A Metamodeling Approach for Bias Estimation of Biological Reference Points

Nick Grunloh

13 August 2024

Outline

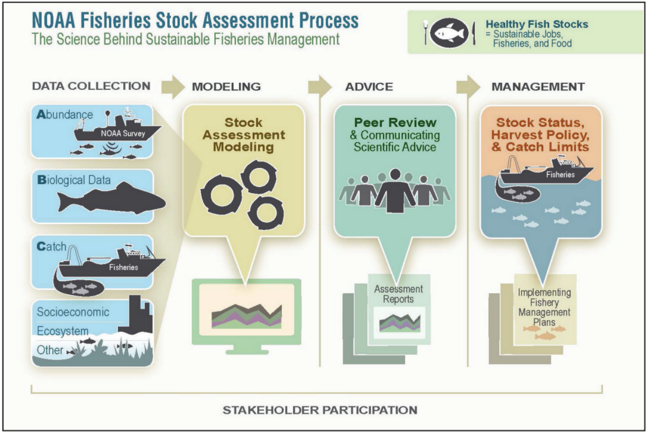
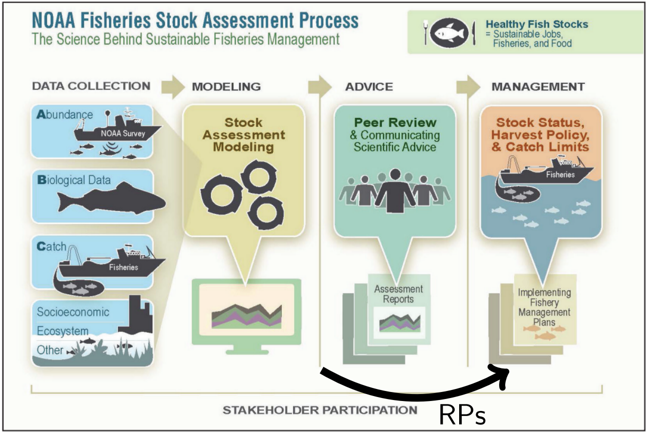


Figure 1: Overview of the stock assessment process from data collection through the provision of scientific advice to fishery managers. Stakeholders and other partners participate in each step of the assessment process. This report captures NOAA Fisheries products associated with the 'Advice' phase of the process. *

*NOAA Fisheries Annual Stock Assessment Report 2020



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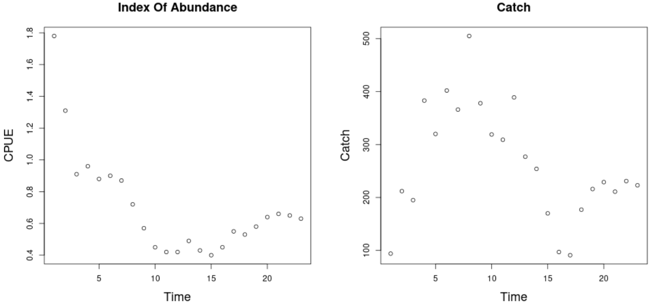
Surplus Production Model General Structure

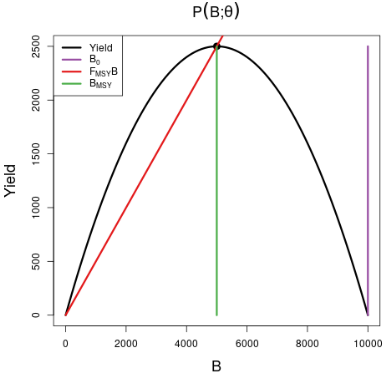


$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

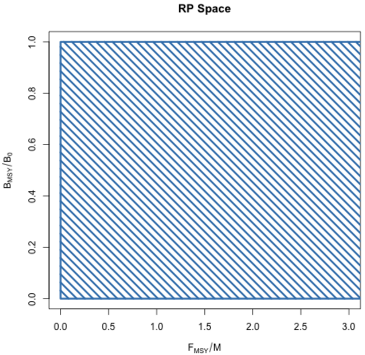
Reference Points:

- Maximum Sustainable Yield (MSY)
- F_{MSY} (or $\frac{F_{MSY}}{M}$): Fishing rate to achieve MSY
- $\frac{B_{MSY}}{B_0}$: Biomass Depletion when at MSY
- Driven by the shape of P as determined by θ.



Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0,1)$$



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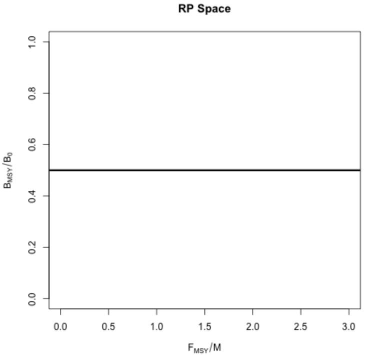
- Schaefer Model:
  $F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{2}$



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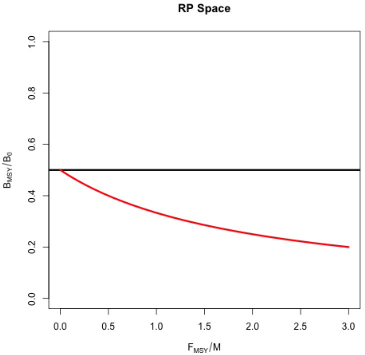

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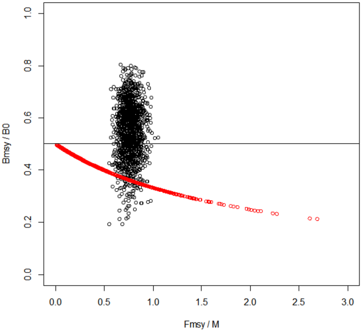
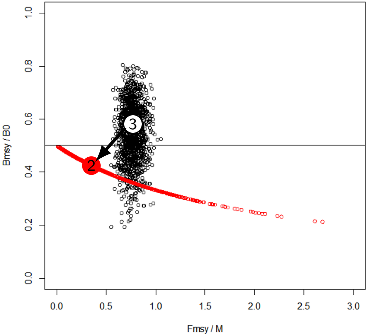
  $\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Models:
  Fox, Ricker, etc...
- Three-Parameter Models Allow Independent RP Estimation



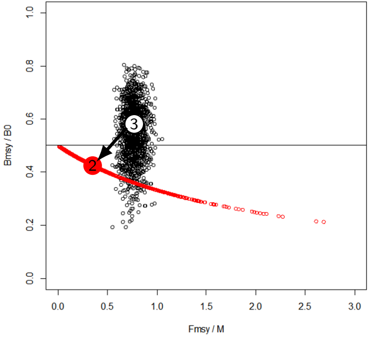
[a]

[a]Mangel et al. 2013, CJFAS

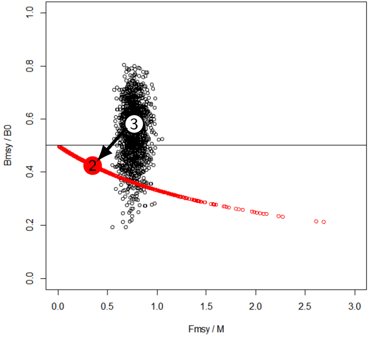
- Two-Parameter models remain standard and can induce biased RP estimation.



