# Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

In collaboration with:
Dr. E.J. Dick
Dr. H. K.H. Lee

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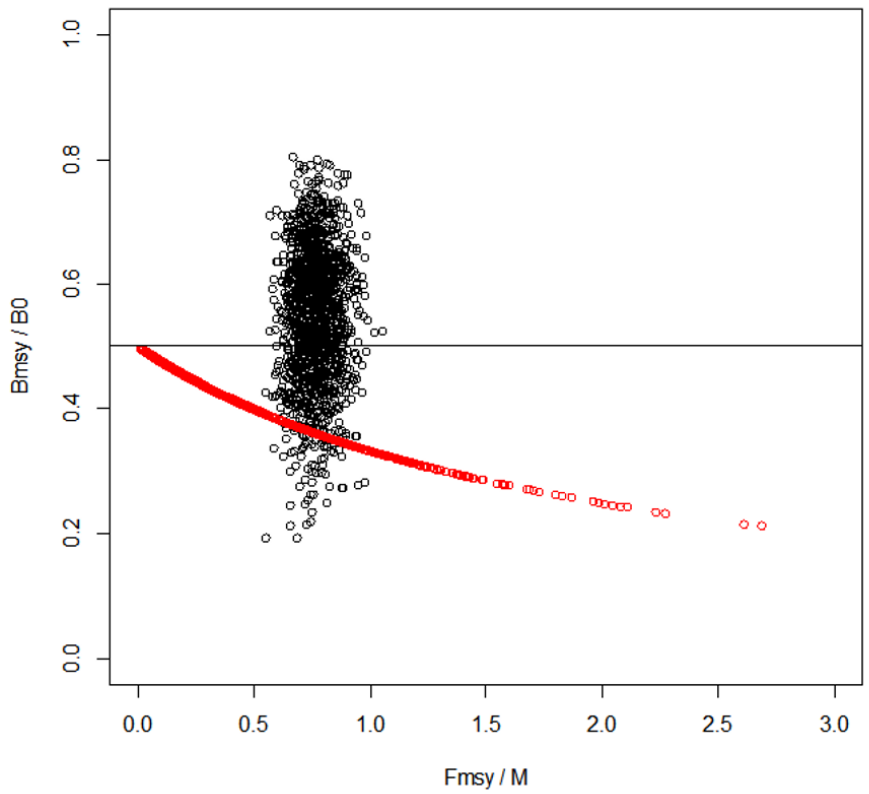
# Mangel et al. 2013, CJFAS

$$\frac{dB(t)}{dt} = \frac{\alpha B(t)}{1+\beta B(t)} - (M + F(t))B(t)$$

$$h = \frac{\frac{\alpha}{M}}{4 + \frac{\alpha}{M}}$$

$$\frac{F^*}{M} = \sqrt{\frac{4h}{1-h}} - 1$$

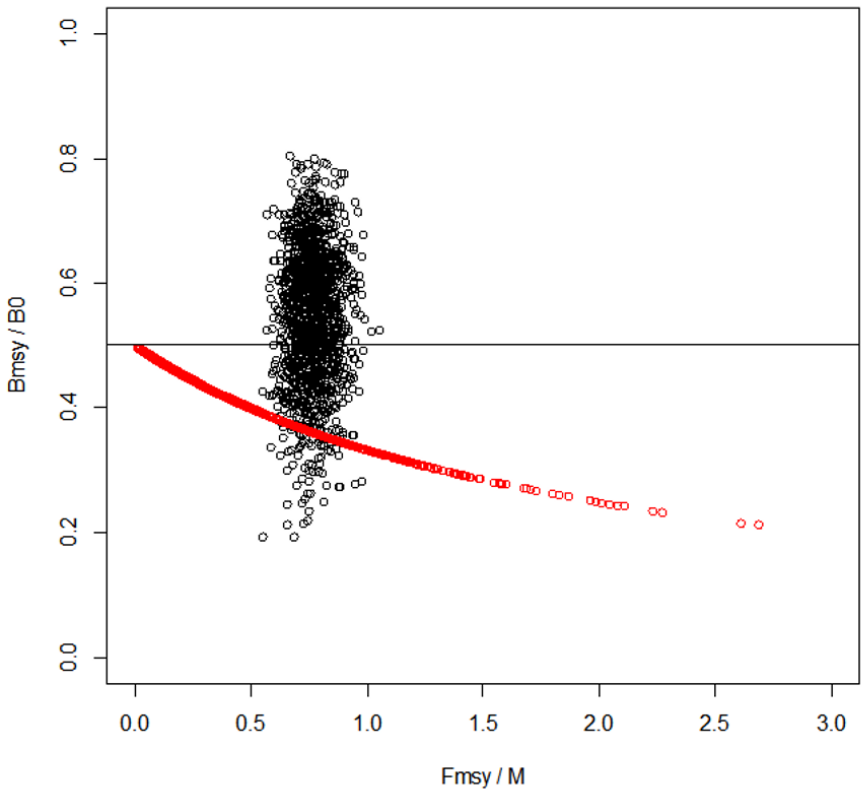
$$\frac{B^*}{B_0} = \frac{\sqrt{\frac{4h}{1-h}} - 1}{\frac{4h}{1-h} - 1}$$



# Mangel et al. 2013, CJFAS

$$\frac{dB(t)}{dt} = \frac{\alpha B(t)}{1 + \beta B(t)^{\frac{1}{\gamma}}} - (M + F(t))B(t)$$

Mangel et al. (2013) suggest exploration of three parameter stock recruit relationships (SRRs) to avoid pre-determined reference points (RP) in assessments



# A'Priori RP Prior Relationships

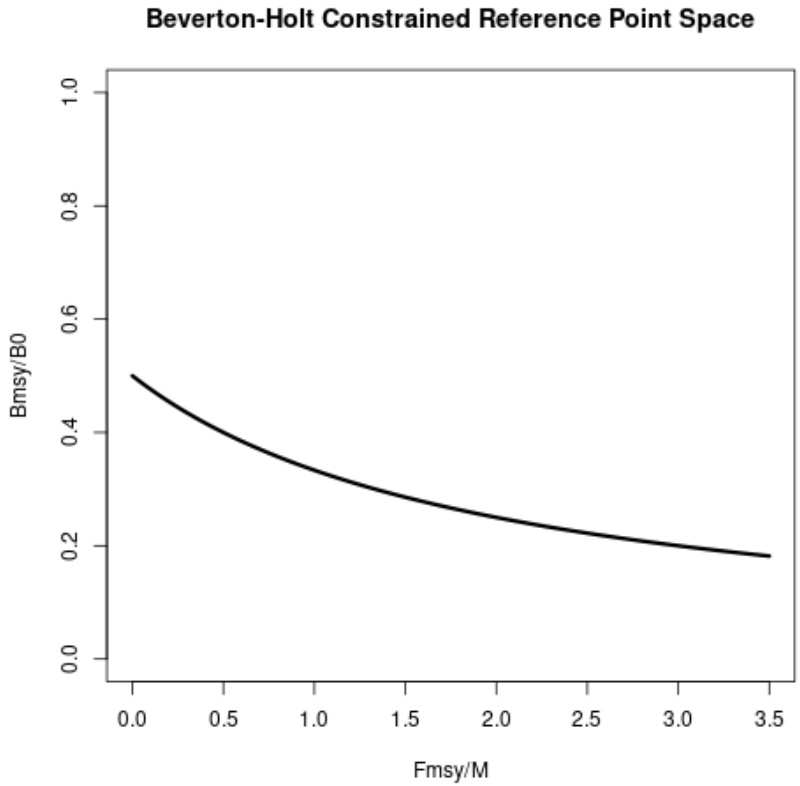
$$\frac{dB(t)}{dt} = \frac{\alpha B(t)}{1+\beta B(t)} - (M + F(t))B(t)$$

$$\frac{B^*}{B_0} = \frac{1}{\frac{F^*}{M} + 2}$$

$$log(F^*) \sim N(\mu, \sigma^2)$$

$$\Updownarrow$$

$$2\frac{B^*}{B_0} \sim \text{logit-N}\left(\log(2M) - \mu, \sigma^2\right)$$



# Pella-Tomlinson Production Model

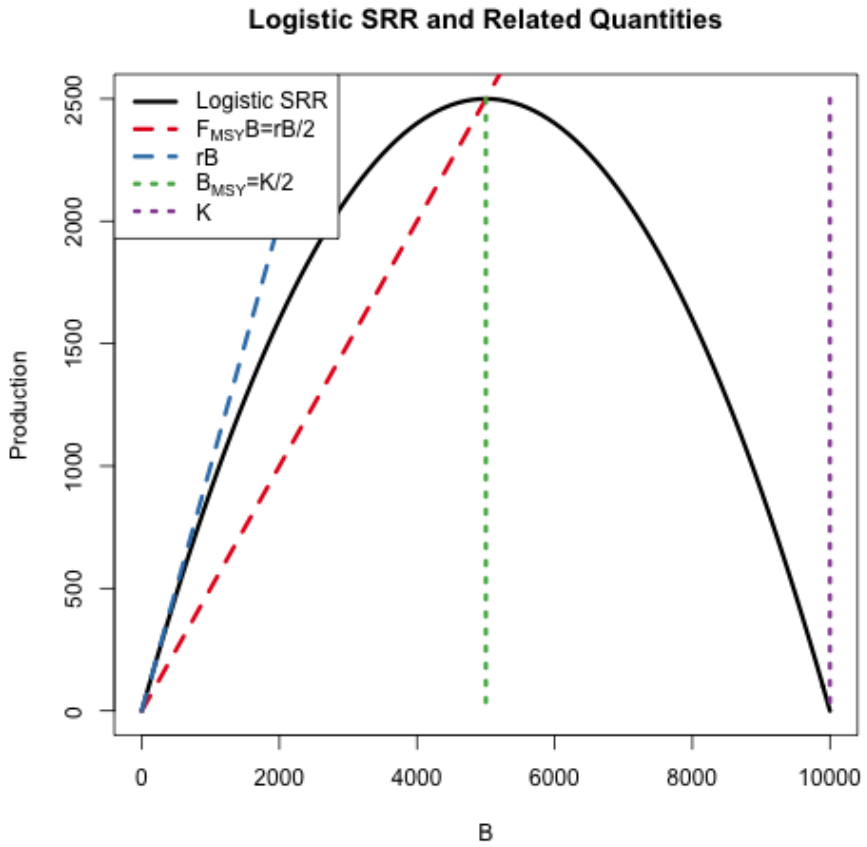
$$I(t) \sim LN(qB(t), \sigma^2)$$

$$\frac{dB(t)}{dt} = R_{\boldsymbol{\theta}}(B(t)) - F(t)B(t)$$

$$R_{\boldsymbol{\theta}}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma - 1}$$

$$\boldsymbol{\theta} = (r, K, \gamma)$$

$$\gamma = 2 \Rightarrow \text{Schaefer Model}$$



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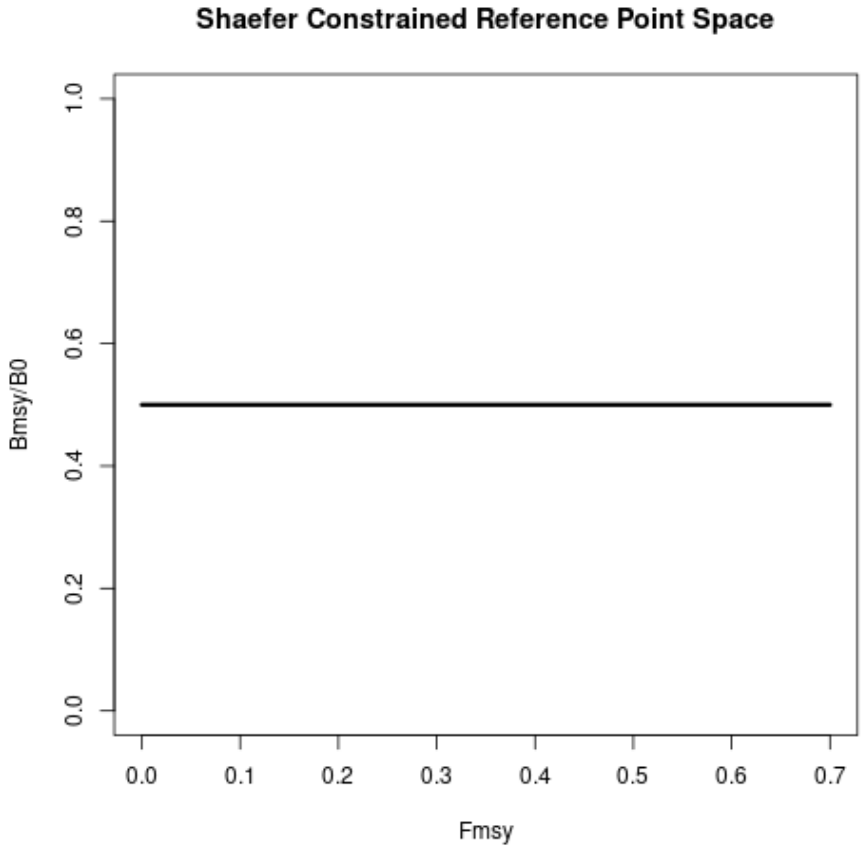
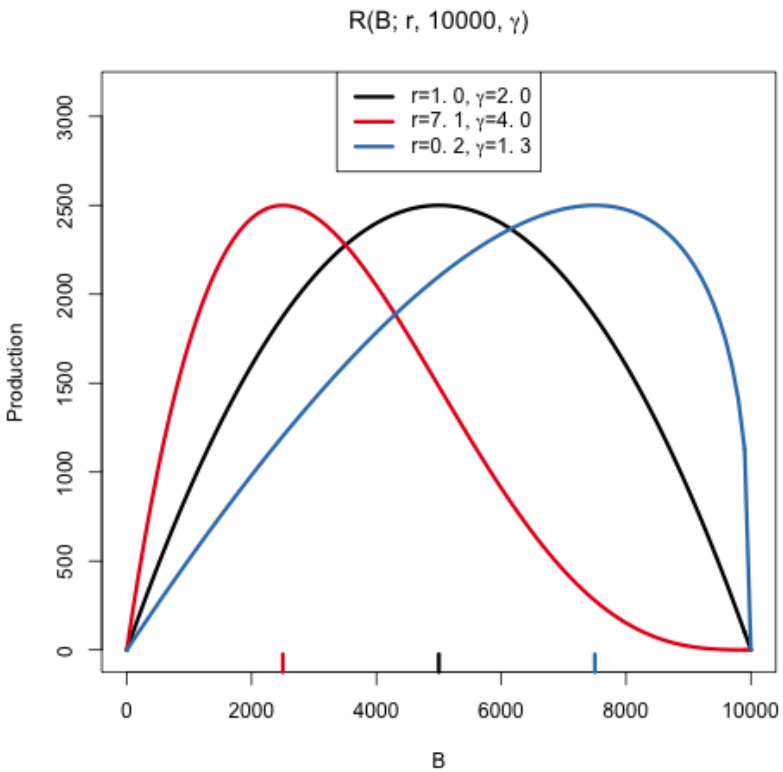
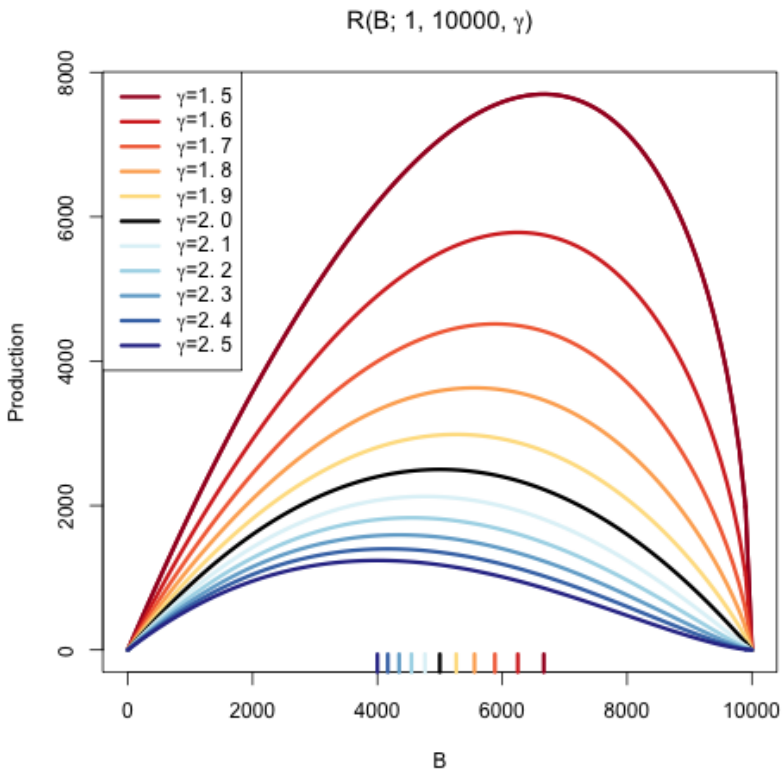
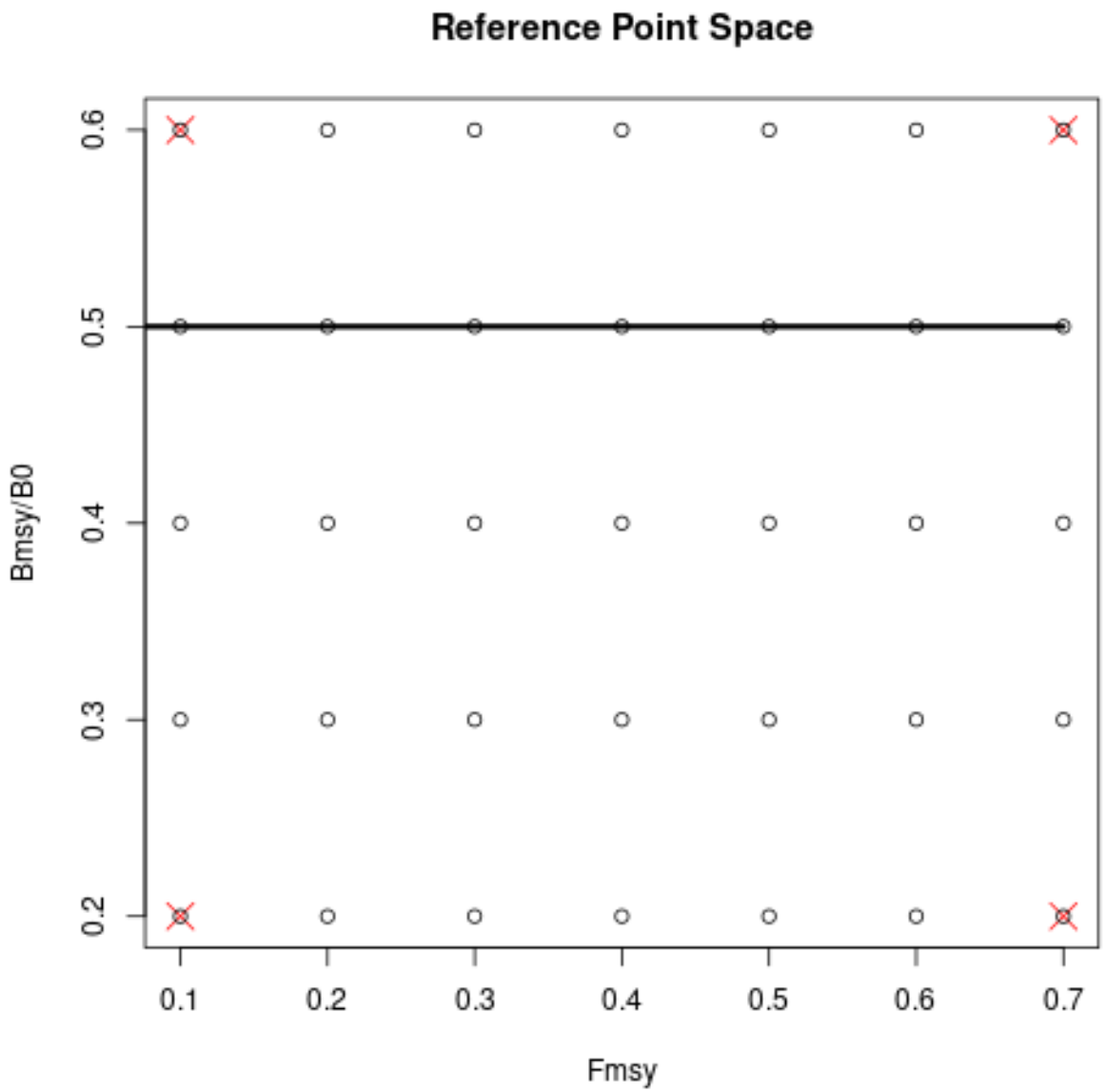


