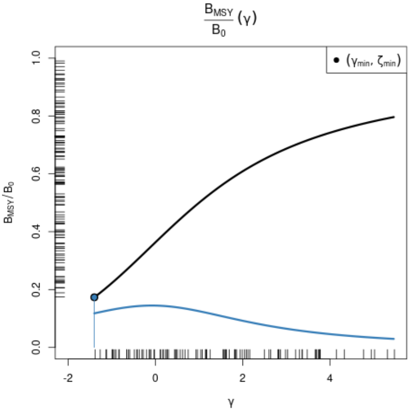
Schnute & Richards (1998). CJFAS.

$$\frac{B^*}{B_0}(\gamma) = \frac{1 - \left(\frac{M+F^*}{\alpha(\gamma)}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha(\gamma)}\right)^{\gamma}}$$



$$\frac{B^*}{B_0}(\gamma) = \frac{1 - \left(\frac{M+F^*}{\alpha(\gamma)}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha(\gamma)}\right)^{\gamma}}$$

$$\gamma' \sim \zeta_{min}\delta(\gamma_{min}) + (1-\zeta_{min})t(\mu,\sigma,\nu)\mathbf{1}_{\gamma>\gamma_{min}}$$



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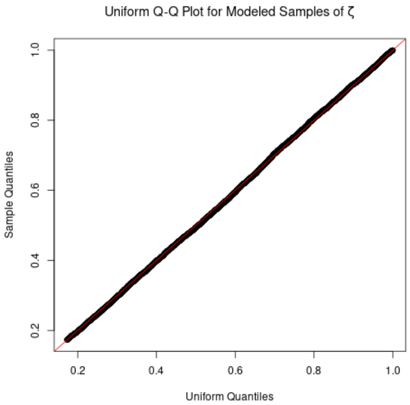


Schnute LHS Design

Logistic

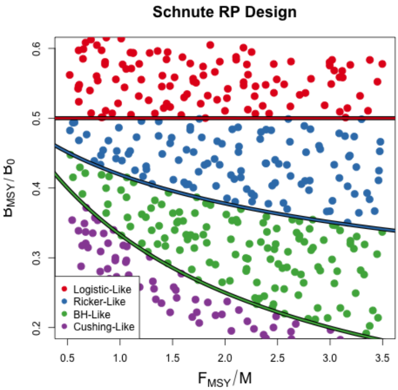
$\gamma = 1$

Ricker

$\gamma \to 0$

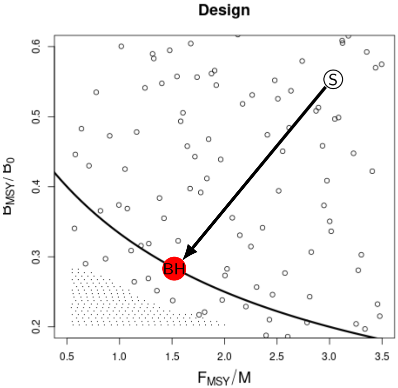
Beverton-Holt

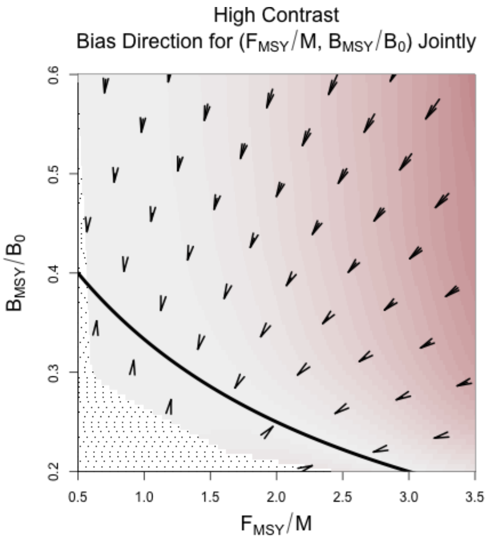
$\gamma = -1$



Schnute LHS Design

$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{GP}{\mapsto} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{1}{\hat{F}_{MSY}/M+2}\right)}_{\text{BH Estimate}}$$




