Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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In collaboration with:
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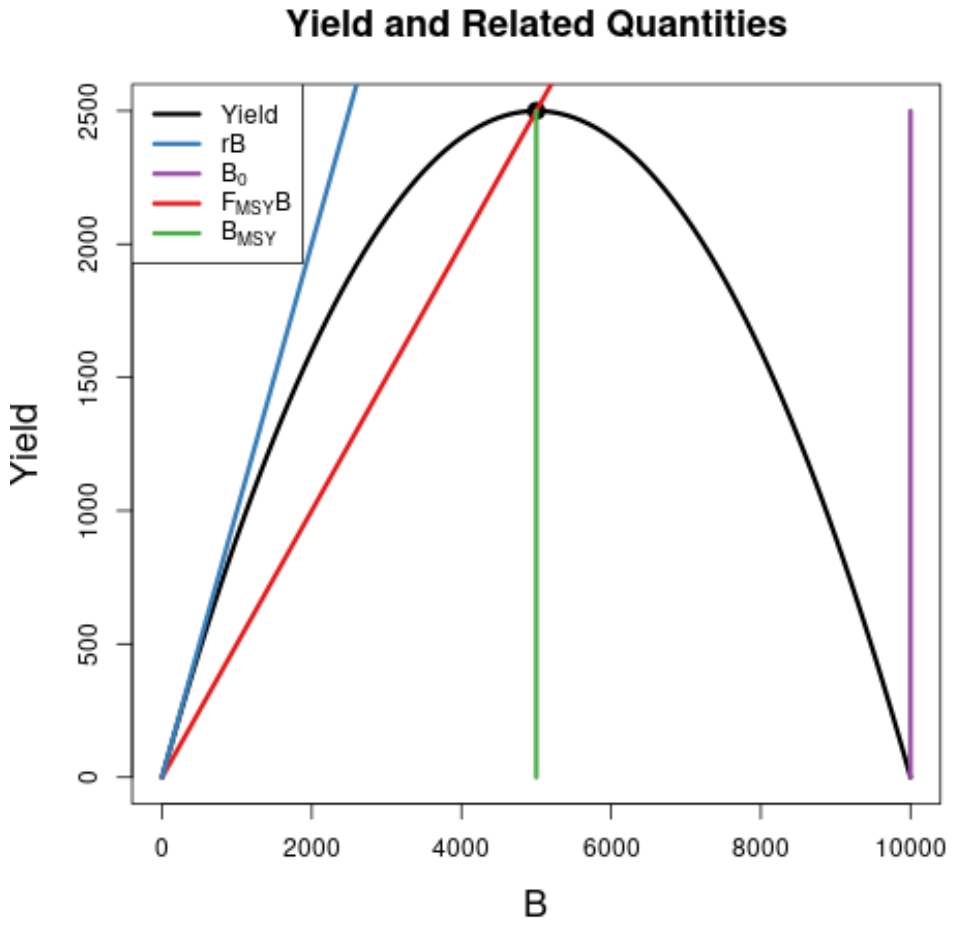
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