- Two-Parameter models remain standard and can induce biased RP estimation.
- Metamodeling RP estimation makes these RP trade-offs explicit.
- Clear description of the induced risk profile of over/under fishing.



- Two-Parameter models remain standard and can induce biased RP estimation.
- Metamodeling RP estimation makes these RP trade-offs explicit.
- Clear description of the induced risk profile of over/under fishing.
- Suggests mechanisms and the types of stocks where RP estimation may fail.
- Demonstrates the utility of adopting more flexible SRR models.



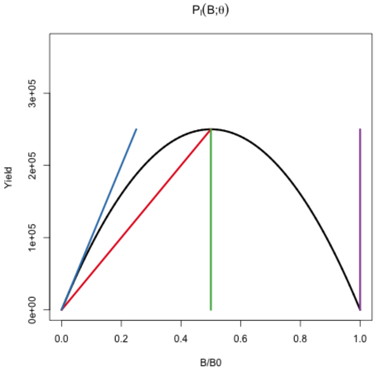
Outline

- P_ℓ is logistic production
- Logistic map in discrete time
- Implicit Natural Mortality
- Explicit Fishing Mortality

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_\ell(B(t); \boldsymbol{\theta}) - F(t)B(t)$$

$$P_\ell(B;[r,K]) = rB\left(1-\left(\frac{B}{K}\right)\right)$$



Reference Points:

$$F^* = \frac{r}{2}$$

$$B^* = \frac{K}{2} \qquad B_0 = K$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

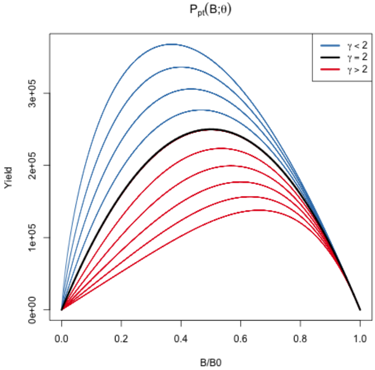
$$P_{pt}(B;[r,K,\gamma]) = \frac{rB}{\gamma-1}\left(1-\left(\frac{B}{K}\right)^{(\gamma-1)}\right)$$

Reference Points:

$$F^* = \frac{r}{\gamma}$$

$$B^* = K\left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}} \quad B_0 = K$$

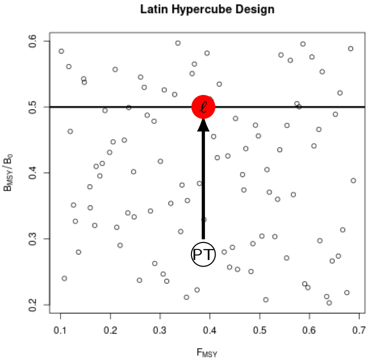
$$\frac{B^*}{B_0} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$



$$F^* = \frac{r}{\gamma} \qquad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

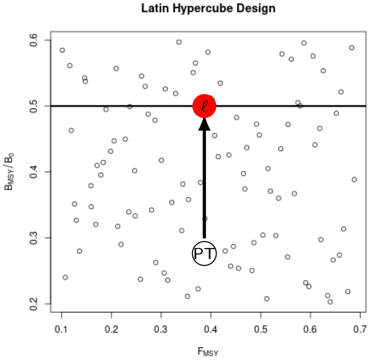
Closed-Form Inversion

$$r = \gamma F^* \quad \gamma = \frac{W\left(\frac{B^*}{\bar{B}(0)}\log\left(\frac{B^*}{\bar{B}(0)}\right)\right)}{\log\left(\frac{B^*}{\bar{B}(0)}\right)}$$

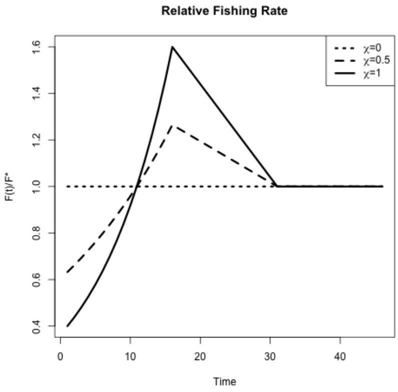


* Lambert W function inverts xe^x $s.t.$ $W(xe^x) = x$

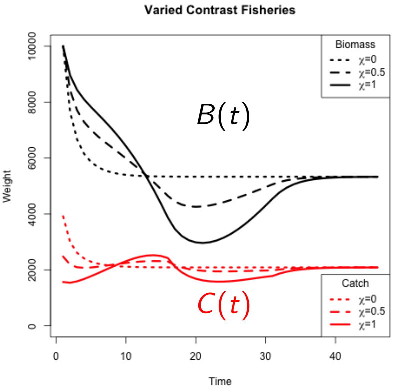
$$\underbrace{\left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{PT Truth}} \overset{GP}{\mapsto} \underbrace{\left(\hat{F}_{MSY}, \frac{1}{2}\right)}_{\text{Schaefer Estimate}}$$

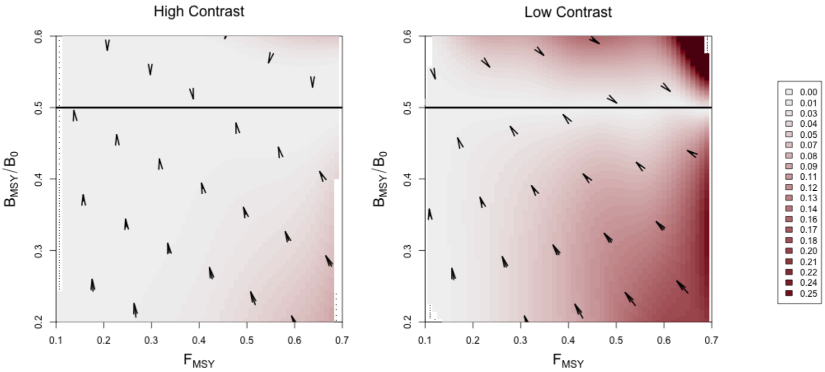


$$\frac{dB}{dt} = P(B(t);\theta) - F(t)B(t)$$
$$= P(B(t);\theta) - F^*_\theta \underbrace{\frac{F(t)}{F^*_\theta}}_{\text{Relative Fishing}} B(t)$$

