

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

14 March 2022

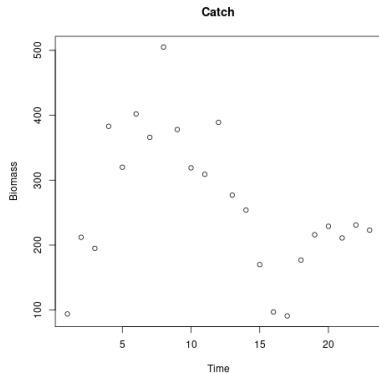
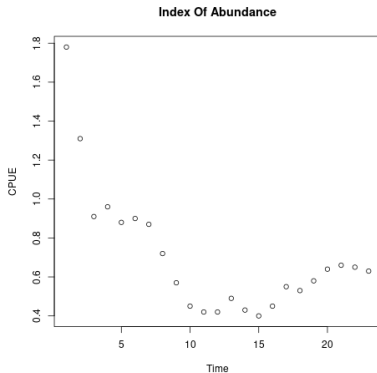


Outline

- 1 Introduction
- 2 Simulation
- 3 Results
 - Low Contrast
 - High Contrast
- 4 Proposals
 - Growth & Productivity
 - Catch Interpolation
- 5 End

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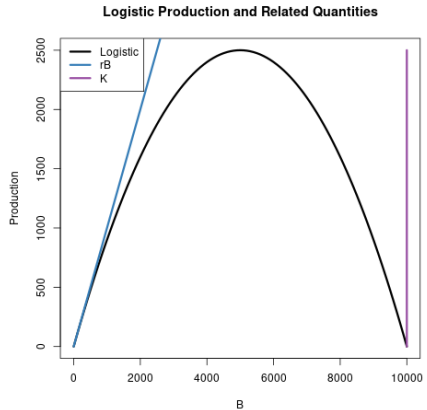
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \theta) - C(t)$$

Schaefer Model

$$P_{\theta}(B) = rB \left(1 - \frac{B}{K}\right)$$

$$\theta = (r, K)$$

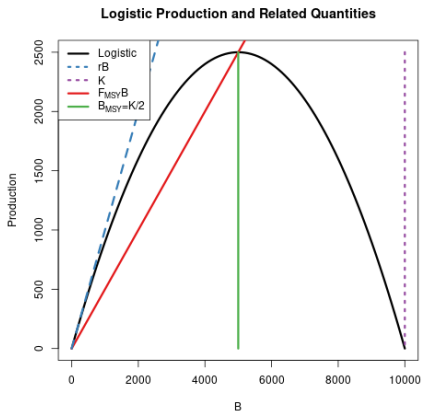


Schaefer Reference Points

$$F^* = \frac{r}{2}$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

$$MSY = \frac{rK}{4}$$



Conceptually:

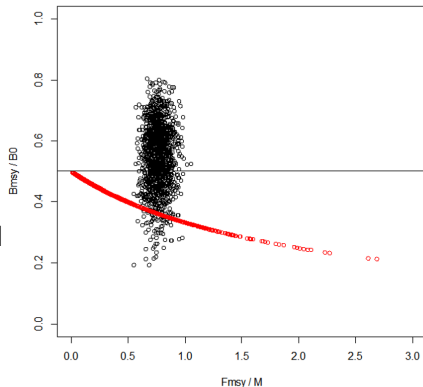
$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

- BH Model:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{F^*/M+2}$$

- Similar Constraint for Ricker and other 2 Parameter Curves



Conceptually:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} \in (0, 1)$$

Mangel et al. 2013, CJFAS:

