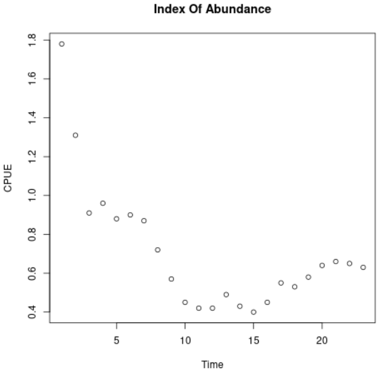
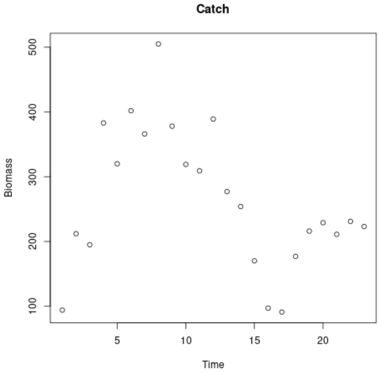
Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh

14 March 2022





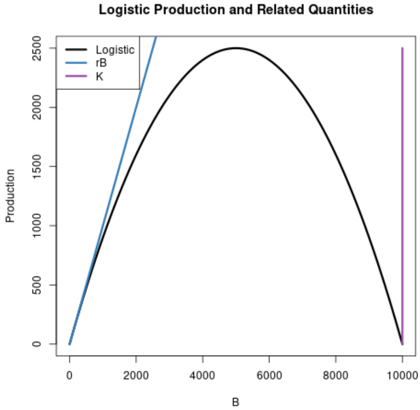
$$I_t = qB_t e^{\epsilon} \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \boldsymbol{\theta}) - C(t)$$

Schaefer Model

$$P_{\boldsymbol{\theta}}(B) = rB\left(1 - \frac{B}{K}\right)$$

$$\boldsymbol{\theta} = (r, K)$$

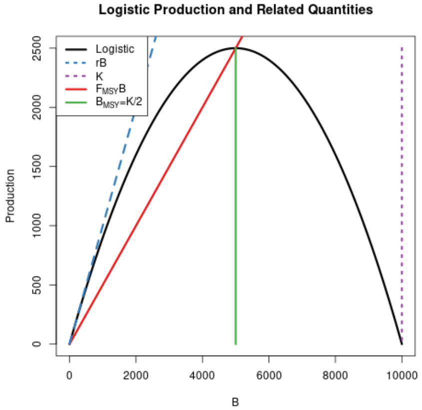


Schaefer Reference Points

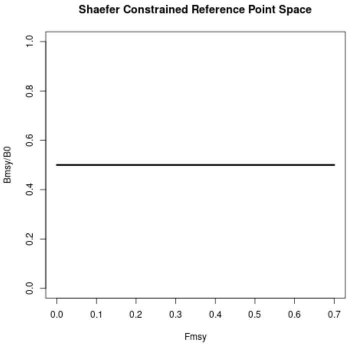
$$F^* = \frac{r}{2}$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$

$$MSY = \frac{rK}{4}$$



Something about Model misspecification of RPs
Three parameter SRR
functional form with oprthogonal RPs

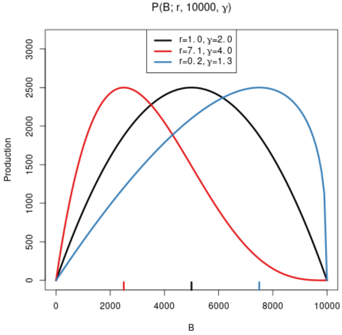


Pella-Tomlinson Production Model

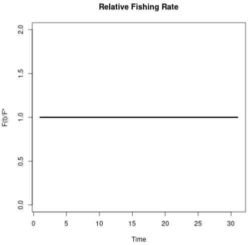
$$I(t) \sim LN(qB(t), \sigma^2)$$
$$\frac{dB(t)}{dt} = P_{\theta}(B(t)) - F(t)B(t)$$

