

# A Metamodeling Approach for Bias Estimation of Biological Reference Points

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# Outline

1 Introduction

2 The Schaefer Model

3 The Beverton-Holt Model

4 Delay Differential Growth Extension

5 End

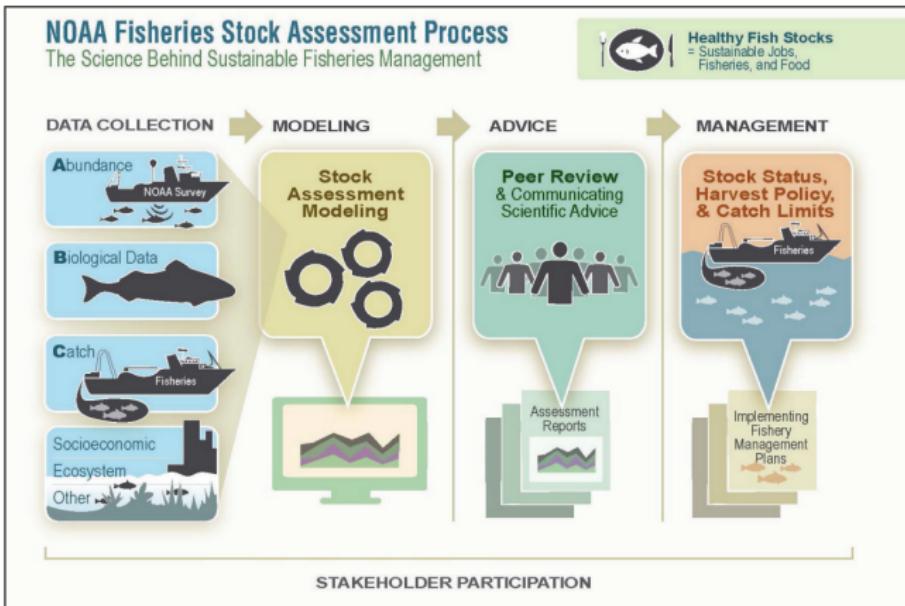
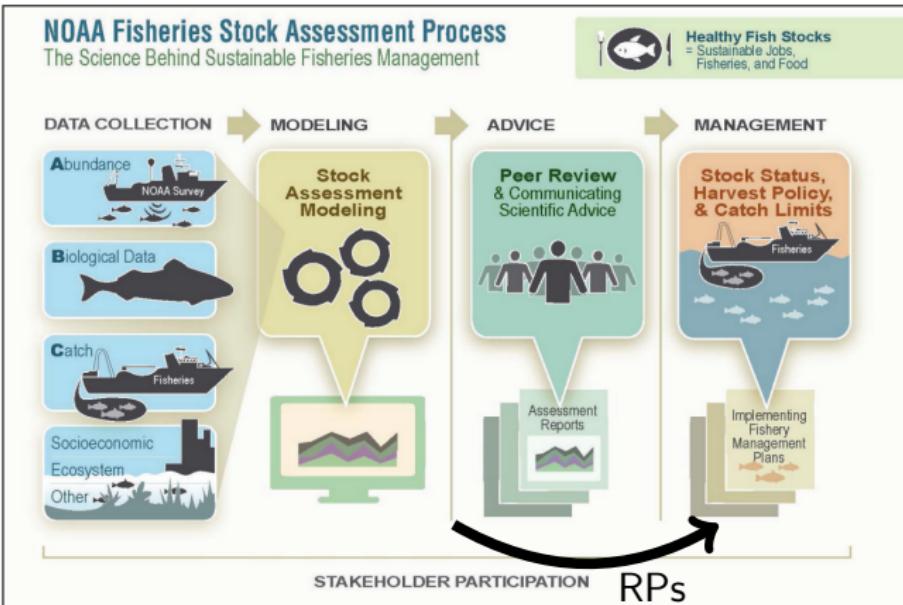


Figure 1: Overview of the stock assessment process from data collection through the provision of scientific advice to fishery managers. Stakeholders and other partners participate in each step of the assessment process. This report captures NOAA Fisheries products associated with the 'Advice' phase of the process.