High Contrast
Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
0.2
0.3
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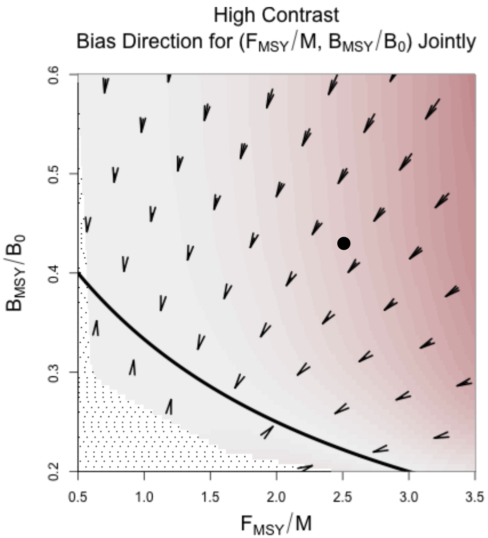
F_{MSY}/M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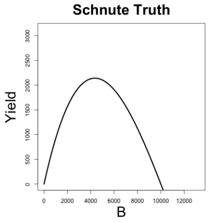
F_{MSY}/M


Schnute Truth
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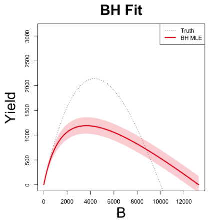
BH Fit
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BH MLE
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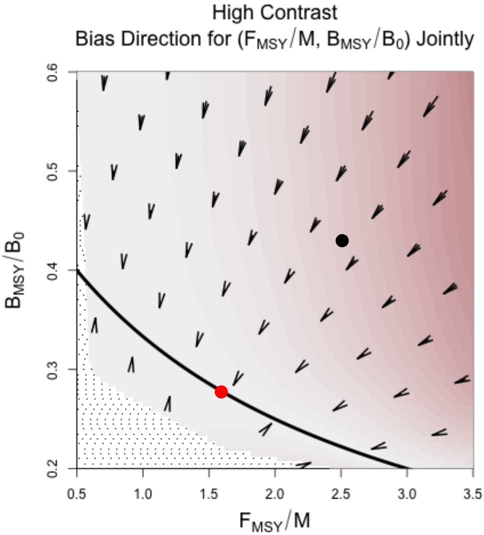
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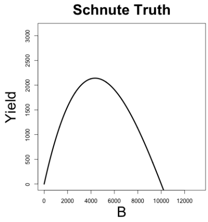
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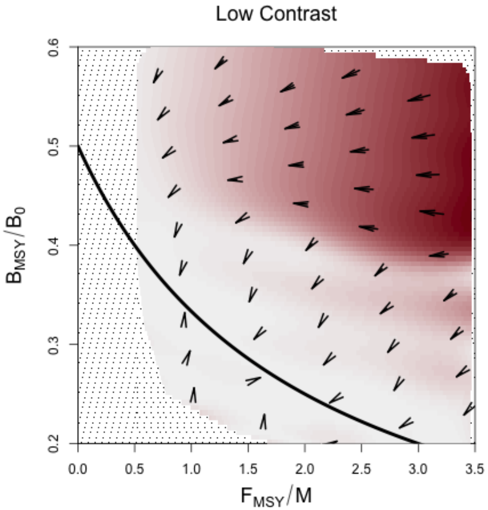
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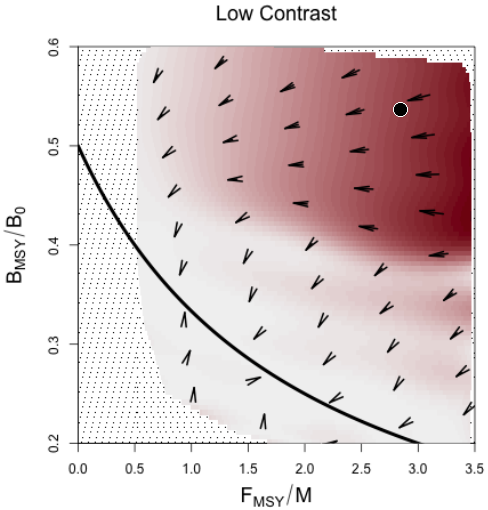
F_{MSY}/M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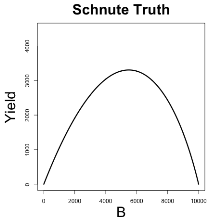
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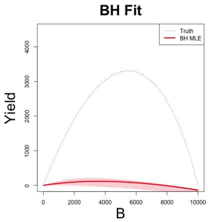
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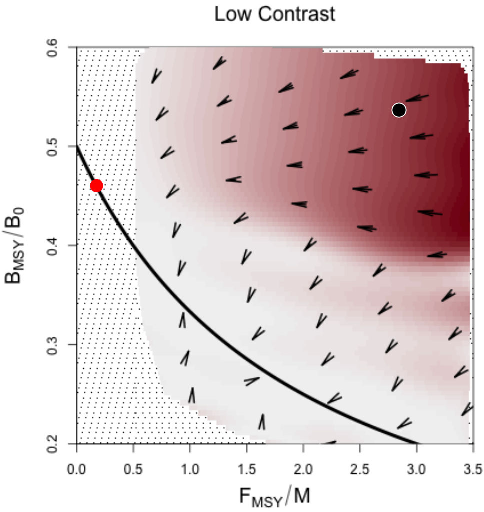
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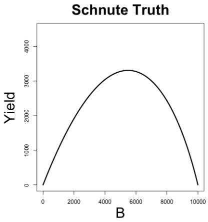
F_{MSY}/M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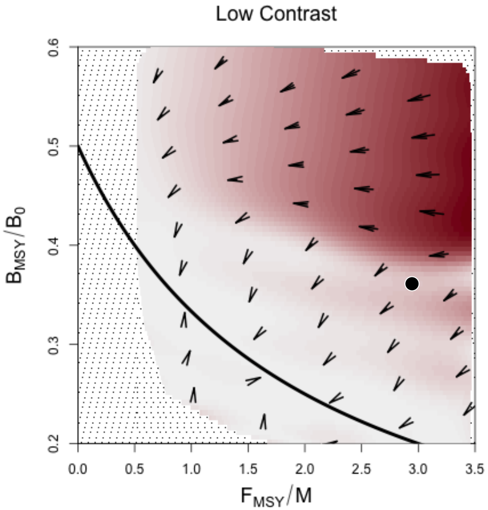
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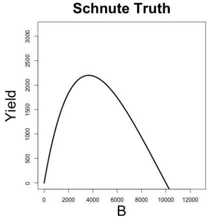
F_{MSY}/M


