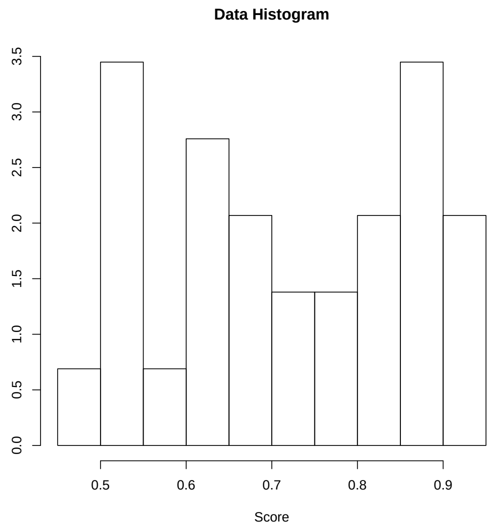
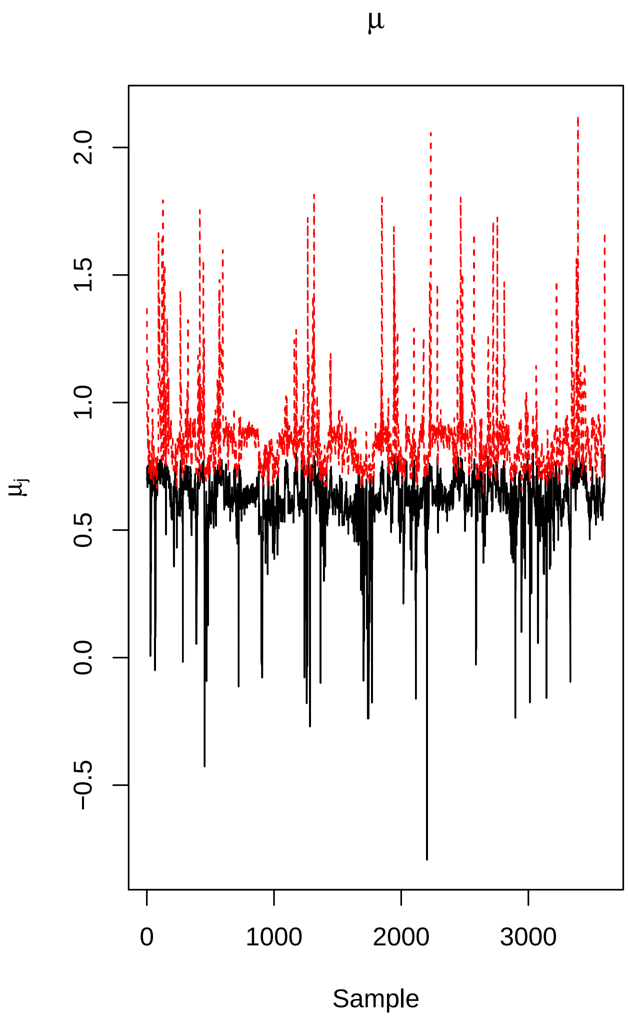