$$RP : MSY, \ \frac{F_{MSY}}{M}, \ \frac{B_{MSY}}{B_0}$$

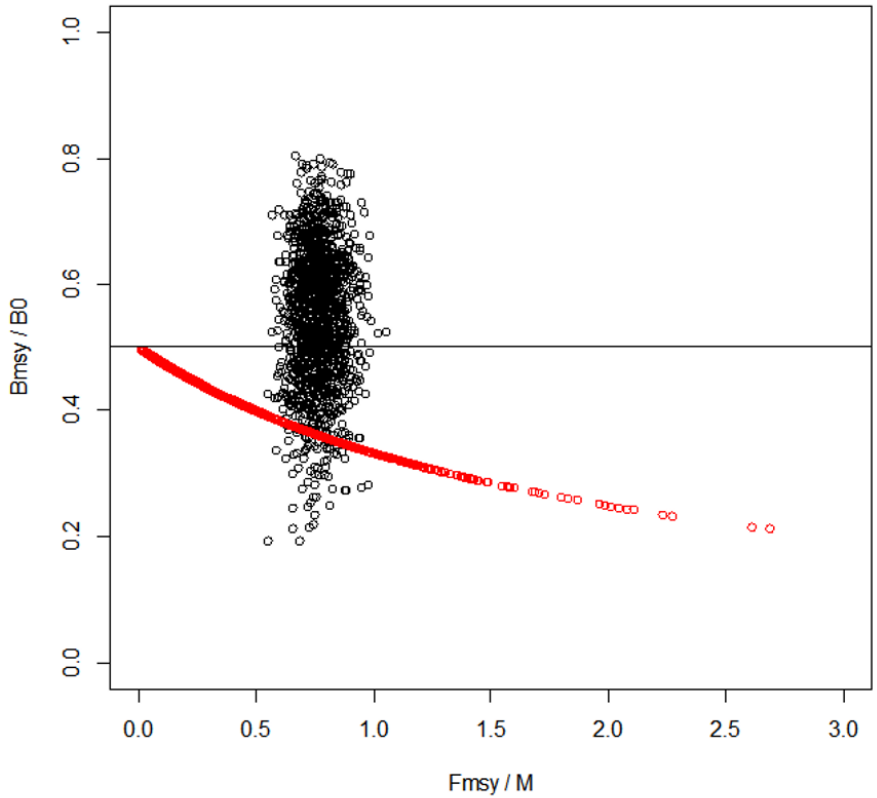


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Curves

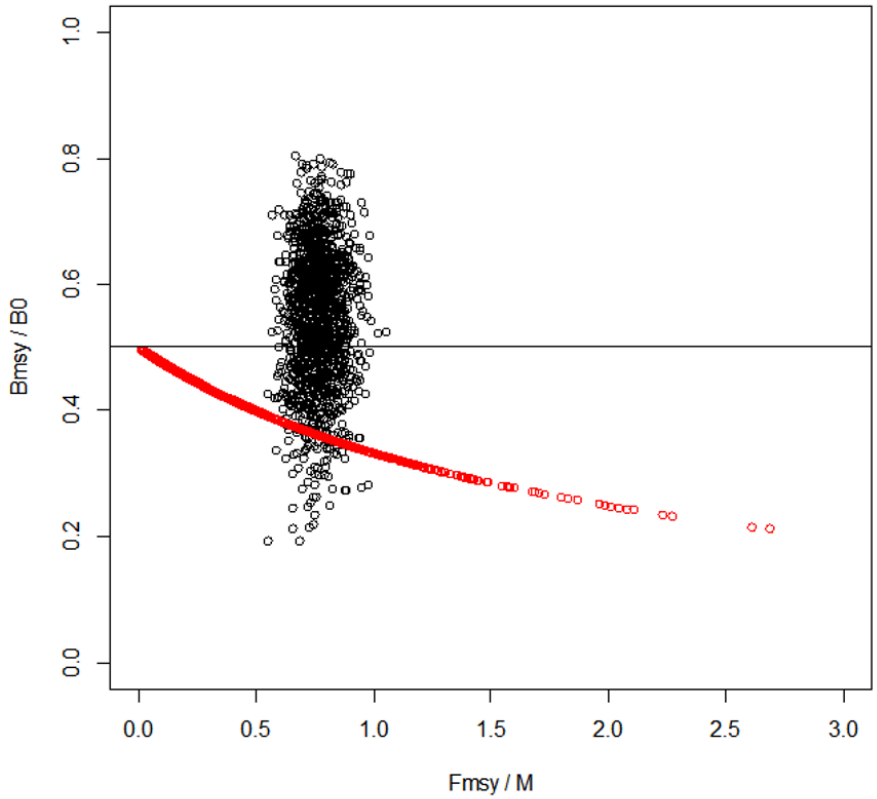


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^{+} \quad \frac{B_{MSY}}{B_0} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F_{MSY} \in \mathbb{R}^{+} \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Curves
- Three-Parameter Relationships Allow Independent RP Estimation