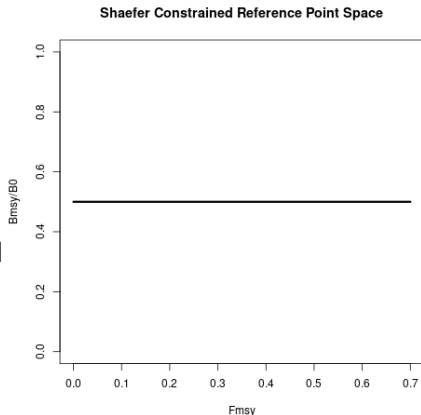
- BH Model:

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- Similar Constraint for Ricker and other 2 Parameter Curves

Schaefer Model:

$$F^* \in \mathbb{R}^+ \quad \frac{B^*}{\bar{B}(0)} = \frac{1}{2}$$



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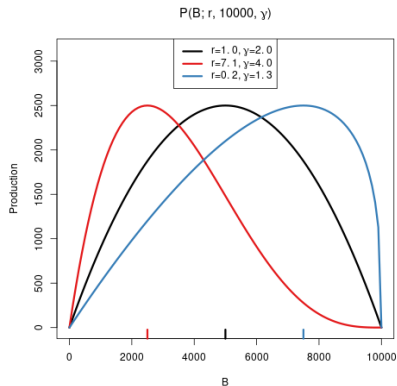
5 End

Pella-Tomlinson Production Model

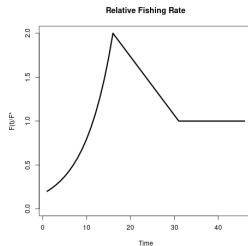
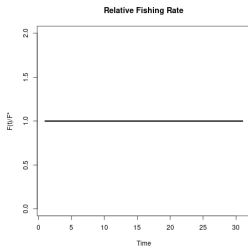
$$I(t) \sim LN(qB(t), \sigma^2)$$
$$\frac{dB(t)}{dt} = P_{\theta}(B(t)) - F(t)B(t)$$

$$P_{\theta}(B) = \frac{rB}{\gamma - 1} \left(1 - \frac{B}{K}\right)^{\gamma-1}$$
$$\theta = (r, K, \gamma)$$

$\gamma = 2 \Rightarrow$ Schaefer Model

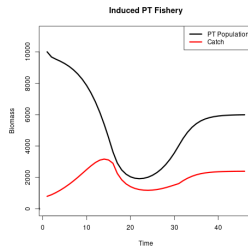
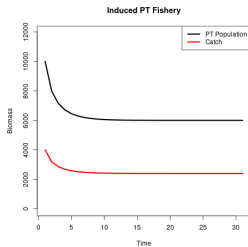
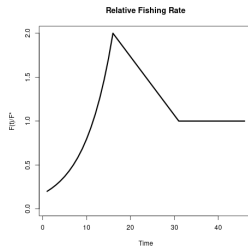
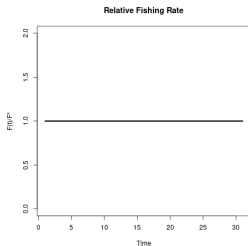


Catch

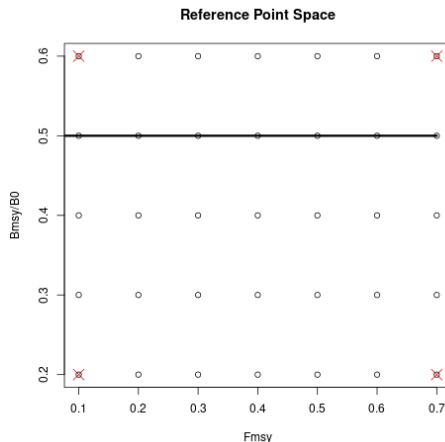


$$\begin{aligned} C(t) &= F(t)B(t) \\ &= F^* \left(\frac{F(t)}{F^*} \right) B(t) \end{aligned}$$