1 Model

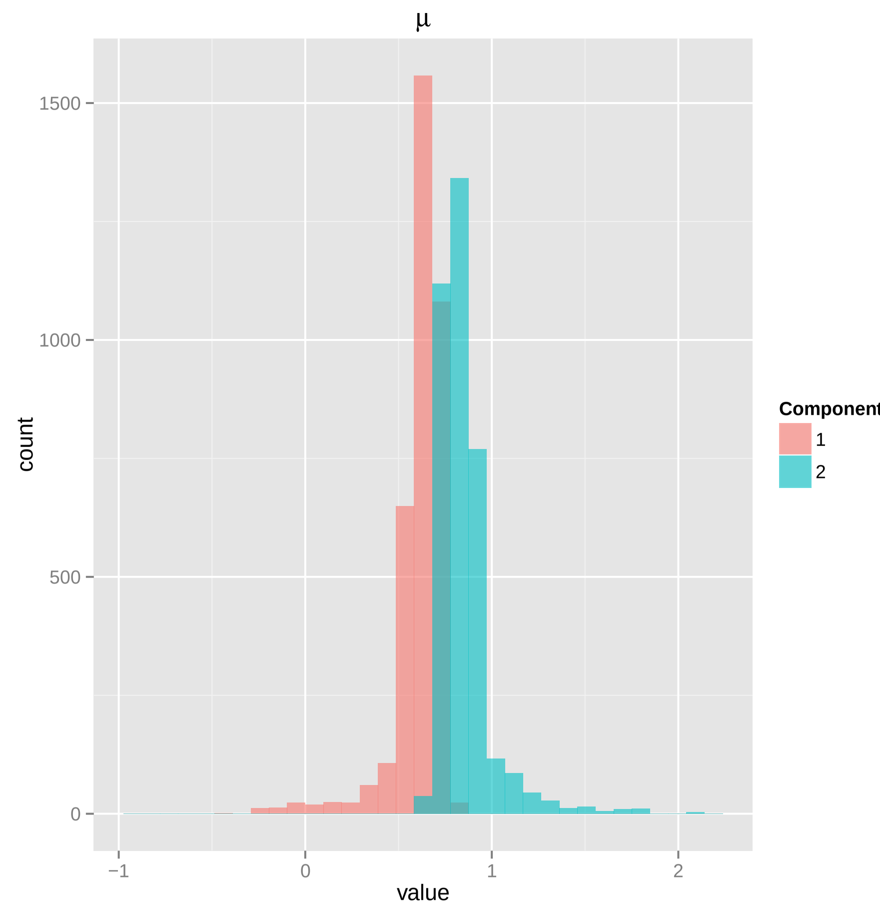
$$\ell(\boldsymbol{y}|\boldsymbol{\mu}, \boldsymbol{\sigma^2}, \boldsymbol{w}, \boldsymbol{z}) \propto \prod_{i=1}^{n}\prod_{j=1}^{k}\left[w_j N(y_i|\mu_j, \sigma_j^2)\right]^{z_{ij}} \qquad \begin{aligned} \mu_j &\sim N(\xi, \kappa^{-1}) \\ \sigma^{-2} &\sim \Gamma(\alpha, \beta) \\ \beta &\sim \Gamma(g, h) \\ \boldsymbol{w} &\sim D(\boldsymbol{\delta}) \\ k &\sim Poisson(\lambda) \end{aligned} \tag{1}$$

