

Metamodeling for Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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Outline

1 Introduction

2 The Schaefer Model

3 The Beverton-Holt Model

4 Delay Differential Growth Extension

5 End

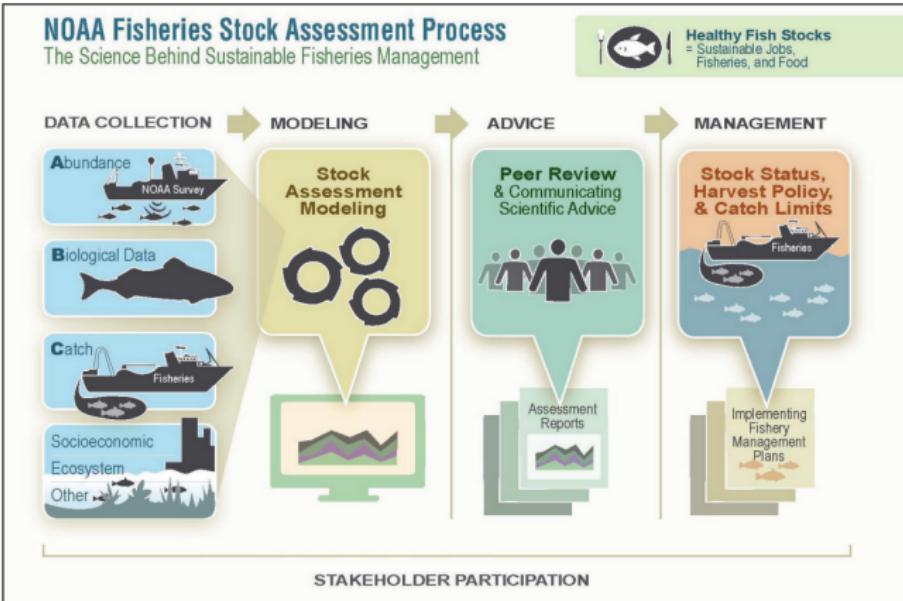


Figure 1: Overview of the stock assessment process from data collection through the provision of scientific advice to fishery managers. Stakeholders and other partners participate in each step of the assessment process. This report captures NOAA Fisheries products associated with the 'Advice' phase of the process.

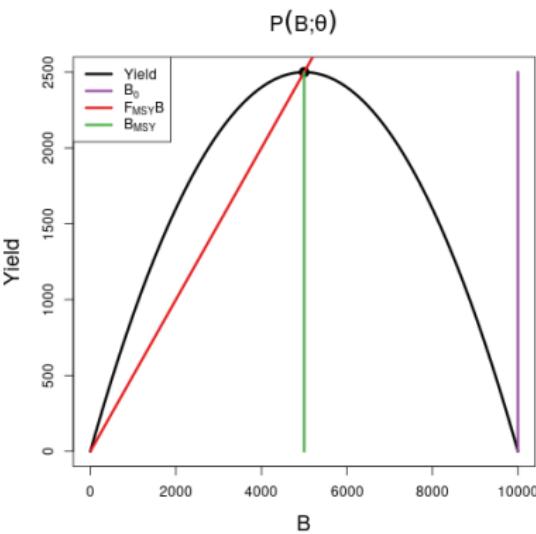
Surplus Production Model General Structure

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2).$$

$$\frac{dB}{dt} = P(B(t); \theta) - Z(t)B(t).$$

Reference Points:

- Maximum sustainable Yield (*MSY*)
 - F_{MSY}^a : Fishing rate to achieve *MSY*
 - $\frac{B_{MSY}}{B_0}$: Biomass Depletion when at *MSY*
 - Driven by the shape of P as determined by θ .



^a or $\frac{F_{MSY}}{M}$

Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

- Schaefer Model:

$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{2}$$

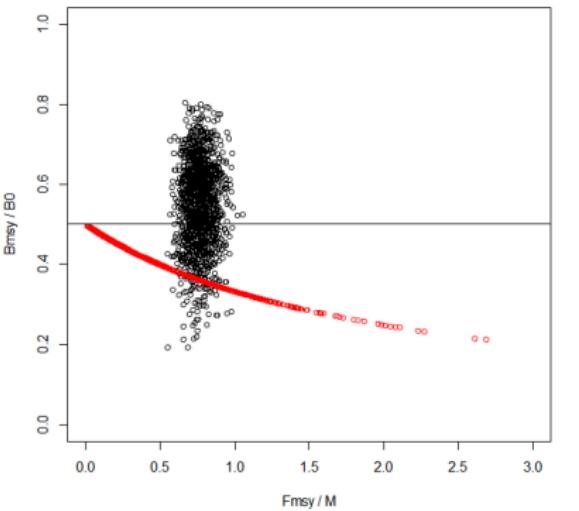
- BH Model:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M + 2}$$

- Similar Constraints for other Two-Parameter Models:

Fox, Ricker, etc...

- Three-Parameter Models Allow Independent RP Estimation



^aMangel et al. 2013, CJFAS

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- P_ℓ is logistic production
- Logistic map in discrete time
- Implicit Natural Mortality
- Explicit Fishing Mortality

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_\ell(B(t); \theta) - F(t)B(t)$$

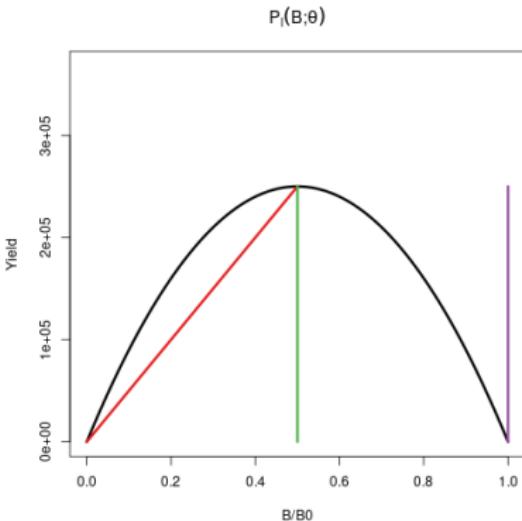
$$P_\ell(B; [r, K]) = rB \left(1 - \left(\frac{B}{K}\right)\right)$$

Reference Points:

$$F^* = \frac{r}{2}$$

$$B^* = \frac{K}{2} \quad B_0 = K$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$



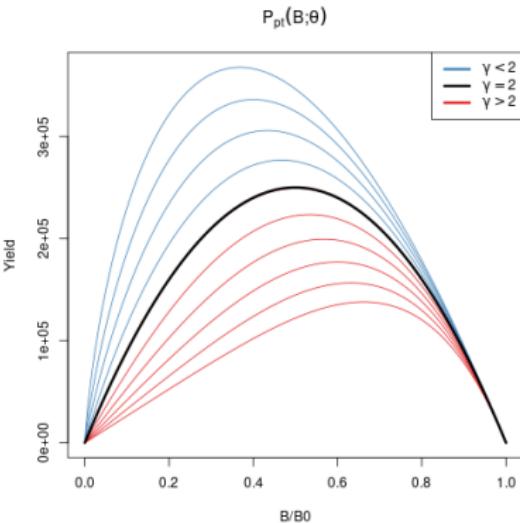
$$P_{pt}(B; [r, K, \gamma]) = \frac{rB}{\gamma - 1} \left(1 - \left(\frac{B}{K} \right)^{(\gamma-1)} \right)$$

Reference Points:

$$F^* = \frac{r}{\gamma}$$

$$B^* = K \left(\frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}} \quad B_0 = K$$