Schnute 1985, CJFAS

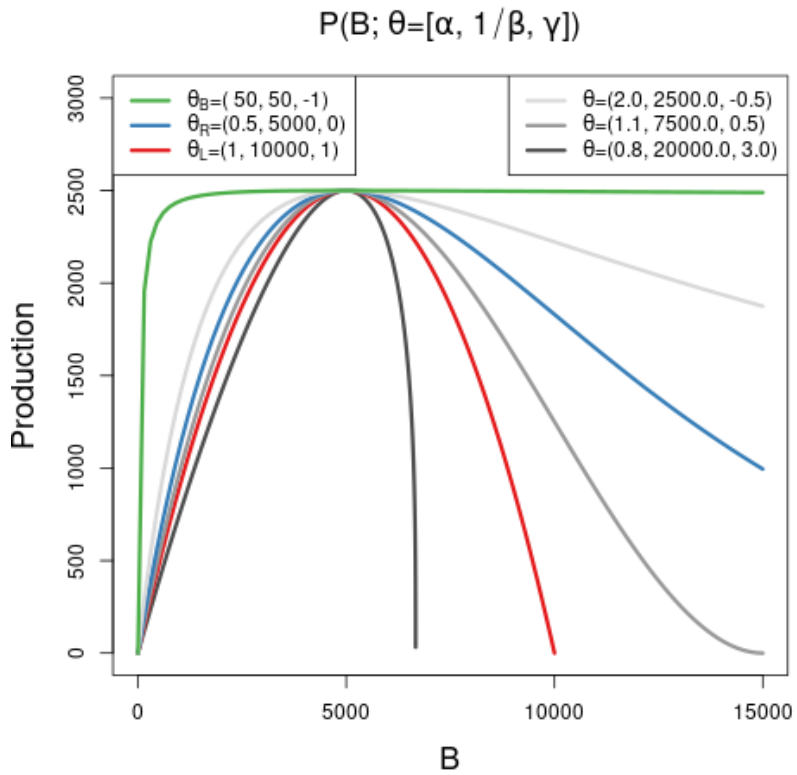
$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma \to 0 \Rightarrow$ Ricker

$\gamma = 1 \Rightarrow$ Logistic



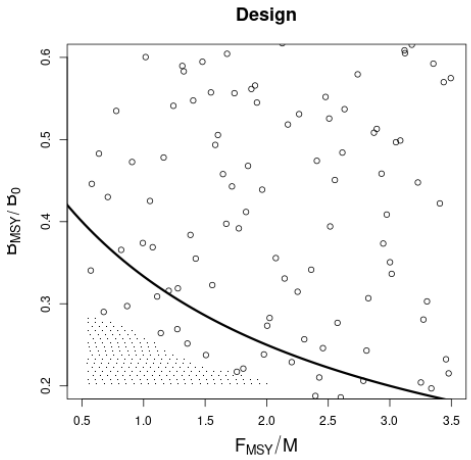
Introish Ideas list

- PT/Schaffer work (link)
- Computational Difficulties
- Schnute Space Filling
- Catch/Contrast

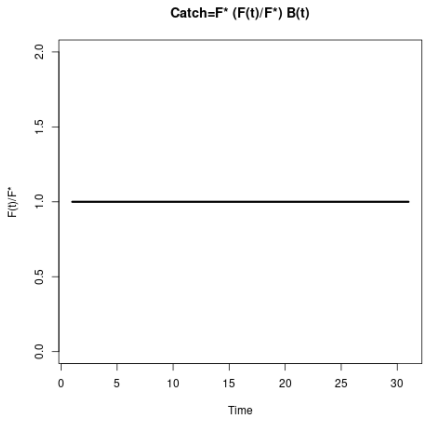
$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M+F_{MSY}}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}$$

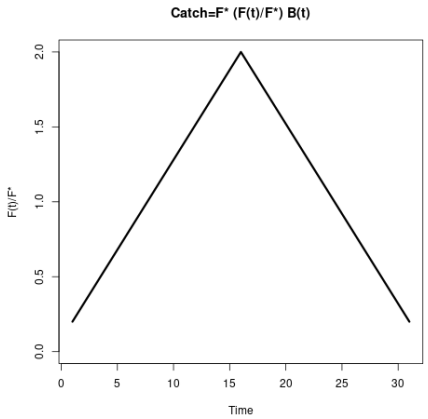
$$\alpha = (M + F_{MSY}) \left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

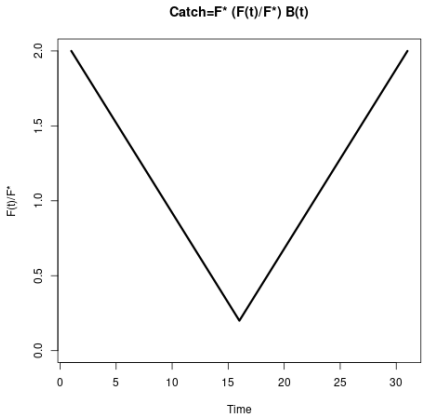
$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