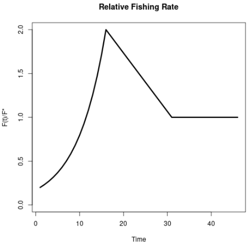
$$P_{\theta}(B) = \frac{rB}{\gamma - 1}\left(1 - \frac{B}{K}\right)^{\gamma - 1}$$
$$\theta = (r, K, \gamma)$$

$$\gamma = 2 \Rightarrow \text{Schaefer Model}$$



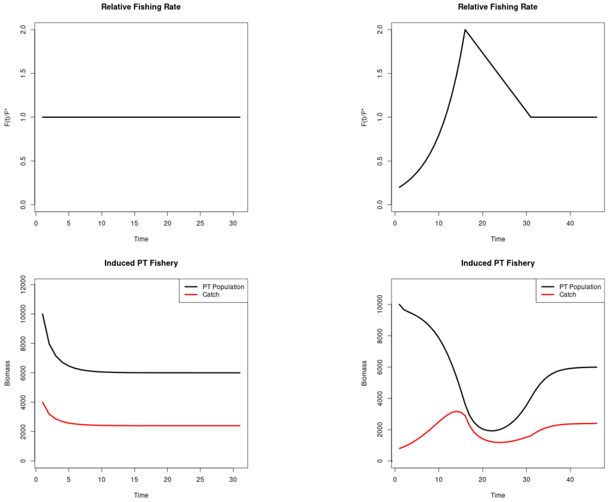
Catch



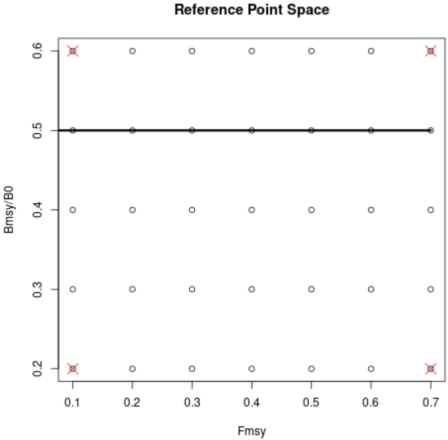


$$C(t) = F^* \left(\frac{F(t)}{F^*} \right) B(t)$$

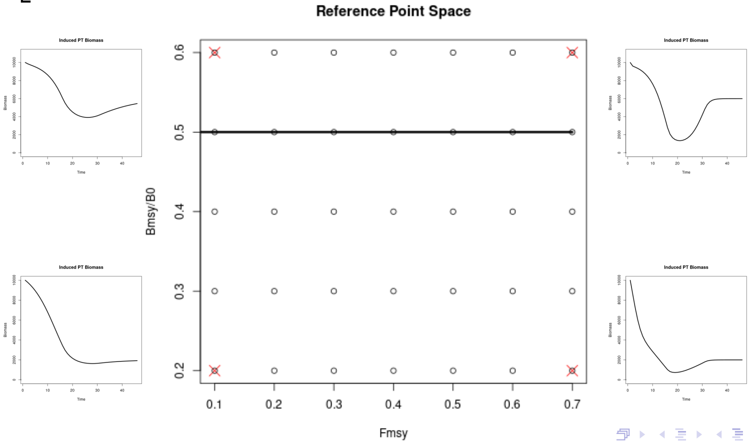
Catch


Relative Fishing Rate
F(t)/F*
Time
Relative Fishing Rate
F(t)/F*
Time
Induced PT Fishery
Biomass
Time
PT Population
Catch
Induced PT Fishery
Biomass
Time
PT Population
Catch

$$\boldsymbol{\theta} = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)},\; K = 10000,\; \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$



$$\boldsymbol{\theta} = \left[r = F^{*} \left(\frac{1 - \frac{B^{*}}{\bar{B}(0)}}{\frac{B^{*}}{\bar{B}(0)}} \right) \left(1 - \frac{B^{*}}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^{*}}{\bar{B}(0)} - 1}{\frac{B^{*}}{\bar{B}(0)}} \right)},\ K = 10000,\ \gamma = \frac{1}{\frac{B^{*}}{\bar{B}(0)}} \right]$$



Metamodel

$$\boldsymbol{x} = \left(F^*, \frac{B^*}{\bar{B}(0)}\right)$$

$$\hat{\mu} = \beta_0 + \boldsymbol{\beta}'\boldsymbol{x} + f(\boldsymbol{x}) + \epsilon$$
$$f(\boldsymbol{x}) \sim \mathrm{GP}(0, \tau^2 R(\boldsymbol{x}, \boldsymbol{x}'))$$
$$\epsilon_i \sim \mathrm{N}(0, \hat{\omega}_i).$$

