Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

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In collaboration with:
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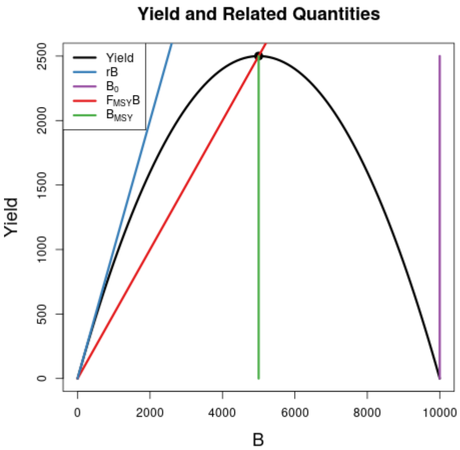
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - Z(t)B(t)$$

$$RP : MSY, \ \frac{F_{MSY}}{M}, \ \frac{B_{MSY}}{B_0}$$

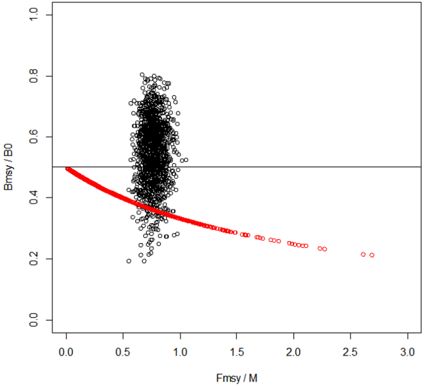


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0,1)$$

Mangel et al. 2013, CJFAS:

- BH Model:
  $F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{\bar{B}(0)} = \frac{1}{F_{MSY}/M+2}$
- Similar Constraints for other Two-Parameter Curves



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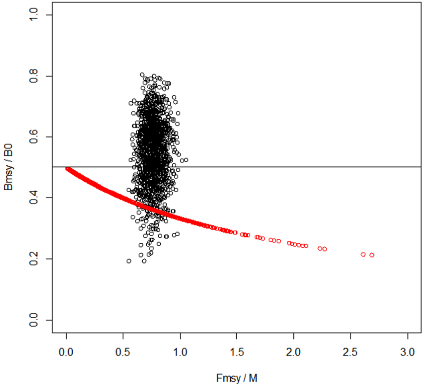
- Similar Constraints for other Two-Parameter Curves
- Three-Parameter Relationships Allow Independent RP Estimation



Schnute 1985, CJFAS

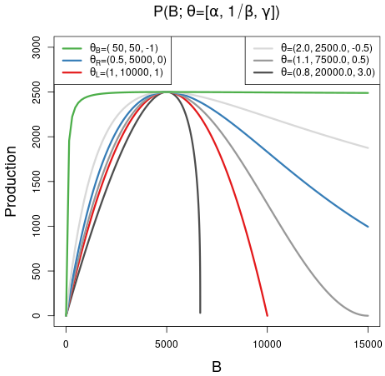
$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ Beverton-Holt

$\gamma \rightarrow 0 \Rightarrow$ Ricker

$\gamma = 1 \Rightarrow$ Logistic



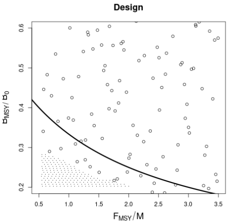
Introish Ideas list

- PT/Schaffer work (link)
- Computational Difficulties
- Schnute Space Filling
- Catch/Contrast

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M+F_{MSY}}{\alpha}\right)^{\gamma}}{1 - \left(\frac{M}{\alpha}\right)^{\gamma}}$$

$$\alpha = (M + F_{MSY})\left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0}\left(1 - \left(\frac{M}{\alpha}\right)^{\gamma}\right)$$