Schnute Truth
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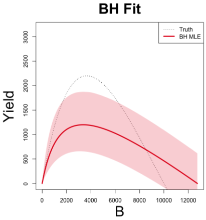
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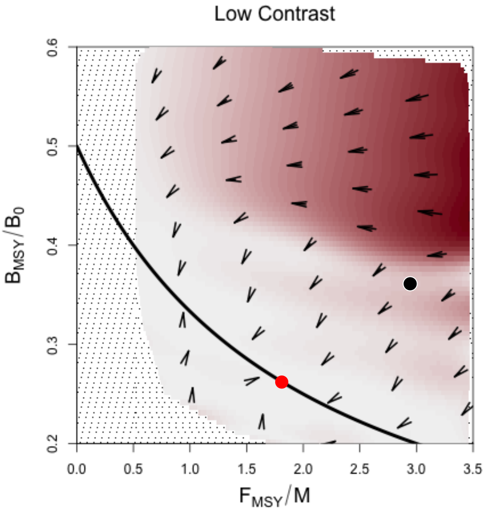
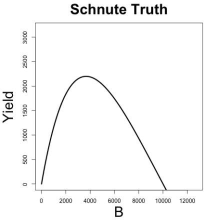
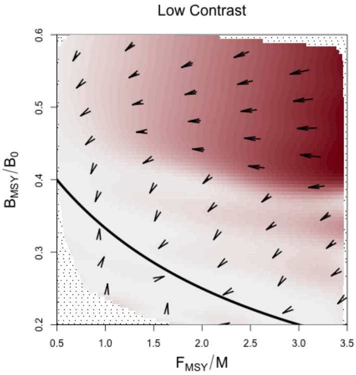
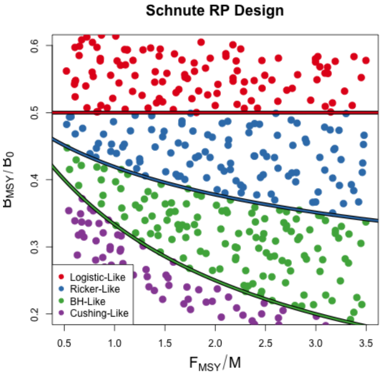

Low Contrast
B_{MSY}/B_0
F_{MSY}/M


Schnute Truth
Yield
B


Low Contrast
0.2
0.3
0.4
0.5
0.6
B_{MSY}/B_0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M