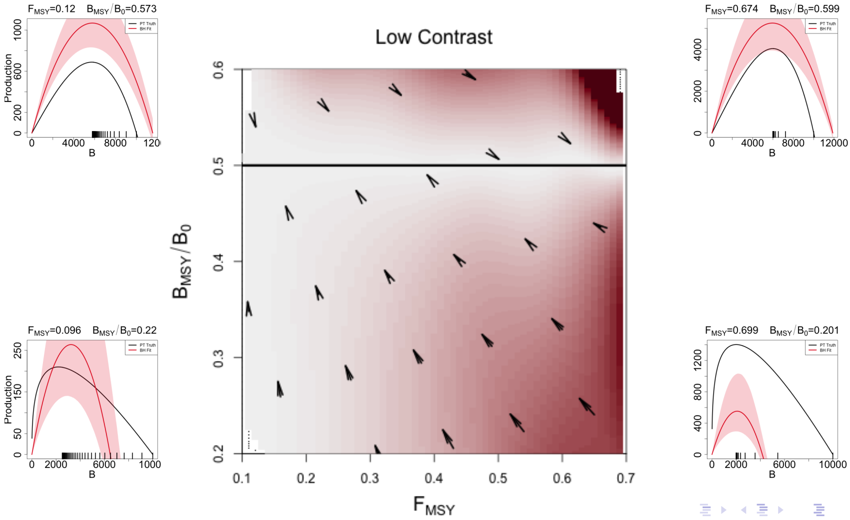


$$\frac{dB}{dt} = P(B(t);\theta) - F(t)B(t)$$
$$= P(B(t);\theta) - \underbrace{F^*_\theta \frac{F(t)}{F^*_\theta} B(t)}_{C(t)}$$



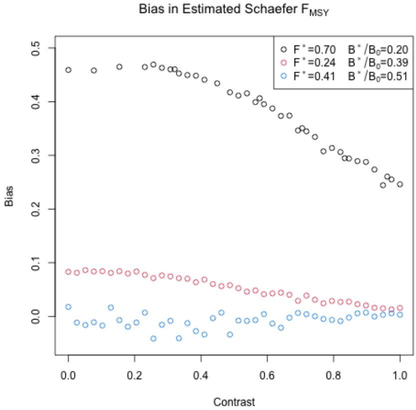

High Contrast
Low Contrast
B_{MSY}/B_0
F_{MSY}
0.1 0.2 0.3 0.4 0.5 0.6 0.7
0.2 0.3 0.4 0.5 0.6
0.00
0.01
0.03
0.04
0.05
0.07
0.08
0.09
0.11
0.12
0.13
0.14
0.16
0.17
0.18
0.20
0.21
0.22
0.24
0.25

Mechanism for Bias in F_{MSY} via Contrast



Mechanism for Bias in F_{MSY} via Contrast

- Low contrast limits the observation window on production.
- As contrast increases F^* bias diminishes.
- Bias in $\frac{B^*}{B_0}$ remains, but these biases are independent of F^*.



Summary

- Closed-form RP designs via three-parameter PT model.
- The useful notion of contrast developed here together with the simplified geometry of the Schaefer model exposes a mechanism for RP bias.
- Metamodel describes a risk of overfishing for stocks coming from $\frac{B^*}{B_0} > \frac{1}{2}$, and overly cautious fishing for $\frac{B^*}{B_0} < \frac{1}{2}$.
- What to do when the simulation design is not analytical?

Outline

$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$
$$\frac{dB}{dt} = P_{BH}(B; [\alpha, \beta]) - (M + F)B$$

$$P_{BH}(B; [\alpha, \beta]) = \frac{\alpha B}{1 + \beta B}$$

$$I_t = qB_te^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \underbrace{P_{BH}(B;[\alpha,\beta]) - MB}_{\text{Surplus Production}} - FB$$

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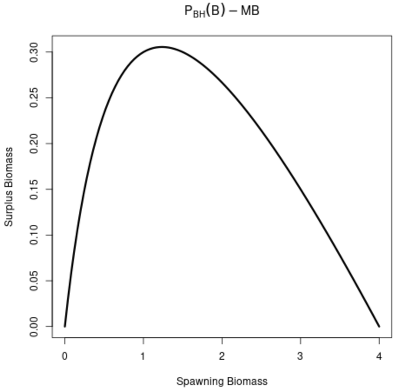

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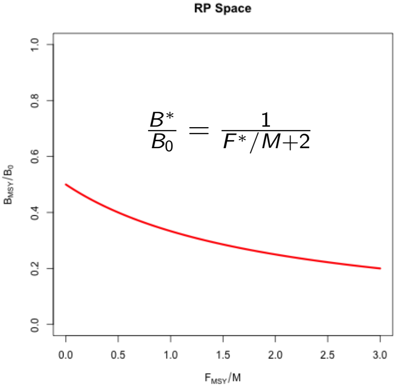


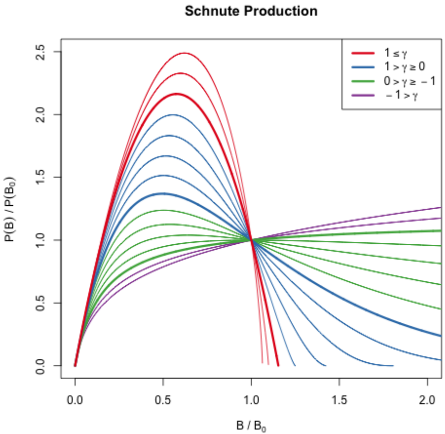
Three Parameter Schnute Generalization

$$P_s(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

Logistic
$\gamma = 1$

Ricker
$\gamma \to 0$

Beverton-Holt
$\gamma = -1$



Schnute RPs Challenges

$$\text{RPs} \mapsto \theta$$

Incomplete
Inversion

$$\theta \mapsto \text{RPs}$$

$$\alpha = (M + F^*)\left(1 + \frac{\gamma F^*}{M + F^*}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$

$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}.$$

Schnute & Richards (1998). CJFAS.

Schnute RPs Challenges

$$\text{RPs} \mapsto \theta$$

Incomplete Inversion

$$\theta \mapsto \text{RPs}$$

$$\alpha(\gamma)$$

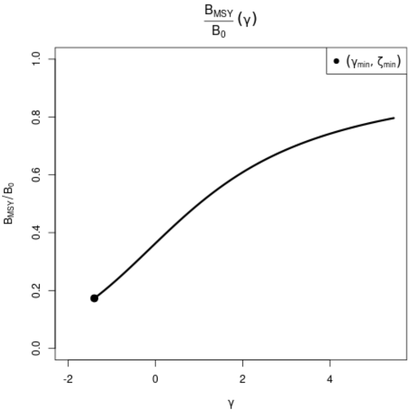
$$\beta(\alpha(\gamma), \gamma)$$

$$\frac{B^*}{B_0}(\alpha(\gamma), \gamma)$$

Schnute & Richards (1998). CJFAS.