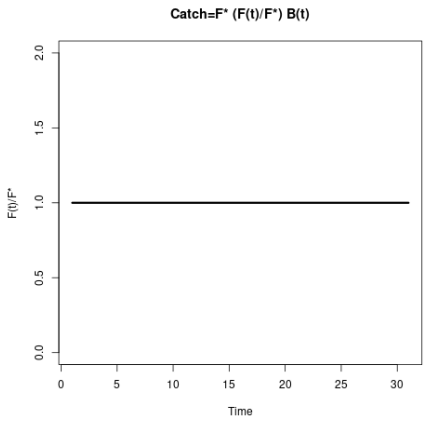
# Pella-Tomlinson Family of Curves

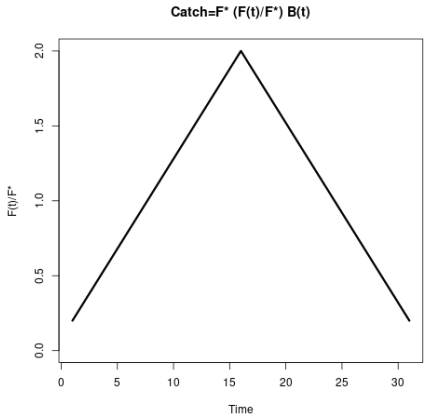


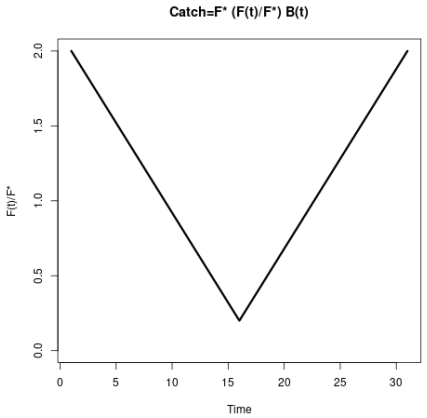



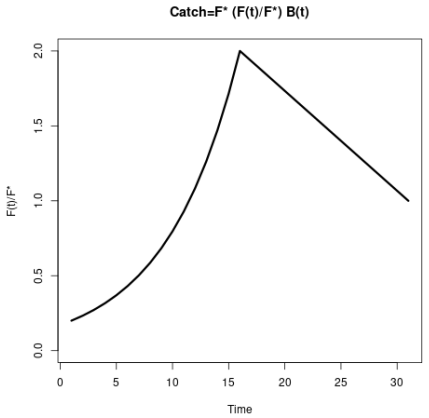

Reference Point Space
Bmsy/B0
0.2
0.3
0.4
0.5
0.6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Fmsy