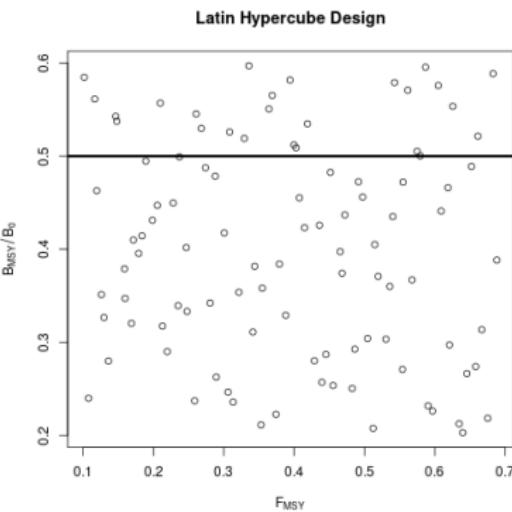
$$\frac{B^*}{B_0} = \left(\frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}}$$



$$F^* = \frac{r}{\gamma} \quad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

Closed Form Inversion

$$r = \gamma F^* \quad \gamma = \frac{W\left(\frac{B^*}{\bar{B}(0)} \log\left(\frac{B^*}{\bar{B}(0)}\right)\right)}{\log\left(\frac{B^*}{\bar{B}(0)}\right)}$$

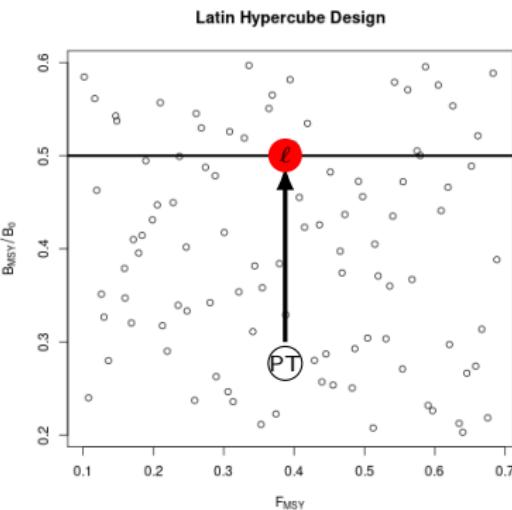


* Lambert W function inverts xe^x s.t. $W(xe^x) = x$

$$F^* = \frac{r}{\gamma} \quad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

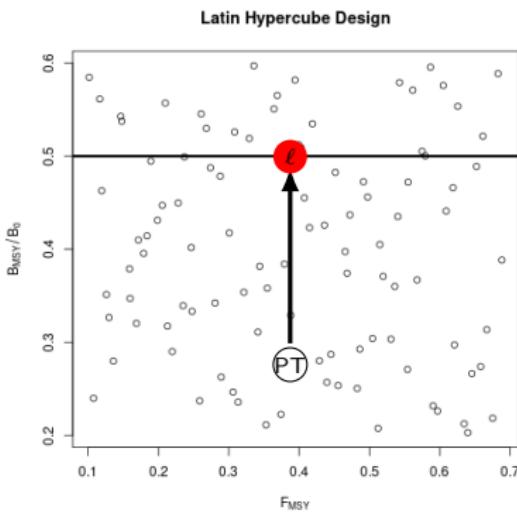
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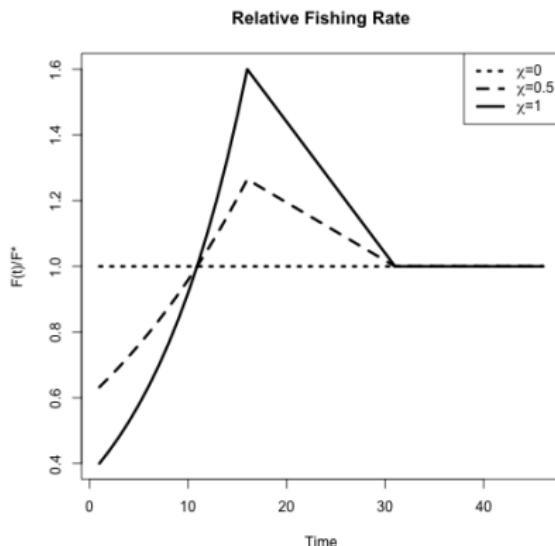


* Lambert W function inverts xe^x s.t. $W(xe^x) = x$

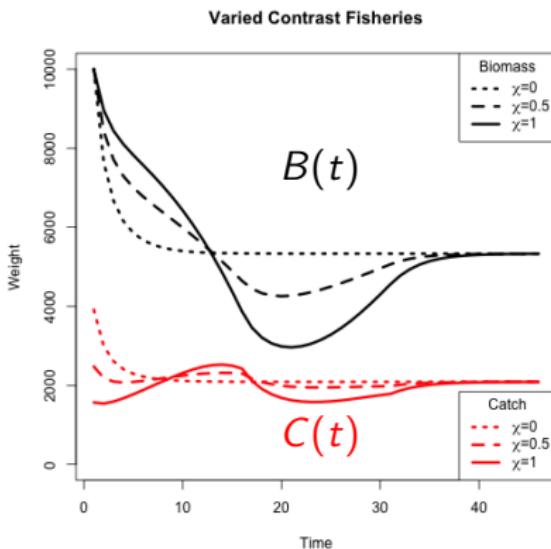
$$\underbrace{\left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{PT Truth}} \xrightarrow{\text{GP}} \underbrace{\left(\hat{F}_{MSY}, \frac{1}{2} \right)}_{\text{Shaefer Estimate}}$$

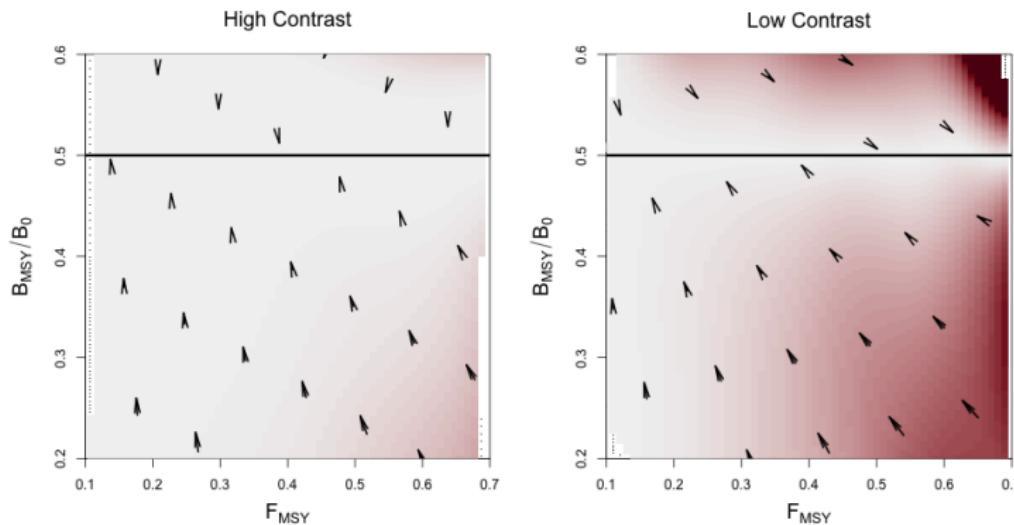


$$\begin{aligned}\frac{dB}{dt} &= P(B(t); \theta) - F(t)B(t) \\ &= P(B(t); \theta) - F_\theta^* \underbrace{\frac{F(t)}{F_\theta^*}}_{\text{Relative Fishing}} B(t)\end{aligned}$$

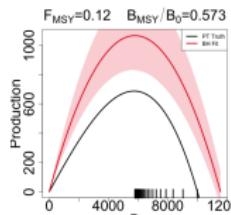


$$\begin{aligned}\frac{dB}{dt} &= P(B(t); \theta) - F(t)B(t) \\ &= P(B(t); \theta) - F_\theta^* \underbrace{\frac{F(t)}{F_\theta^*} B(t)}_{C(t)}\end{aligned}$$