Sudo Inverse Inverse-CDF Sampling

$$\frac{B^*}{B_0}(\gamma) = \frac{1-\left(\frac{M+F^*}{\alpha(\gamma)}\right)^{\gamma}}{1-\left(\frac{M}{\alpha(\gamma)}\right)^{\gamma}}$$



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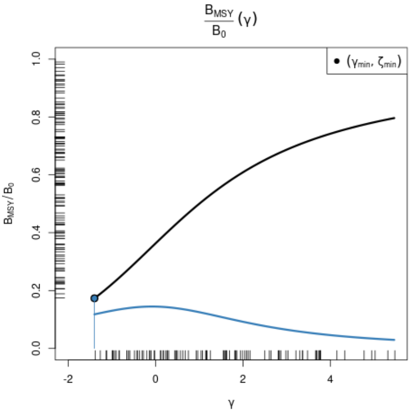
$$\gamma' \sim \zeta_{min}\delta(\gamma_{min}) + (1-\zeta_{min})t(\mu,\sigma,\nu)\mathbf{1}_{\gamma>\gamma_{min}}$$



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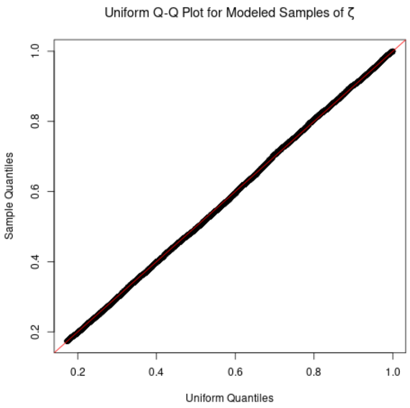


Schnute LHS Design

Logistic

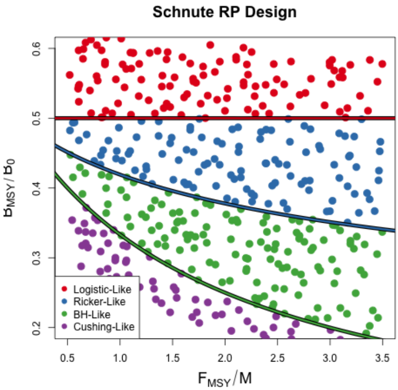
$$\gamma = 1$$

Ricker

$$\gamma \to 0$$

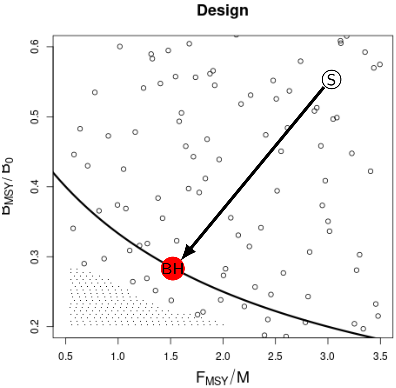
Beverton-Holt

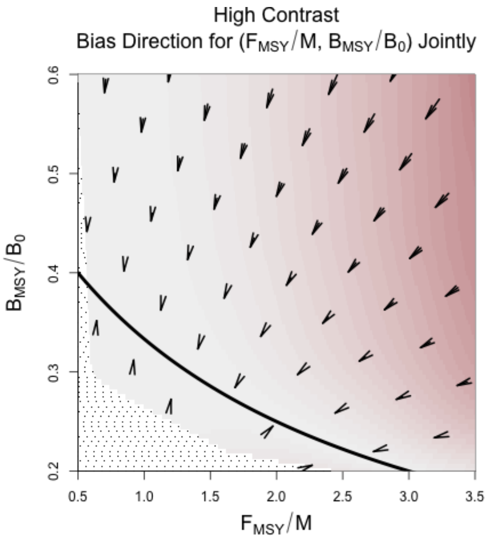
$$\gamma = -1$$

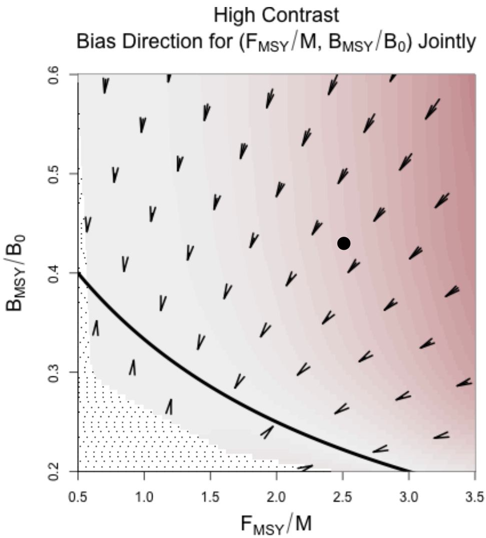


Schnute LHS Design

$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{GP}{\mapsto} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{1}{\hat{F}_{MSY}/M+2}\right)}_{\text{BH Estimate}}$$




High Contrast
Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
0.6
0.5
0.4
0.3
0.2
B_{MSY}/B_0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M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B_{MSY}/B_0
0.2
0.3
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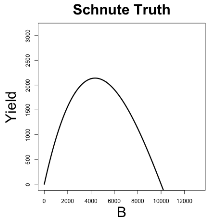
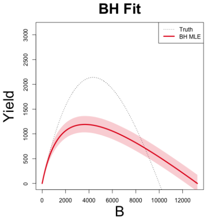
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M


Schnute Truth
Yield
0
500
1000
1500
2000
2500
3000
0
2000
4000
6000
8000
10000
12000
B


BH Fit
Truth
BH MLE
Yield
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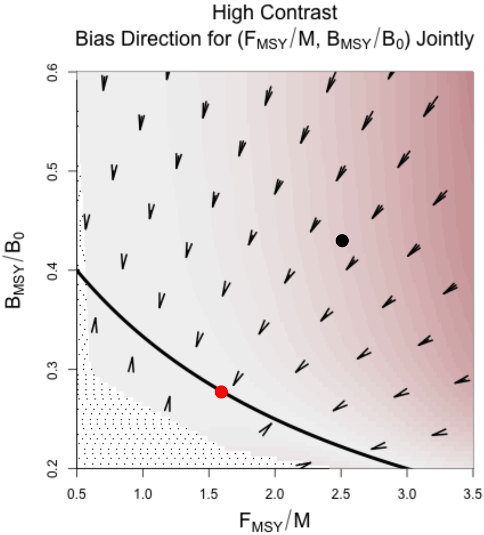
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Bias Direction for (F_{MSY}/M, B_{MSY}/B_0) Jointly
B_{MSY}/B_0
F_{MSY}/M
0.2
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0.4
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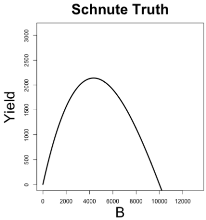
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5