# Catch

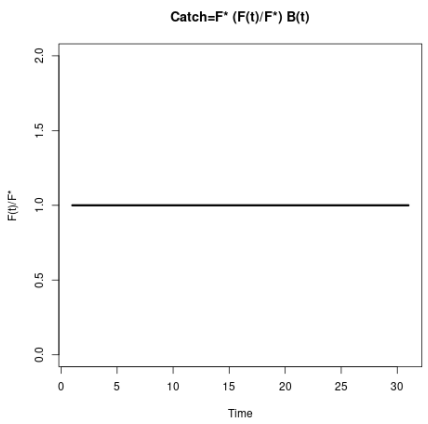

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

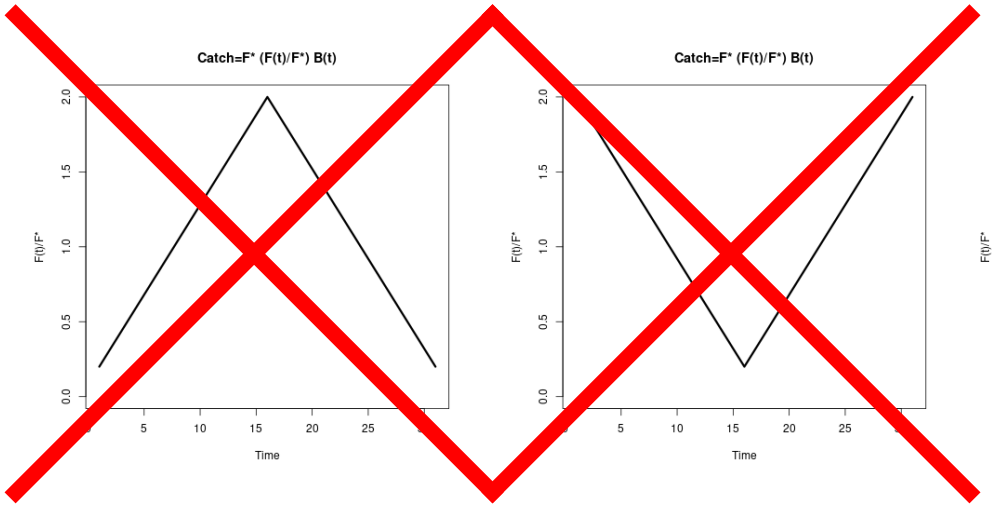

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

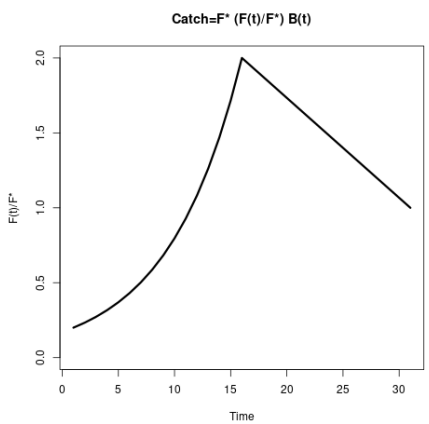

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

# Catch

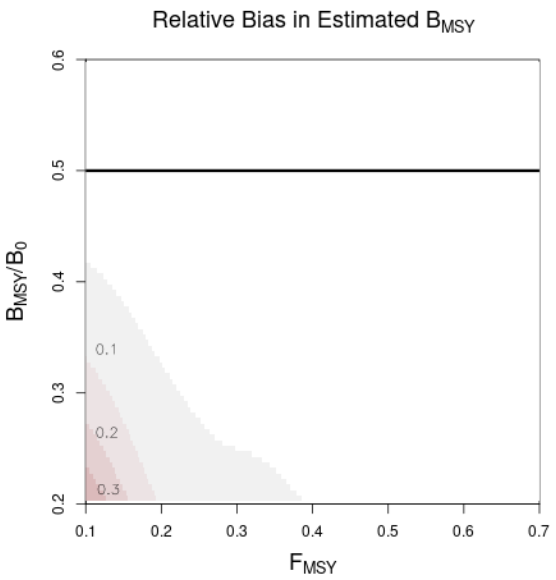


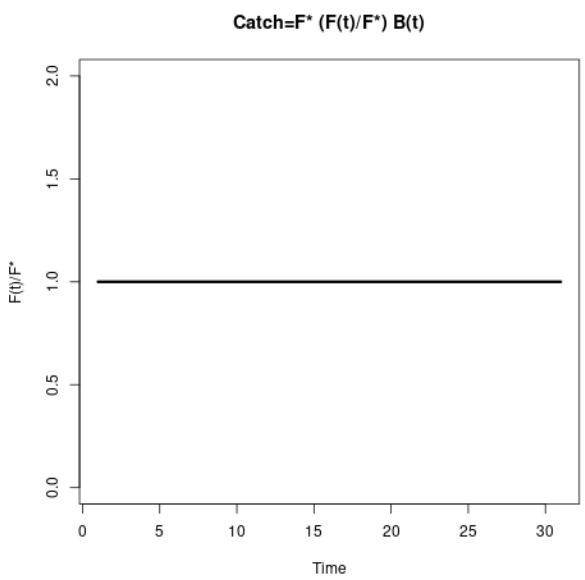

Flat

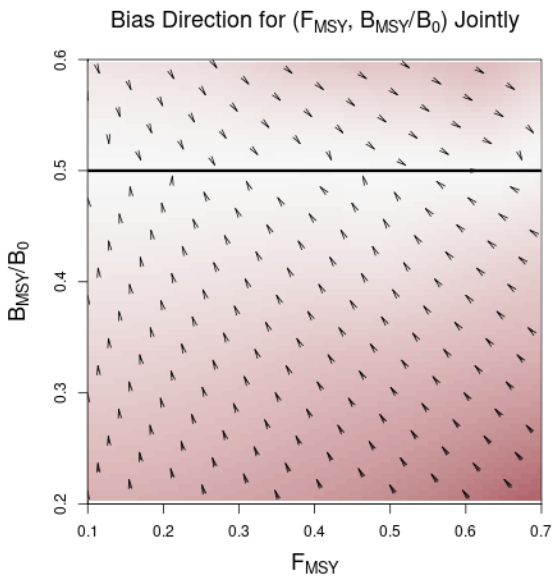


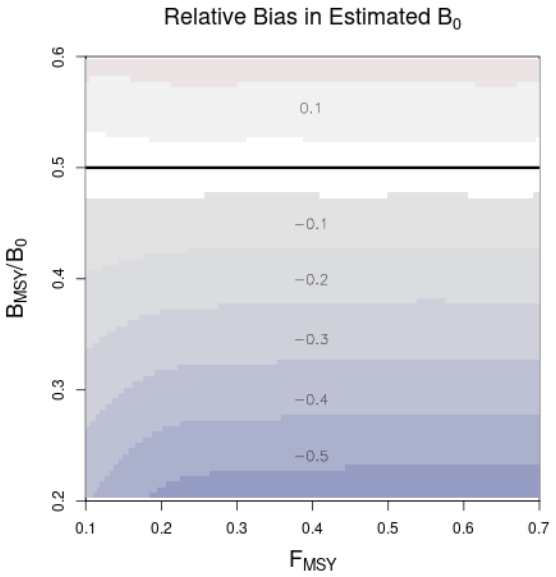



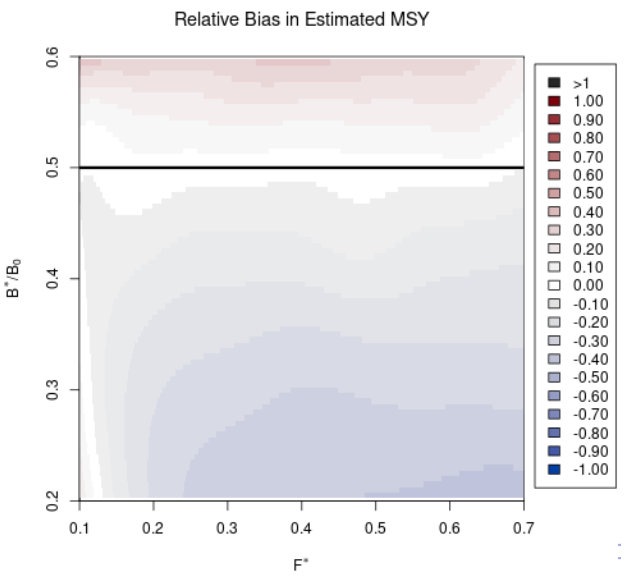

Contrast

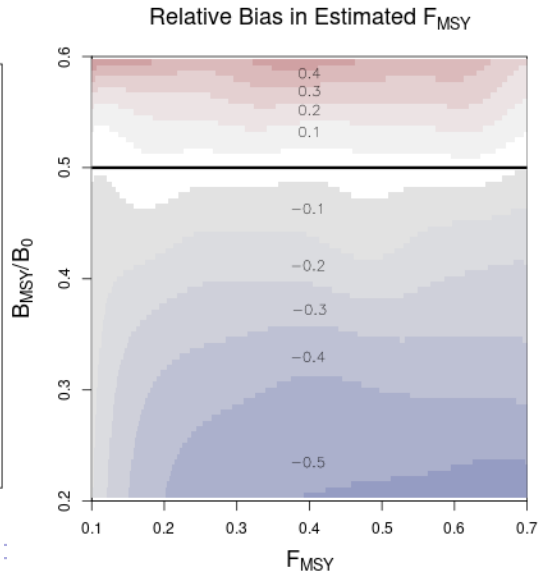

Relative Bias in Estimated $B_{MSY}$
$B_{MSY}/B_0$
$F_{MSY}$
0.1
0.2
0.3

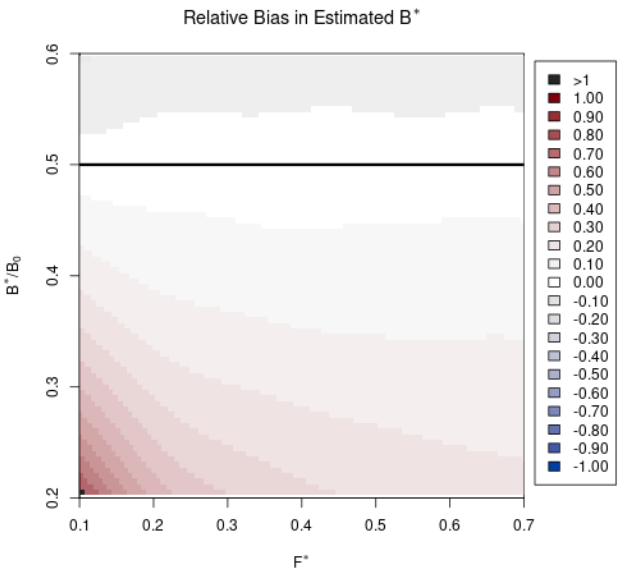

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

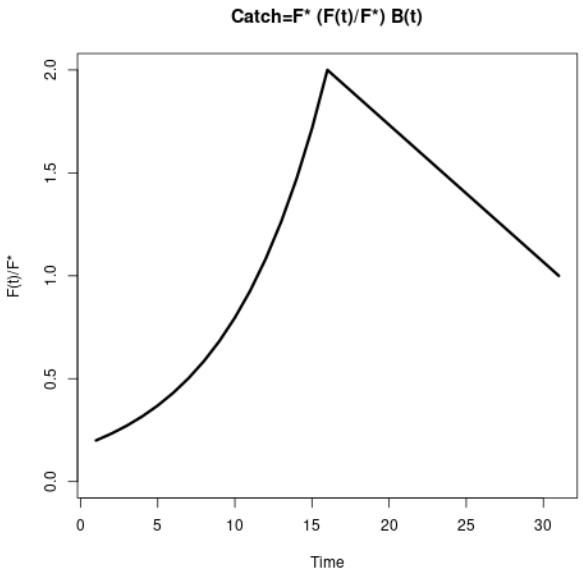

Bias Direction for ($F_{MSY}$, $B_{MSY}/B_0$) Jointly
$B_{MSY}/B_0$
$F_{MSY}$

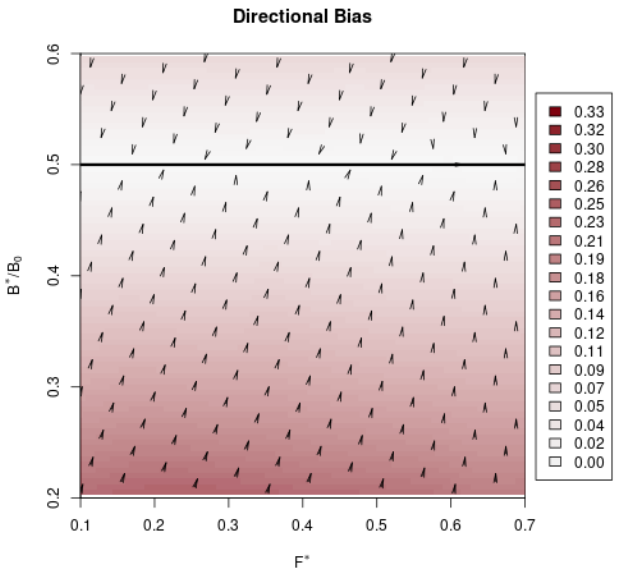

Relative Bias in Estimated $B_0$
$B_{MSY}/B_0$
$F_{MSY}$
0.1
−0.1
−0.2
−0.3
−0.4
−0.5

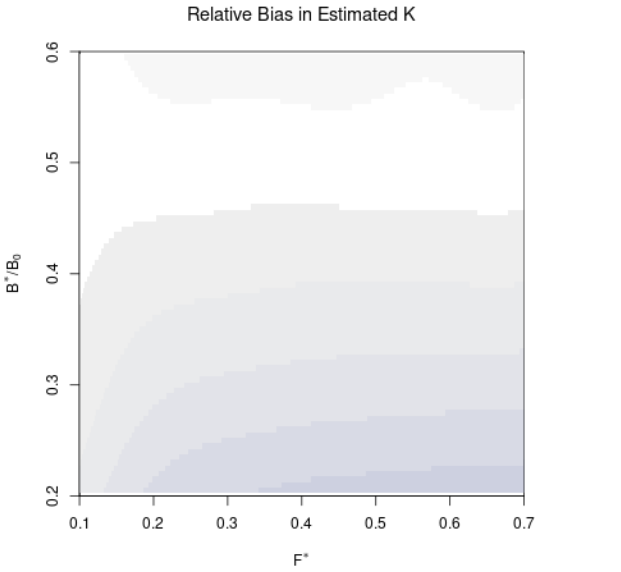

Relative Bias in Estimated MSY
$B^*/B_0$
$F^*$
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

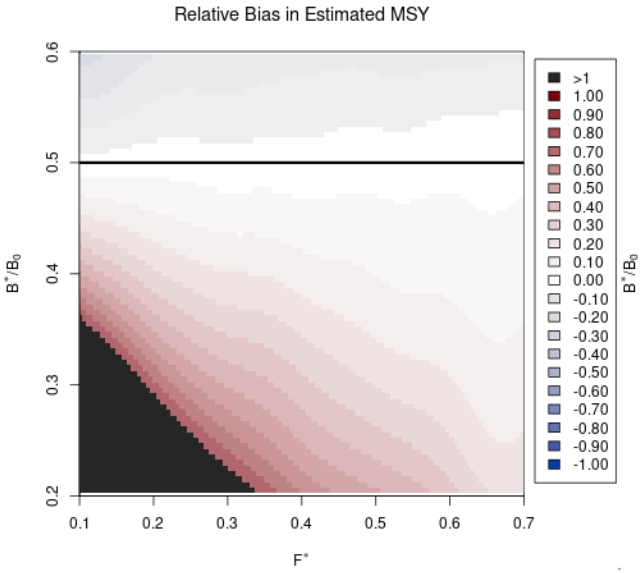

Relative Bias in Estimated $F_{MSY}$
$B_{MSY}/B_0$
$F_{MSY}$
0.4
0.3
0.2
0.1
−0.1
−0.2
−0.3
−0.4
−0.5

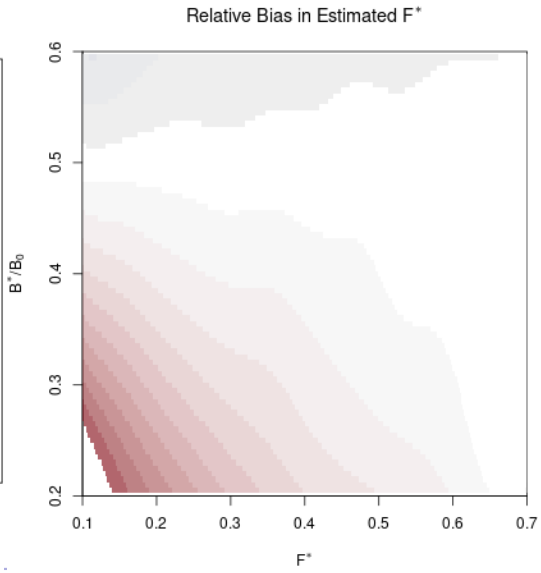

Relative Bias in Estimated B*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

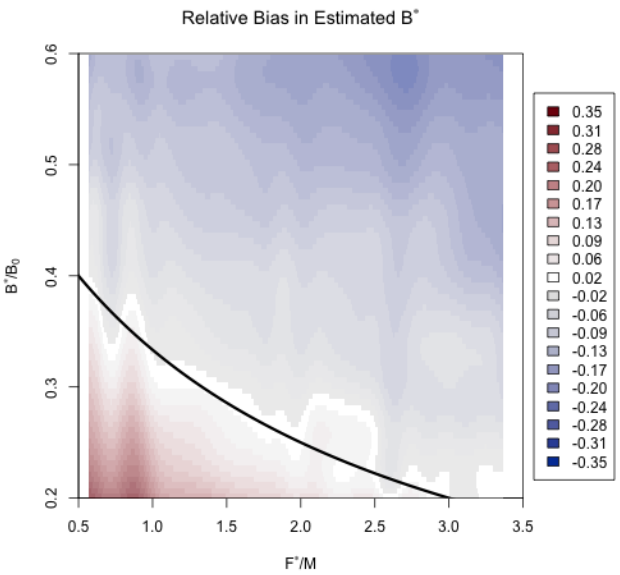

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

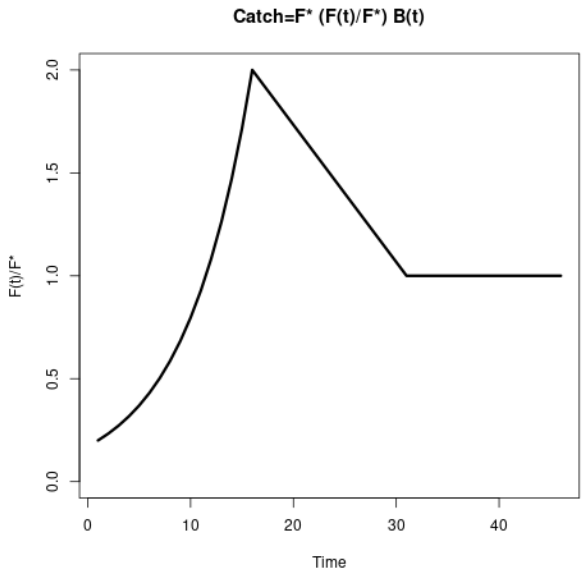

Directional Bias
B*/B0
F*
0.33
0.32
0.30
0.28
0.26
0.25
0.23
0.21
0.19
0.18
0.16
0.14
0.12
0.11
0.09
0.07
0.05
0.04
0.02
0.00

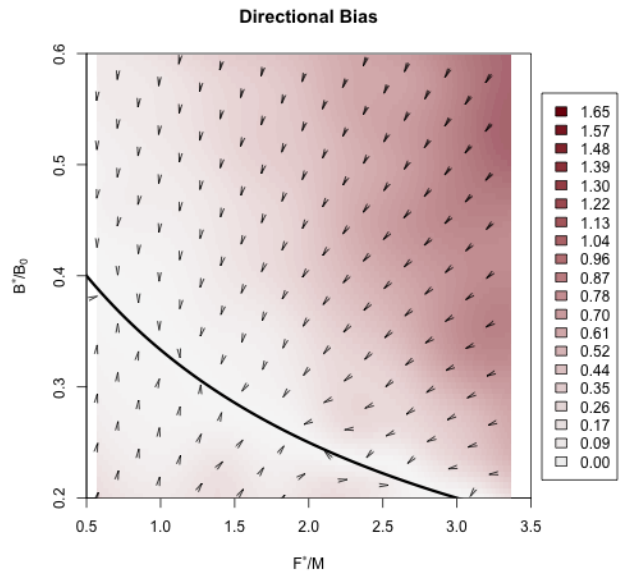

Relative Bias in Estimated K
B*/B0
F*

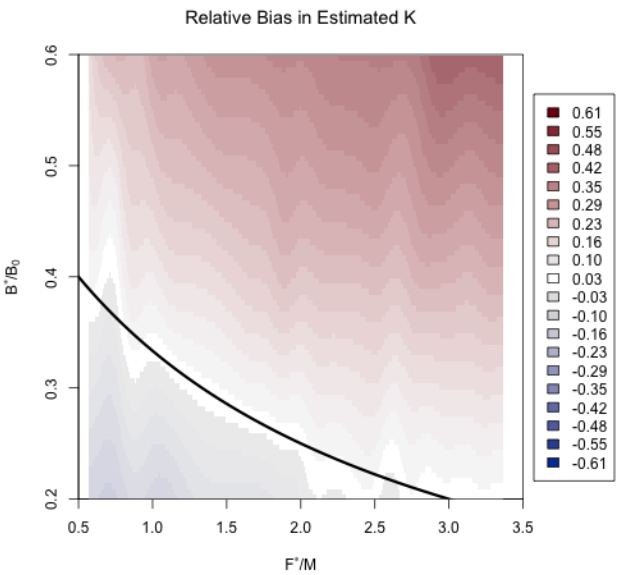

Relative Bias in Estimated MSY
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

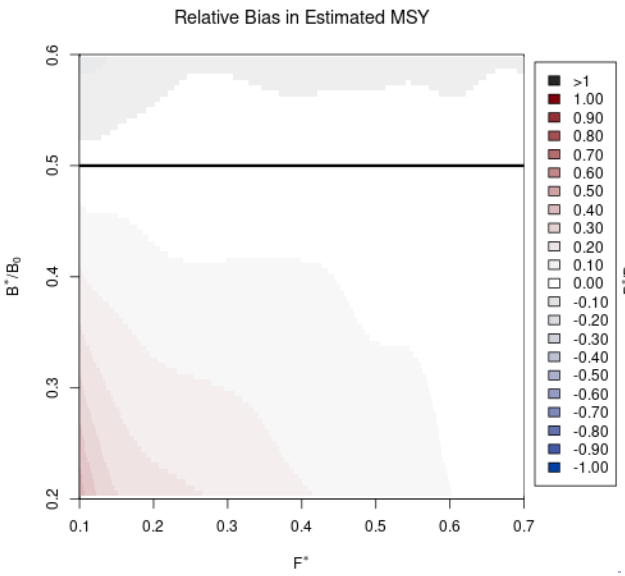

Relative Bias in Estimated F*
B*/B0
F*

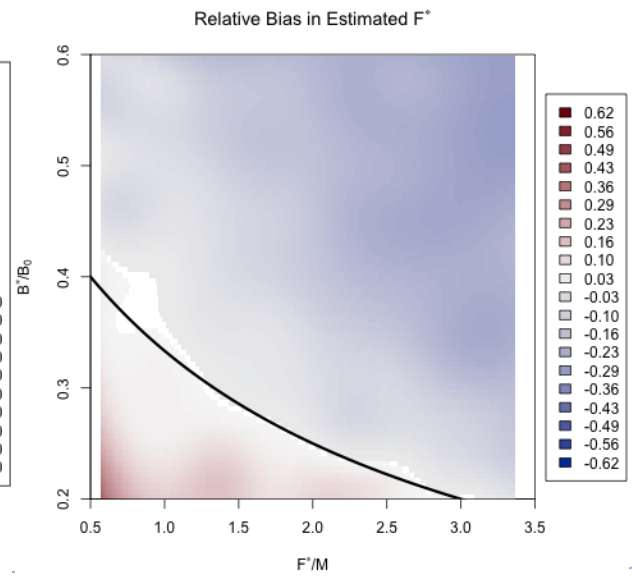

Relative Bias in Estimated B*
B*/B0
F*/M
0.35
0.31
0.28
0.24
0.20
0.17
0.13
0.09
0.06
0.02
-0.02
-0.06
-0.09
-0.13
-0.17
-0.20
-0.24
-0.28
-0.31
-0.35