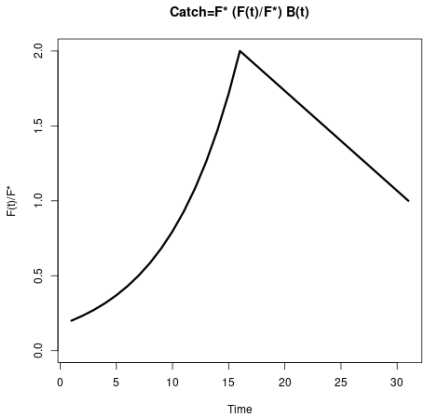


Catch

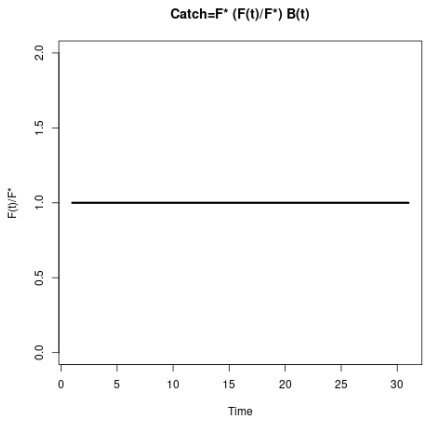

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

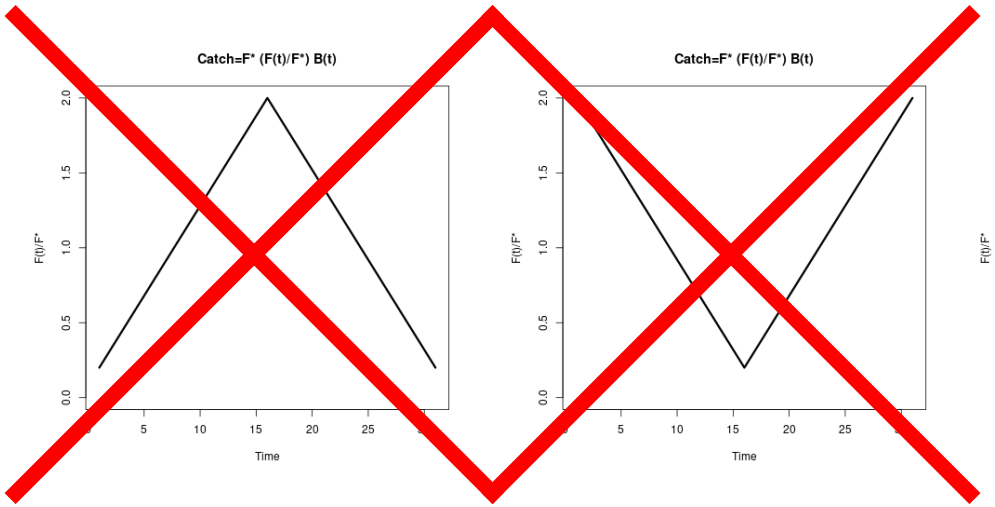

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

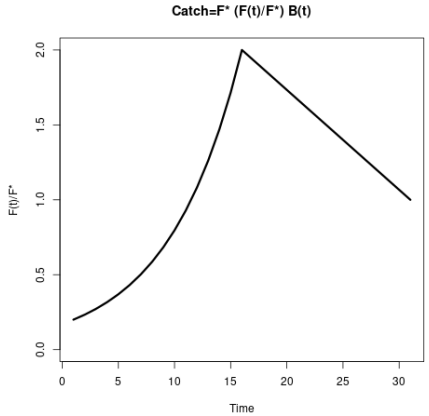

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

Catch