Schnute RP Design
0.2
0.3
0.4
0.5
0.6
B_{MSY}/B_0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M
Logistic-Like
Ricker-Like
BH-Like
Cushing-Like

Quick Summary and Transition

Outline

1 Introduction

2 The Schaefer Model

3 The Beverton-Holt Model

4 Delay Differential Growth Extension

5 End

General Modeling Structure

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \overbrace{w(a_s)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

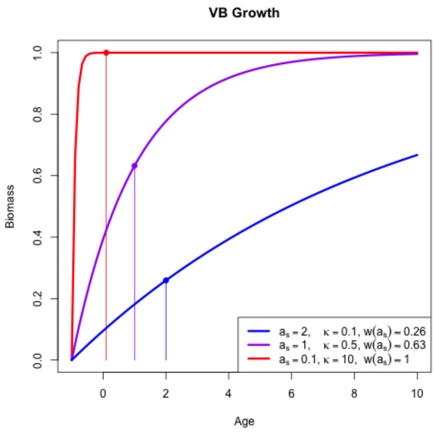
Individual Growth

$$w(a) = w_\infty(1 - e^{-\kappa(a-a_0)})$$

a_s : Lagged Maturity & Knife-Edge Selectivity

κ : Individual Growth

- Instant Growth: (Production Model)
  $a_s \to 0 \quad \kappa \to \infty$



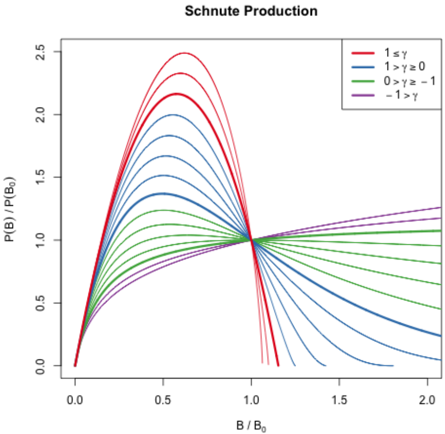
Schnute Recruitment

$$R(B;\alpha,\beta,\gamma) = \alpha B_{t-a_s}(1-\beta\gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic
$\gamma = 1$

Ricker
$\gamma \to 0$

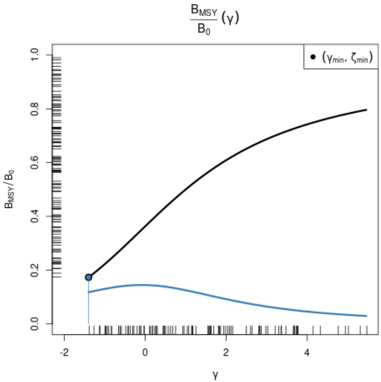
Beverton-Holt
$\gamma = -1$

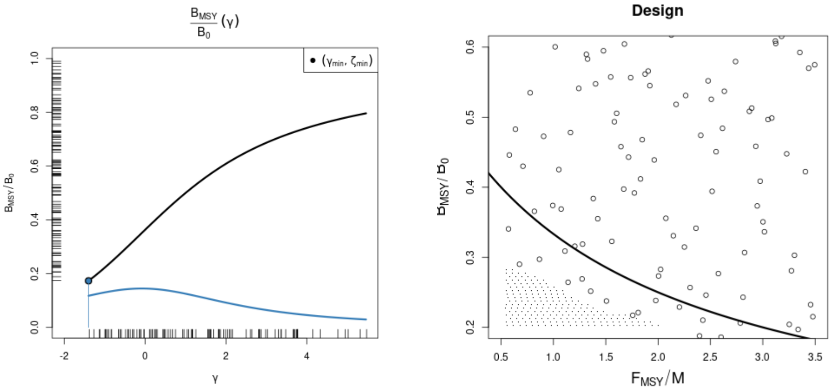


$$\alpha(\gamma) : \alpha = \left[\left(\frac{Z^*(Z^* + \kappa)}{w(a_s)(Z^* + \frac{\kappa w_\infty}{w(a_s)})} \right)^{\gamma} + \left(\frac{\gamma F^*}{w(a_s)} \right) \left(\frac{Z^*(Z^* + \kappa)}{w(a_s)(Z^* + \frac{\kappa w_\infty}{w(a_s)})} \right)^{\gamma - 1} \left(1 + \frac{\left(\frac{\kappa w_\infty}{w(a_s)}\right)\left(\kappa - \frac{\kappa w_\infty}{w(a_s)}\right)}{(Z^* + \frac{\kappa w_\infty}{w(a_s)})^2} \right) \right]^{\frac{1}{\gamma}}$$

$$\beta(\alpha(\gamma), \gamma) : \quad \beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M(M + \kappa)}{\alpha w(a_s)(M + \frac{\kappa w_\infty}{w(a_s)})} \right)^{\gamma} \right)$$