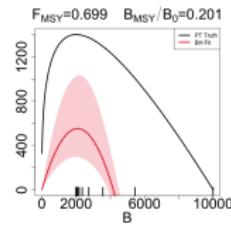
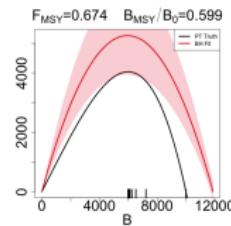
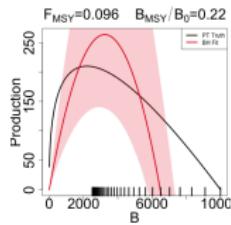
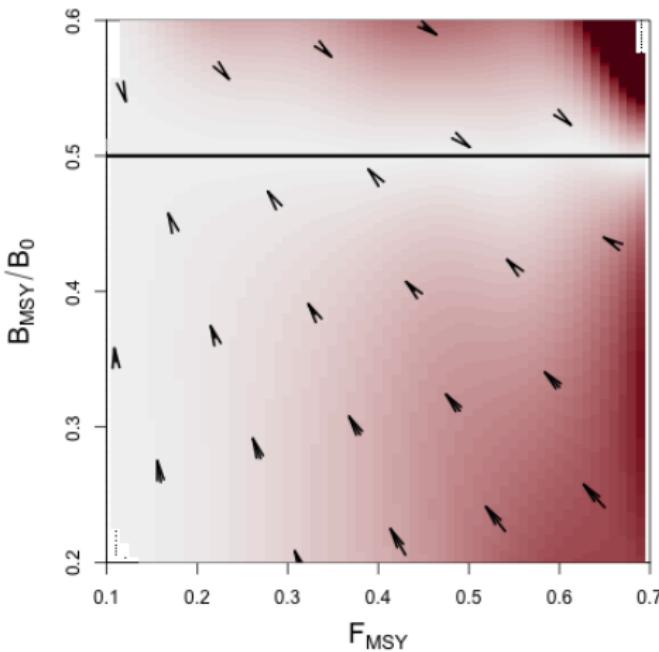




Mechanism for Bias in F_{MSY} via Contrast

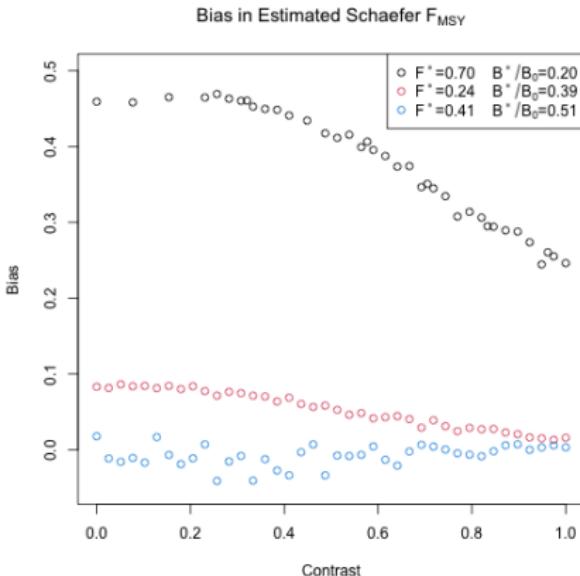


Low Contrast



Mechanism for Bias in F_{MSY} via Contrast

- Only observe upper half
- learn about slope at origin from upper biomass range
- as contrast increases biases perspective diminishes



Summary

- A quick summary
- just a few points will suffice
- the quicker and more concise the better
- transition to BH

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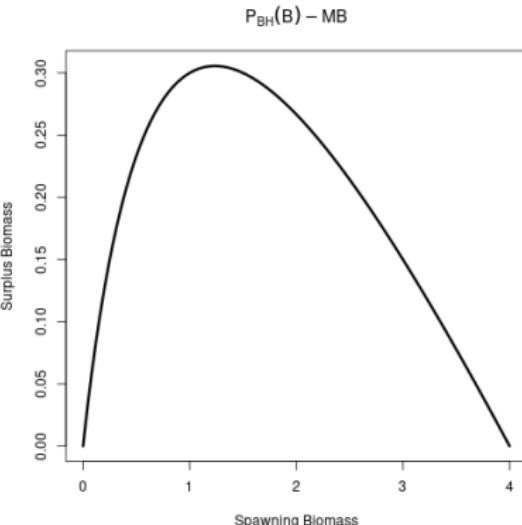
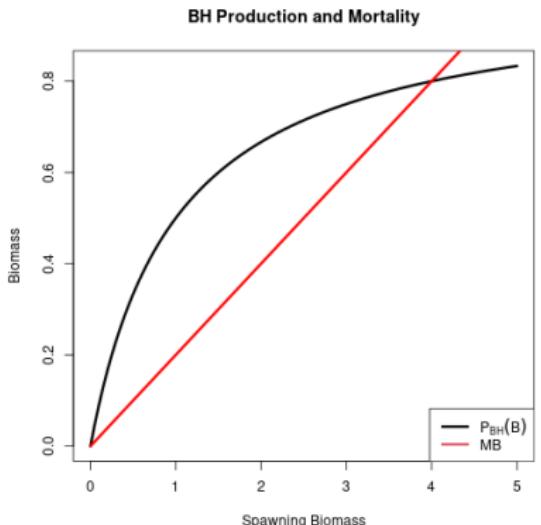
- Beverton-Holt production
 - Production asymptotes
 - Explicit Natural Mortality
 - Explicit Fishing Mortality

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_{BH}(B(t); \theta) - (M + F(t))B(t)$$

$$P_{BH}(B; [\alpha, \beta]) = \frac{\alpha B}{1 + \beta B}.$$

$$\frac{dB}{dt} = P_{BH}(B; [\alpha, \beta]) - MB - FB$$



$$P_s(B; [\alpha, \beta, \gamma]) = \alpha B (1 - \beta \gamma B)^{\frac{1}{\gamma}}$$

Logistic

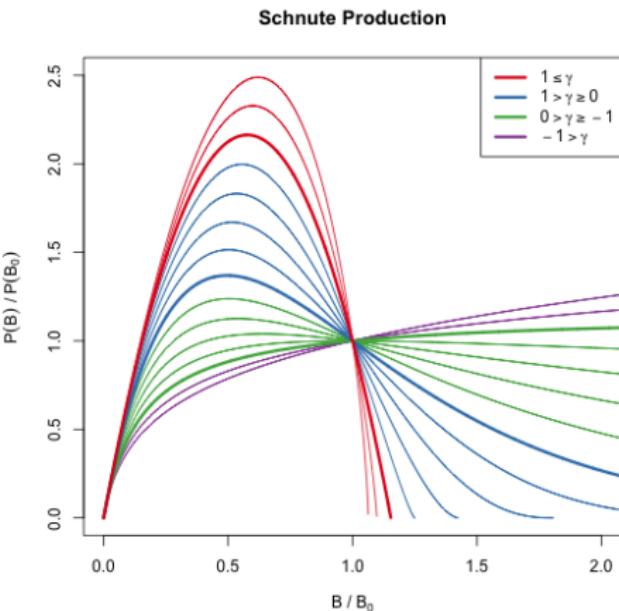
$$\gamma = 1$$

Ricker

$$\gamma \rightarrow 0$$

Beverton-Holt

$$\gamma = -1$$

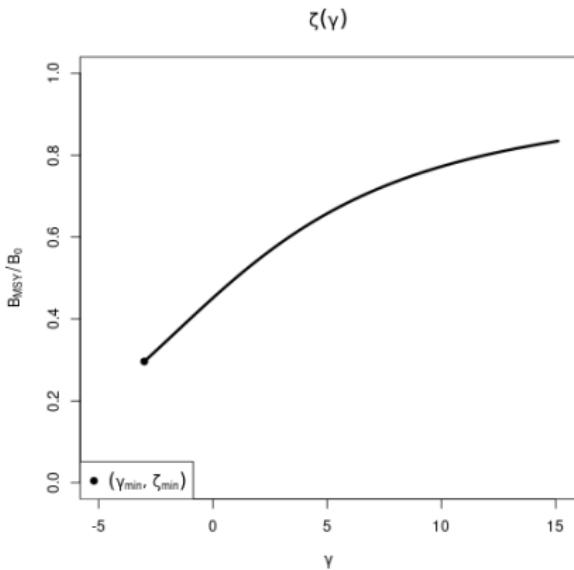


some how softly introduce calculation via F^* , B_0 , and $\frac{B^*}{B_0}$. Show the partial solution and link to Schnute and Richards citation.
Setup ζ for next slide.