Flat

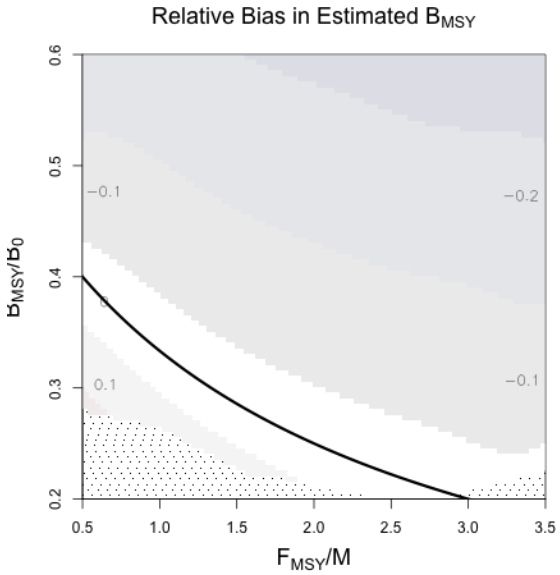


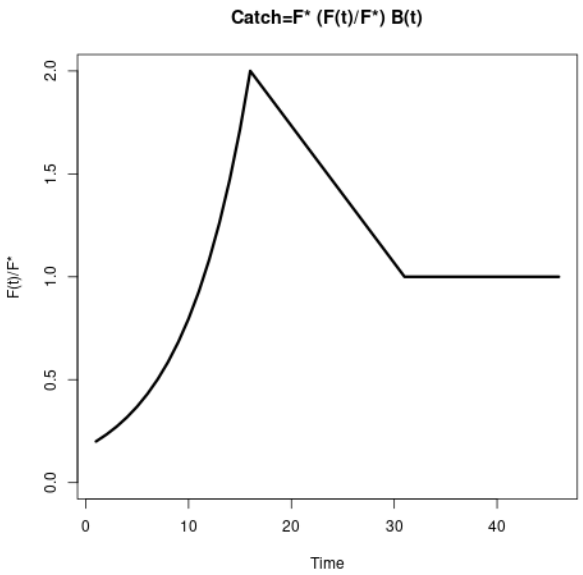


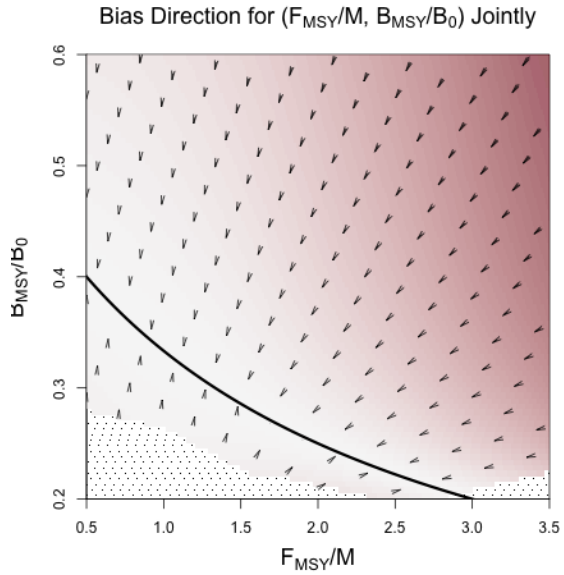
Contrast

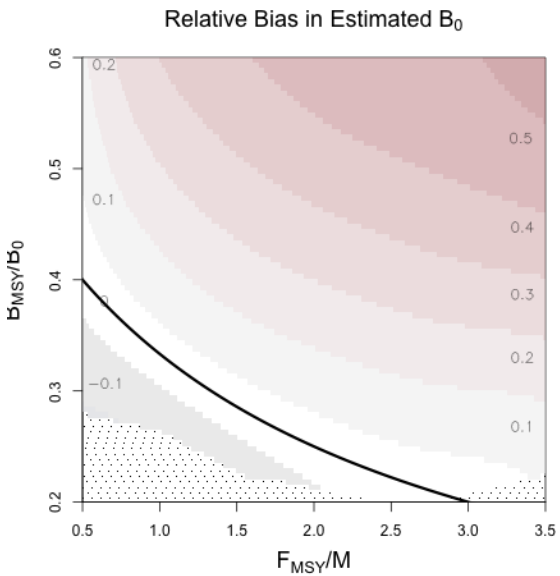
Results Idea List

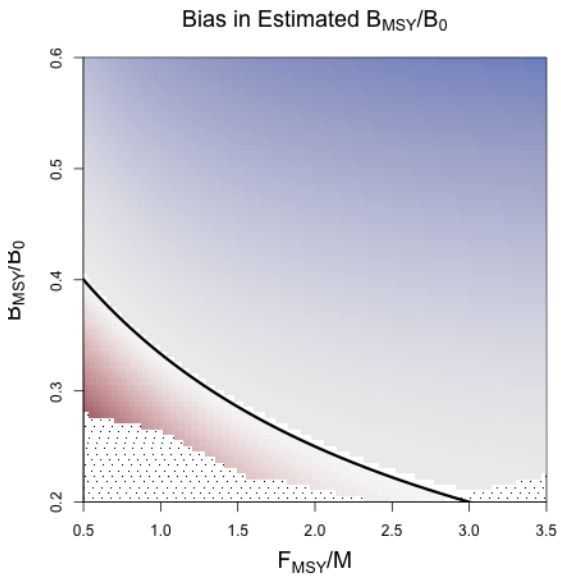
- contrast
  - components
  - animated arrows and yeild curves
- flat
  - animated arrows and yeild curves