Schnute Truth
Yield
B
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500
1000
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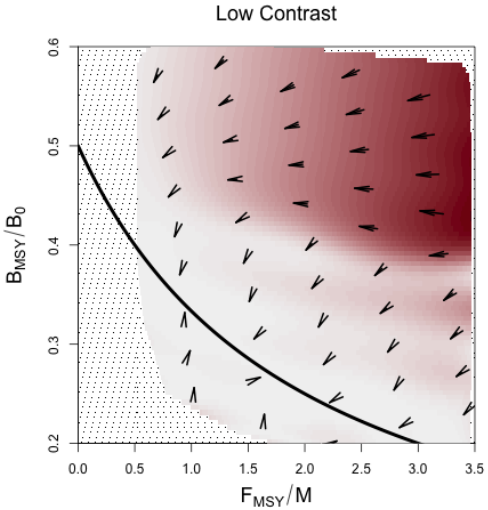
Low Contrast
B_{MSY}/B_0
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2.5
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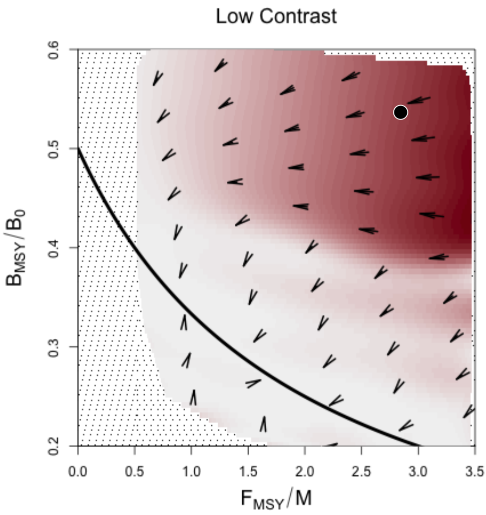
3.5
F_{MSY}/M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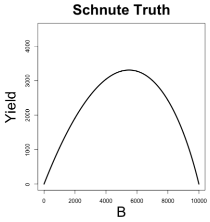
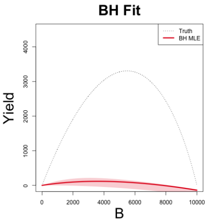
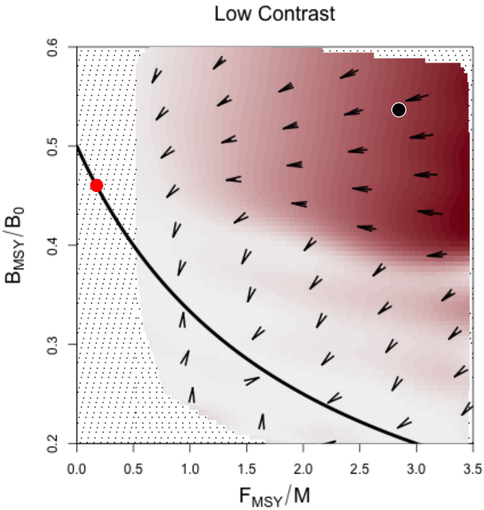
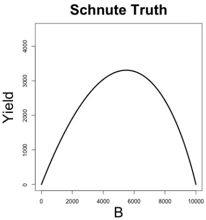
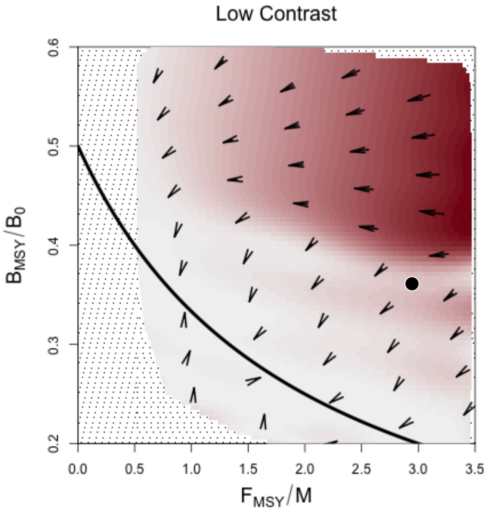
F_{MSY}/M

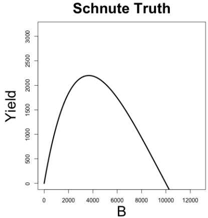

Schnute Truth
Yield
0
1000
2000
3000
4000
0
2000
4000
6000
8000
10000
B

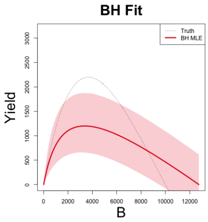

BH Fit
Yield
B
Truth
BH MLE
0
1000
2000
3000
4000
0
2000
4000
6000
8000
10000

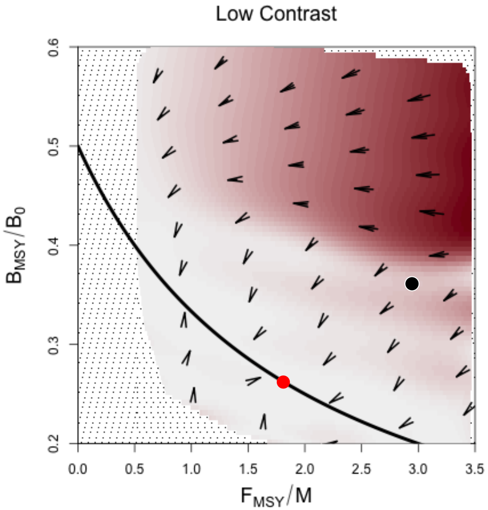

Low Contrast
B_{MSY}/B_0
F_{MSY}/M
0.2
0.3
0.4
0.5
0.6
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

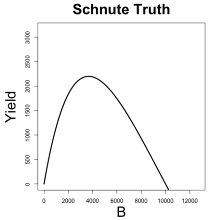

Schnute Truth
Yield
B
0
1000
2000
3000
4000
0
2000
4000
6000
8000
10000

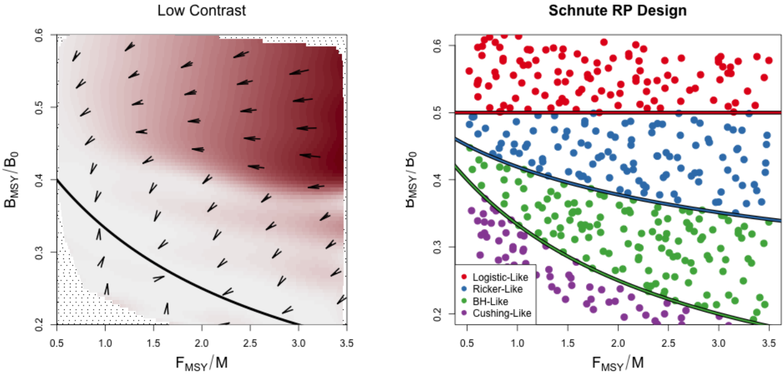

Low Contrast
0.2
0.3
0.4
0.5
0.6
B_{MSY}/B_0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M


Schnute Truth
Yield
0
500
1000
1500
2000
2500
3000
0
2000
4000
6000
8000
10000
12000
B


BH Fit
Truth
BH MLE
Yield
B


Low Contrast
B_{MSY}/B_0
F_{MSY}/M


Schnute Truth
Yield
B


Low Contrast
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5
B_{MSY}/B_0
F_{MSY}/M
Schnute RP Design
Logistic-Like
Ricker-Like
BH-Like
Cushing-Like

Summary

- Schnute three-parameter generalizing model is very attractive since it interpolates the most common models of productivity.
  - Logistic, Ricker, Beverton-Holt
- A stable method of generating simulation designs despite non-analytical and numerically treacherous RPs$\leftrightarrow$ θ.
- GP metamodel demonstrates that misspecified BH models enjoy some sense of optimality in RP estimation.
  - Nearly shortest distance RP mapping as mediated by contrast.
  - but misspecified BH models induce a risk structure in RPs.
- Can more complex biological dynamics help RP estimation?