$$\begin{aligned}\alpha &= (M + F^*) \left(1 + \frac{\gamma F^*}{M + F^*}\right)^{1/\gamma} \\ \beta &= \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^\gamma\right) \\ \frac{B^*}{B_0} &= \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}.\end{aligned}\tag{1}$$

Show approximated t with rugs. explain/relabel ζ . show qq plot.

$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}$$



Show approximated t with rugs. explain/relabel ζ . show qq plot.

$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}$$

$$\gamma' \sim \zeta_{min} \delta(\gamma_{min}) + t(\mu, \sigma, \nu) \mathbf{1}_{\gamma > \gamma_{min}}$$



Show approximated t with rugs. explain/relabel ζ . show qq plot.

$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}$$

.../.../gpBias/qqPlot.png

$$\gamma' \sim \zeta_{min} \delta(\gamma_{min}) + t(\mu, \sigma, \nu) \mathbf{1}_{\gamma > \gamma_{min}}$$

Schnute LHS Design

Logistic

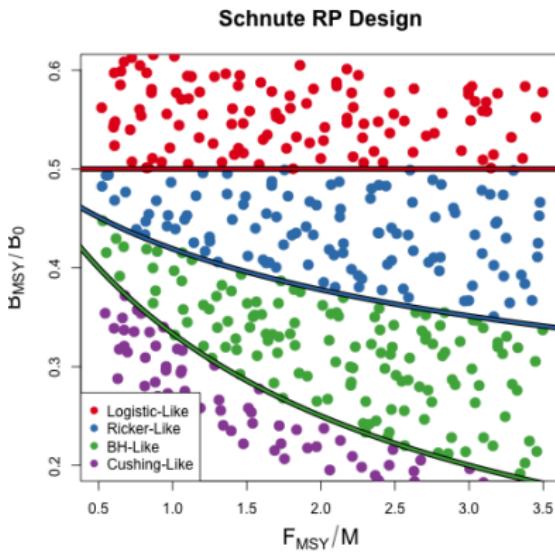
$$\gamma = 1$$

Ricker

$$\gamma \rightarrow 0$$

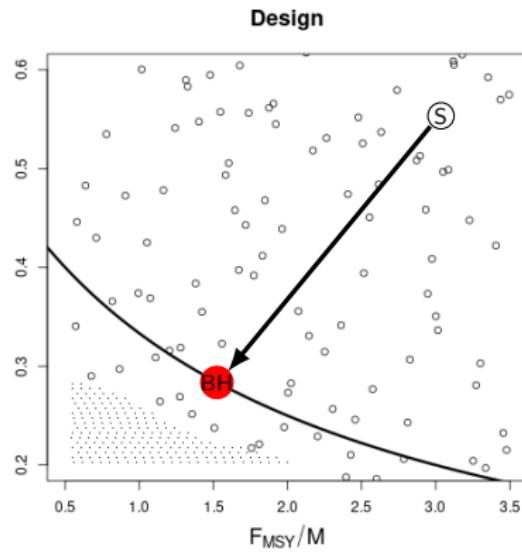
Beverton-Holt

$$\gamma = -1$$

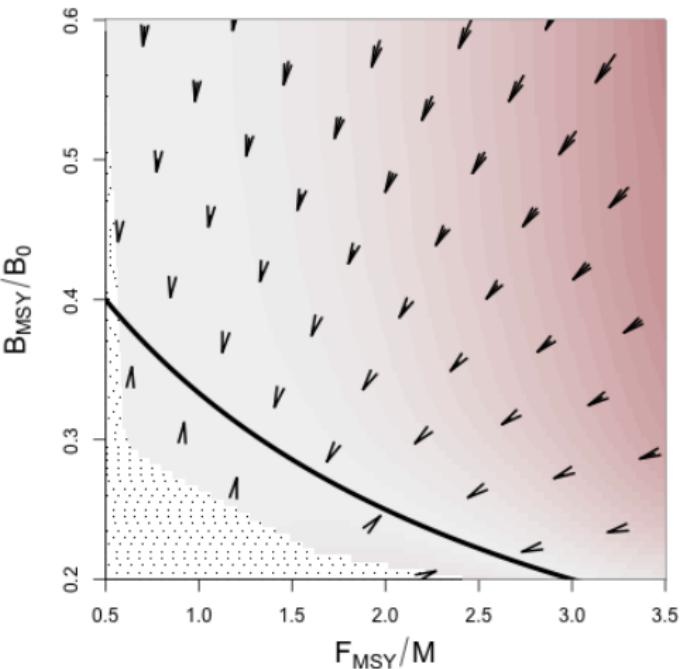


Schnute LHS Design

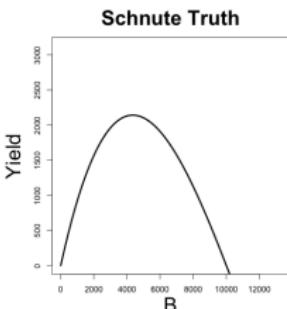
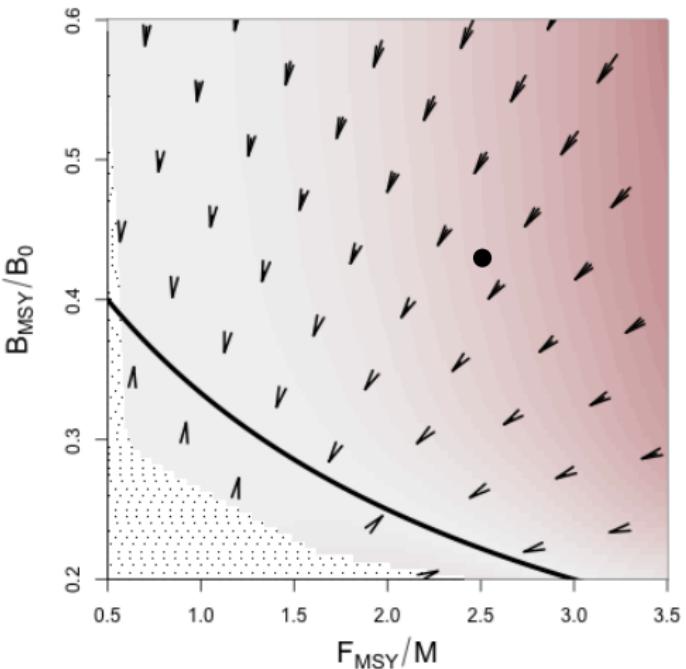
$$\underbrace{\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{Schnute Truth}} \xrightarrow{\text{GP}} \underbrace{\left(\frac{\hat{F}_{MSY}}{M}, \frac{1}{\hat{F}_{MSY}/M + 2} \right)}_{\text{BH Estimate}}$$