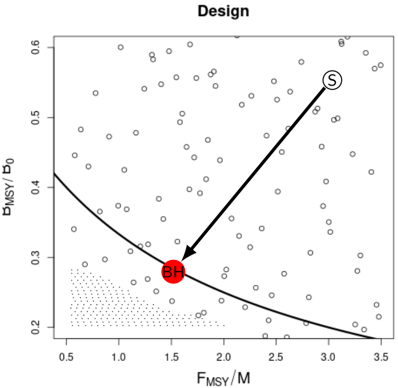
$$\frac{B^*}{B_0}(\alpha(\gamma), \gamma) : \quad \frac{B^*}{B_0} = \frac{1 - \left(\frac{(F^* + M)(F^* + M + \kappa)}{\alpha w(a_s)(F^* + M + \frac{\kappa w_\infty}{w(a_s)})} \right)^{\gamma}}{1 - \left(\frac{M(M + \kappa)}{\alpha w(a_s)(M + \frac{\kappa w_\infty}{w(a_s)})} \right)^{\gamma}}$$


$\frac{B_{MSY}}{B_0}(\gamma)$
$(\gamma_{min}, \zeta_{min})$
B_{MSY}/B_0
0.0
0.2
0.4
0.6
0.8
1.0
-2
0
2
4
γ

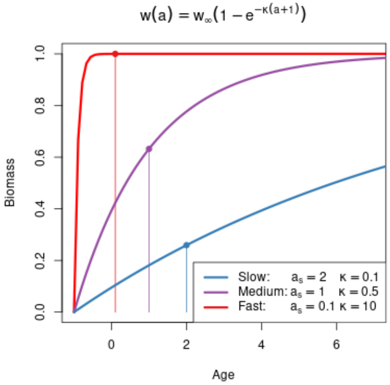


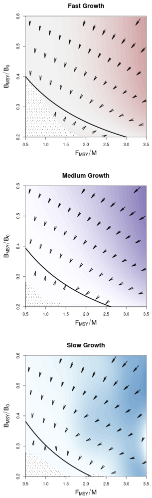
BMSY/B0 (γ)
(γmin, ζmin)
BMSY/B0
γ
-2 0 2 4
0.0 0.2 0.4 0.6 0.8 1.0
Design
BMSY/B0
FMSY/M
0.5 1.0 1.5 2.0 2.5 3.0 3.5
0.2 0.3 0.4 0.5 0.6

$$\underbrace{\left(\frac{F^*}{M}, \frac{B^*}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{\text{GP}}{\mapsto} \underbrace{\left(\frac{\hat{F}^*}{M}, \frac{B^*}{B_0}(-1; \hat{F}^*)\right)}_{\text{BH Estimate}}$$