Catch

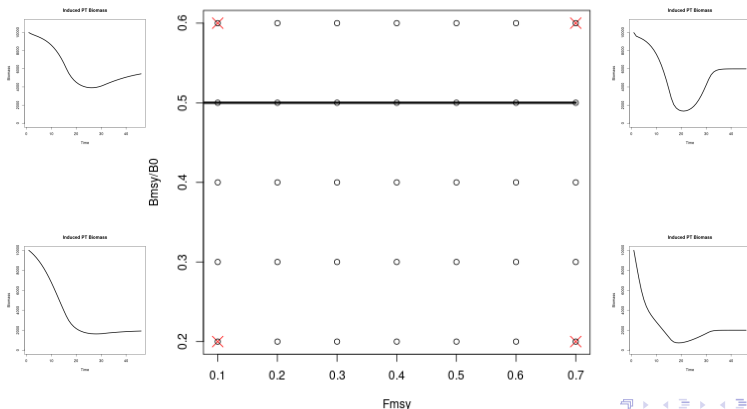


$$\theta = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)}, K = 10000, \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$



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Reference Point Space



Metamodel

$$\underbrace{\left(F^*, \frac{B^*}{\bar{B}(0)} \right)}_{\text{PT Truth}} \xrightarrow{\text{GP}} \underbrace{\left(\hat{F}^*, \frac{\hat{B}^*}{\bar{B}(0)} \right)}_{\text{Shaefer Estimate}}$$

- GP interpolates over degrees of RP model misspecification.
- Propagation of estimate uncertainty smooths bias estimation.
- Explicitly highlights trade-offs induced in RPs.

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2 Simulation

3 Results

■ Low Contrast

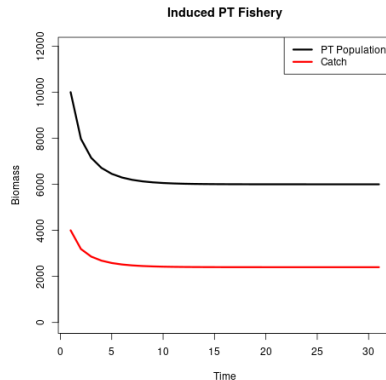
■ High Contrast

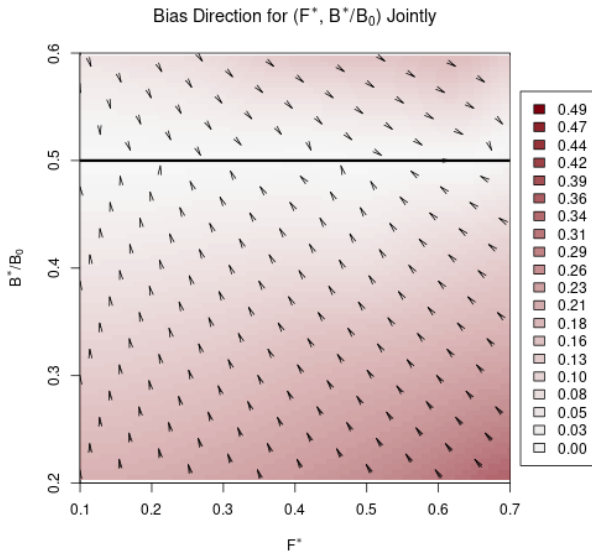
4 Proposals

■ Growth & Productivity

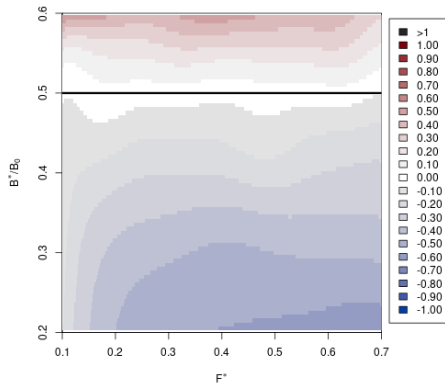
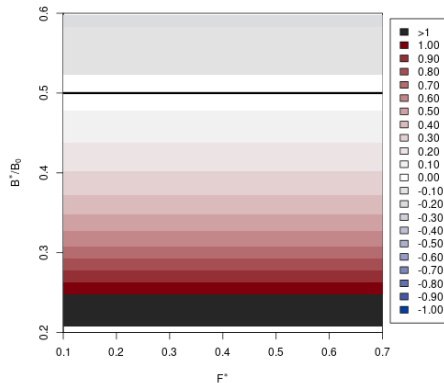
■ Catch Interpolation