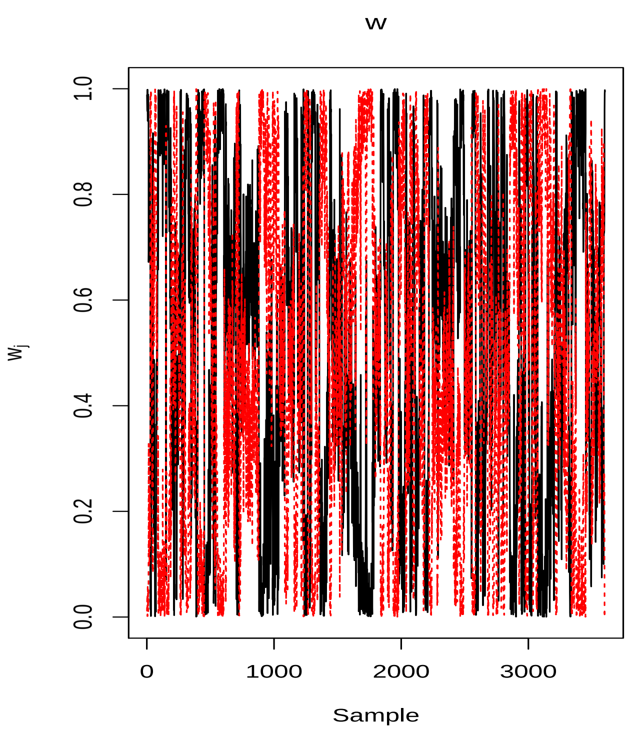
2 MCMC

- Initialize $\boldsymbol{w}, \boldsymbol{\mu}, \boldsymbol{\sigma^2}, \beta$
- for $m \in \{1, 2, ..., M\}$
  - $\boldsymbol{w}^{(m+1)}|... \sim D(\delta + n_1, ..., \delta + n_k)$
  - $\mu_j^{(m+1)}|... \sim N\left(\frac{\sigma_j{}^{-2}\sum_{i:z_i=j} y_i + \kappa\xi}{\sigma_j{}^{-2} n_j + \kappa},\ \frac{1}{\sigma_j{}^{-2} n_j + \kappa}\right) \qquad j \in \{1, 2, ..., k^{(m)}\}$
  - $\sigma_j^{-2(m+1)}|... \sim \Gamma\left(\alpha + \frac{n_j}{2},\ \beta + \frac{1}{2}\sum_{i:z_i=j}(y_i - \mu_j)^2\right) \qquad j \in \{1, 2, ..., k^{(m)}\}$
  - Allocate y_i's to a $j \in \{1, 2, ..., k^{(m)}\}$ according to $p(z_i = j|...) \propto \frac{w_j}{\sigma_j}\exp\left\{-\frac{(y_i-\mu_j)^2}{2\sigma_j^2}\right\}$.
  - $\beta^{(m+1)}|... \sim \Gamma\left(g + k\alpha,\ h + \sum_j \sigma_j^{-2}\right)$
  - Propose a Merge or Split:
    - Transform components to get $\boldsymbol{\theta}^* = \left\{\boldsymbol{w}^*, \boldsymbol{\mu}^*, \boldsymbol{\sigma^2}^*, k^*\right\}$.
    - Reallocate y_i's to a $j \in \{1, 2, ..., k^*\}$
    - Calculate acceptance probability, ρ_1.
    - $\boldsymbol{\theta}^{(m+1)} = \boldsymbol{\theta}^*$ with probability $\min(1, \rho_1)$.
  - Propose a Death or Birth:
    - Transform components to get $\boldsymbol{\theta}^* = \left\{\boldsymbol{w}^*, \boldsymbol{\mu}^*, \boldsymbol{\sigma^2}^*, k^*\right\}$.
    - Calculate acceptance probability, ρ_2.
    - $\boldsymbol{\theta}^{(m+1)} = \boldsymbol{\theta}^*$ with probability $\min(1, \rho_2)$.
  - $\ell^{(m+1)} = \ell(\boldsymbol{y}|\boldsymbol{\mu}^{(m+1)}, \boldsymbol{\sigma^2}^{(m+1)}, \boldsymbol{w}^{(m+1)}, \boldsymbol{z}^{(m+1)})$
  - $P^{(m+1)} = P(\boldsymbol{\theta}^{(m+1)}|\boldsymbol{y})$