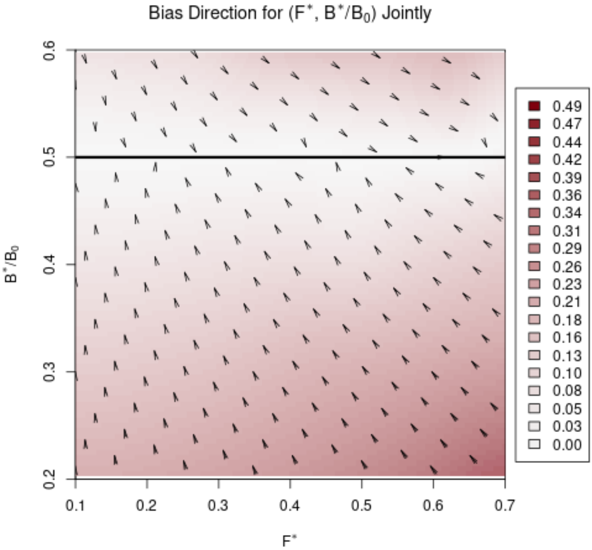
$$R(\boldsymbol{x}, \boldsymbol{x}') = \exp\left(\sum_{j=1}^{2} \frac{-(x_j - x_j')^2}{2\ell_j^2}\right)$$

define μ

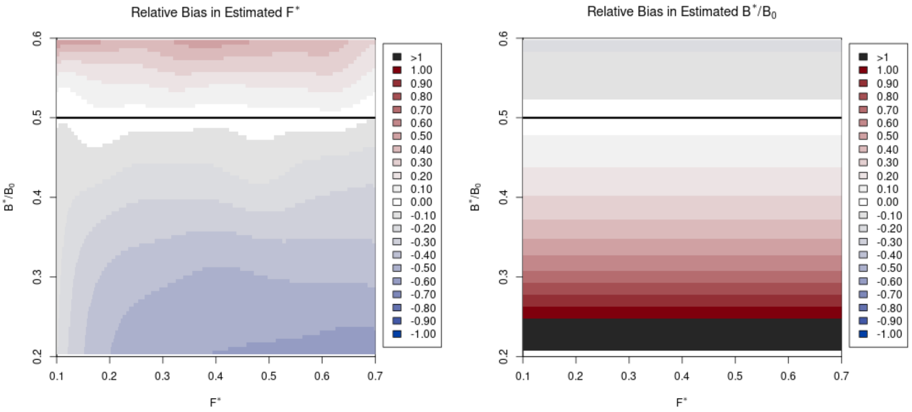
$$\check{\mu}(\check{s}) = \beta_0 + \boldsymbol{x}(\check{s})\boldsymbol{\beta} + R_{\ell}(\check{s}, s)R_{\ell}^{-1}(s, s)\Big(\hat{\mu}(s) - \big(\beta_0 + \boldsymbol{x}(s)\boldsymbol{\beta}\big)\Big)$$

$$\check{B}^* = \frac{\check{K}}{2} \qquad \check{F}^* = \frac{\check{r}}{2}.$$

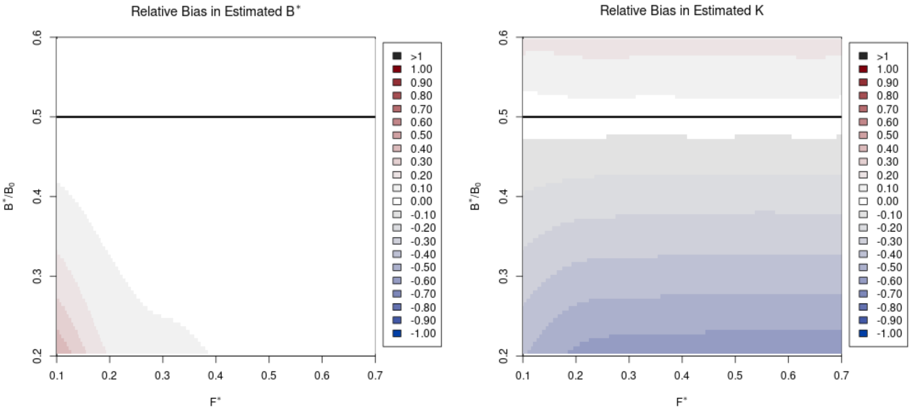
$$\text{Relative Bias} = \frac{\check{RP} - RP}{RP}$$