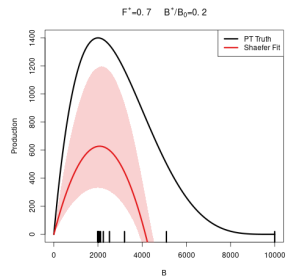
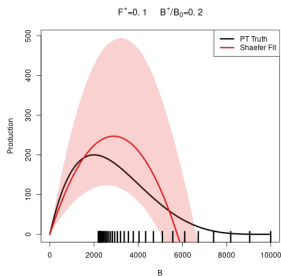
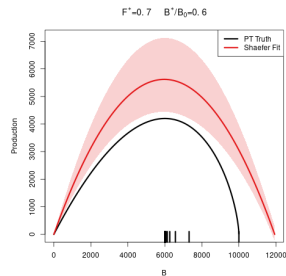
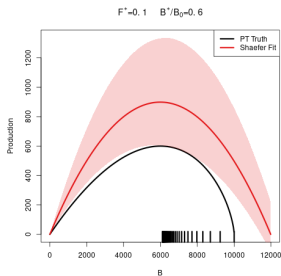
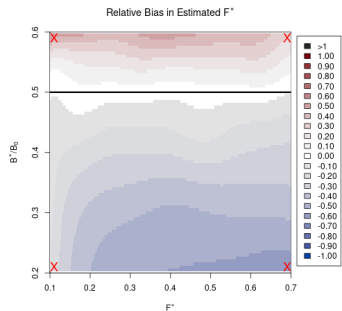
5 End

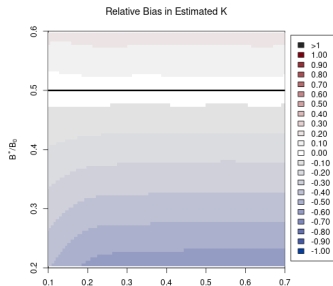
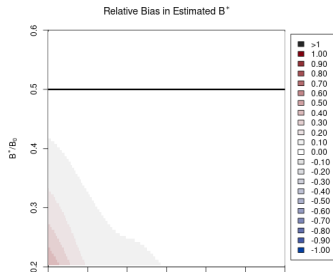
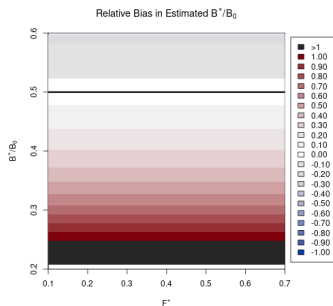




Components of Bias

Relative Bias in Estimated F^* Relative Bias in Estimated B^*/B_0 





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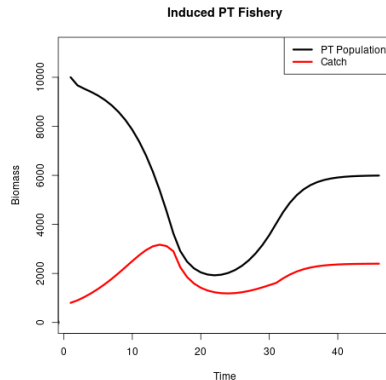
■ High Contrast