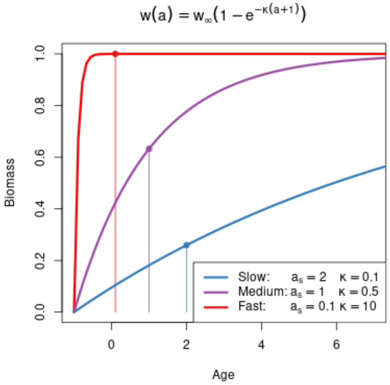


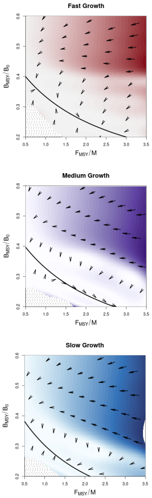
High Contrast



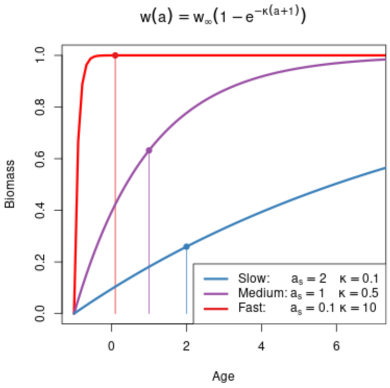


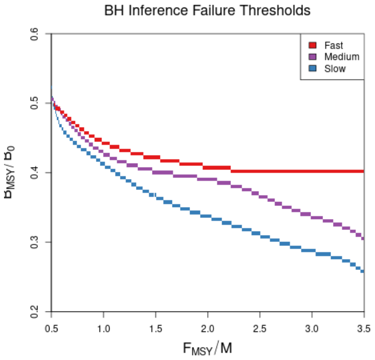
Low Contrast



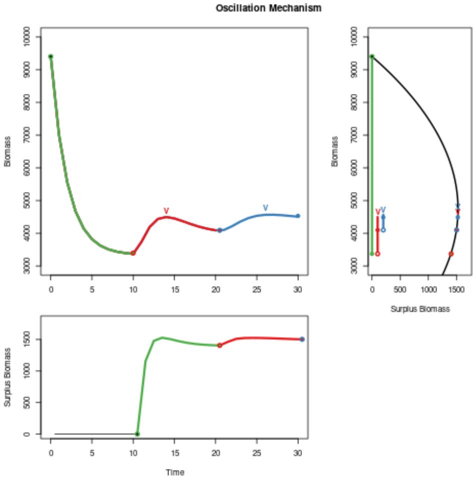


Low Contrast





Oscillation

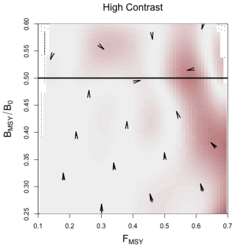


General Conclusions and future work.

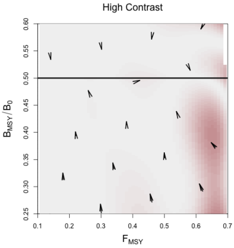
Many Thanks:

- UCSC Advisors
- Collaborators at NOAA
- NMFS Sea Grant

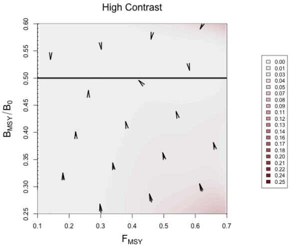
High Contrast PT $\sigma = 0.12$ Data



1x Samples



2x Samples



4x Samples

Metamodel Details

$$\mathbf{y} = \widehat{log(F_{MSY})} \quad - or - \quad \mathbf{y} = \widehat{log(B_0)}$$

$$\boldsymbol{x} = \left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)$$

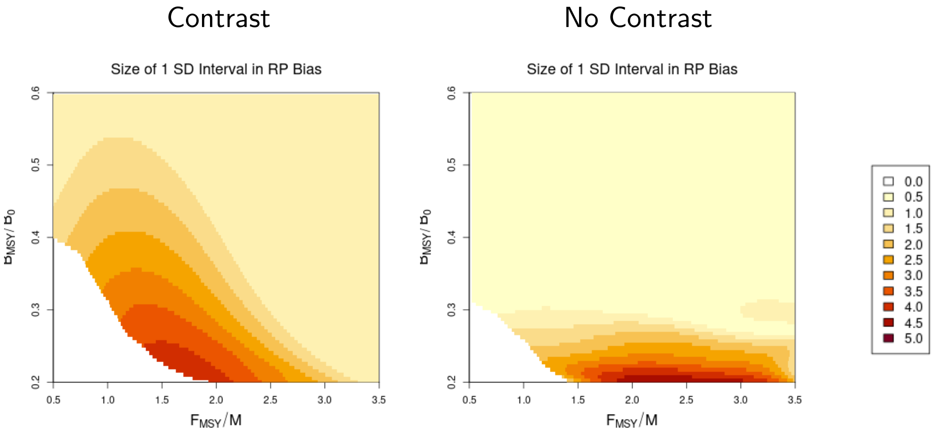
$$\begin{aligned} \mathbf{y} &= \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \boldsymbol{\epsilon} \\ f(\boldsymbol{x}) &\sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x'})) \\ \boldsymbol{\epsilon} &\sim \mathrm{N}(0, \boldsymbol{\omega}'\boldsymbol{I}). \end{aligned}$$

$$R(\boldsymbol{x}, \boldsymbol{x'}) = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2}\right)$$