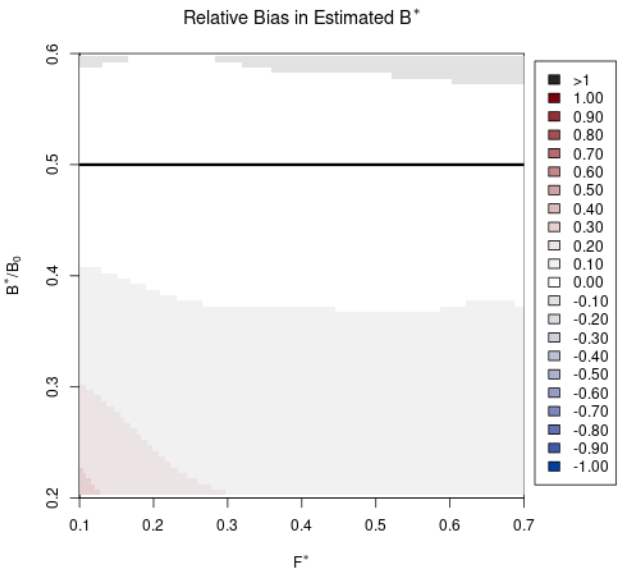

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

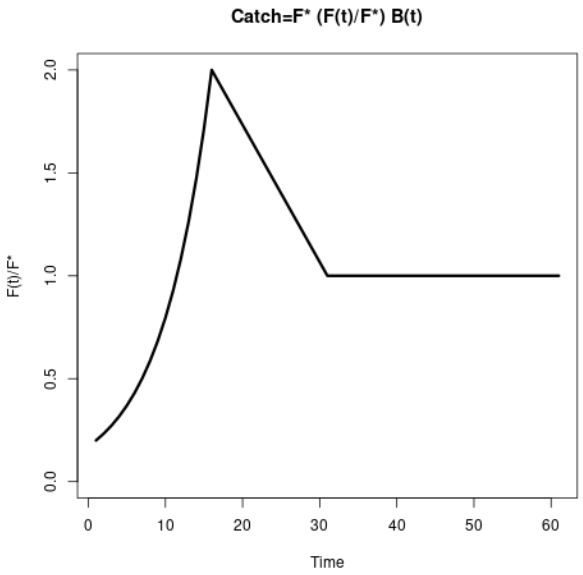

Directional Bias
B*/B0
F*/M
1.65
1.57
1.48
1.39
1.30
1.22
1.13
1.04
0.96
0.87
0.78
0.70
0.61
0.52
0.44
0.35
0.26
0.17
0.09
0.00

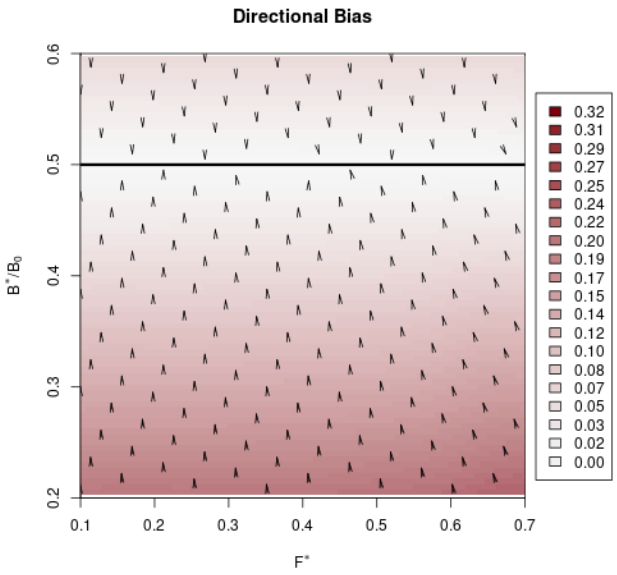

Relative Bias in Estimated K
B*/B0
F*/M
0.61
0.55
0.48
0.42
0.35
0.29
0.23
0.16
0.10
0.03
-0.03
-0.10
-0.16
-0.23
-0.29
-0.35
-0.42
-0.48
-0.55
-0.61

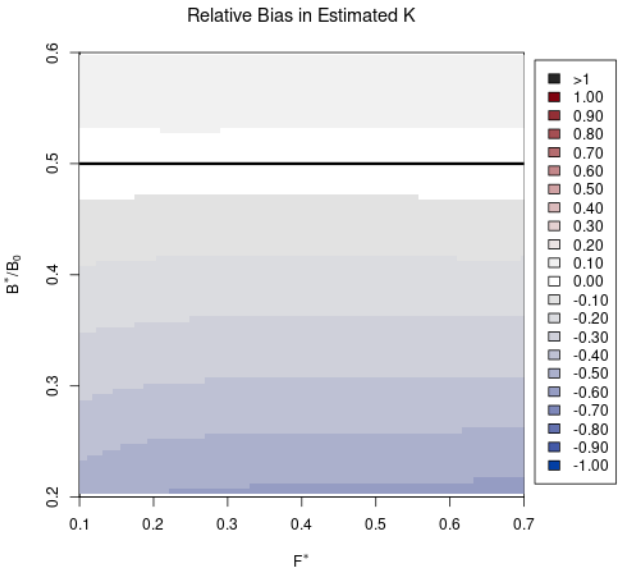

Relative Bias in Estimated MSY
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

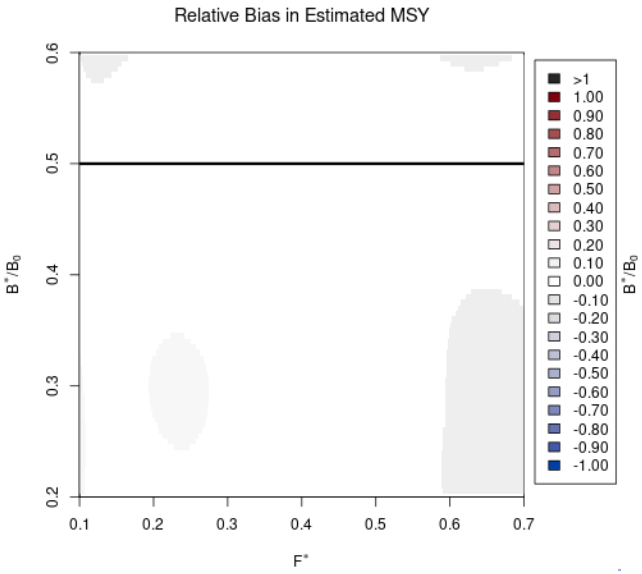

Relative Bias in Estimated F*
B*/B0
F*/M
0.62
0.56
0.49
0.43
0.36
0.29
0.23
0.16
0.10
0.03
-0.03
-0.10
-0.16
-0.23
-0.29
-0.36
-0.43
-0.49
-0.56
-0.62

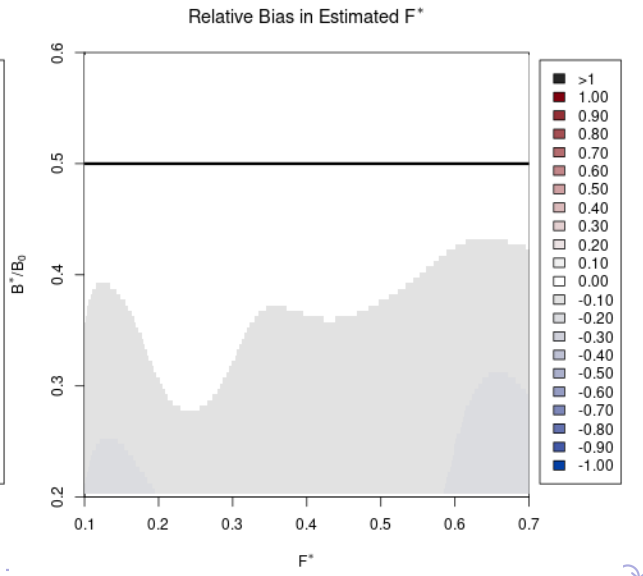

Relative Bias in Estimated B*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

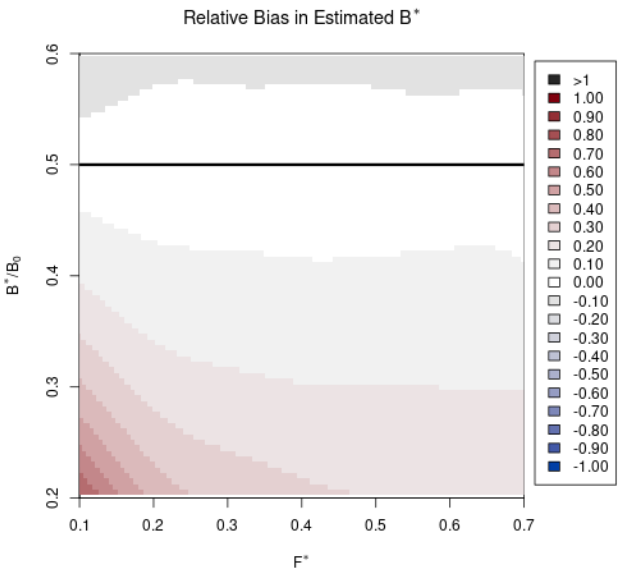

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

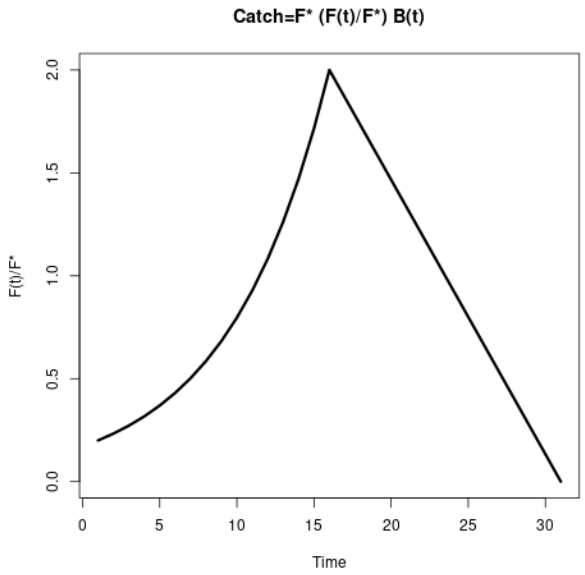

Directional Bias
B*/B0
F*
0.32
0.31
0.29
0.27
0.25
0.24
0.22
0.20
0.19
0.17
0.15
0.14
0.12
0.10
0.08
0.07
0.05
0.03
0.02
0.00

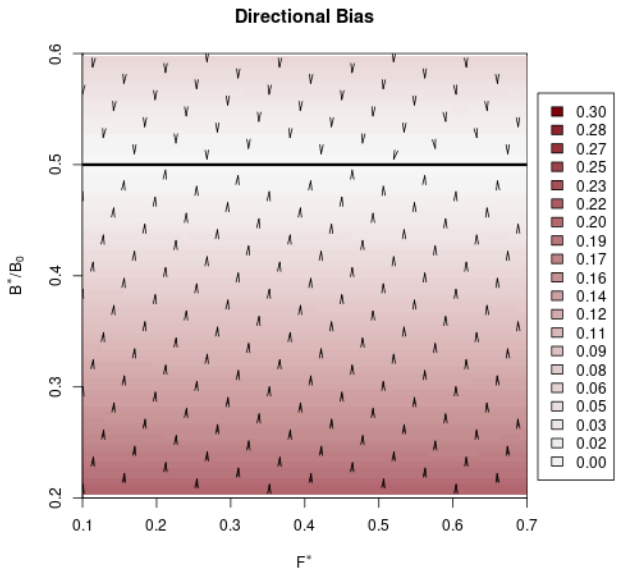

Relative Bias in Estimated K
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

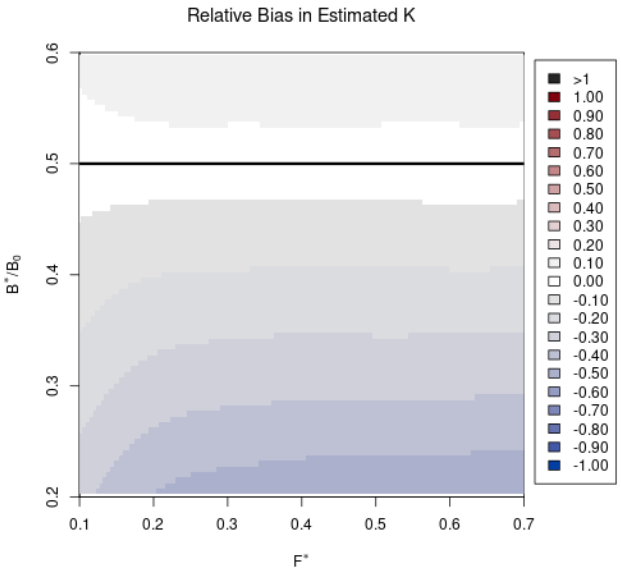

Relative Bias in Estimated MSY
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

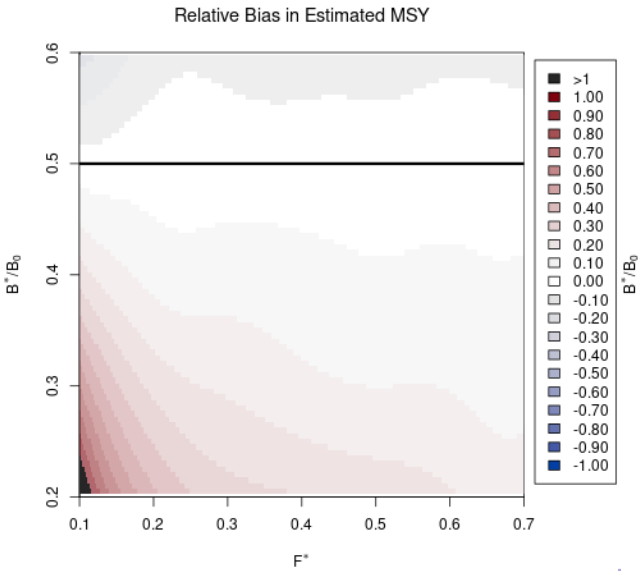

Relative Bias in Estimated F*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

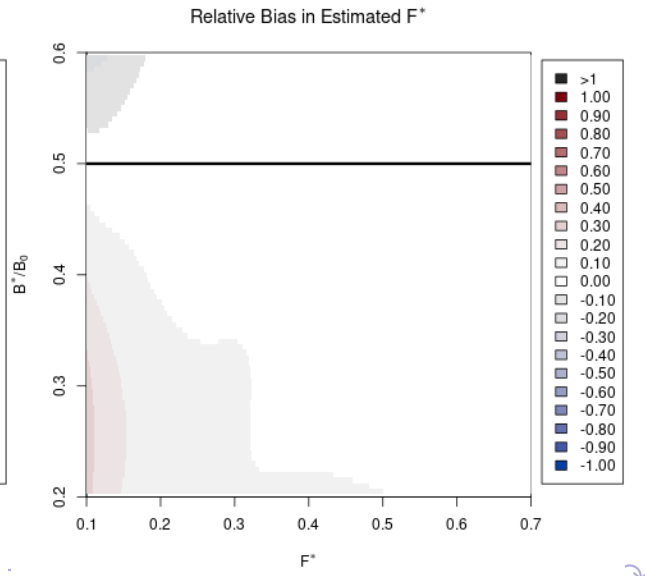

Relative Bias in Estimated B*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

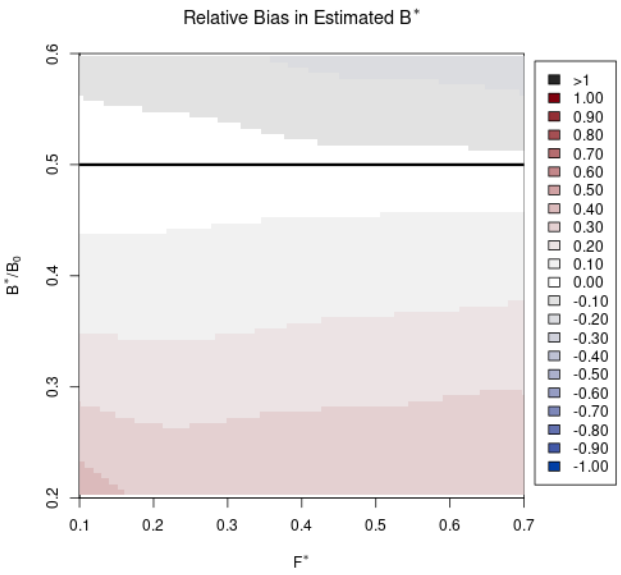

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

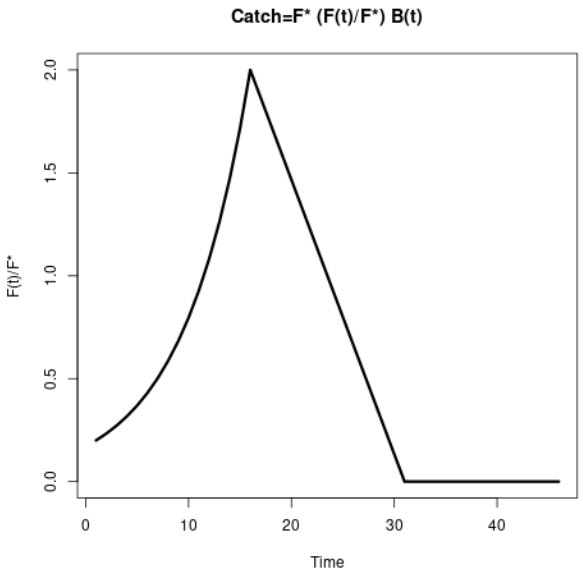

Directional Bias
B*/B0
F*
0.30
0.28
0.27
0.25
0.23
0.22
0.20
0.19
0.17
0.16
0.14
0.12
0.11
0.09
0.08
0.06
0.05
0.03
0.02
0.00