4 Proposals

■ Growth & Productivity

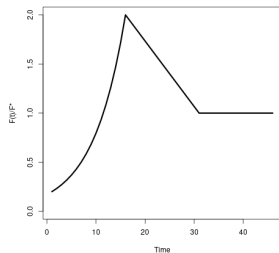
■ Catch Interpolation

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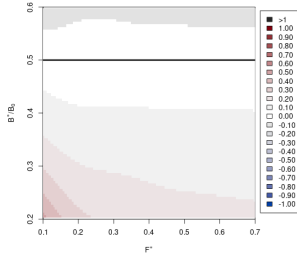


High Contrast

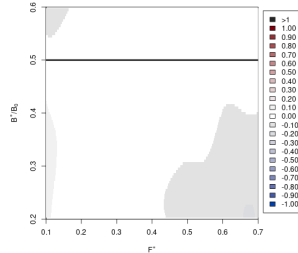
Relative Fishing Rate



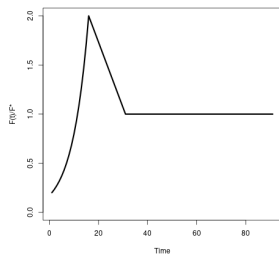
Relative Bias in Estimated B*



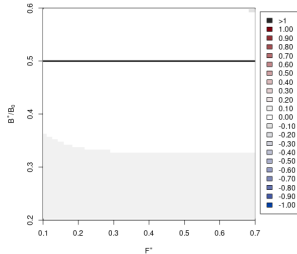
Relative Bias in Estimated F*



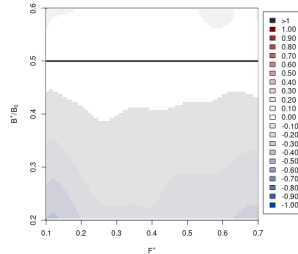
Relative Fishing Rate



Relative Bias in Estimated B*



Relative Bias in Estimated F*



Summary

- In practice the Schaefer model is at best only likely to estimate one of either B^* or F^* reasonably.
- The observed contrast serves to distribute the available information among B^* and F^* .
 - ⇒ Improved models of catch could contextulize interpretation of RP estimation.
- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
 - ⇒ Productivity Extensions
 - ⇒ Individual growth and maturity dynamics

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Productivity Extension

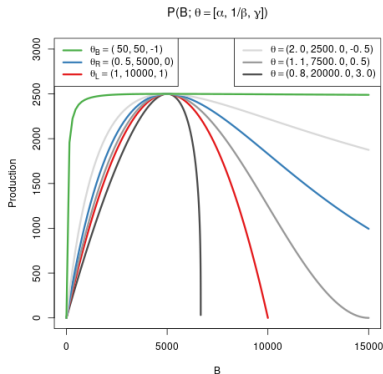
$$\frac{dB}{dt} = P(B; \theta) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B(1 - \beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma = 1 \Rightarrow$ Logistic

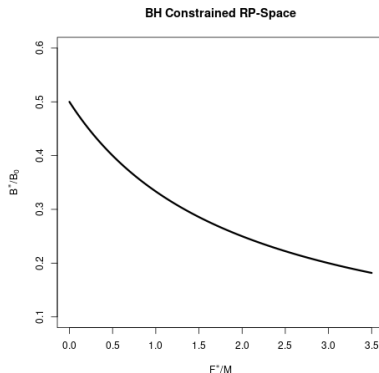
$\gamma \rightarrow 0 \Rightarrow$ Ricker



Productivity Extension

$$P_{\text{BH}}(B; [\alpha, \beta, -1]) = \frac{\alpha B}{(1 + \beta B)}$$

$$\frac{B^*}{\bar{B}(0)} = \frac{1}{\frac{F^*}{M} + 2}$$



Growth Extension

$$\frac{dB}{dt} = \overbrace{w(a_0)R(B; \theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa [w_\infty N - B]}^{\text{Net Growth}} - \overbrace{(M + F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B; \theta) - (M + F)N$$

$$R(B; [\alpha, \beta, \gamma]) = \alpha B(t - a_0)(1 - \beta \gamma B(t - a_0))^{\frac{1}{\gamma}}$$

$$w(a) = w_\infty(1 - e^{-\kappa a})$$

$$\theta' = [\alpha, \beta, \gamma] \quad \text{Species Properties: } a_0, \kappa, w_\infty, M$$