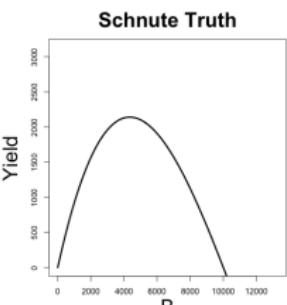
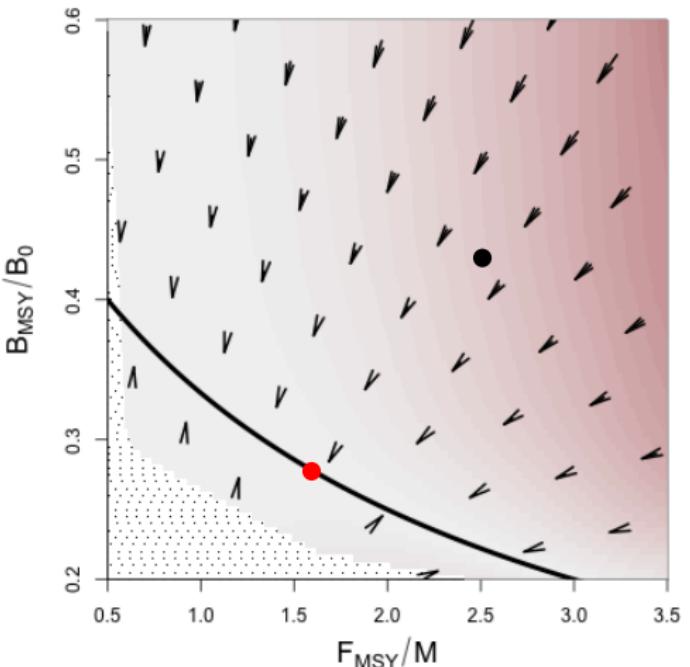
High Contrast



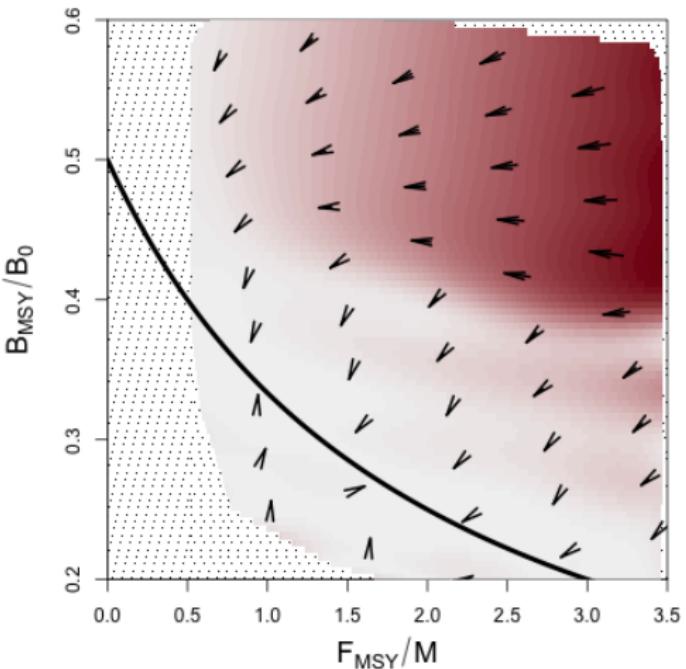
High Contrast



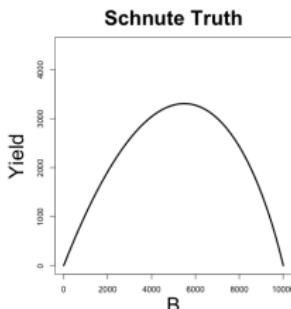
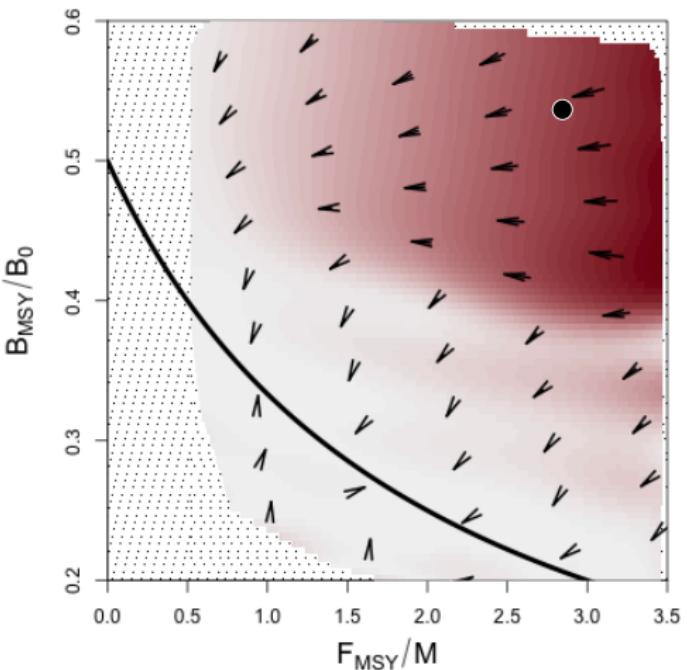
High Contrast



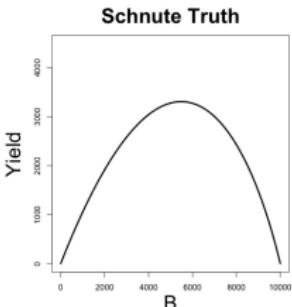
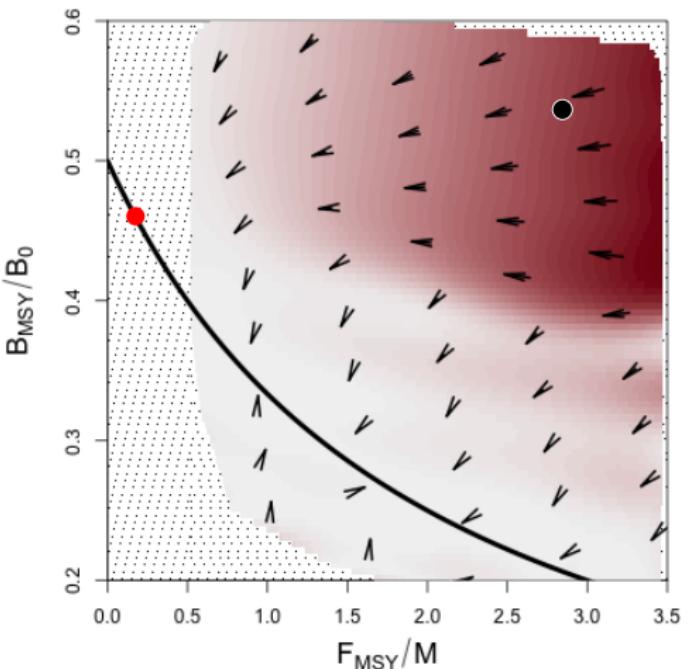
Low Contrast