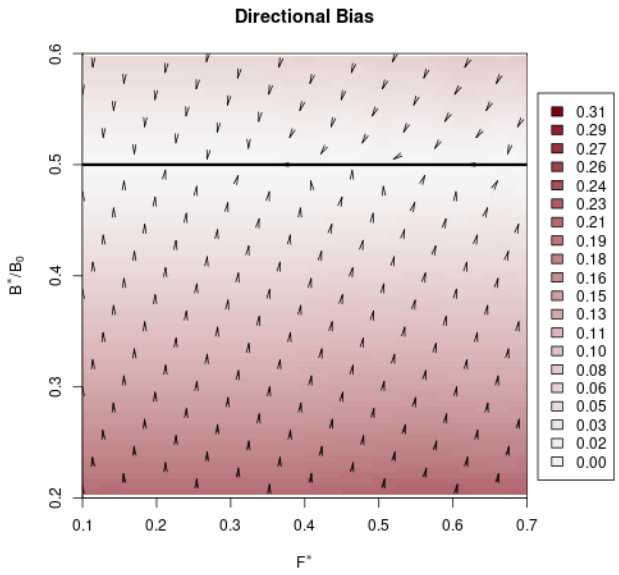

Relative Bias in Estimated K
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

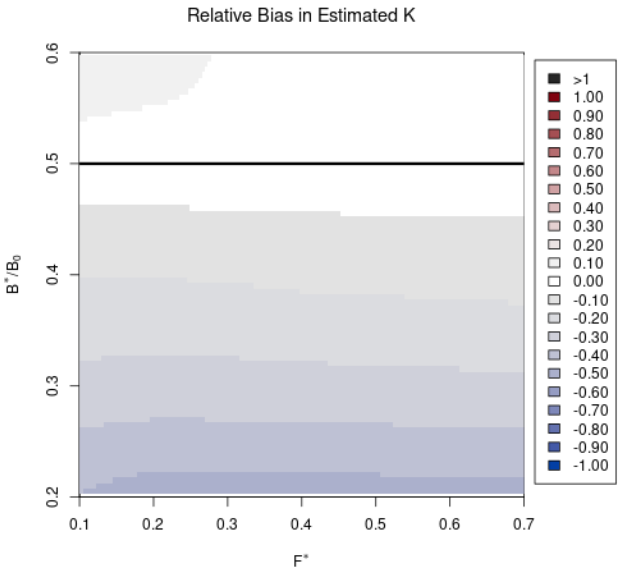

Relative Bias in Estimated MSY
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

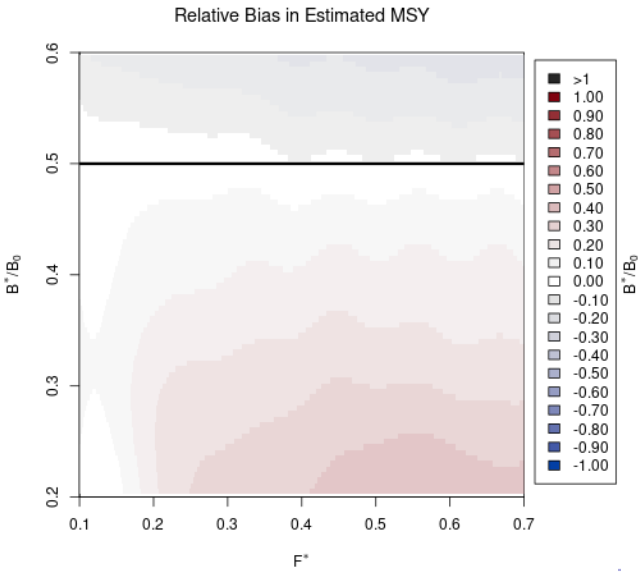

Relative Bias in Estimated F*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

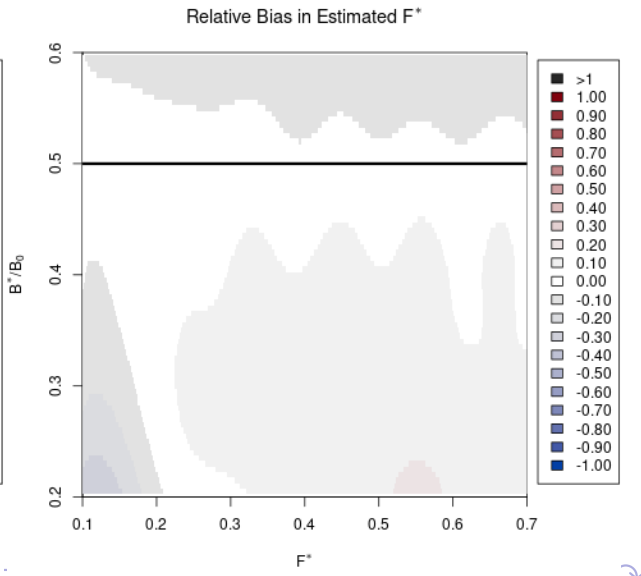

Relative Bias in Estimated B*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

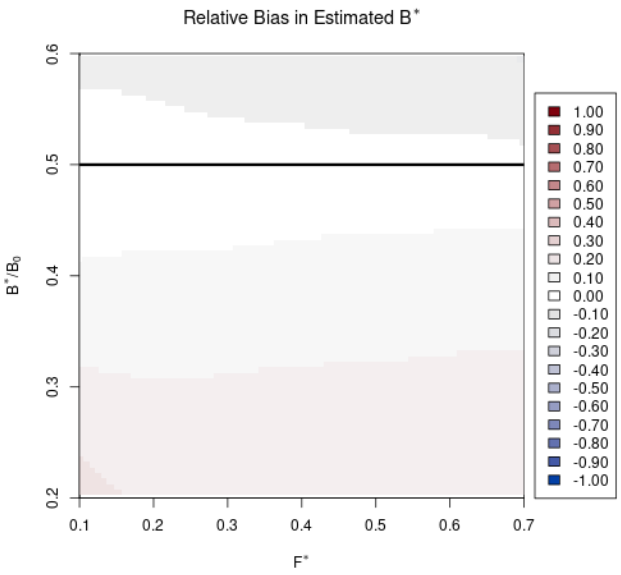

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

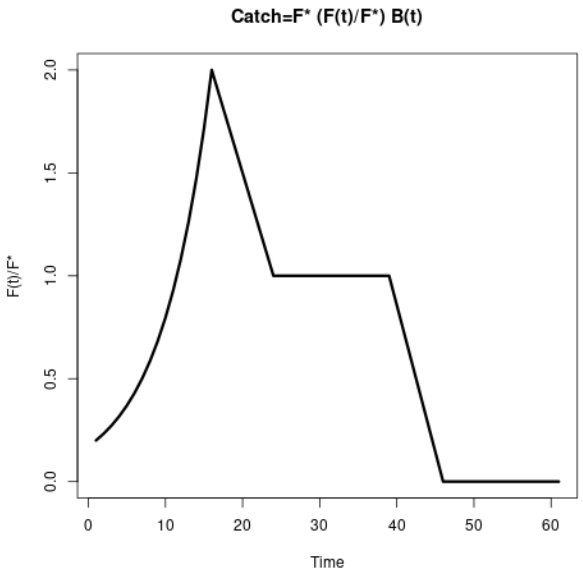

Directional Bias
B*/B0
F*
0.31
0.29
0.27
0.26
0.24
0.23
0.21
0.19
0.18
0.16
0.15
0.13
0.11
0.10
0.08
0.06
0.05
0.03
0.02
0.00

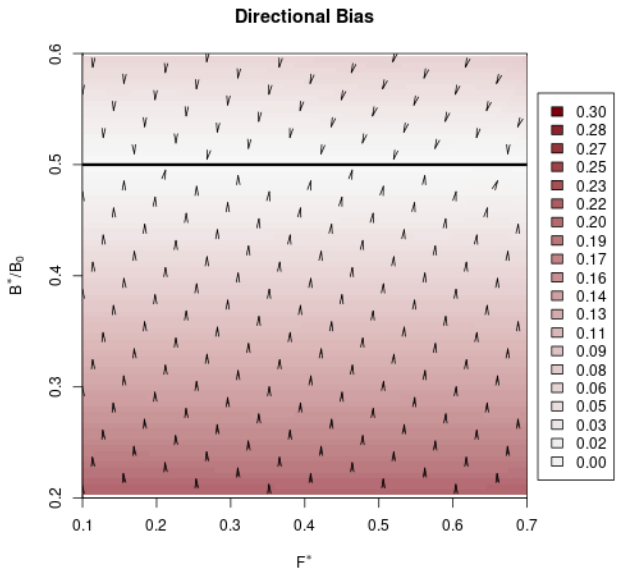

Relative Bias in Estimated K
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

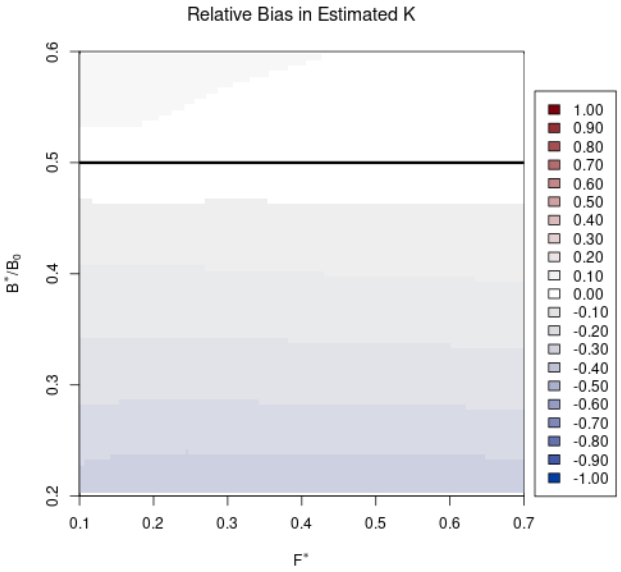

Relative Bias in Estimated MSY
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

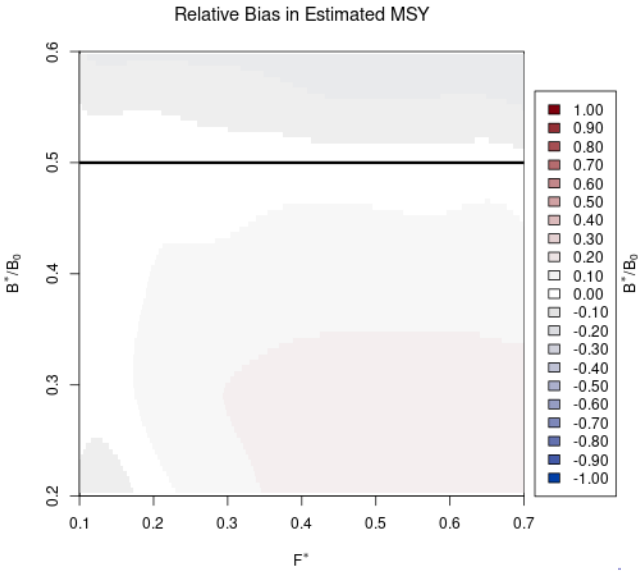

Relative Bias in Estimated F*
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

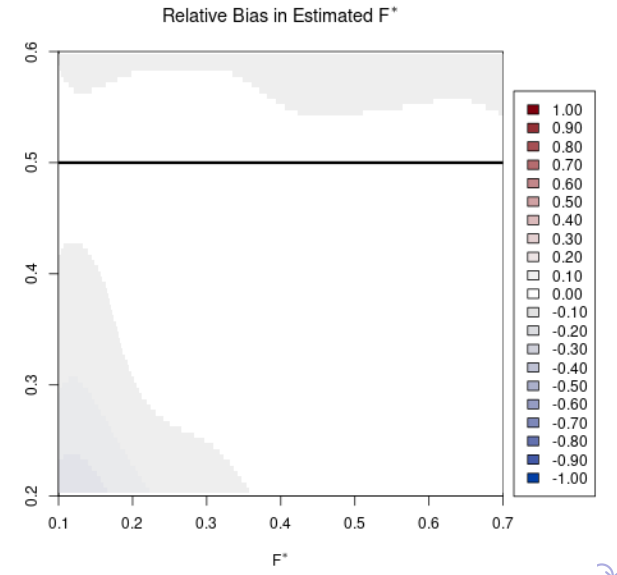

Relative Bias in Estimated B*
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

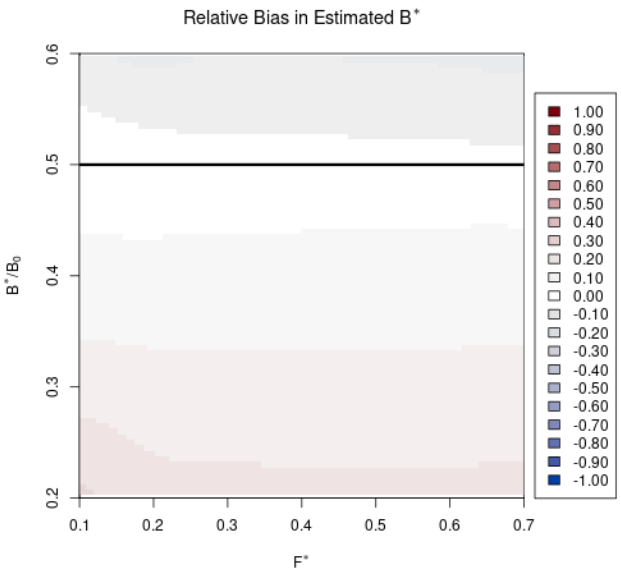

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

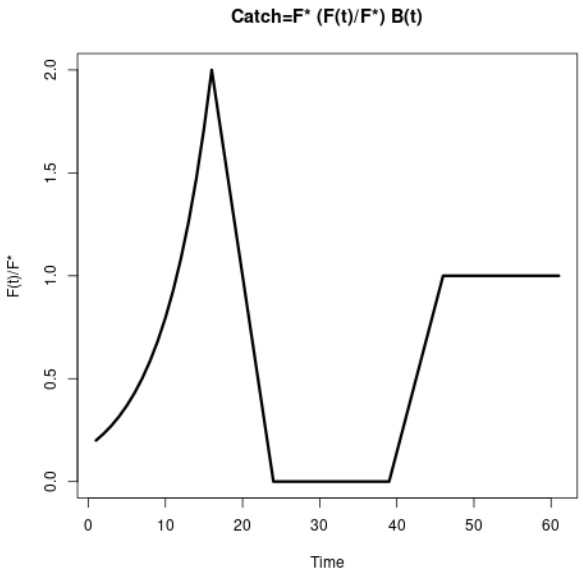

Directional Bias
B*/B0
F*
0.30
0.28
0.27
0.25
0.23
0.22
0.20
0.19
0.17
0.16
0.14
0.13
0.11
0.09
0.08
0.06
0.05
0.03
0.02
0.00

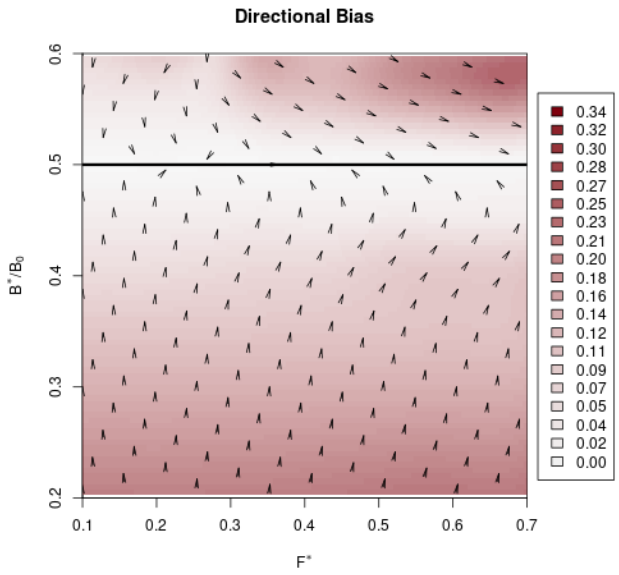

Relative Bias in Estimated K
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