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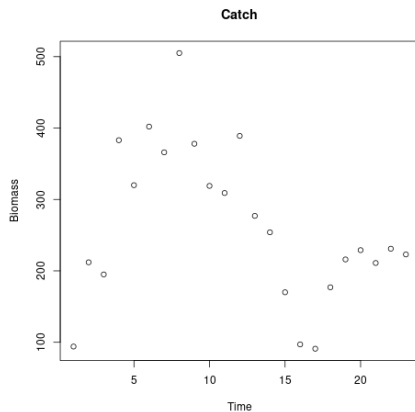
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Catch Interpolation

$$t \in \mathbb{R}^+ \quad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[y(t)] = \int_{\tau}^t x(t^*) dt^*$$

$$x(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbf{1}_{t > \tau_j}$$

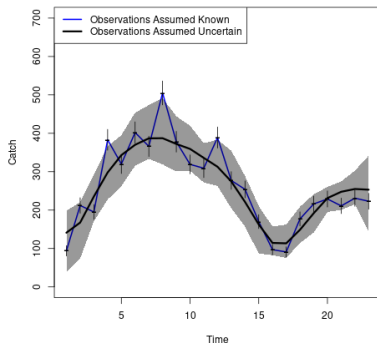


Catch Interpolation

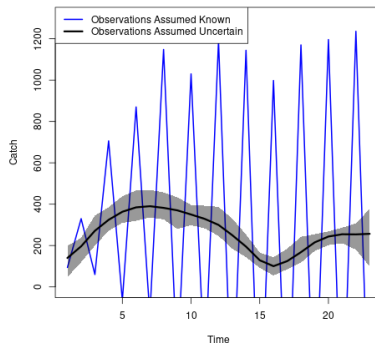
$$y(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \quad \phi \sim \text{Half-Cauchy}(0, 1) \quad \epsilon_i \sim N(0, \sigma_i^2)$$

Observed Catch with Predictive Interpolations



Interpolated Instantaneous Catch

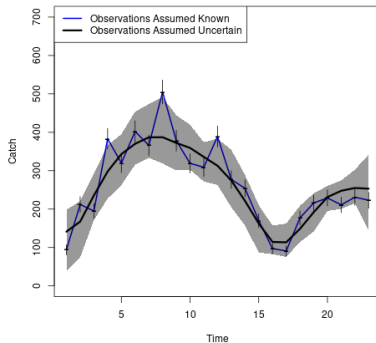


Catch Interpolation

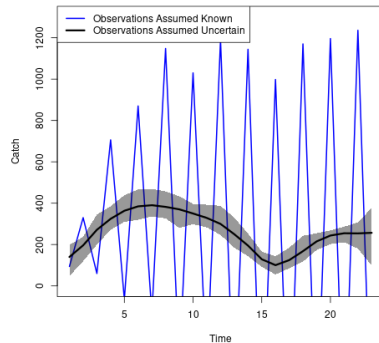
$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - C(\tau)$$

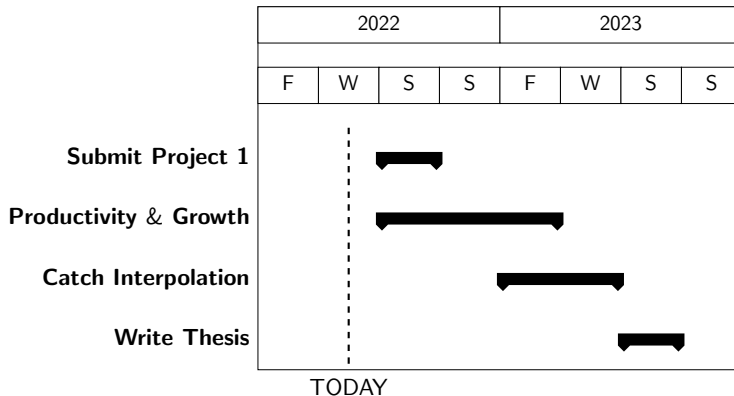
Observed Catch with Predictive Interpolations