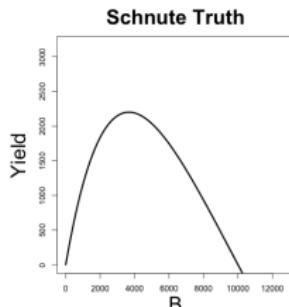
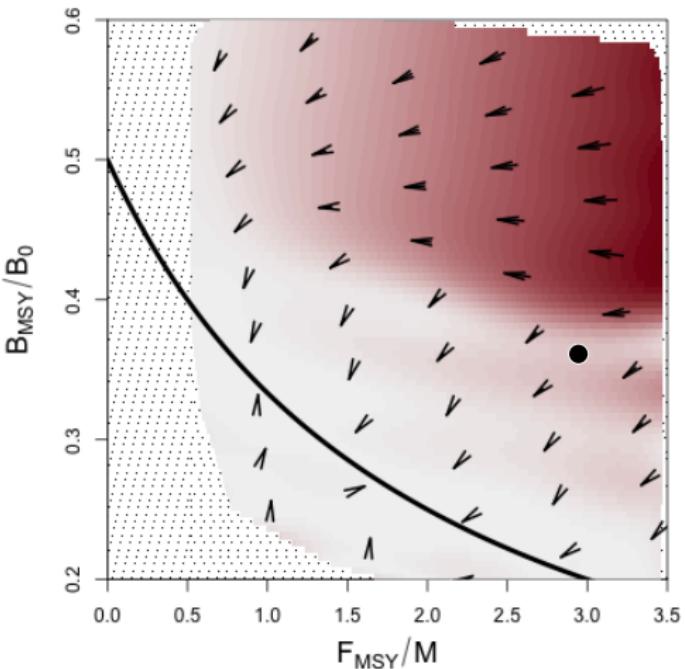
Low Contrast



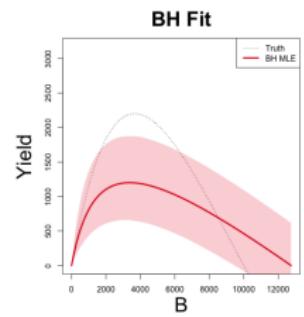
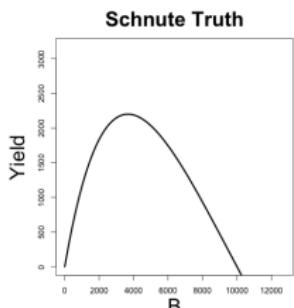
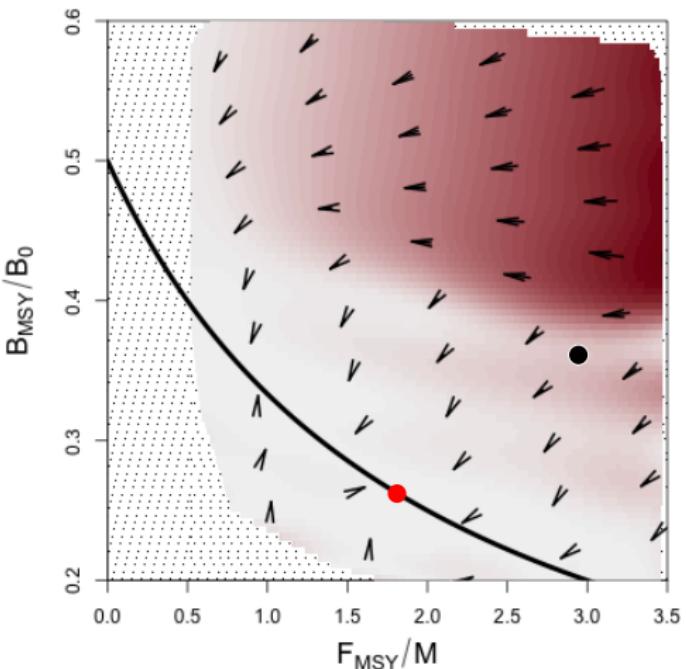
Low Contrast

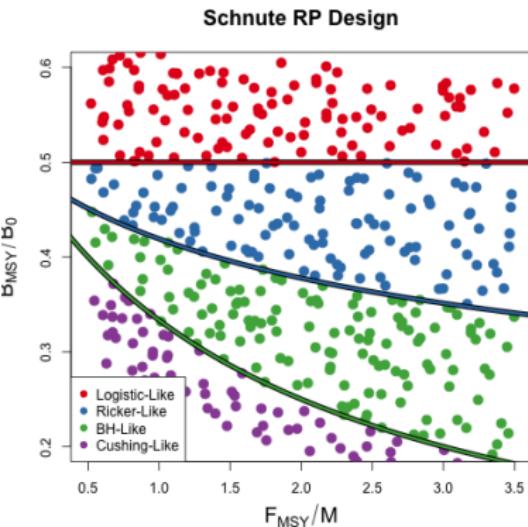
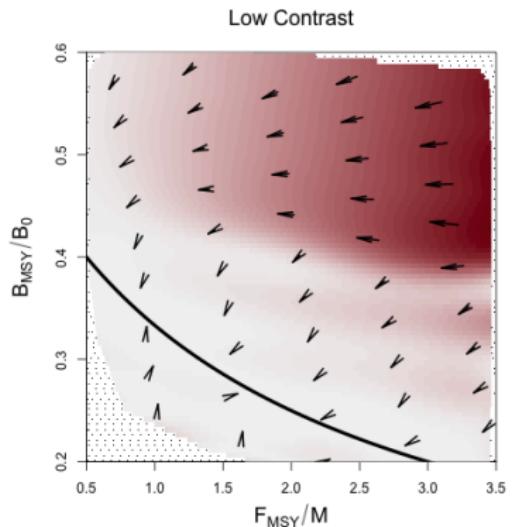


Low Contrast



Low Contrast





Quick Summary and Transition

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General Modeling Structure

$$\begin{aligned}\frac{dB}{dt} &= \overbrace{w(a_s)R(B; \theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa [w_\infty N - B]}^{\text{Net Growth}} - \overbrace{(M + F)B}^{\text{Mortality}} \\ \frac{dN}{dt} &= R(B; \theta) - (M + F)N\end{aligned}$$

$R(B_{t-a_s}; \alpha, \beta, \gamma)$: Schnute Three Parameter Recruitment

$w(a)$: Von Bertalanffy Individual Growth

Knife-Edge Selectivity at age a_s

Schnute Recruitment

$$R(B; \alpha, \beta, \gamma) = \alpha B_{t-a_s} (1 - \beta \gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic

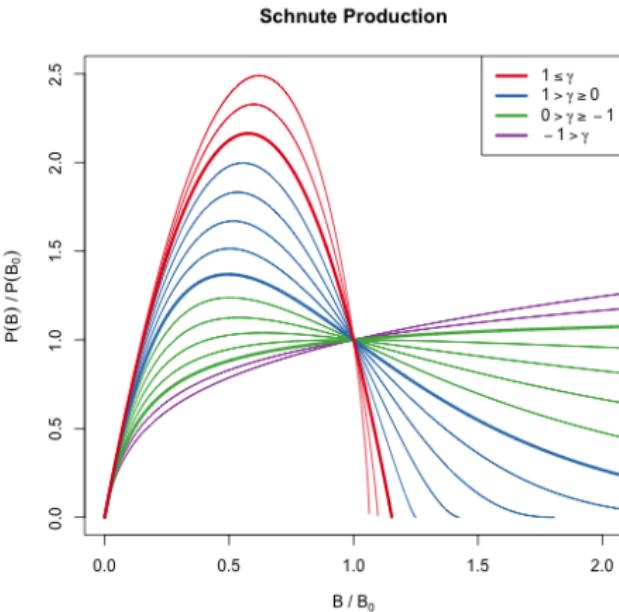
$$\gamma = 1$$

Ricker

$$\gamma \rightarrow 0$$

Beverton-Holt

$$\gamma = -1$$



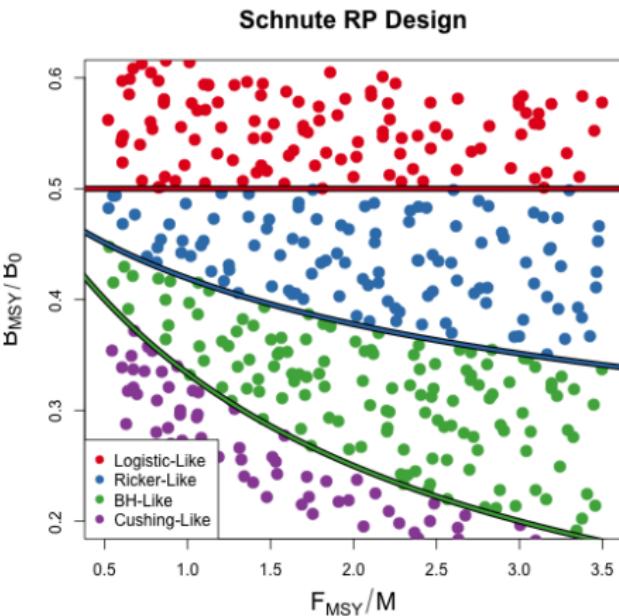
Schnute Recruitment

$$R(B; \alpha, \beta, \gamma) = \alpha B_{t-a_s} (1 - \beta \gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic
 $\gamma = 1$