Pella-Tomlinson Production Model

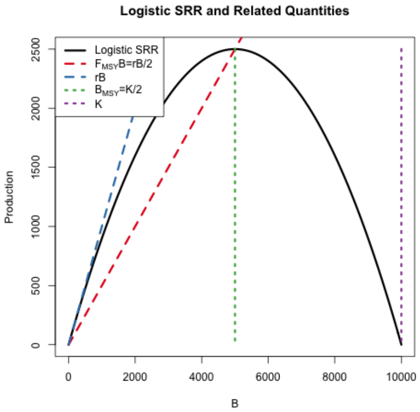
$$I(t) \sim LN(qB(t), \sigma^2)$$

$$\frac{dB(t)}{dt} = R_{\boldsymbol{\theta}}(B(t)) - F(t)B(t)$$

$$R_{\boldsymbol{\theta}}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma-1}$$

$$\boldsymbol{\theta} = (r, K, \gamma)$$

$$\gamma = 2 \Rightarrow \text{Schaefer Model}$$



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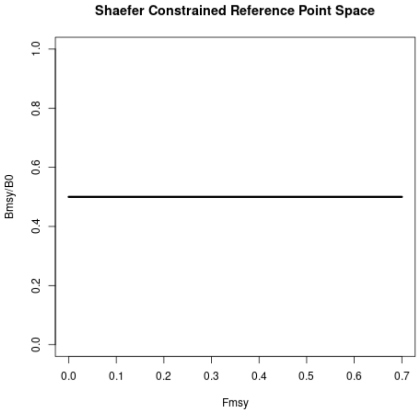
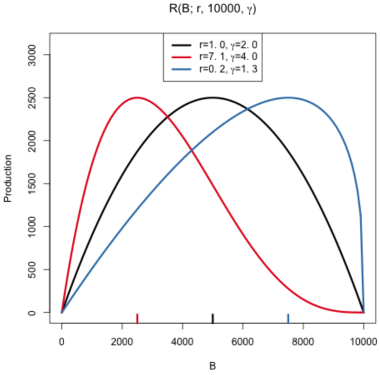
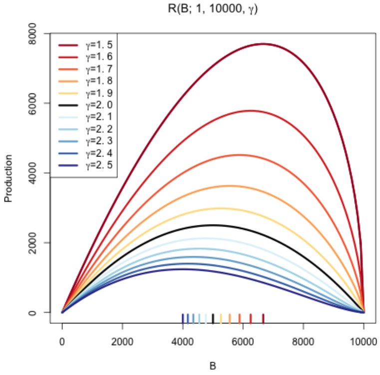
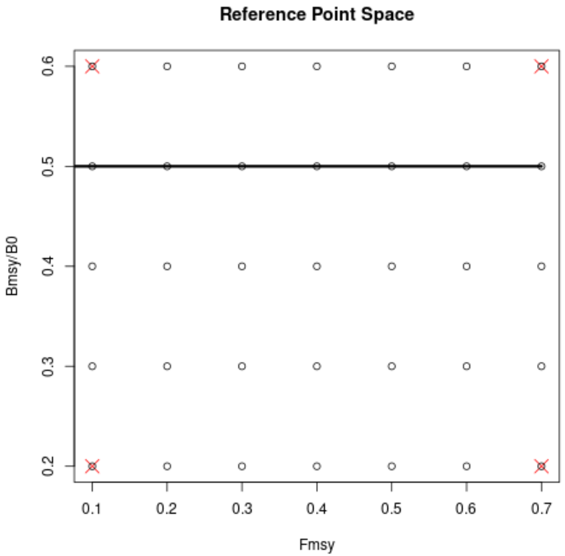


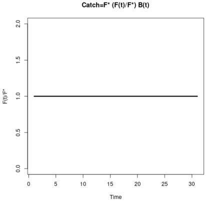
Pella-Tomlinson Family of Curves

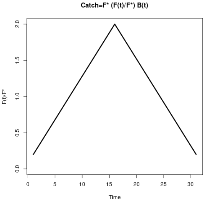





Reference Point Space
Bmsy/B0
0.2
0.3
0.4
0.5
0.6
Fmsy
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Catch


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time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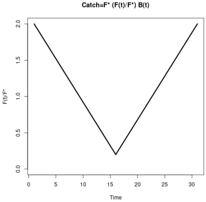
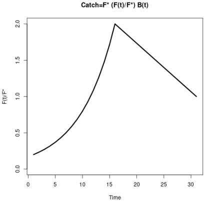
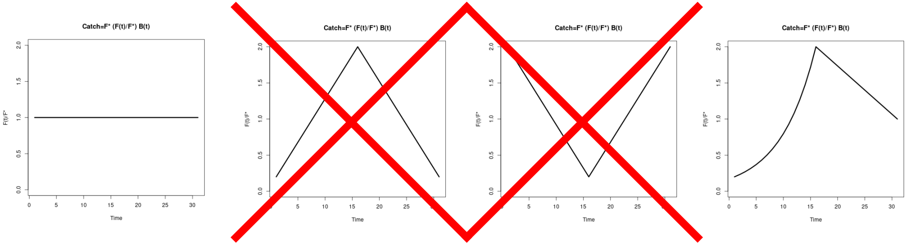
Time