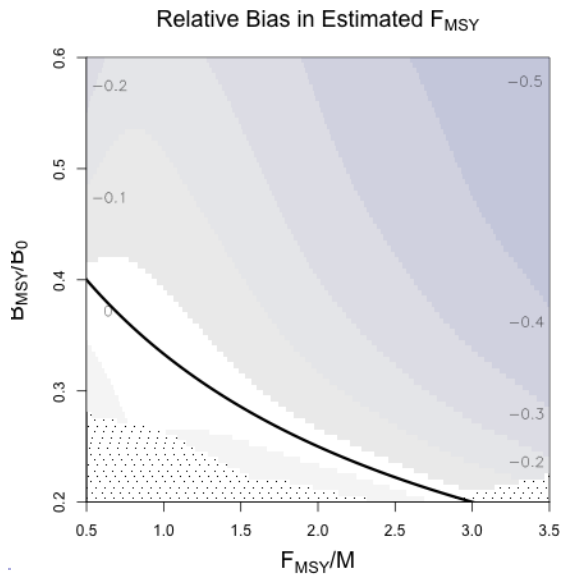

Relative Bias in Estimated B_{MSY}
B_{MSY}/B_0
F_{MSY}/M
−0.1
−0.2
0
0.1

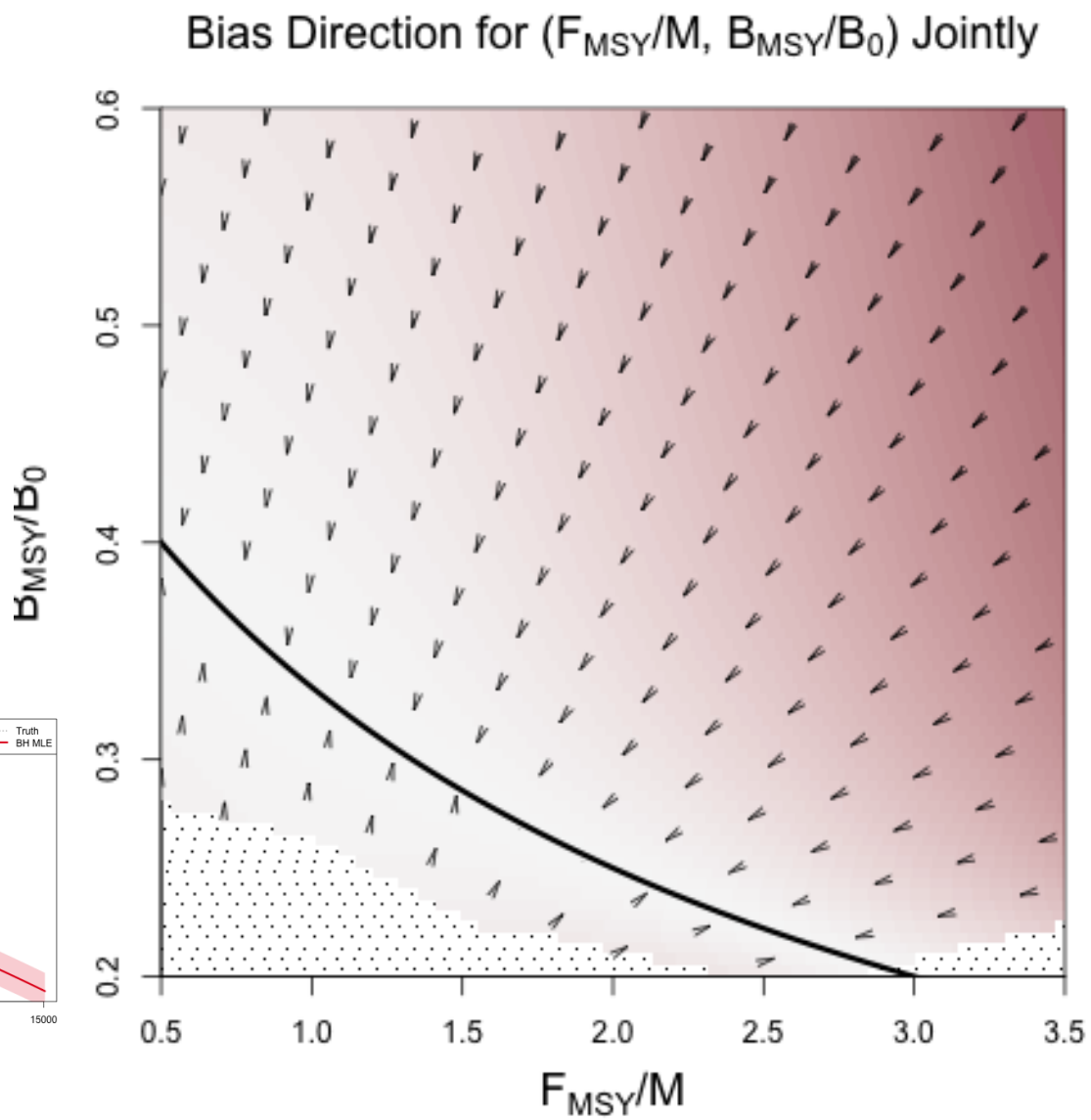

Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

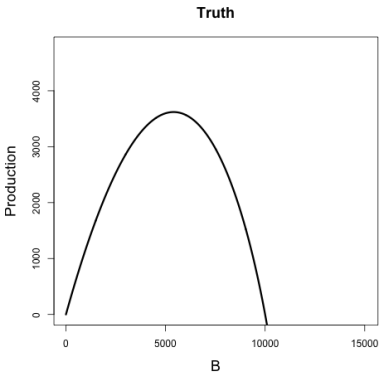

Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M

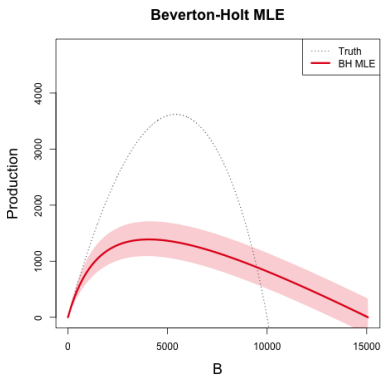

Relative Bias in Estimated B_0
B_{MSY}/B_0
F_{MSY}/M
0.2
0.1
0.5
0.4
0.3
0.2
0.1
−0.1
0