Outline

General Modeling Structure

$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \overbrace{w(a_s)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_{\infty}N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$
$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

Walters, C. J. (2020). The continuous time Schnute-Deriso delay difference model for age-structured population dynamics, with example application to the Peru anchoveta stock. University of British Columbia.

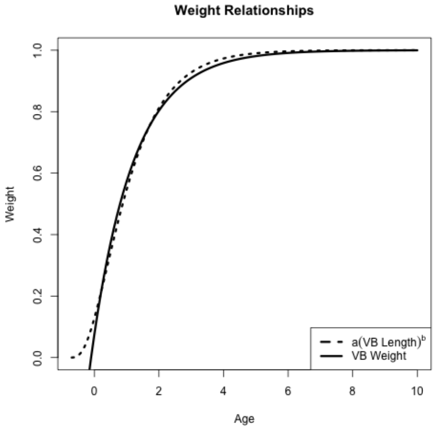
Individual Growth

$$w(a) = w_\infty(1 - e^{-\kappa(a-a_0)})$$

a_s : Lagged Maturity & Knife-Edge Selectivity

κ : Individual Growth

- Instant Growth: (Production Model)
  $a_s \to 0 \quad \kappa \to \infty$



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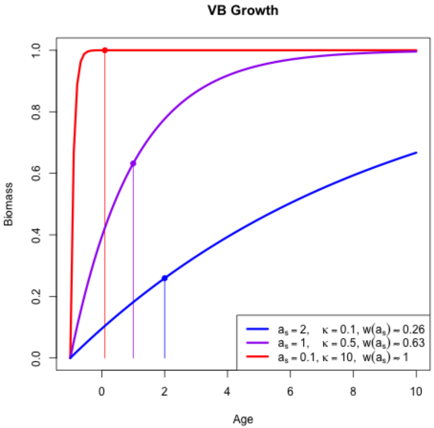


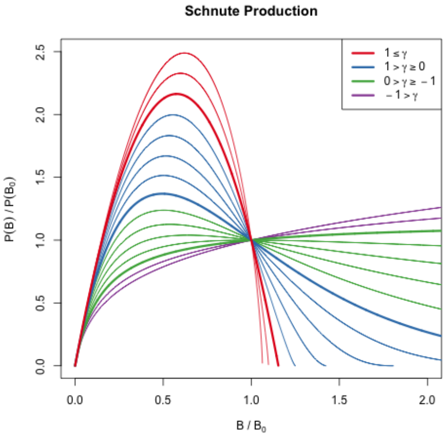
Schnute Recruitment

$$R(B;\alpha,\beta,\gamma) = \alpha B_{t-a_s}(1-\beta\gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic
$\gamma = 1$

Ricker
$\gamma \to 0$

Beverton-Holt
$\gamma = -1$

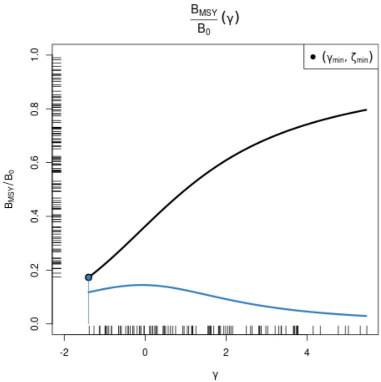


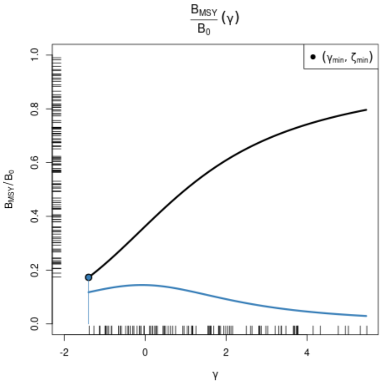
Growth Further Complicates RPs

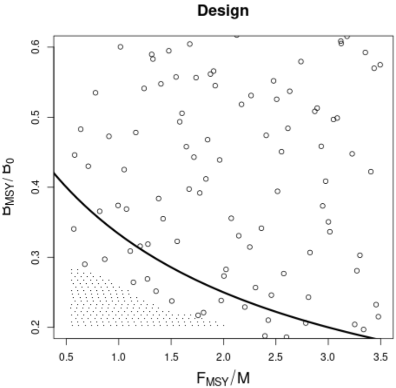
$\alpha(\gamma)$: $$\alpha = \left[\left(\frac{Z^*(Z^*+\kappa)}{w(a_s)(Z^*+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma} + \left(\frac{\gamma F^*}{w(a_s)}\right)\left(\frac{Z^*(Z^*+\kappa)}{w(a_s)(Z^*+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma-1}\left(1+\frac{\left(\frac{\kappa w_\infty}{w(a_s)}\right)\left(\kappa-\frac{\kappa w_\infty}{w(a_s)}\right)}{(Z^*+\frac{\kappa w_\infty}{w(a_s)})^2}\right)\right]^{\frac{1}{\gamma}}$$

$\beta(\alpha(\gamma),\gamma)$: $$\beta = \frac{1}{\gamma B_0}\left(1-\left(\frac{M(M+\kappa)}{\alpha w(a_s)(M+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma}\right)$$

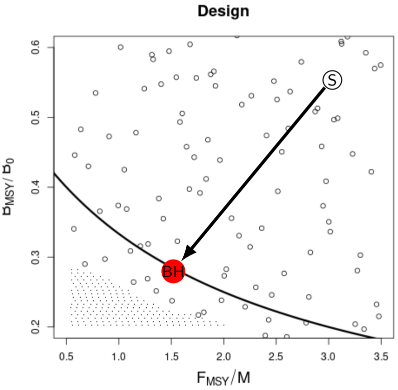
$\frac{B^*}{B_0}(\alpha(\gamma),\gamma)$: $$\frac{B^*}{B_0} = \frac{1-\left(\frac{(F^*+M)(F^*+M+\kappa)}{\alpha w(a_s)(F^*+M+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma}}{1-\left(\frac{M(M+\kappa)}{\alpha w(a_s)(M+\frac{\kappa w_\infty}{w(a_s)})}\right)^{\gamma}}$$


$\frac{B_{MSY}}{B_0}(\gamma)$
$(\gamma_{min}, \zeta_{min})$
B_{MSY}/B_0
0.0
0.2
0.4
0.6
0.8
1.0
-2
0
2
4
γ

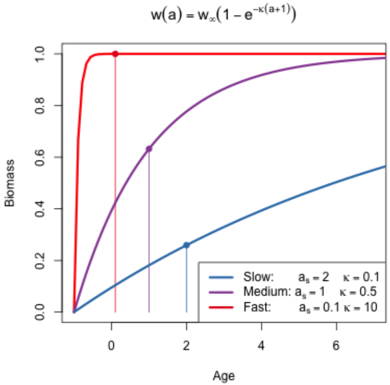

$\frac{B_{MSY}}{B_0}(\gamma)$
● (γ_{min}, ζ_{min})
B_{MSY}/B_0
0.0
0.2
0.4
0.6
0.8
1.0
-2
0
2
4
γ