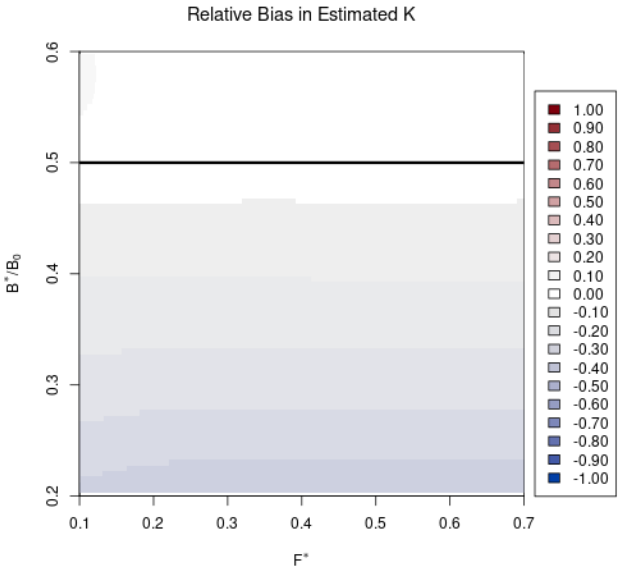

Relative Bias in Estimated MSY
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

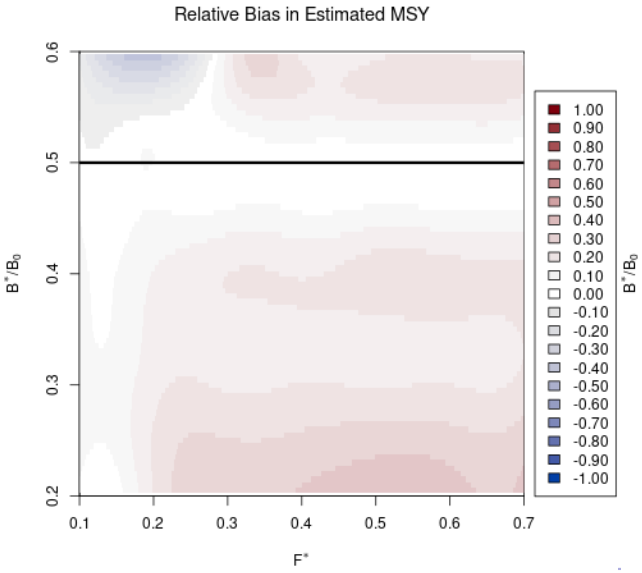

Relative Bias in Estimated F*
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

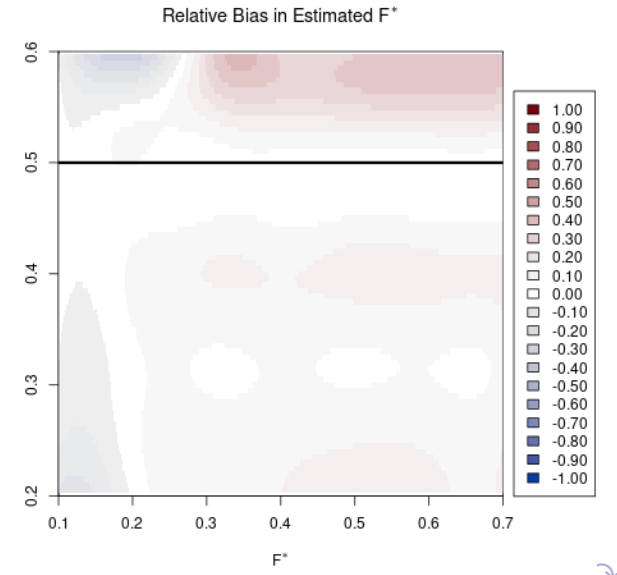

Relative Bias in Estimated B*
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

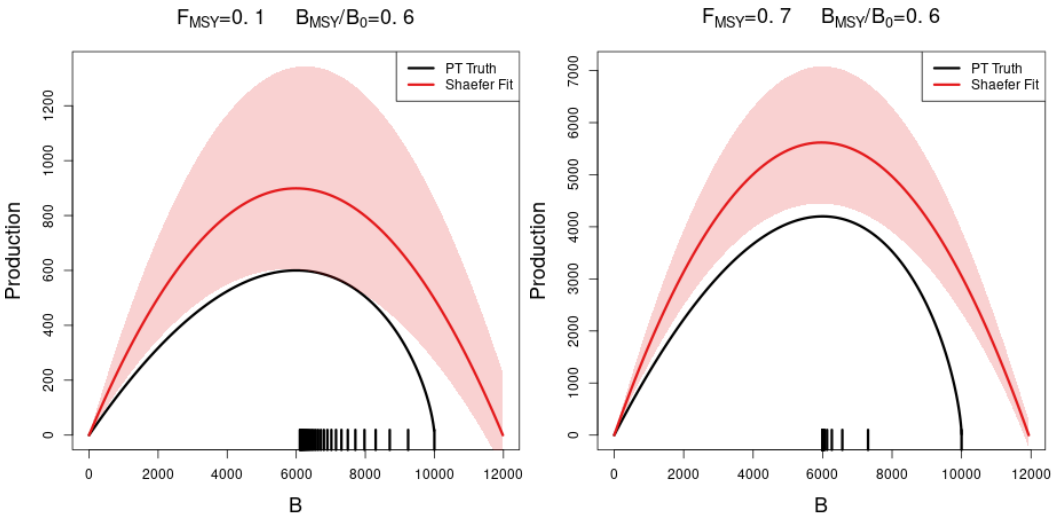

Directional Bias
B*/B0
F*
0.34
0.32
0.30
0.28
0.27
0.25
0.23
0.21
0.20
0.18
0.16
0.14
0.12
0.11
0.09
0.07
0.05
0.04
0.02
0.00


Relative Bias in Estimated K
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00


Relative Bias in Estimated MSY
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

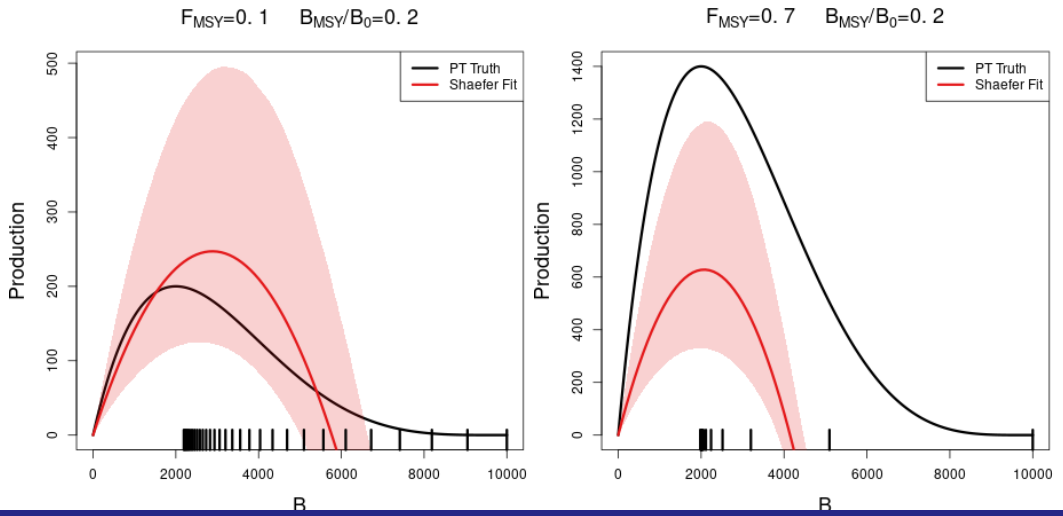

Relative Bias in Estimated F*
B*/B0
F*
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00