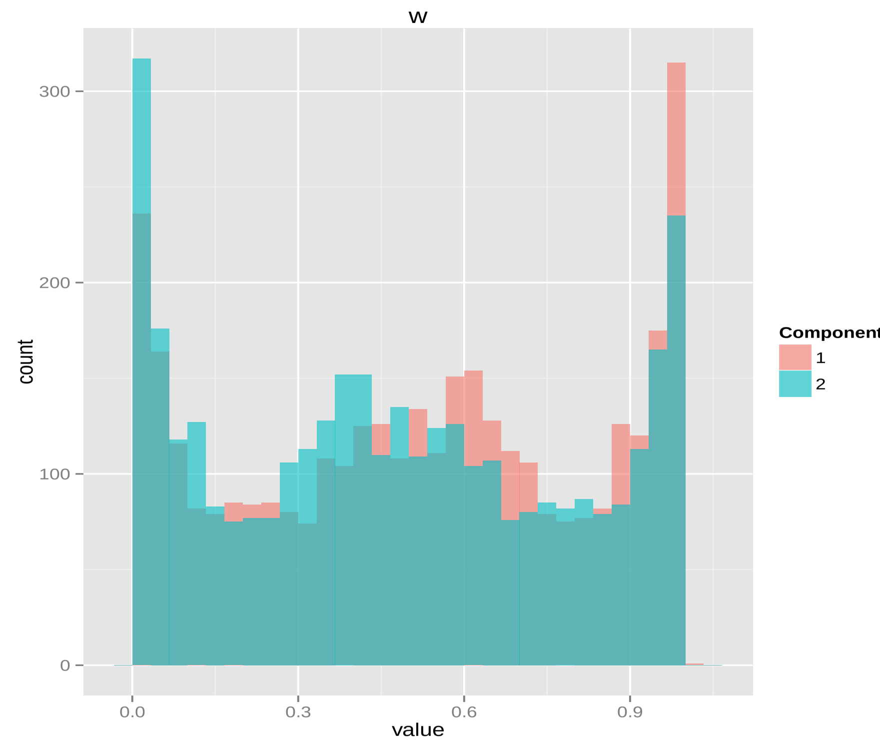


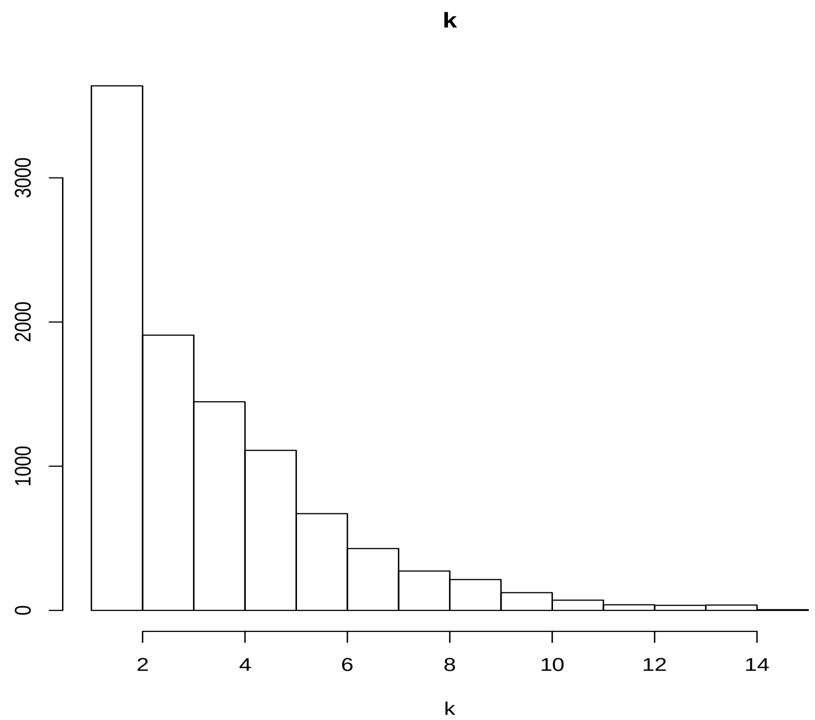
$\hat{w} \approx [0.61, 0.85]$

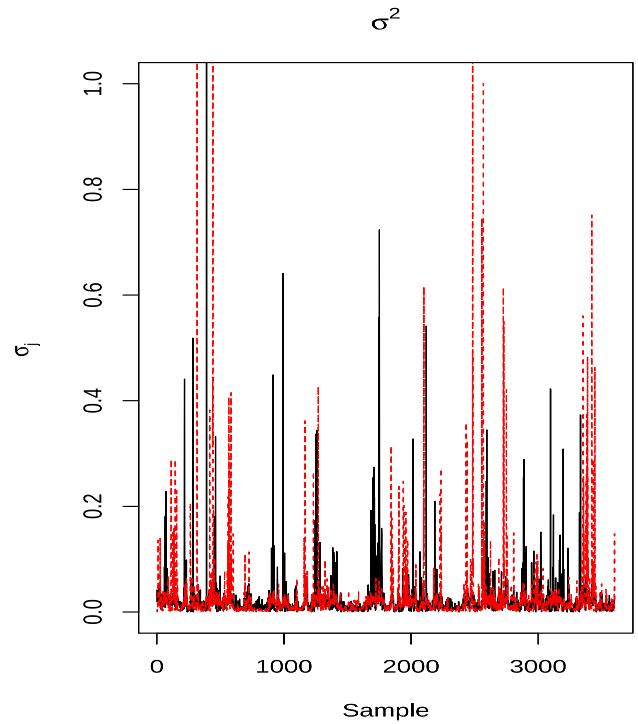




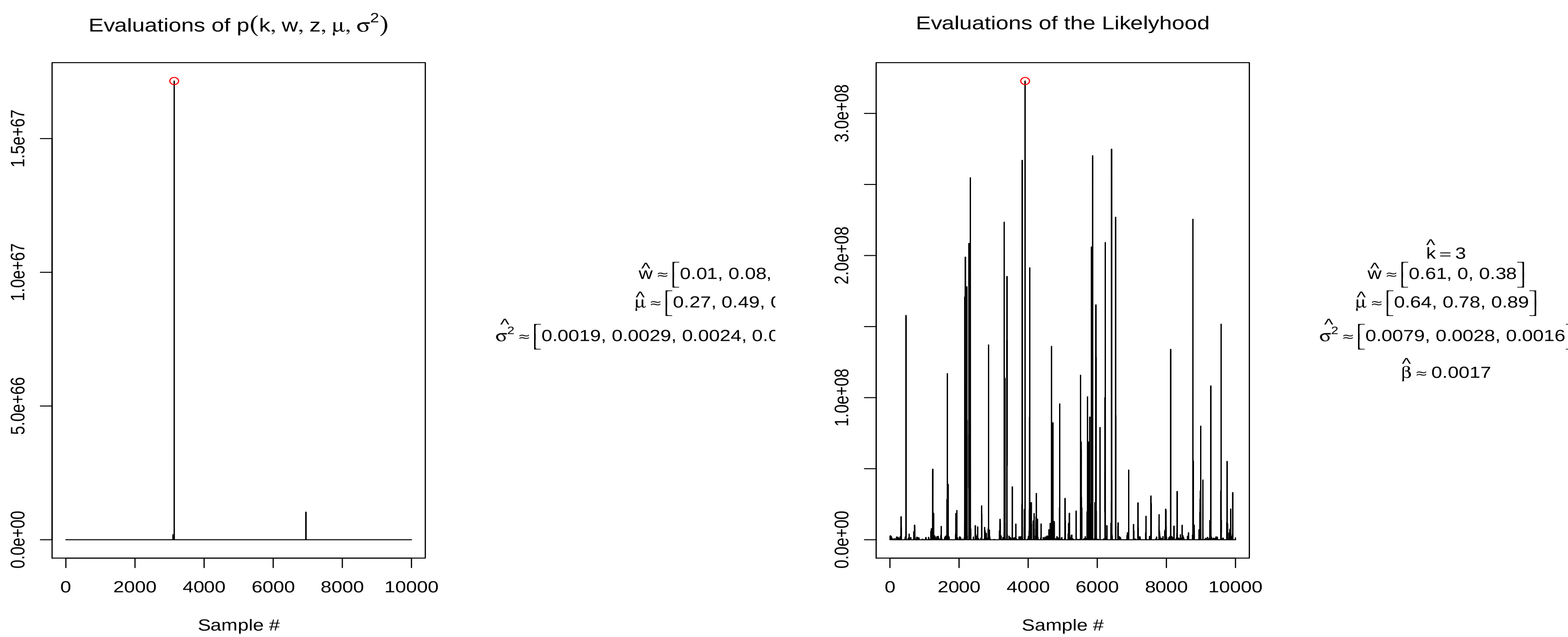
$\hat{\mu} \approx [0.52, 0.48]$







$\hat{\sigma}^2 \approx [0.0235, 0.024]$



Evaluations of p(k, w, z, μ, σ²)
0.0e+00
5.0e+66
1.0e+67
1.5e+67
0
2000
4000
6000
8000
10000
Sample #
$\hat{w} \approx [0.01, 0.08,$
$\hat{\mu} \approx [0.27, 0.49,$
$\hat{\sigma}^2 \approx [0.0019, 0.0029, 0.0024,$
Evaluations of the Likelyhood
0.0e+00
1.0e+08
2.0e+08
3.0e+08
0
2000
4000
6000
8000
10000
Sample #
$\hat{k} = 3$
$\hat{w} \approx [0.61, 0, 0.38]$
$\hat{\mu} \approx [0.64, 0.78, 0.89]$
$\hat{\sigma}^2 \approx [0.0079, 0.0028, 0.0016]$
$\hat{\beta} \approx 0.0017$