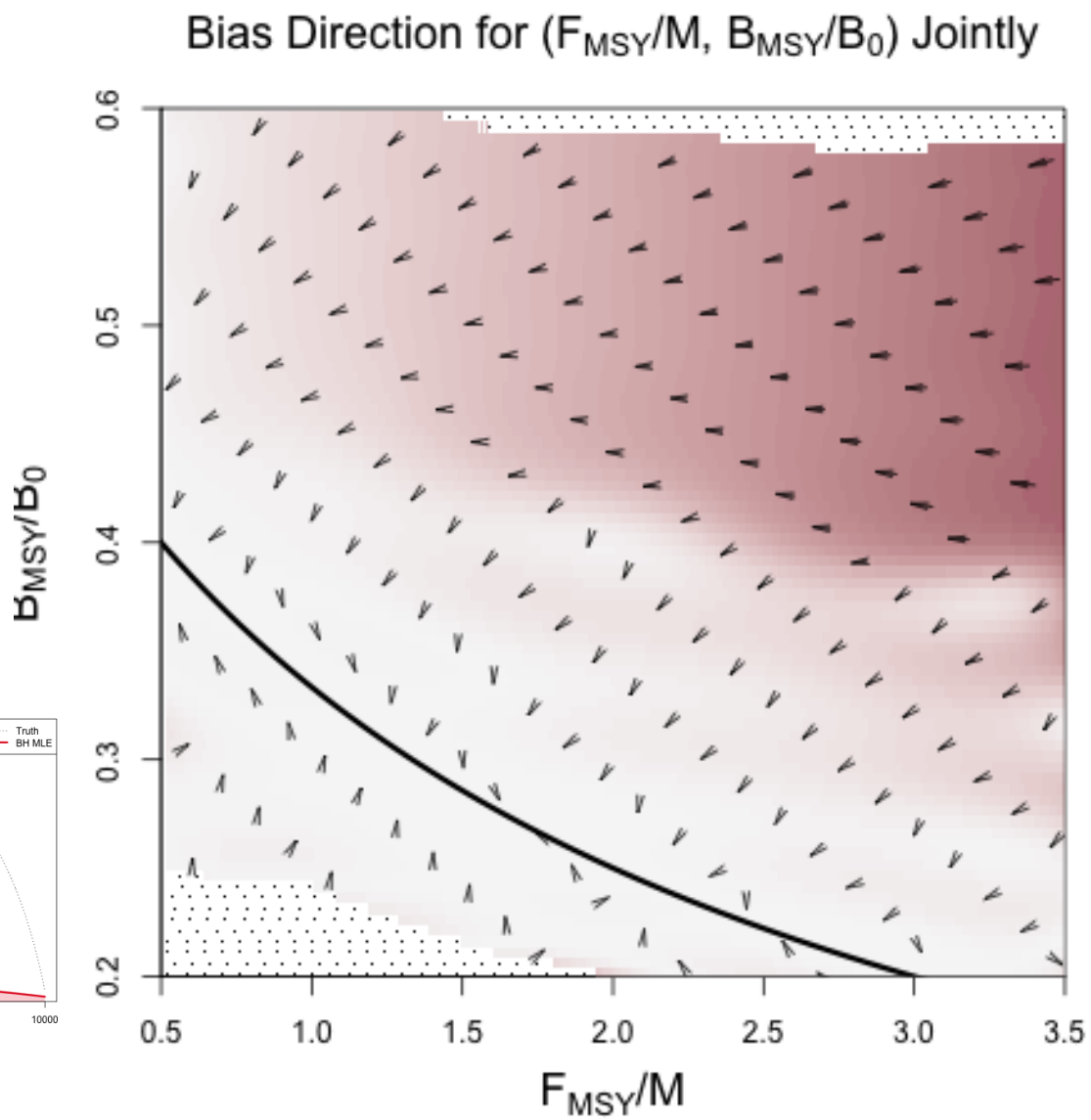

Bias in Estimated B_{MSY}/B_0
B_{MSY}/B_0
F_{MSY}/M

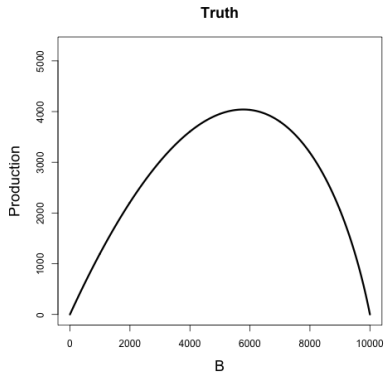

Relative Bias in Estimated F_{MSY}
B_{MSY}/B_0
F_{MSY}/M
−0.2
−0.1
0
−0.5
−0.4
−0.3
−0.2

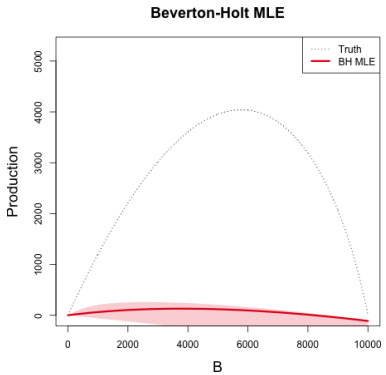

Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5


Truth
Production
B
0
1000
2000
3000
4000
0
5000
10000
15000


Beverton-Holt MLE
Truth
BH MLE
Production
B
0
1000
2000
3000
4000
0
5000
10000
15000


Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5


Truth
Production
B
0
1000
2000
3000
4000
5000
0
2000
4000
6000
8000
10000


Beverton-Holt MLE
Truth
BH MLE
Production
B
0
1000
2000
3000
4000
5000
0
2000
4000
6000
8000
10000

Conclusions

- Contrast story
- Importance of getting the computaional details correct for moving to analysis of Delay Difference and age structure