# Flat: Misspecified SRR Biomass Depletion



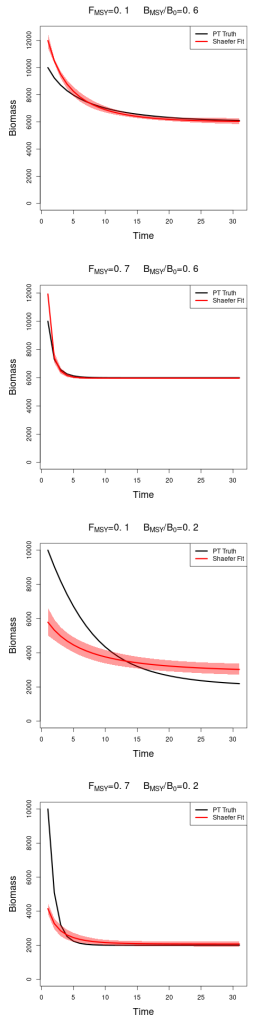


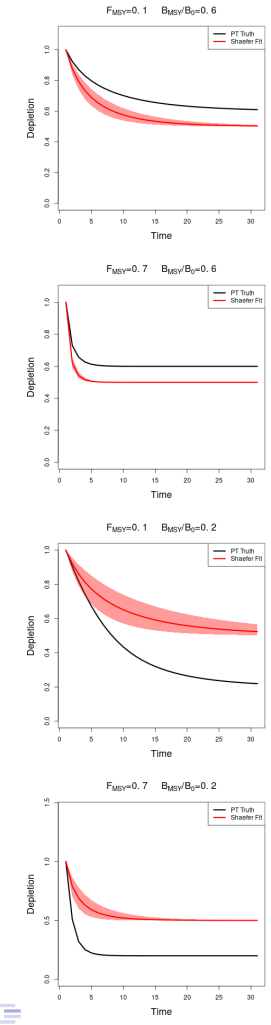




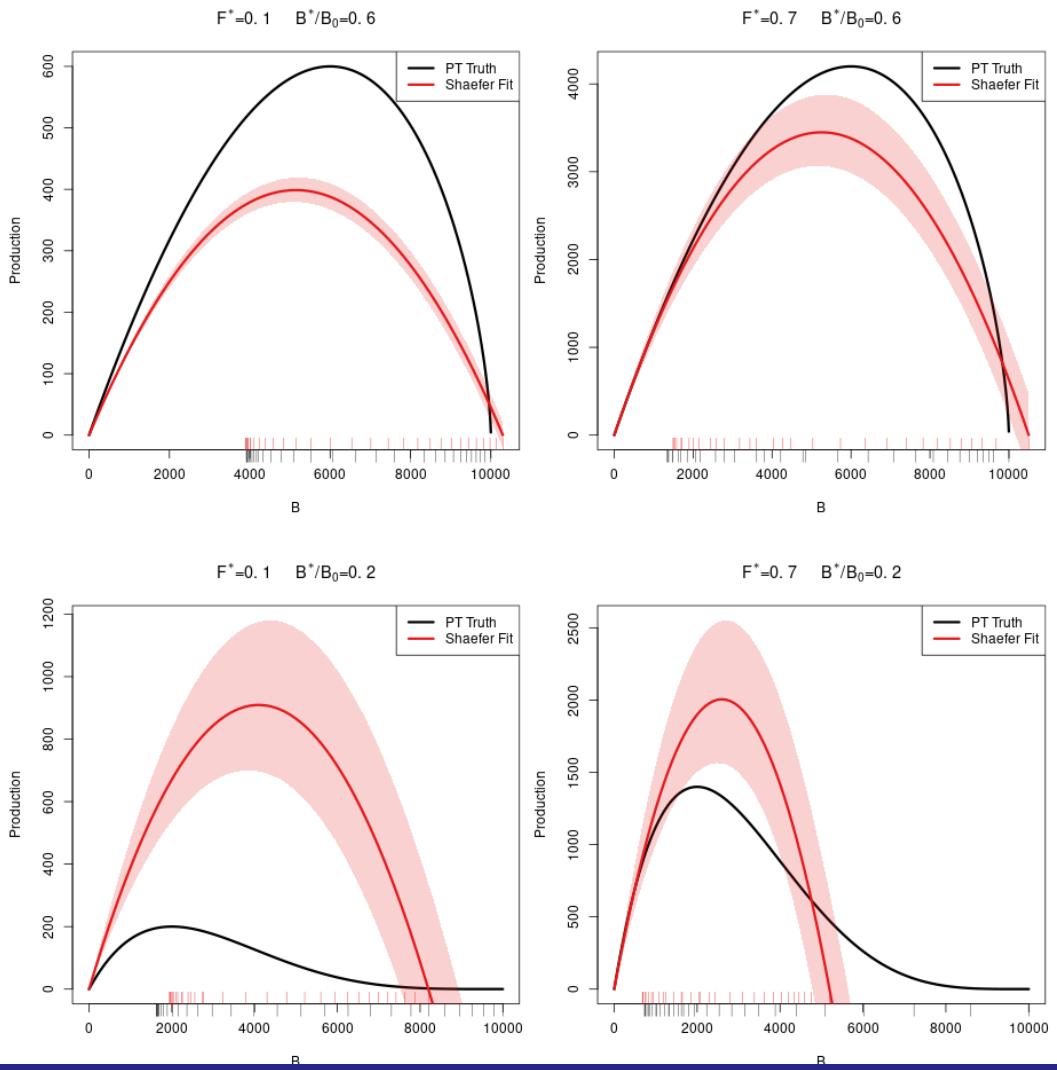


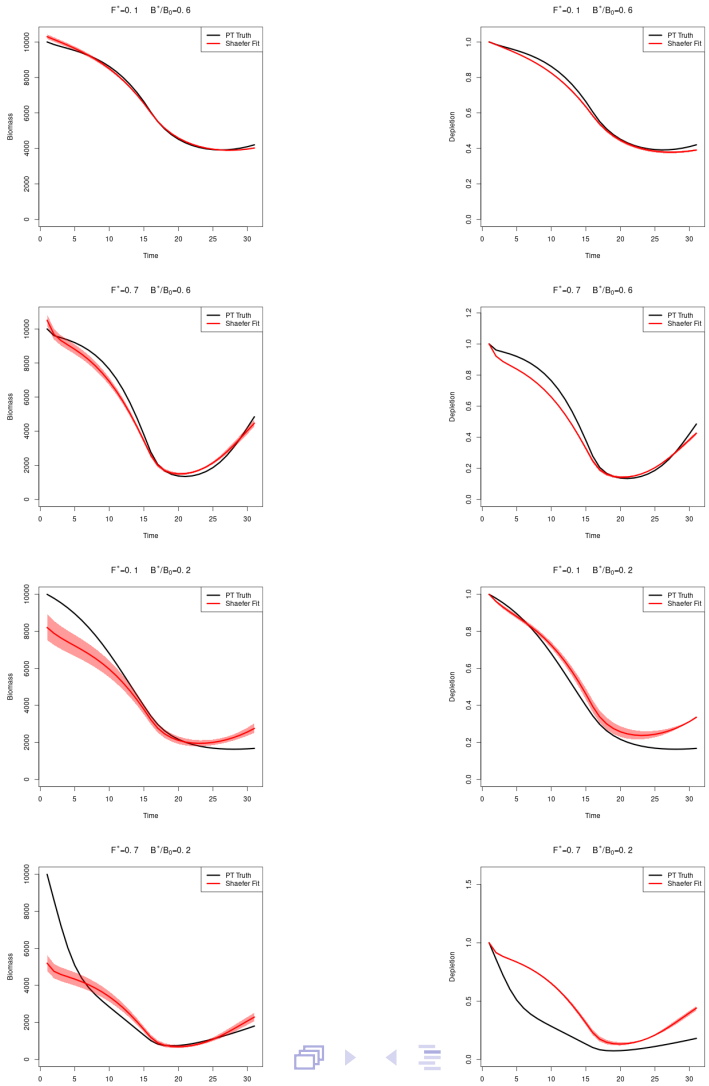




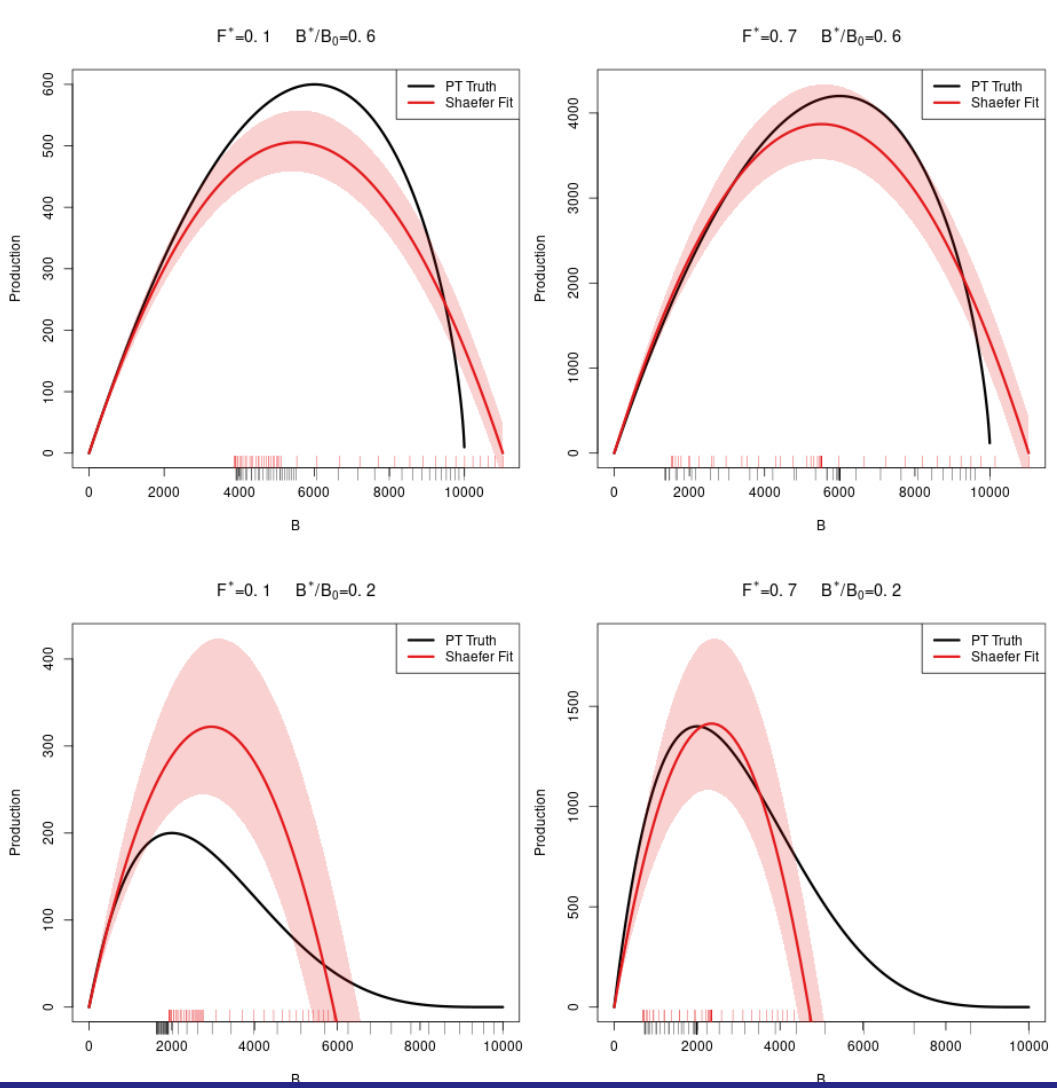



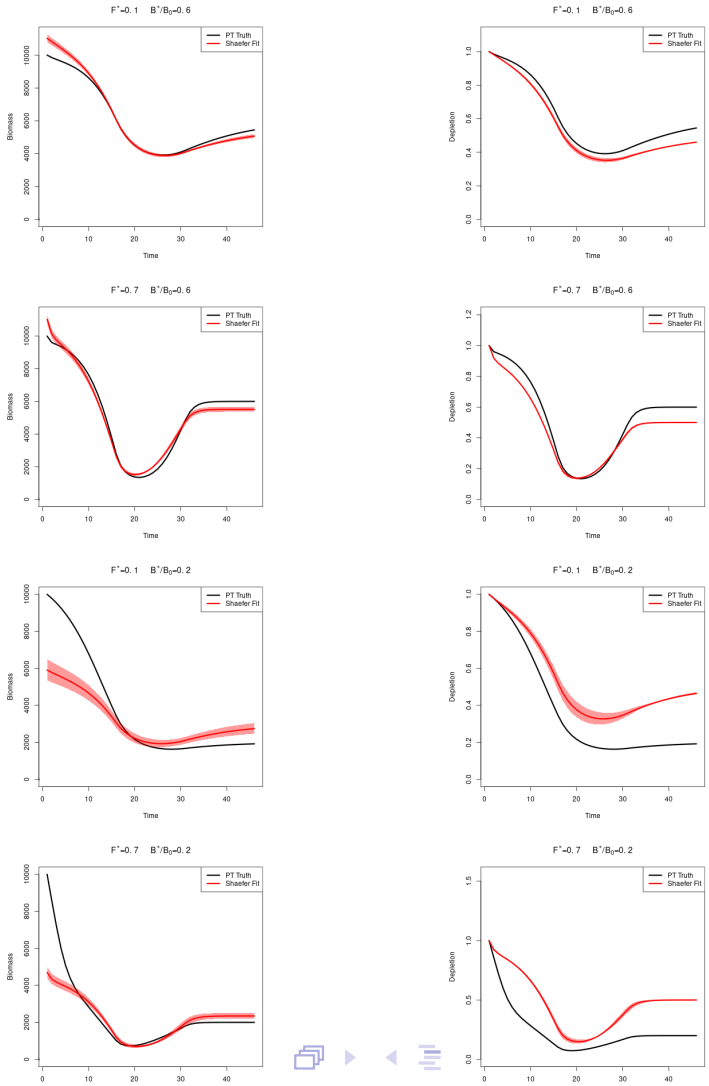
# Contrast: Misspecified SRR Biomass Depletion



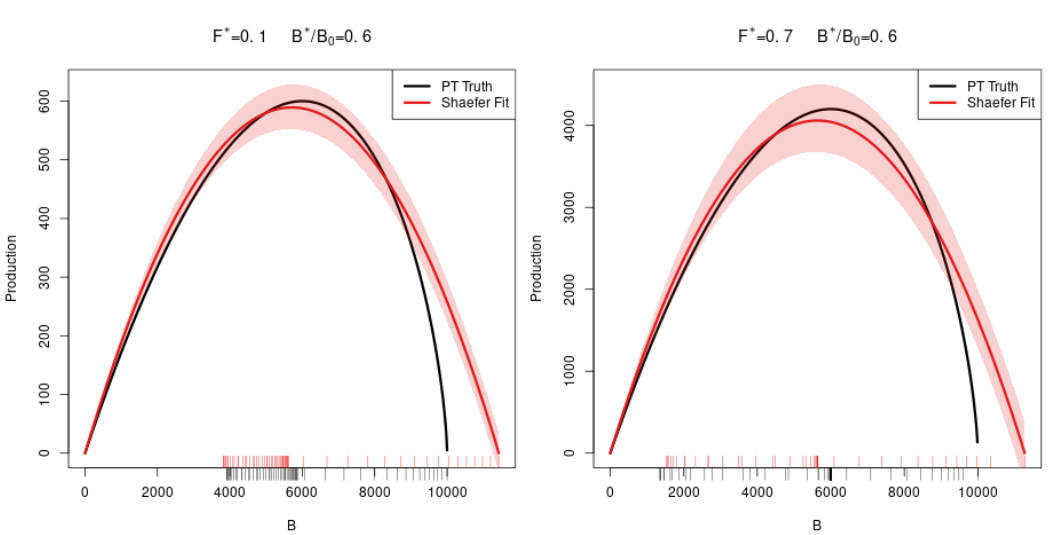



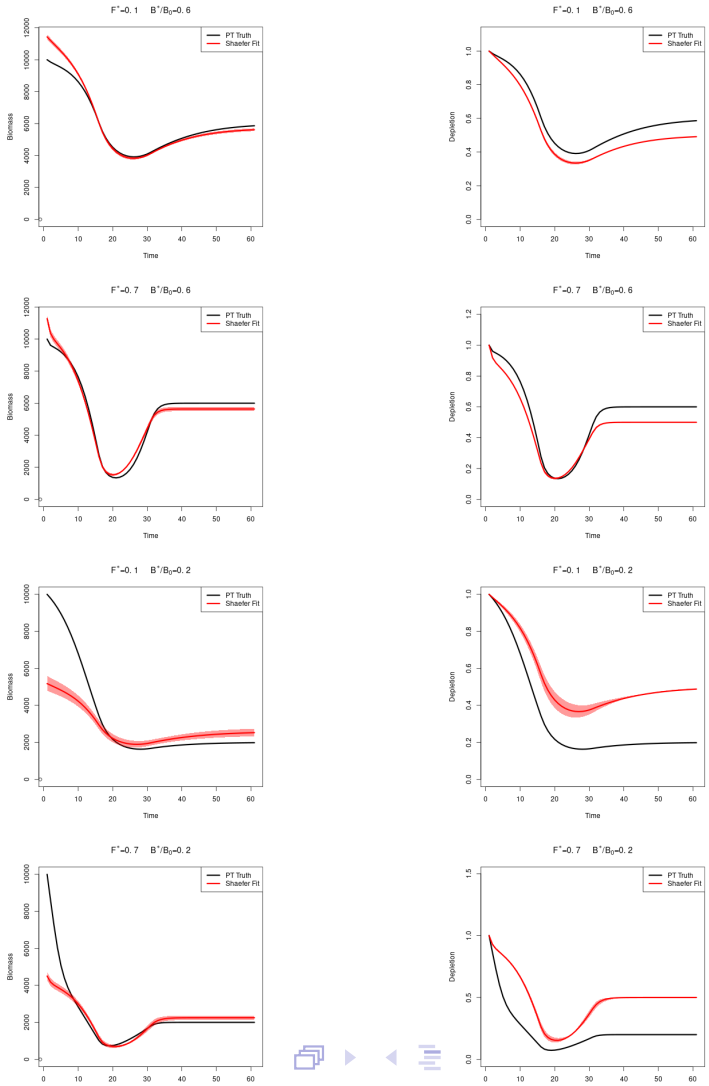
# ContrastT45: Misspecified SRR Biomass Depletion