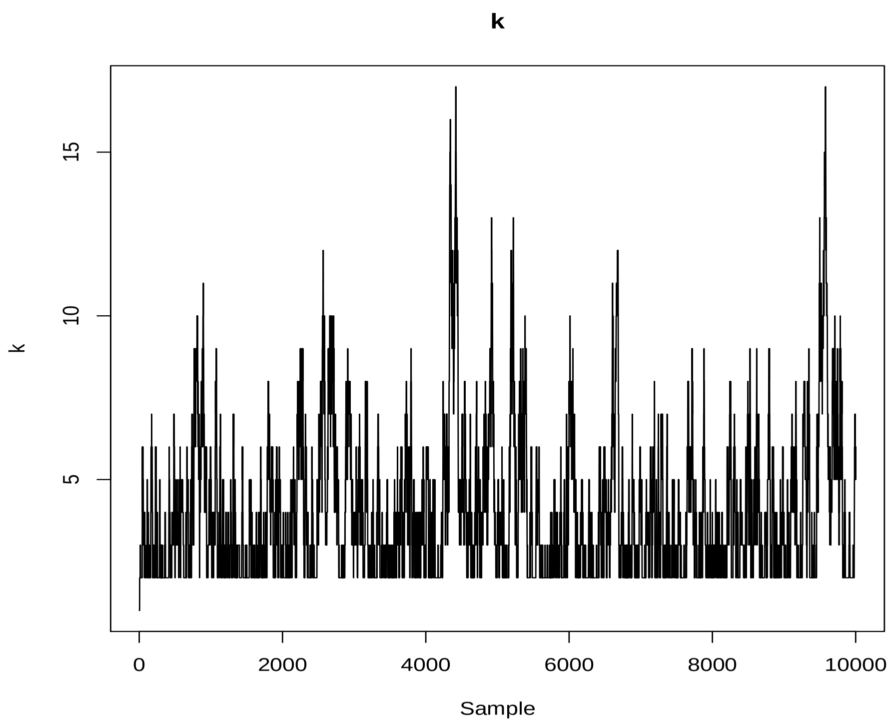


Figure 1: Merge/Split and Death/Birth

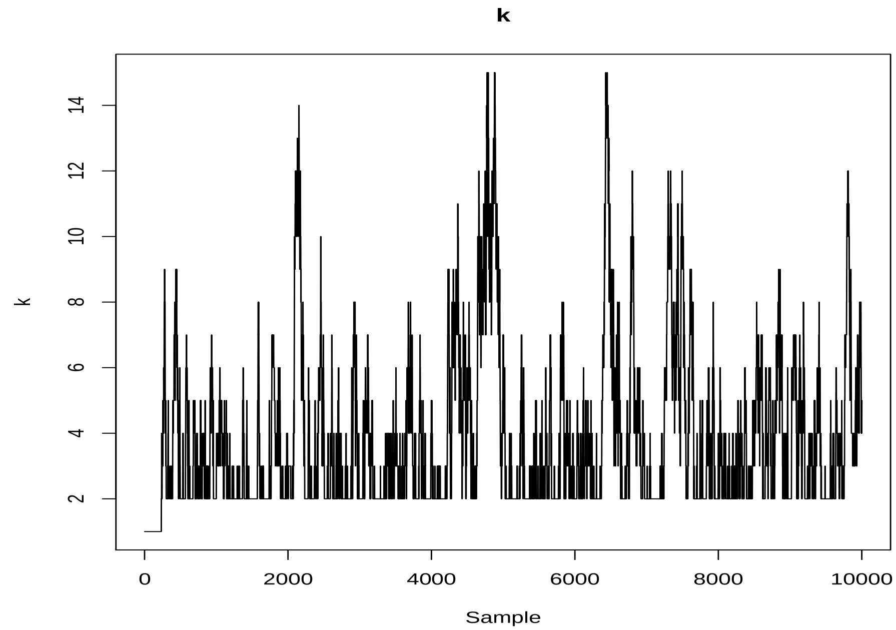


Figure 2: Merge/Split Only