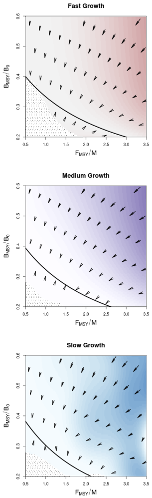

Design
B_{MSY}/B_0
0.2
0.3
0.4
0.5
0.6
0.5
1.0
1.5
2.0
2.5
3.0
3.5
F_{MSY}/M

$$\underbrace{\left(\frac{F^*}{M}, \frac{B^*}{\bar{B}(0)}\right)}_{\text{Schnute Truth}} \overset{\text{GP}}{\mapsto} \underbrace{\left(\frac{\hat{F}^*}{M}, \frac{B^*}{B_0}(-1;\hat{F}^*)\right)}_{\text{BH Estimate}}$$

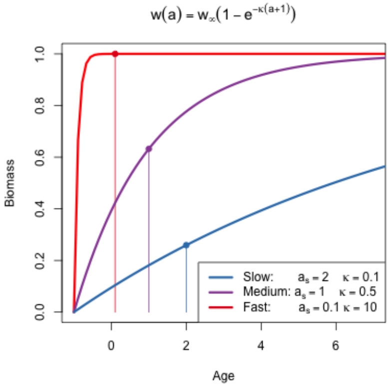


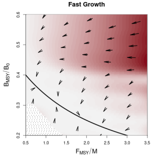
High Contrast

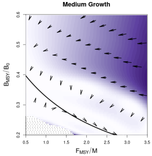


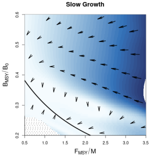


Low Contrast

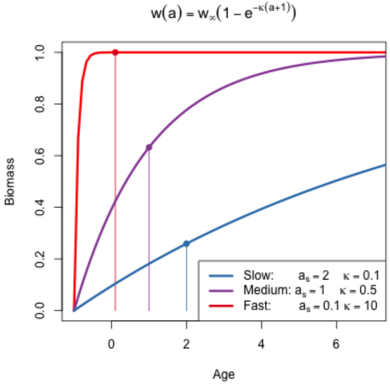


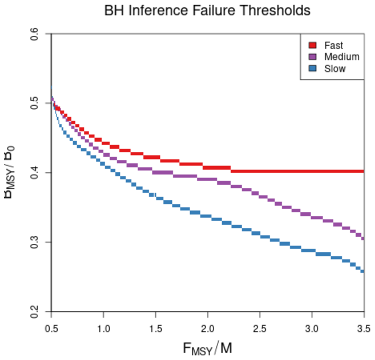






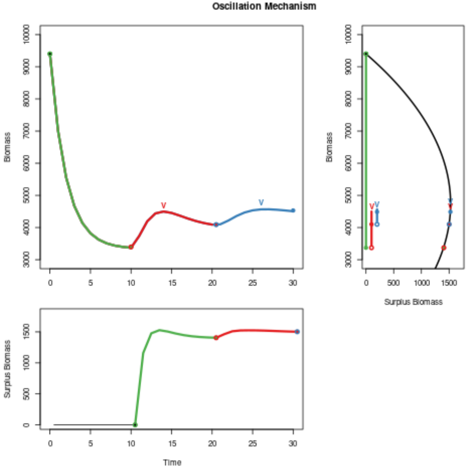
Low Contrast





Oscillation

- Large a_s window.
- Fishing shocks within a_s window.
- Repeated shocks can lead to chaos.



Conclusions

- Metamodeling controls for RP model misspecification and makes trade-offs in RP estimation explicit.
- RP map is largely dictated by the geometry of RPs under two-parameter models as mediated by contrast.
- Similar RP mapping under BH-DDM as the BH-SPM. Emphasizing growth dynamics makes BH more brittle.
- Emphasizes the need to use more flexible SRR models.

Products:

- Metamodeling framework and sampling methodology.
- Analytical methods for estimating γ based on proxies.
- DDM software and RP calculations.

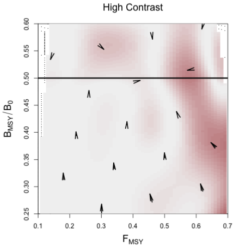
Future Work:

- Additional Biologies
  - Straightforward application to ASMs
  - Recruitment Deviations
- Further metamodeling techniques can expedite the simulation into more challenging simulation applications.
  - Targeted acquisition functions
  - Non-stationarity metamodels

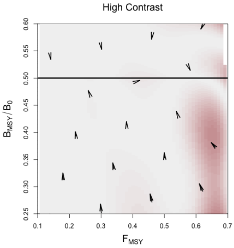
Many Thanks:

- UCSC Advisors
- Collaborators at NOAA
- NMFS Sea Grant

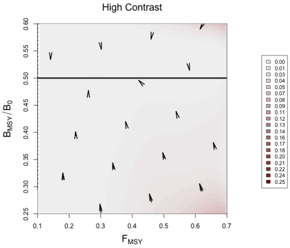
High Contrast PT $\sigma = 0.12$ Data



1x Samples



2x Samples



4x Samples

Metamodel Details

$$\mathbf{y} = \widehat{log(F_{MSY})} \quad - or - \quad \mathbf{y} = \widehat{log(B_0)}$$

$$\boldsymbol{X} = \left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\mathbf{y} = \beta_0 + \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{v} + \boldsymbol{\epsilon}$$

$$\boldsymbol{v} \sim N_n(\mathbf{0}, \tau^2 \boldsymbol{R}_\ell)$$

$$\boldsymbol{\epsilon} \sim N_n(\mathbf{0}, \boldsymbol{\omega}' \boldsymbol{I})$$

$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2} \right)$$

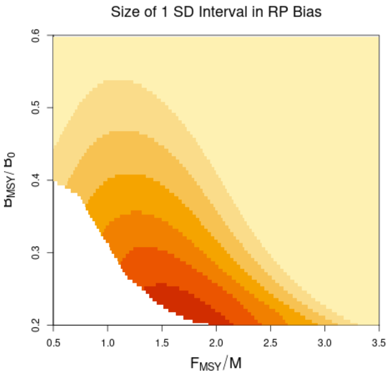
Prediction

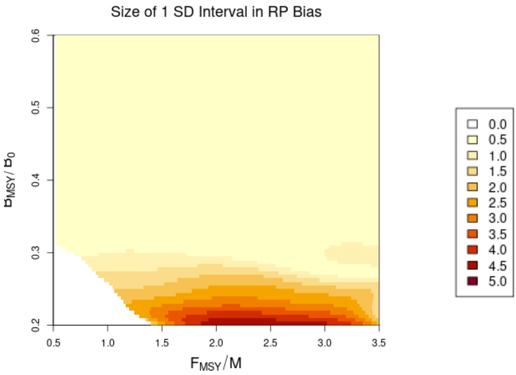
$$\hat{y}(\mathbf{x}^\star) = \beta_0 + \mathbf{x}^\star\boldsymbol{\beta} + \mathbf{r}(\mathbf{x}^\star)'\boldsymbol{R}_\ell^{-1}\Big(\mathbf{y} - (\beta_0 + \boldsymbol{X}\boldsymbol{\beta})\Big)$$

$$\hat{\sigma}^2(\mathbf{x}^\star) = \mathbf{R}(\mathbf{x}^\star, \mathbf{x}^\star) - \mathbf{r}(\mathbf{x}^\star)'\boldsymbol{R}_\ell^{-1}\mathbf{r}(\mathbf{x}^\star)$$