Ricker
 $\gamma \rightarrow 0$

Beverton-Holt
 $\gamma = -1$

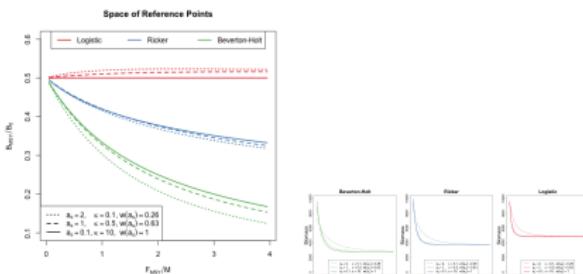


Von Bertalanffy Individual Growth

$$w(a) = w_\infty(1 - e^{-\kappa(a-a_0)})$$

- Instant Growth:
(Production Model)
 $a_s \rightarrow 0$ $\kappa \rightarrow \infty$

- Dynamic Growth:
 a_s : Not Instant
 κ : Peakness of Dynamics



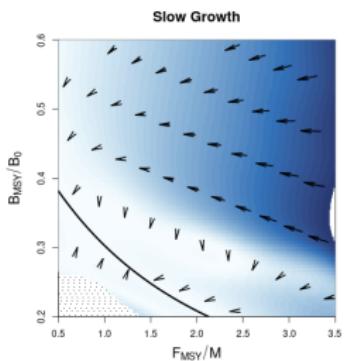
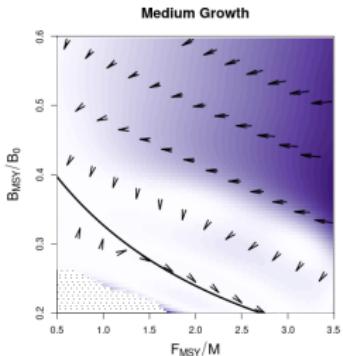
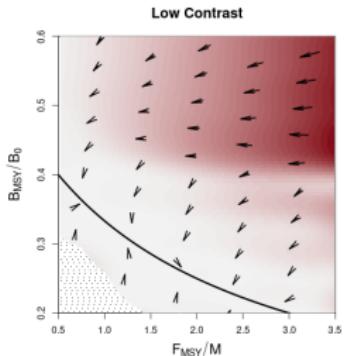
$$\bar{B}(F) = \frac{1}{\beta\gamma} \left(1 - \left(\frac{(F+M)(F+M+\kappa)}{\alpha w(a_s)(F+M+\frac{\kappa w_\infty}{w(a_s)})} \right)^\gamma \right) \quad (2)$$

$$\bar{N}(F) = \frac{\alpha \bar{B}(F)(1 - \beta\gamma \bar{B}(F))^{1/\gamma}}{F + M} \quad (3)$$

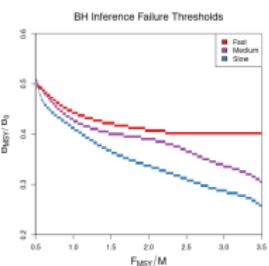
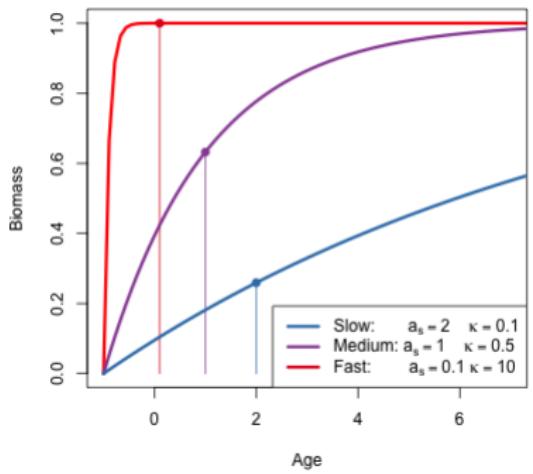
$$\alpha = \left[\left(\frac{Z^*(Z^* + \kappa)}{w(a_S)(Z^* + \frac{\kappa w_\infty}{w(a_S)})} \right)^\gamma + \left(\frac{\gamma F^*}{w(a_S)} \right) \left(\frac{Z^*(Z^* + \kappa)}{w(a_S)(Z^* + \frac{\kappa w_\infty}{w(a_S)})} \right)^{\gamma-1} \left(1 + \frac{\left(\frac{\kappa w_\infty}{w(a_S)} \right) (\kappa - \frac{\kappa w_\infty}{w(a_S)})}{(Z^* + \frac{\kappa w_\infty}{w(a_S)})^2} \right) \right]^{\frac{1}{\gamma}}$$

$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M(M + \kappa)}{\alpha w(a_s)(M + \frac{\kappa w_\infty}{w(a_s)})} \right)^\gamma \right) \quad (5)$$

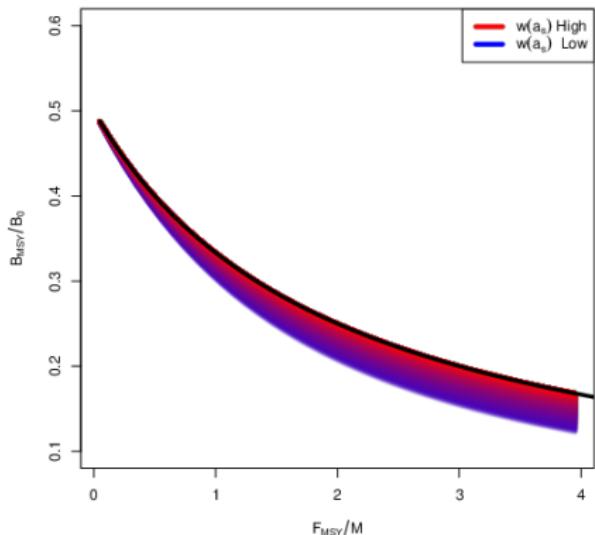
$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{(F^* + M)(F^* + M + \kappa)}{\alpha w(a_s)(F^* + M + \frac{\kappa w_\infty}{w(a_s)})} \right)^\gamma}{1 - \left(\frac{M(M + \kappa)}{\alpha w(a_s)(M + \frac{\kappa w_\infty}{w(a_s)})} \right)^\gamma} \quad (6)$$



$$w(a) = w_\infty(1 - e^{-\kappa(a+1)})$$



Space of BH Reference Points



General Conclusions and future work.