Interpolated Instantaneous Catch



Timeline



Thanks and Acknowledgements NOAA, Sea Grant Ecetra

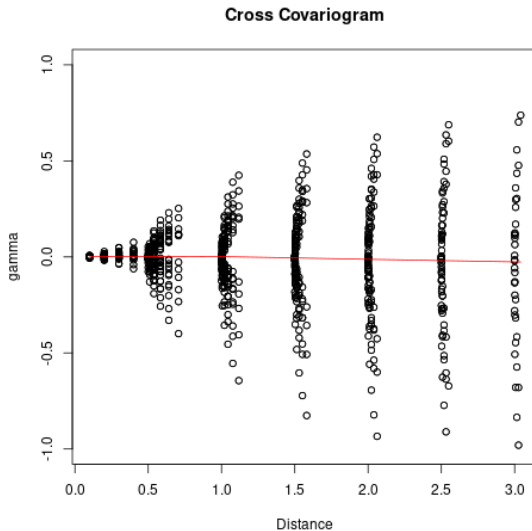
Metamodel Details

$$\hat{\mu} = \widehat{\log(r)} \quad - \text{ or } - \quad \hat{\mu} = \widehat{\log(K)}$$

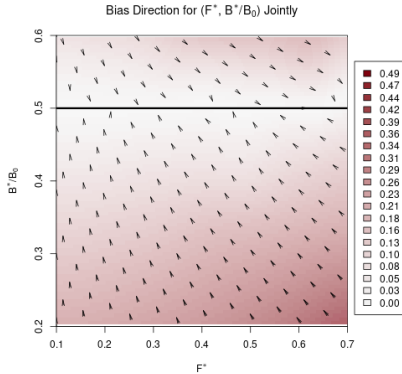
$$\mathbf{x} = \left(F^*, \frac{B^*}{\bar{B}(0)} \right)$$

$$\begin{aligned} \hat{\mu} &= \beta_0 + \beta' \mathbf{x} + f(\mathbf{x}) + \epsilon \\ f(\mathbf{x}) &\sim \text{GP}(0, \tau^2 R(\mathbf{x}, \mathbf{x}')) \\ \epsilon_i &\sim \text{N}(0, \hat{\omega}_i). \end{aligned}$$

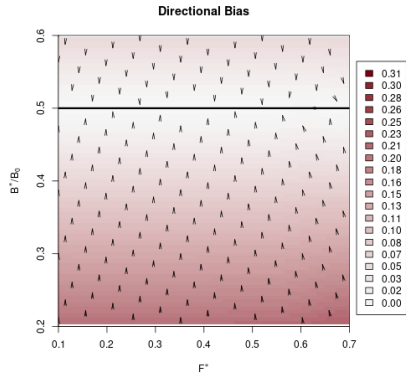
$$R(\mathbf{x}, \mathbf{x}') = \exp \left(\sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2} \right)$$



Low Contrast



High Contrast



$$\frac{B^*}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F^*}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$
$$\alpha = (M + F^*) \left[1 - \frac{1}{\gamma} \left(\frac{F^*}{M + F^*} \right) \right]^{-\gamma}$$
$$\beta = \frac{1}{\gamma \bar{B}(0)} \left(1 - \left(\frac{\alpha}{M} \right)^{\frac{1}{\gamma}} \right)$$

Common Discretization

$$\frac{dB}{dt} = P_{\theta}(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_{\theta}(B(\tau)) - C(\tau)$$