Contrast

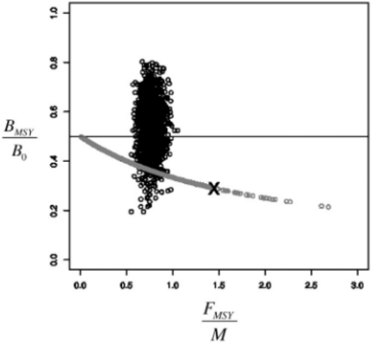
No Contrast





Mangel et al.

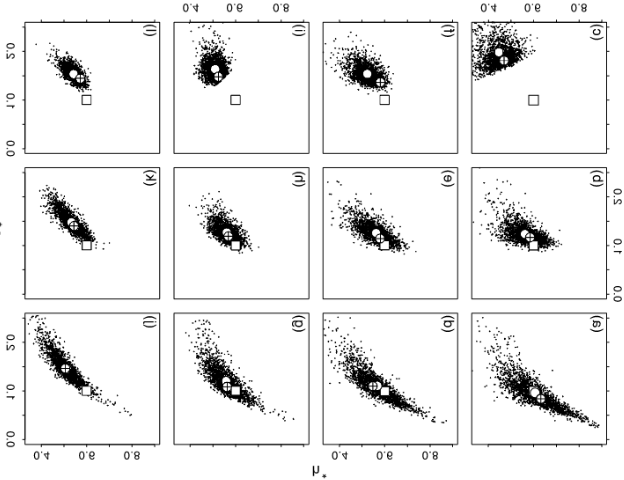
Fig. 4. DeYoreo et al. (2012) used both a BH-SRR and three-parameter SRR, similar to the S-SRR in a stock assessment of cowcod (*Sebastes levis*). We show samples from posterior distributions arising from different values of steepness. Unlike most stock assessments, we plot B_{MSY}/B_0 versus F_{MSY}/M. The grey circles show the results for the BH-SRR. This curve is another way of representing the constraint placed on a stock assessment by using a BH-SRR and specifying steepness — results must lie along this curve. The black circles represent the outcome of the three-parameter SRR. The black X represents the result when steepness is asserted to be 0.6.



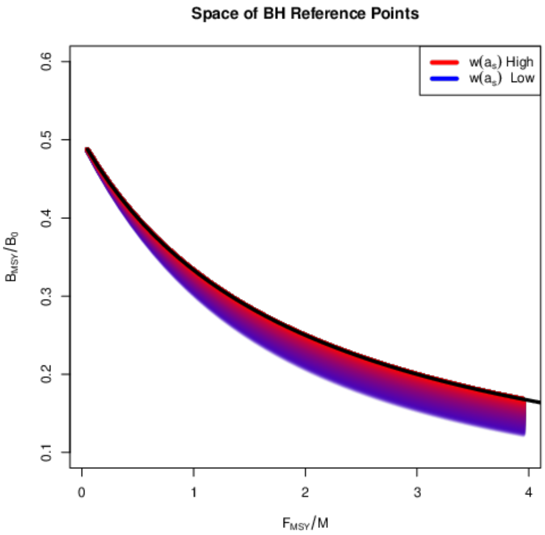
Logistic

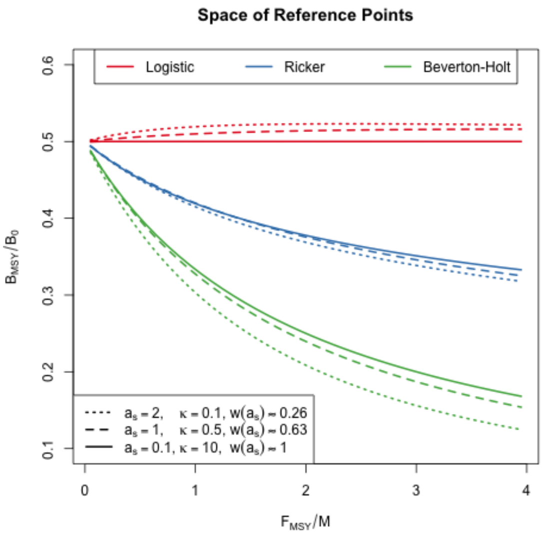
Ricker

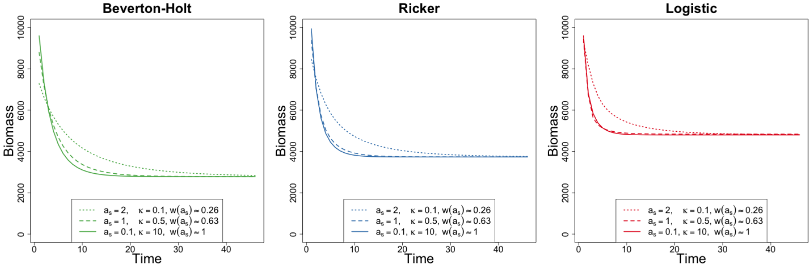
BH



Schnute, J. T., & Kronlund, A. R. (2002). Estimating salmon stock recruitment relationships from catch and escapement data. Canadian Journal of Fisheries and Aquatic Sciences, 59(3), 433–449.


Space of BH Reference Points
$w(a_s)$ High
$w(a_s)$ Low
B_{MSY}/B_0
0.1
0.2
0.3
0.4
0.5
0.6
F_{MSY}/M
0
1
2
3
4


Space of Reference Points
Logistic
Ricker
Beverton-Holt
$a_s = 2$, $\kappa = 0.1$, $w(a_s) \approx 0.26$
$a_s = 1$, $\kappa = 0.5$, $w(a_s) \approx 0.63$
$a_s = 0.1$, $\kappa = 10$, $w(a_s) \approx 1$
B_{MSY}/B_0
F_{MSY}/M
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4


Beverton-Holt
Ricker
Logistic
Biomass
Time
0
2000
4000
6000
8000
10000
0
10
20
30
40
$a_s = 2$, $\kappa = 0.1$, $w(a_s) \approx 0.26$
$a_s = 1$, $\kappa = 0.5$, $w(a_s) \approx 0.63$
$a_s = 0.1$, $\kappa = 10$, $w(a_s) \approx 1$