Many Thanks:

- Dr. Marc Mangel
- Collaborators at NOAA
- NMFS Sea Grant



$$\hat{\mu} = \widehat{\log(r)} \quad -\text{or}- \quad \hat{\mu} = \widehat{\log(K)}$$

$$x = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

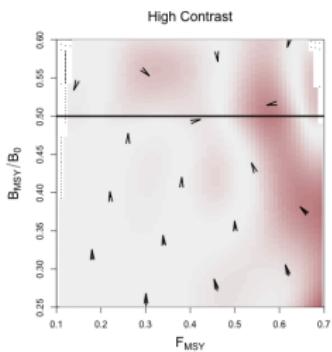
$$\hat{\mu} = \beta_0 + \boldsymbol{\beta}' \mathbf{x} + f(\mathbf{x}) + \epsilon$$

$$f(\mathbf{x}) \sim \text{GP}(0, \tau^2 R(\mathbf{x}, \mathbf{x}'))$$

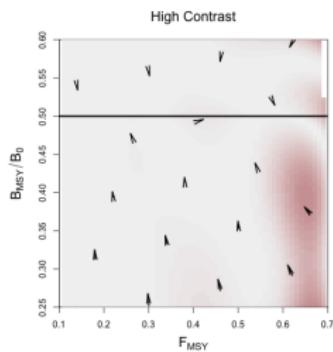
$$\epsilon_i \sim N(0, \hat{\omega}_i).$$

$$R(\mathbf{x}, \mathbf{x}') = \exp\left(\sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2}\right)$$

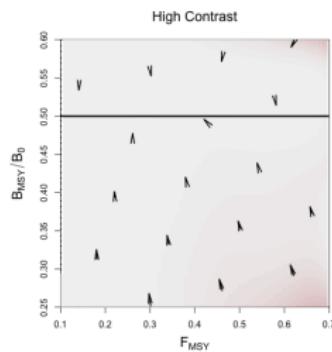
High Contrast PT $\sigma = 0.12$ Data



1x Samples



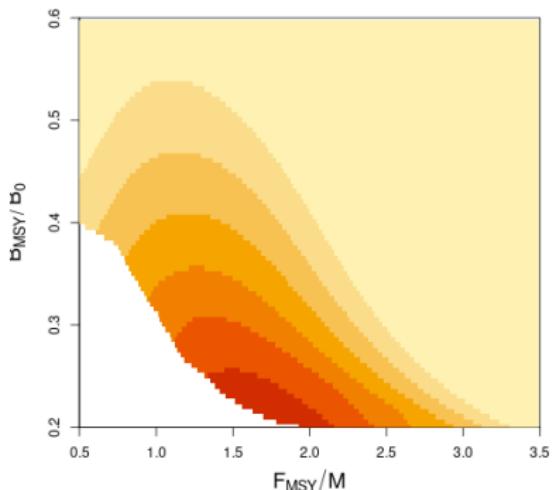
2x Samples



4x Samples

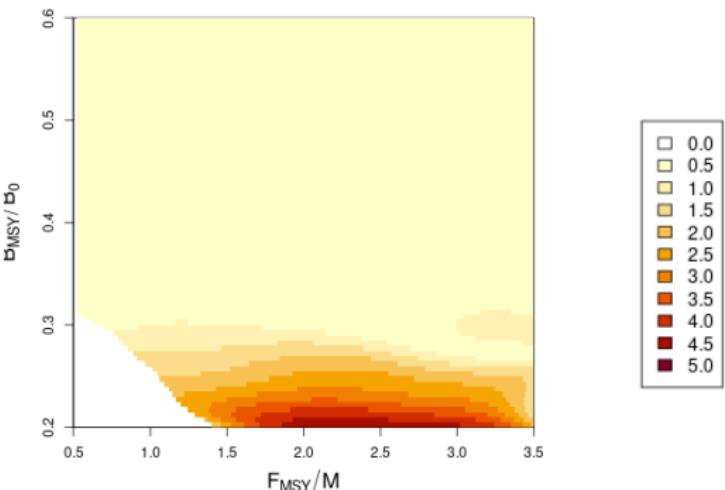
Contrast

Size of 1 SD Interval in RP Bias



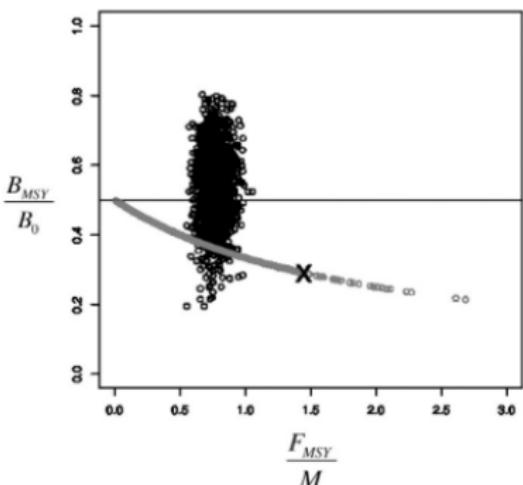
No Contrast

Size of 1 SD Interval in RP Bias

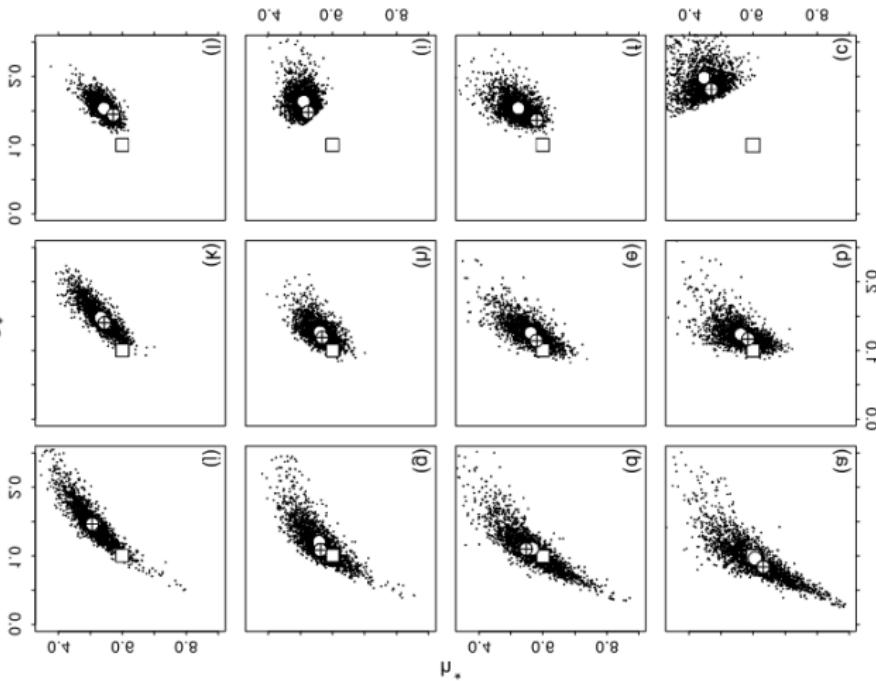


Mangel et al.

Fig. 4. DeYoreo et al. (2012) used both a BH-SRR and three-parameter SRR, similar to the S-SRR in a stock assessment of cowcod (*Sebastodes levis*). We show samples from posterior distributions arising from different values of steepness. Unlike most stock assessments, we plot B_{MSY}/B_0 versus F_{MSY}/M . The grey circles show the results for the BH-SRR. This curve is another way of representing the constraint placed on a stock assessment by using a BH-SRR and specifying steepness — results must lie along this curve. The black circles represent the outcome of the three-parameter SRR. The black X represents the result when steepness is asserted to be 0.6.



Logistic



Schnute, J. T., & Kronlund, A. R. (2002). Estimating salmon stock recruitment relationships from catch and escape-
ment data. Canadian Journal of Fisheries and Aquatic Sciences, 59(3), 433–449.