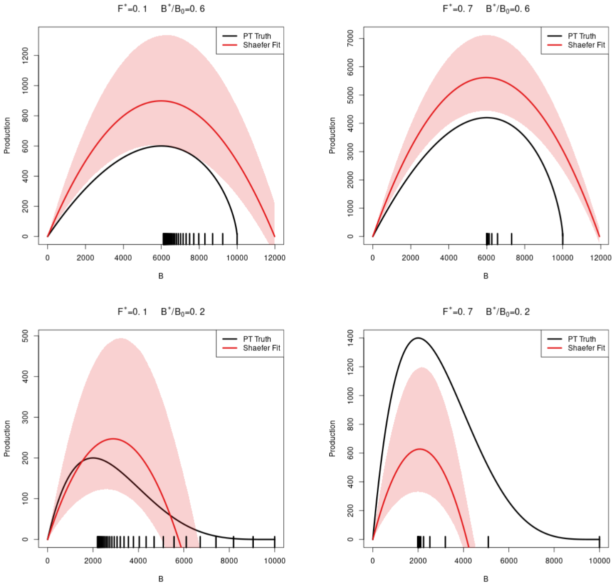

Bias Direction for (F^*, B^*/B_0) Jointly
B^*/B_0
0.2
0.3
0.4
0.5
0.6
0.1
0.2
0.3
0.4
0.5
0.6
0.7
F^*
0.49
0.47
0.44
0.42
0.39
0.36
0.34
0.31
0.29
0.26
0.23
0.21
0.18
0.16
0.13
0.10
0.08
0.05
0.03
0.00

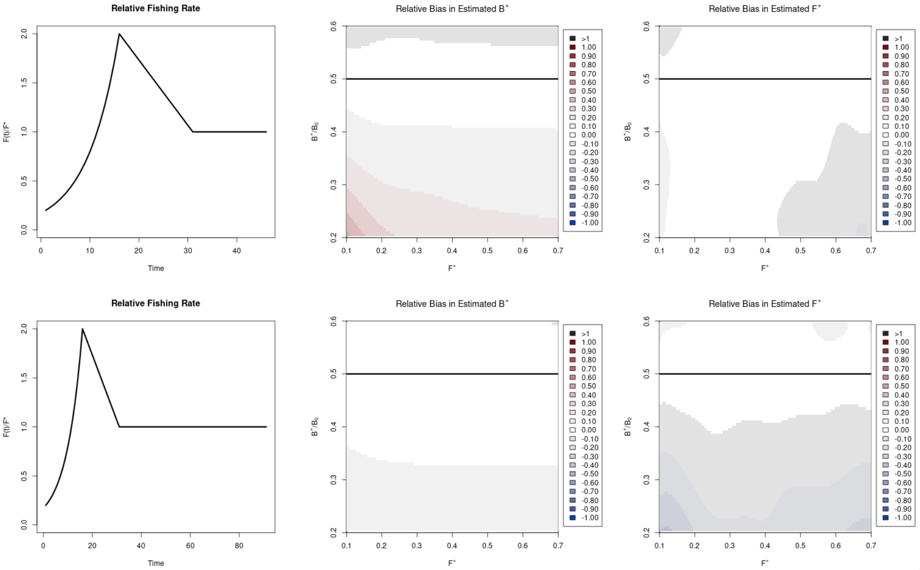
Components of Bias



Split $\frac{B^*}{B_0}$




F*=0.1 B*/B0=0.6
F*=0.7 B*/B0=0.6
F*=0.1 B*/B0=0.2
F*=0.7 B*/B0=0.2
PT Truth
Shaefer Fit
Production
B


Relative Fishing Rate
F(t)/F*
Time
Relative Bias in Estimated B*
B*/B0
F*
Relative Bias in Estimated F*
>1
1.00
0.90
0.80
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
-0.60
-0.70
-0.80
-0.90
-1.00
Relative Fishing Rate
Relative Bias in Estimated B*
Relative Bias in Estimated F*

Conclusions

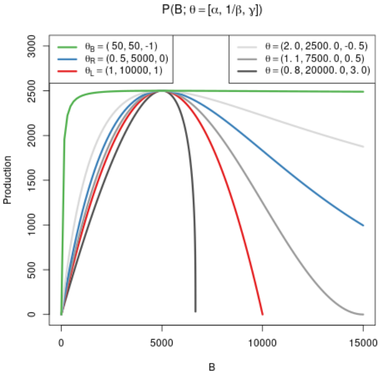
- F^*, B^*, B_0 are not directly observable, but rather modeled quantities
  - Model Misspecification, Posterior Uncertainty, Bias
- RP bias can be very large when production function assumptions are wrong
  - As model misspecification increases, RP biases tend to increase
  - B^* often appears relatively less sensative to model misspecification than either F^* or B_0
- F^* bias is strongly catch dependent
  - Bias depends on how similar the modeled and true production functions can be at the observed biomasses
- A rich simulation-based method for describing global RP bias and a stepping stone for understanding other models
  - BH and Ricker SRRs
  - Age-Structured and Delay Difference Models