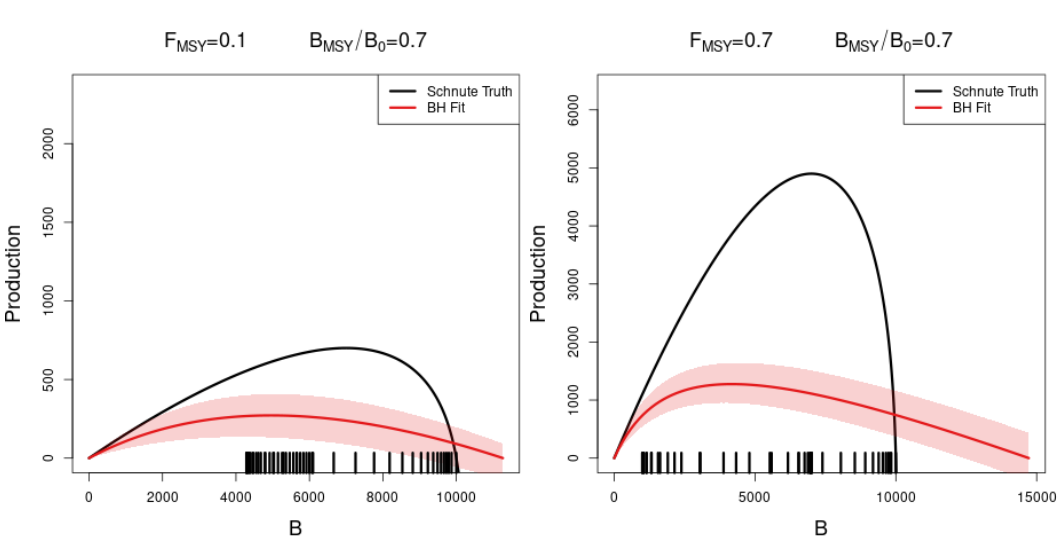



# ContrastT60: Misspecified SRR Biomass Depletion



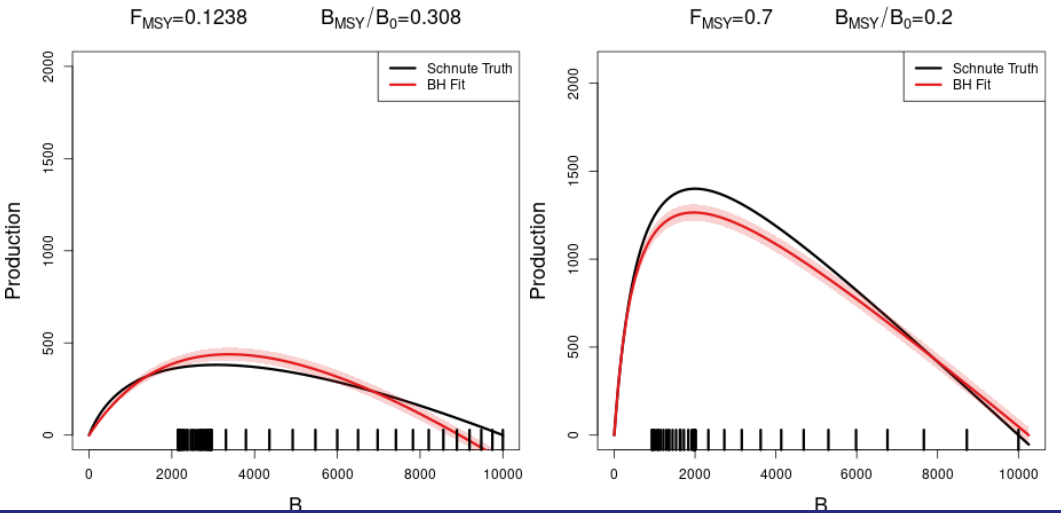



# SchnuteExpT45: Misspecified SRR Biomass Depletion



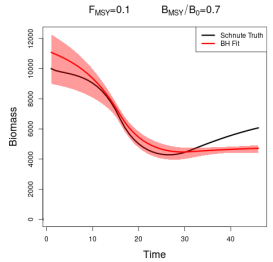


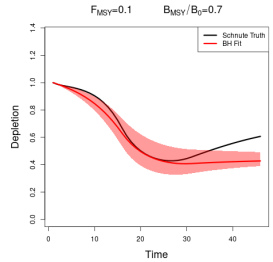


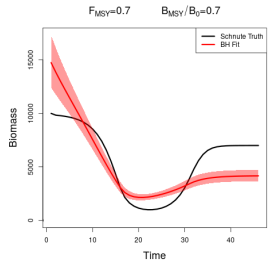


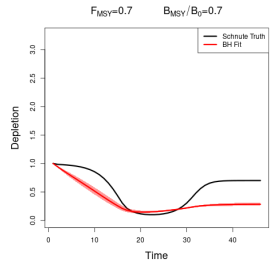


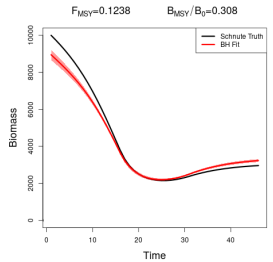


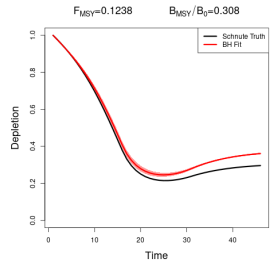


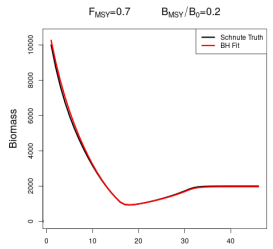



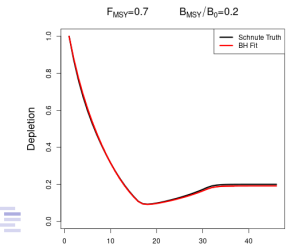



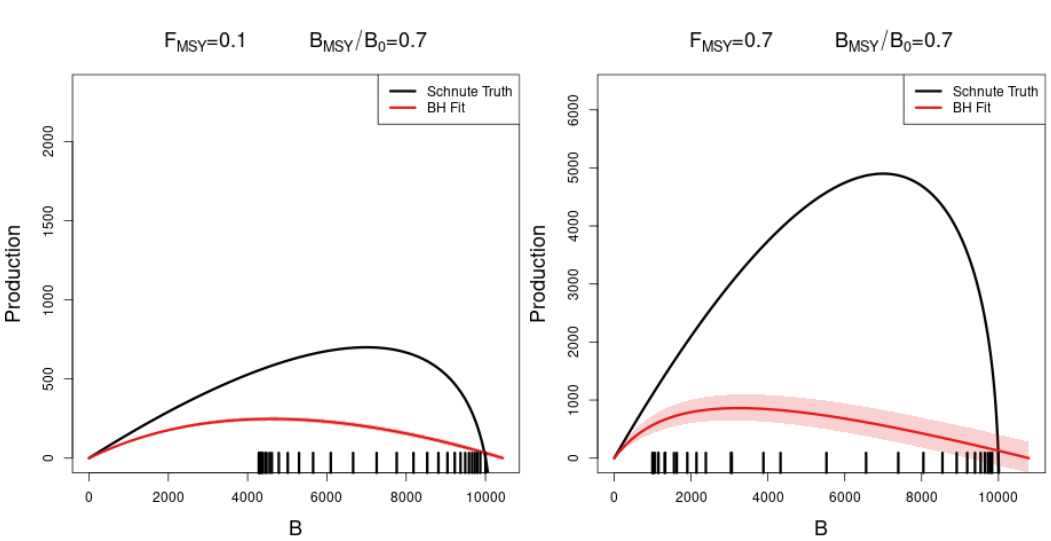



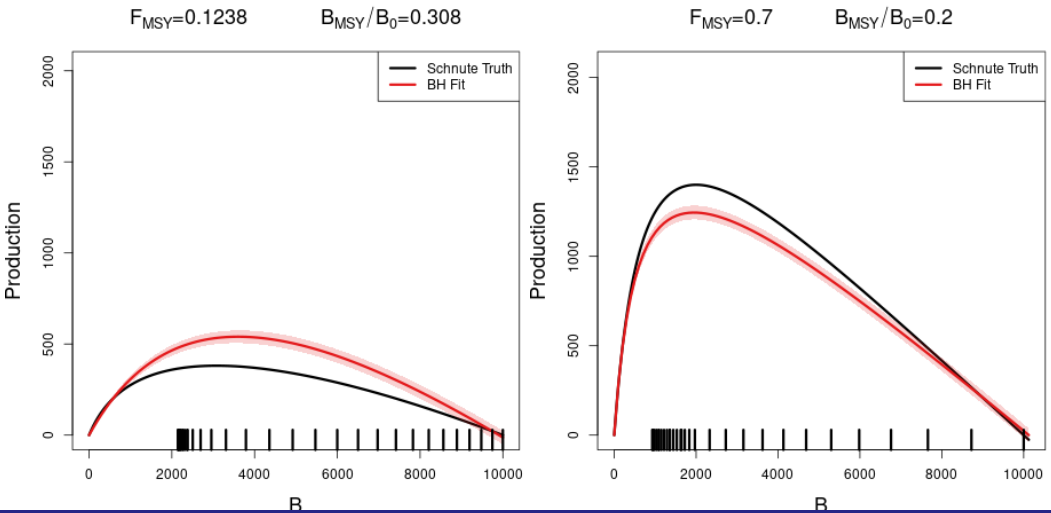



# SchnuteExpT30: Misspecified SRR Biomass Depletion



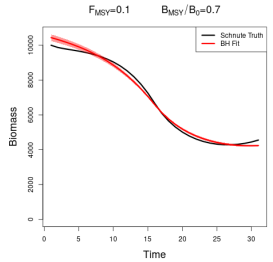


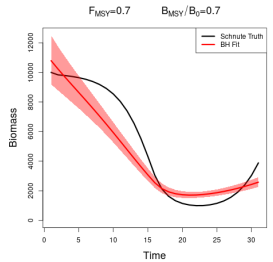


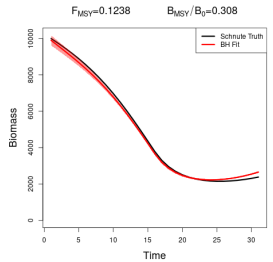


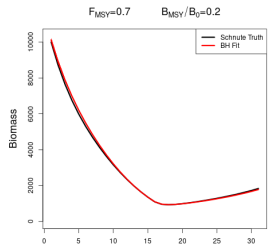


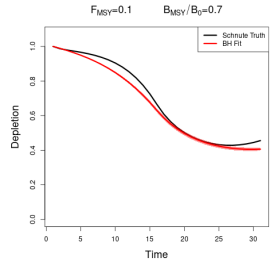


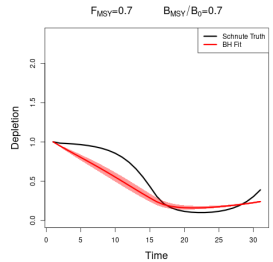


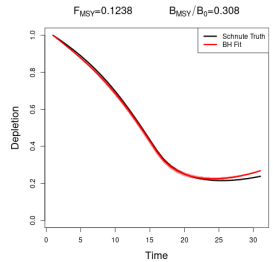



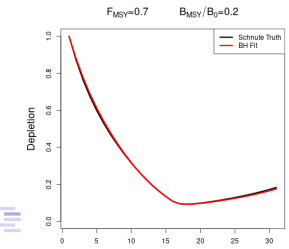



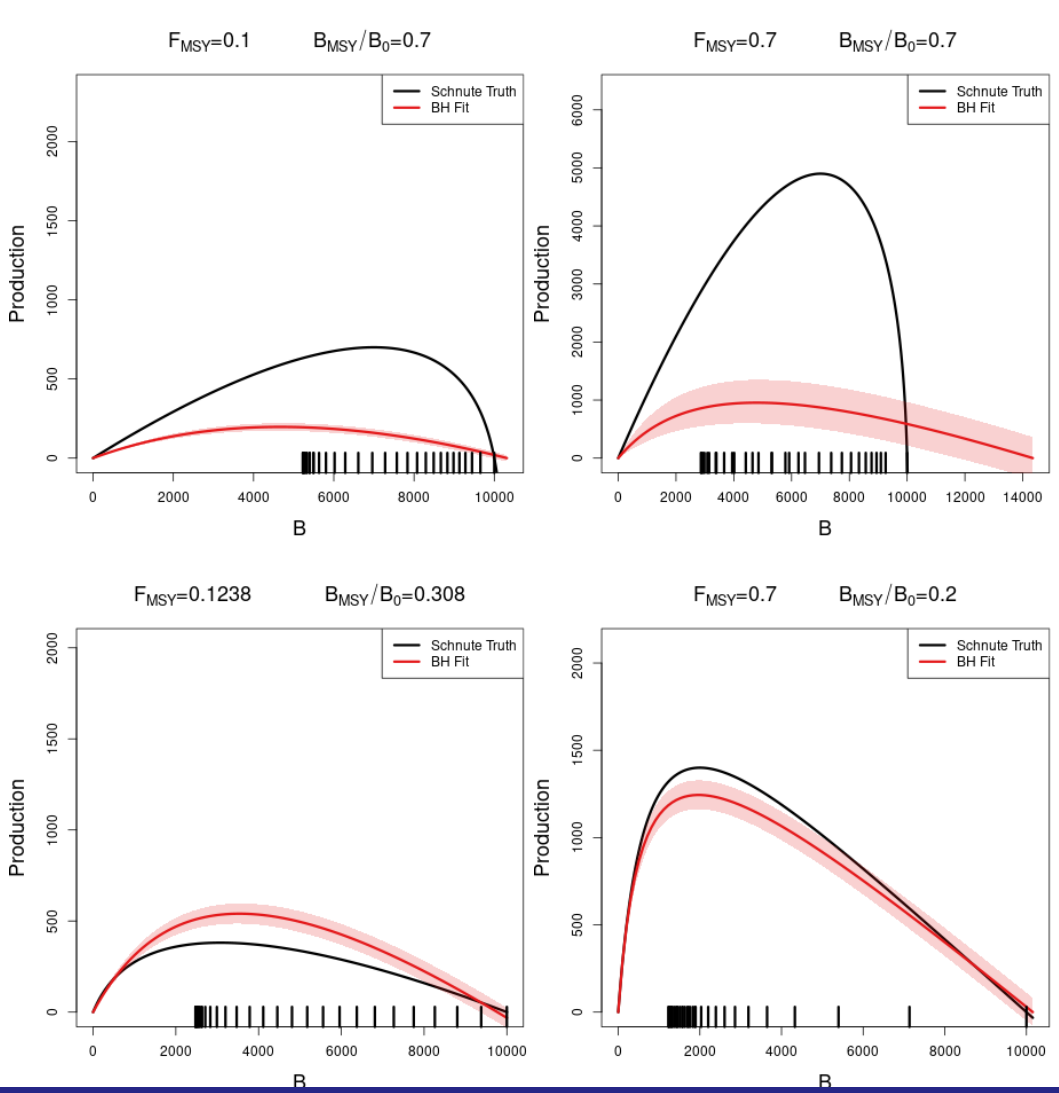



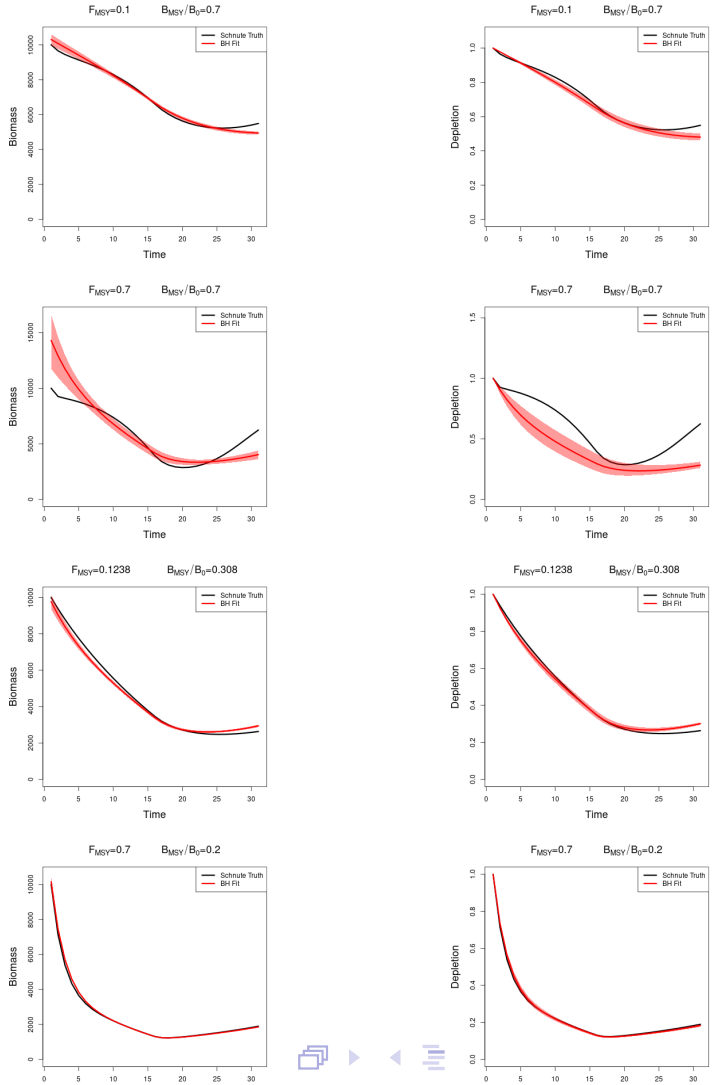



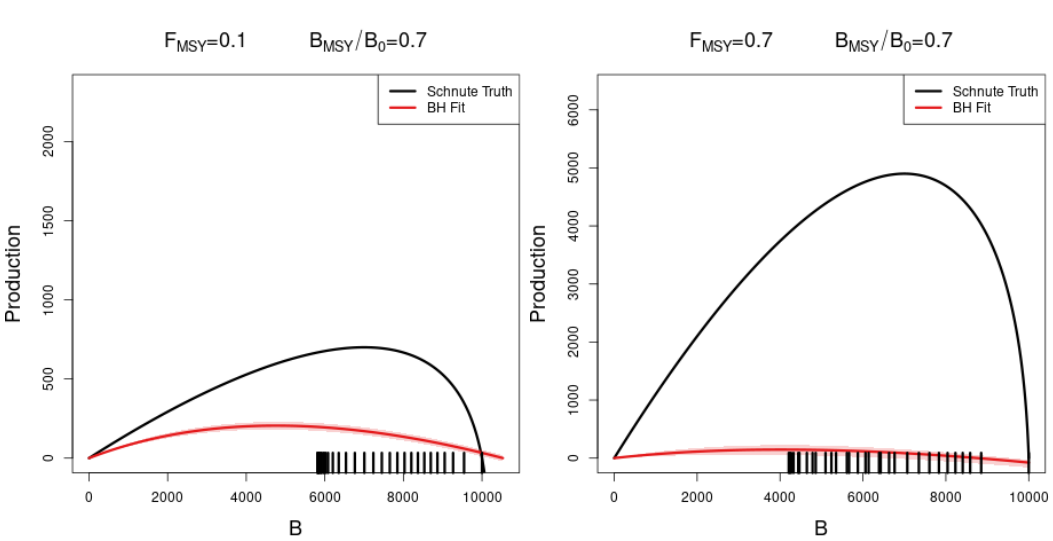



# SchnuteExpT30L2: SRR Biomass Depletion



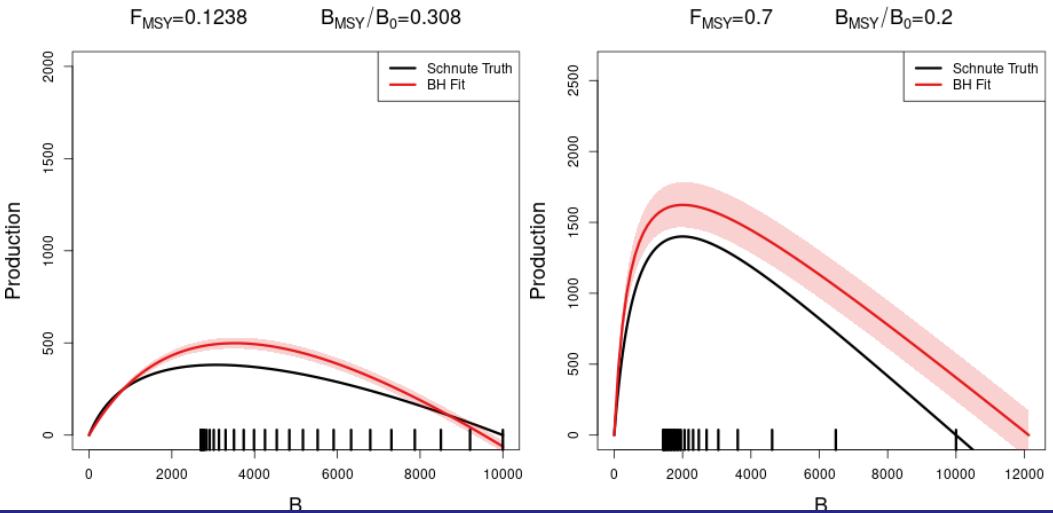



# SchnuteExpT30L3: SRR Biomass Depletion



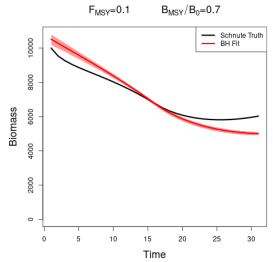


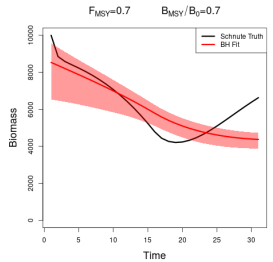


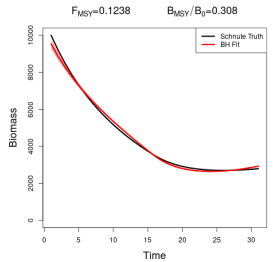


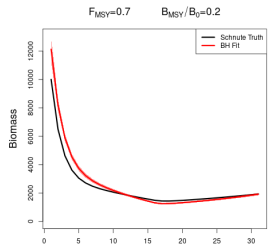


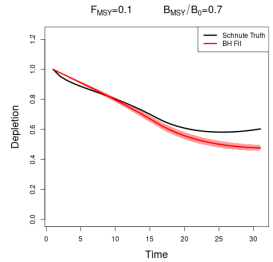


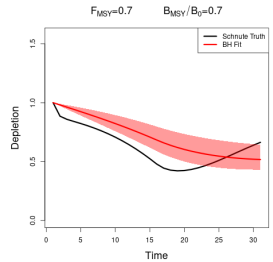


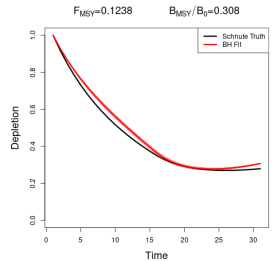



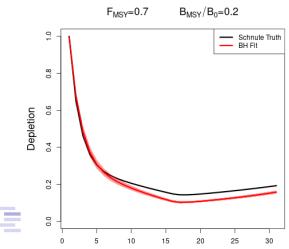



# SchnuteFlatT30: SRR Biomass Depletion