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time


Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time

Catch

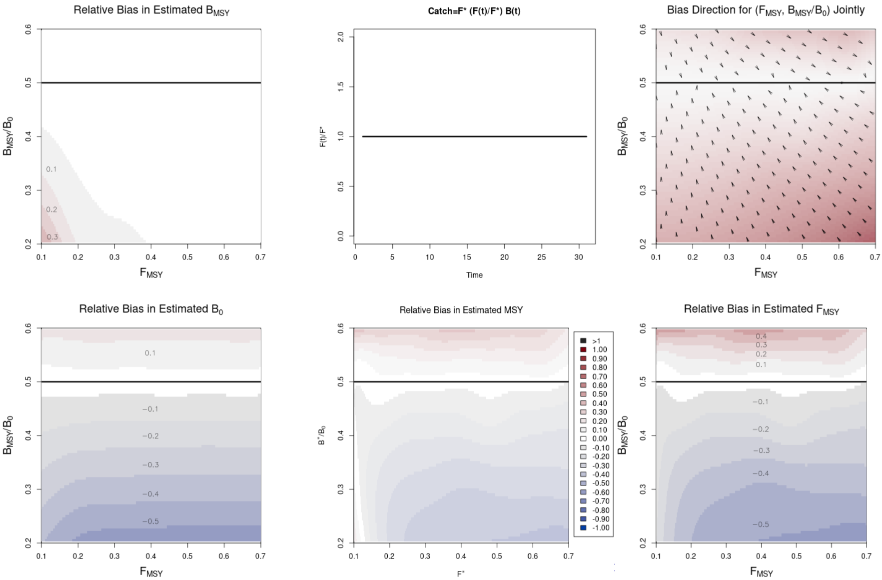


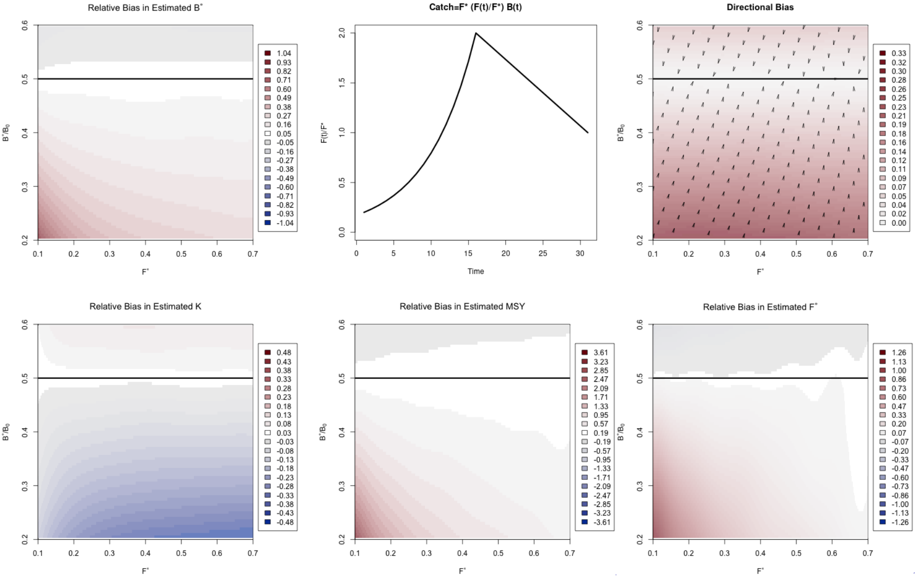
Flat

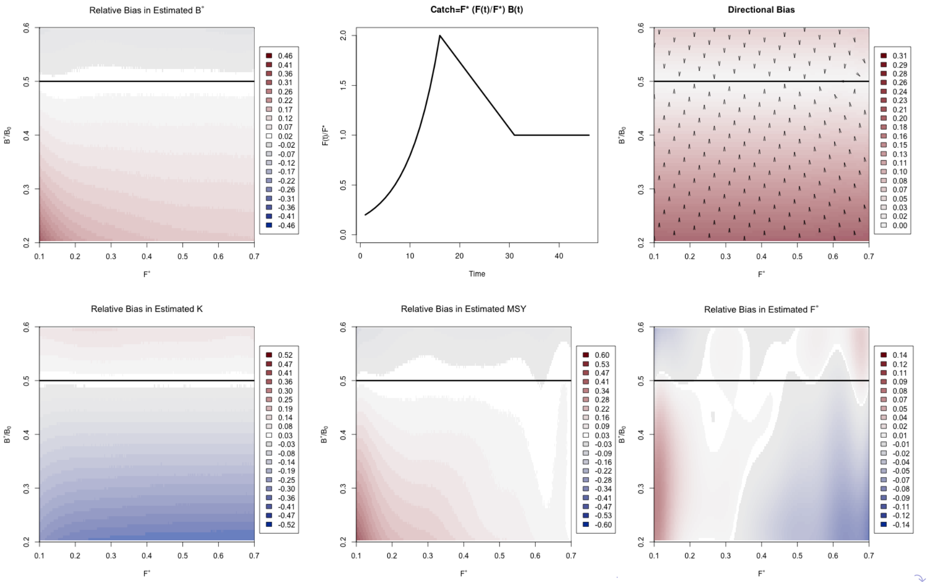
Contrast

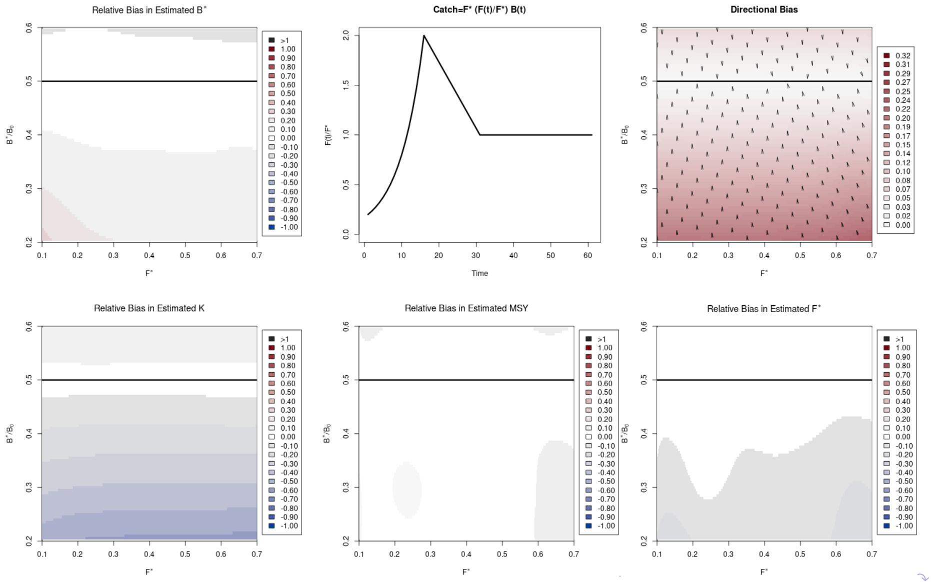
Results Idea List

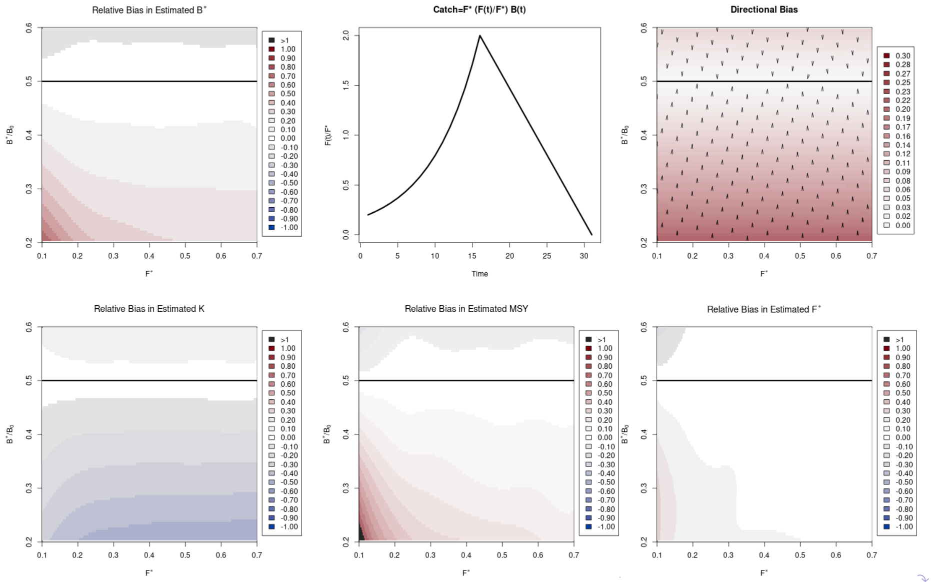
- contrast
    - components
    - animated arrows and yeild curves
- flat
    - animated arrows and yeild curves


Relative Bias in Estimated B_{MSY}
Catch=F* (F(t)/F*) B(t)
Bias Direction for (F_{MSY}, B_{MSY}/B_0) Jointly
Relative Bias in Estimated B_0
Relative Bias in Estimated MSY
Relative Bias in Estimated F_{MSY}
B_{MSY}/B_0
F_{MSY}
F(t)/F*
Time
B*/B0
F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00


Relative Bias in Estimated B*
Catch=F* (F(t)/F*) B(t)
Directional Bias
Relative Bias in Estimated K
Relative Bias in Estimated MSY
Relative Bias in Estimated F*
B*/B0
F*
F(t)/F*
Time


Relative Bias in Estimated B*
Catch=F* (F(t)/F*) B(t)
Directional Bias
Relative Bias in Estimated K
Relative Bias in Estimated MSY
Relative Bias in Estimated F*
B*/B0
F*
F(t)/F*
Time