Prediction

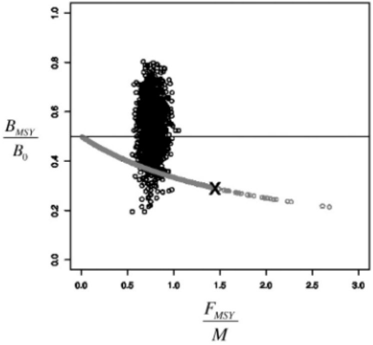
$$\hat{y}(\mathbf{x}^\star) = \beta_0 + \mathbf{x}^\star\boldsymbol{\beta} + \mathbf{r}(\mathbf{x}^\star)'\boldsymbol{R}_\ell^{-1}\Big(\mathbf{y} - \big(\beta_0 + \boldsymbol{X}\beta\big)\Big)$$

$$\hat{\sigma}^2(\mathbf{x}^\star) = \mathbf{R}(\mathbf{x}^\star, \mathbf{x}^\star) - \mathbf{r}(\mathbf{x}^\star)'\boldsymbol{R}_\ell^{-1}\mathbf{r}(\mathbf{x}^\star)$$


Contrast
No Contrast
Size of 1 SD Interval in RP Bias
Size of 1 SD Interval in RP Bias
B_{MSY}/B_0
F_{MSY}/M
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0

Mangel et al.

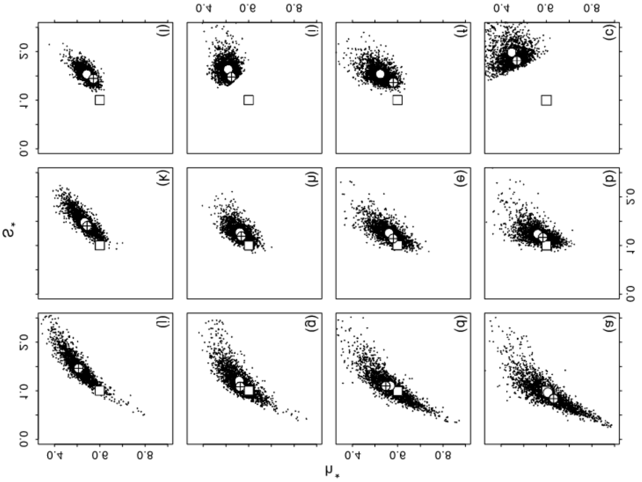
Fig. 4. DeYoreo et al. (2012) used both a BH-SRR and three-parameter SRR, similar to the S-SRR in a stock assessment of cowcod (*Sebastes levis*). We show samples from posterior distributions arising from different values of steepness. Unlike most stock assessments, we plot B_{MSY}/B_0 versus F_{MSY}/M. The grey circles show the results for the BH-SRR. This curve is another way of representing the constraint placed on a stock assessment by using a BH-SRR and specifying steepness — results must lie along this curve. The black circles represent the outcome of the three-parameter SRR. The black X represents the result when steepness is asserted to be 0.6.



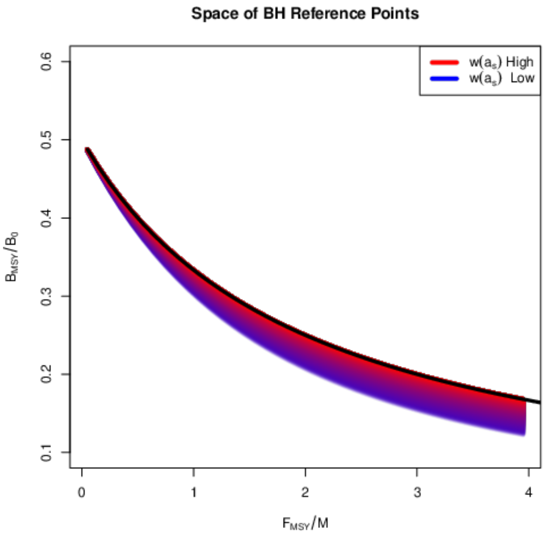
Logistic

Ricker

BH



Schnute, J. T., & Kronlund, A. R. (2002). Estimating salmon stock recruitment relationships from catch and escapement data. Canadian Journal of Fisheries and Aquatic Sciences, 59(3), 433–449.


Space of BH Reference Points
w(a_s) High
w(a_s) Low
0.1
0.2
0.3
0.4
0.5
0.6
B_MSY/B_0
0
1
2
3
4
F_MSY/M