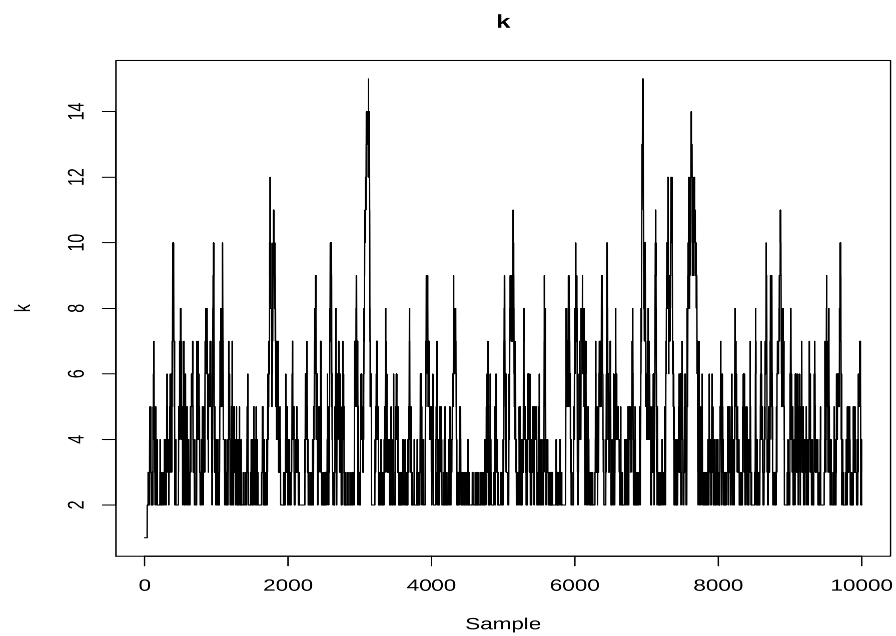
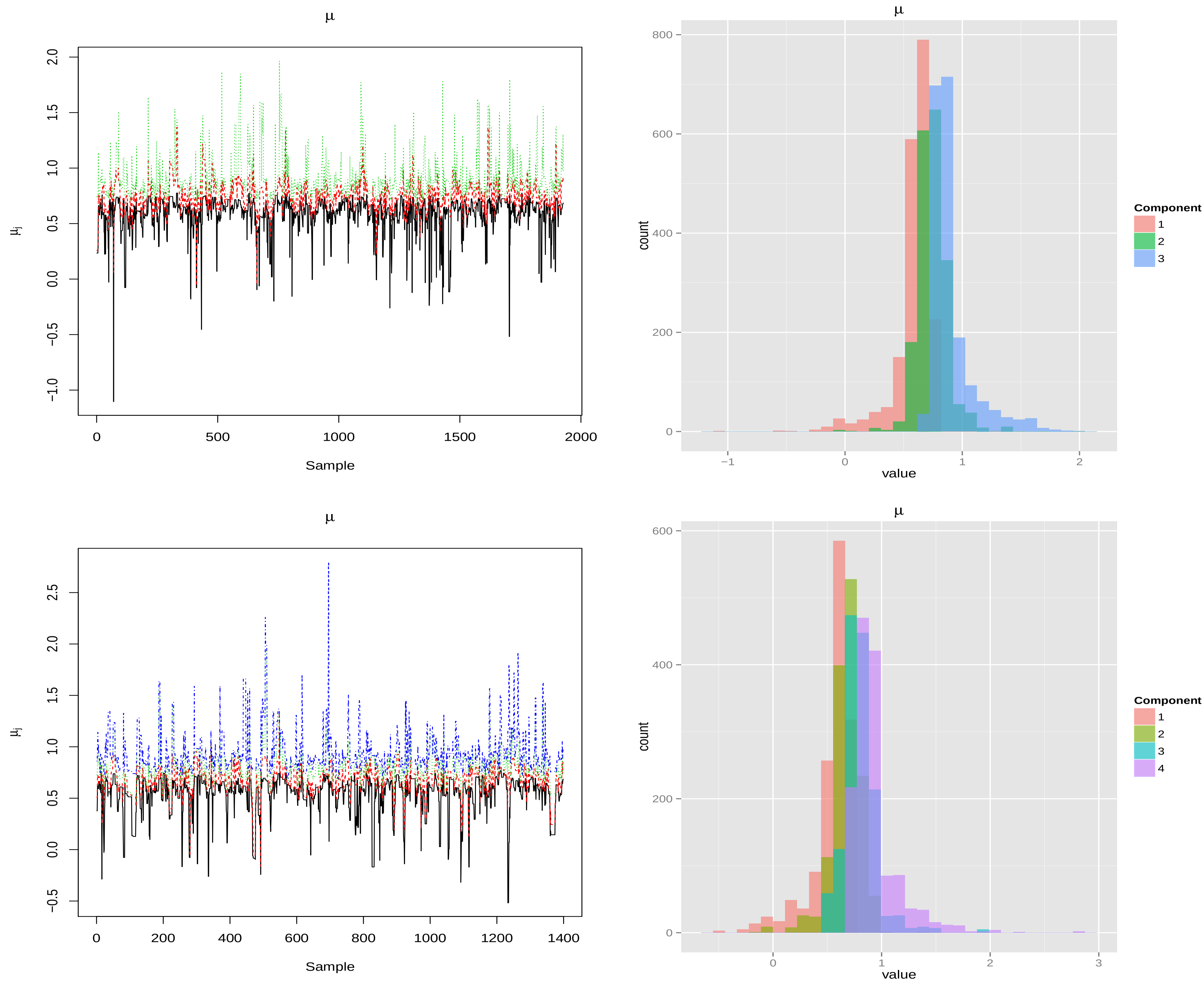


Figure 3: Death/Birth Only



μ
μ_j
Sample
count
value
Component
1
2
3
4