Relative Bias in Estimated B*
Catch=F* (F(t)/F*) B(t)
Directional Bias
Relative Bias in Estimated K
Relative Bias in Estimated MSY
Relative Bias in Estimated F*
B*/B0
F*
F(t)/F*
Time
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
0.32
0.31
0.29
0.27
0.25
0.24
0.22
0.20
0.19
0.17
0.15
0.14
0.12
0.10
0.08
0.07
0.05
0.03
0.02
0.00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Directional Bias
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Relative Bias in Estimated MSY
Relative Bias in Estimated F*
B*/B0
F*
F(t)/F*
Time
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
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0.28
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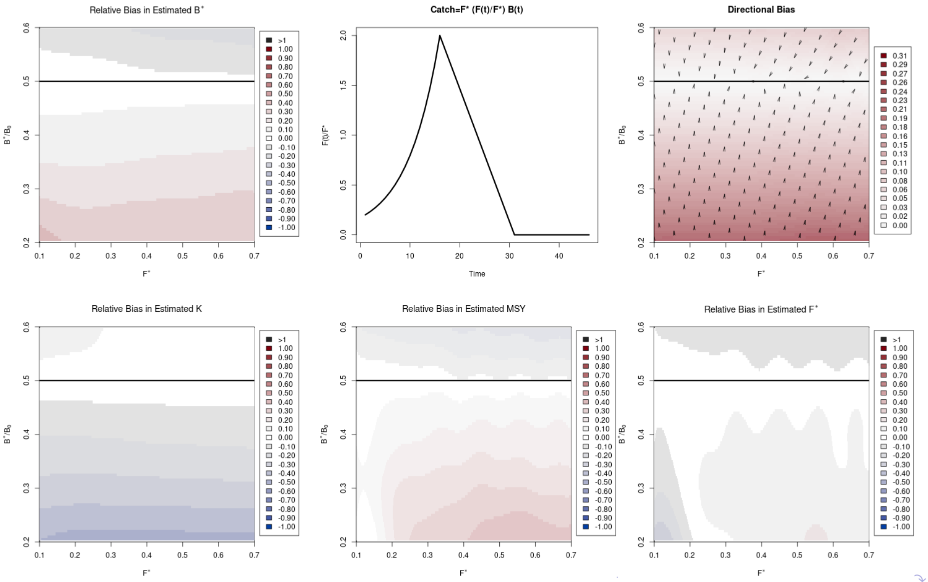