Productivity Extension

$$\frac{dB}{dt} = P(B;\boldsymbol{\theta}) - (M+F)B$$

$$P(B;[\alpha,\beta,\gamma]) = \alpha B(1-\beta\gamma B)^{\frac{1}{\gamma}}$$

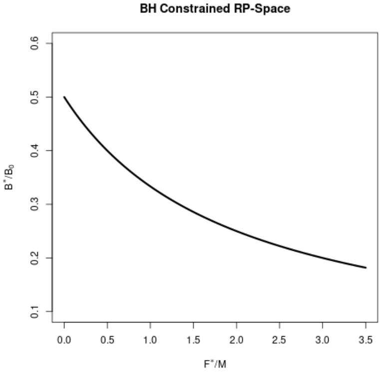
$$\gamma = -1 \Rightarrow \text{Beverton-Holt}$$
$$\gamma = 1 \Rightarrow \text{Logistic}$$
$$\gamma \to 0 \Rightarrow \text{Ricker}$$



Productivity Extension

$$P_{\mathrm{BH}}(B;[\alpha,\beta,-1]) = \frac{\alpha B}{(1+\beta B)}$$

$$\frac{B^*}{\bar{B}(0)} = \frac{1}{\frac{F^*}{M}+2}$$



Growth Extension

$$\frac{dB}{dt} = \overbrace{w(a_0)R(B;\theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa\left[w_\infty N - B\right]}^{\text{Net Growth}} - \overbrace{(M+F)B}^{\text{Mortality}}$$

$$\frac{dN}{dt} = R(B;\theta) - (M+F)N$$

$$R(B;[\alpha,\beta,\gamma]) = \alpha B(t-a_0)(1-\beta\gamma B(t-a_0))^{\frac{1}{\gamma}}$$

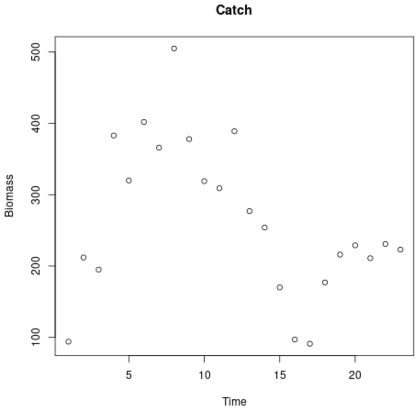
$$w(a) = w_\infty(1-e^{-\kappa a})$$

Catch Interpolation

$$t \in \mathbb{R}^+ \qquad \tau = \lceil t \rceil - 1$$

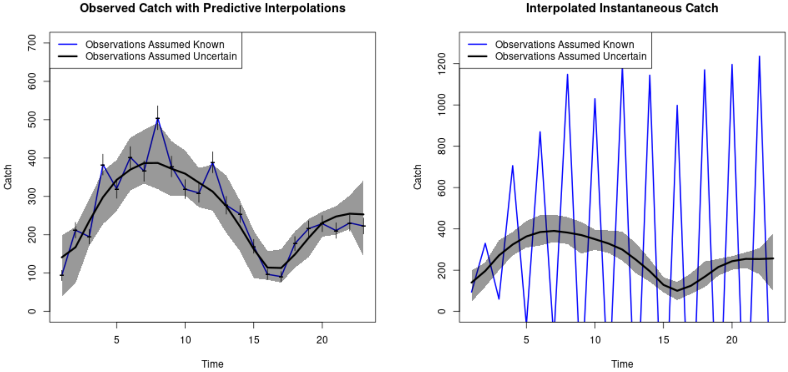
$$\mathbb{E}[y(t)] = \int_{\tau}^{t} x(t^*)dt^*$$

$$x(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



$$y(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi) \qquad \phi \sim \text{Half-Cauchy}(0, 1) \qquad \epsilon_i \sim N(0, \sigma_i^2)$$



Thanks and Acknowldgements NOAA, Sea Grant Ecetra

$$\frac{B^*}{\bar{B}(0)} = \frac{\left(\frac{\alpha}{M+F^*}\right)^{\frac{1}{\gamma}} - 1}{\left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}} - 1}$$

$$\alpha = (M + F^*)\left[1 - \frac{1}{\gamma}\left(\frac{F^*}{M + F^*}\right)\right]^{-\gamma}$$

$$\beta = \frac{1}{\gamma \bar{B}(0)}\left(1 - \left(\frac{\alpha}{M}\right)^{\frac{1}{\gamma}}\right)$$