0.02
0.00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Relative Bias in Estimated MSY
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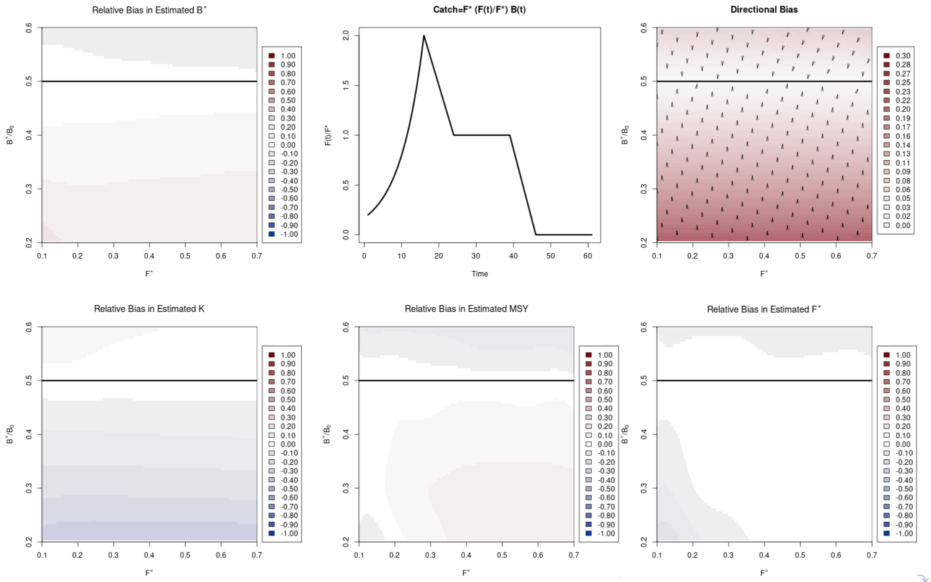
Time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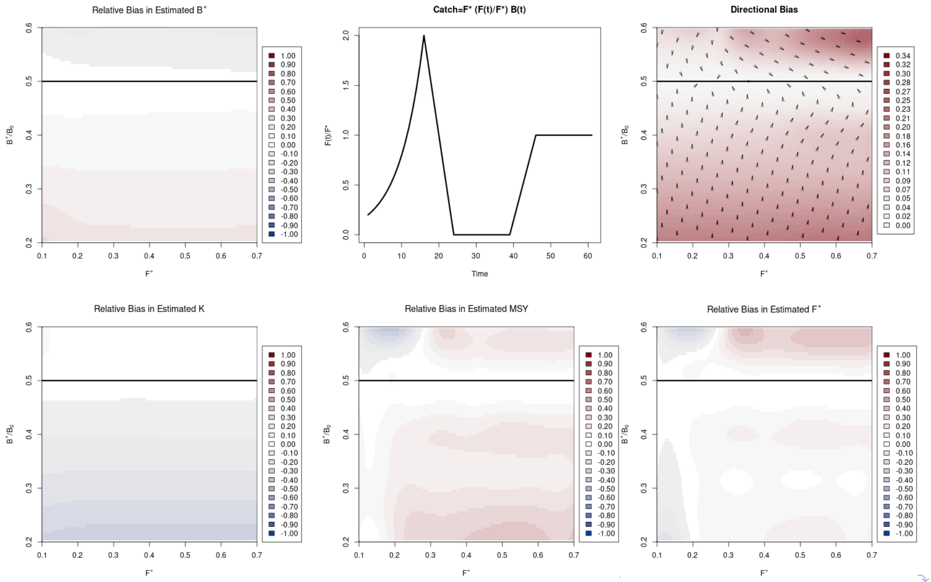
Directional Bias
Relative Bias in Estimated K
Relative Bias in Estimated MSY
Relative Bias in Estimated F*
B*/B0
F*
F(t)/F*
Time


Relative Bias in Estimated B*
B*/B0
F*
Catch=F* (F(t)/F*) B(t)
F(t)/F*
Time
Directional Bias
Relative Bias in Estimated K
Relative Bias in Estimated MSY
Relative Bias in Estimated F*

Conclusions

- Contrast story
- Importance of getting the computaional details correct for moving to analysis of Delay Difference and age structure