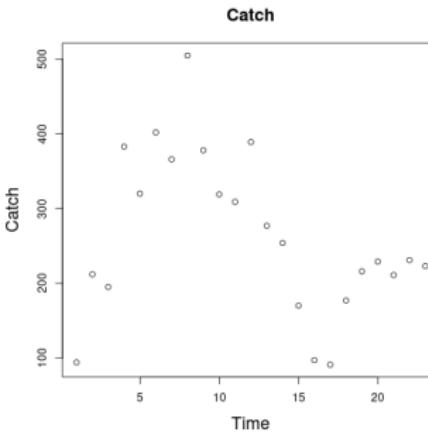
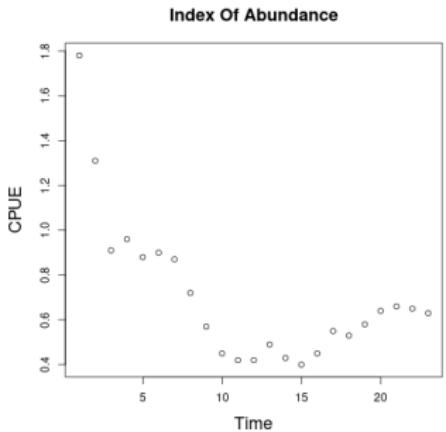
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**Figure 1:** Overview of the stock assessment process from data collection through the provision of scientific advice to fishery managers. Stakeholders and other partners participate in each step of the assessment process. This report captures NOAA Fisheries products associated with the 'Advice' phase of the process.

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## Surplus Production Model General Structure

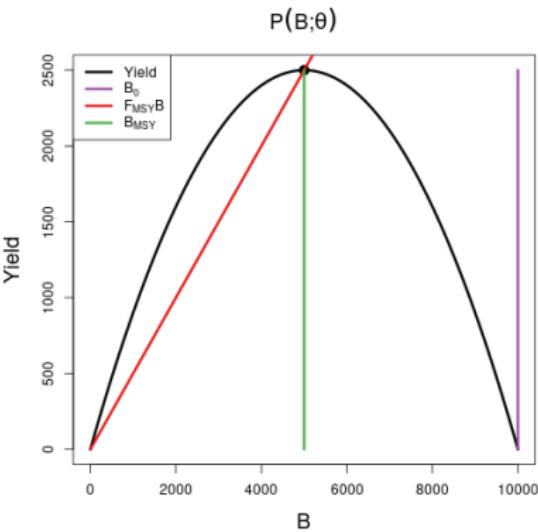


$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P(B(t); \theta) - Z(t)B(t)$$

## Reference Points:

- Maximum Sustainable Yield (*MSY*)
- $F_{MSY}^a$ : Fishing rate to achieve *MSY*
- $\frac{B_{MSY}}{B_0}$ : Biomass Depletion when at *MSY*
- Driven by the shape of  $P$  as determined by  $\theta$ .

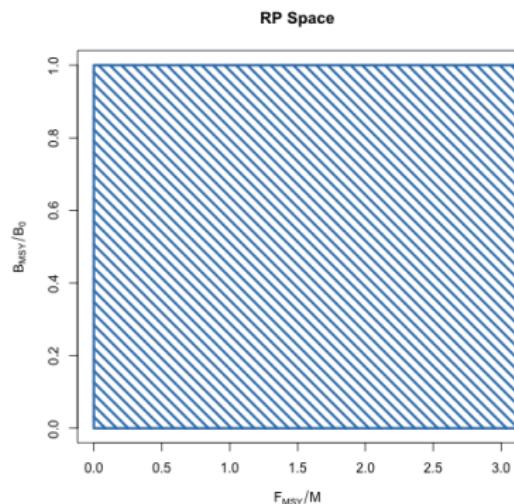


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<sup>a</sup>or  $\frac{F_{MSY}}{M}$

Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

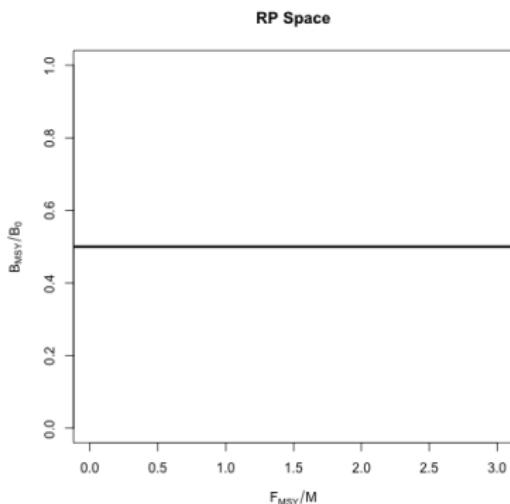


Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

■ Schaefer Model:

$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{2}$$



Conceptually:

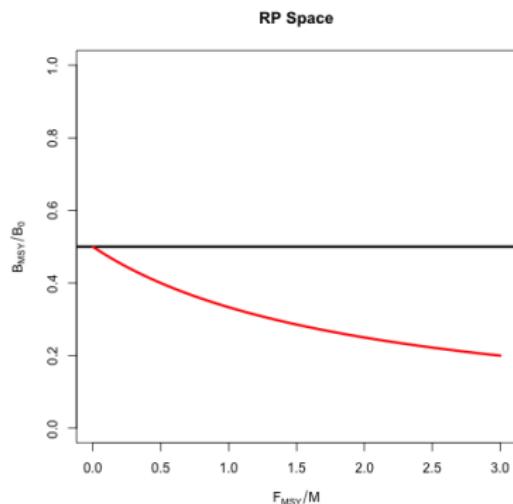
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$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{2}$$

■ BH Model:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M + 2}$$



Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

■ Schaefer Model:

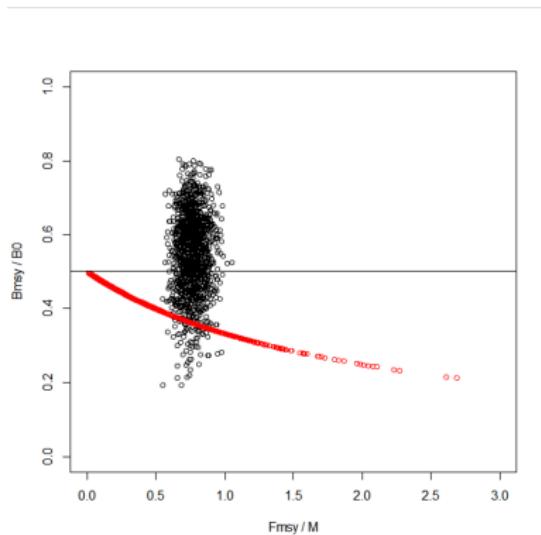
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■ BH Model:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M + 2}$$

■ Similar Constraints for other  
Two-Parameter Models:  
Fox, Ricker, etc...

■ Three-Parameter Models Allow  
Independent RP Estimation



<sup>a</sup>Mangel et al. 2013, CJFAS

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- $P_\ell$  is logistic production
  - Logistic map in discrete time
  - Implicit Natural Mortality
  - Explicit Fishing Mortality

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_\ell(B(t); \theta) - F(t)B(t)$$

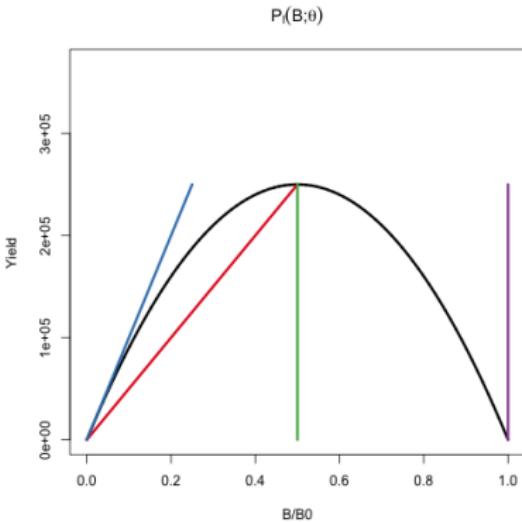
$$P_\ell(B; [r, K]) = rB \left(1 - \left(\frac{B}{K}\right)\right)$$

### Reference Points:

$$F^* = \frac{r}{2}$$

$$B^* = \frac{K}{2} \quad B_0 = K$$

$$\frac{B^*}{B_0} = \frac{1}{2}$$



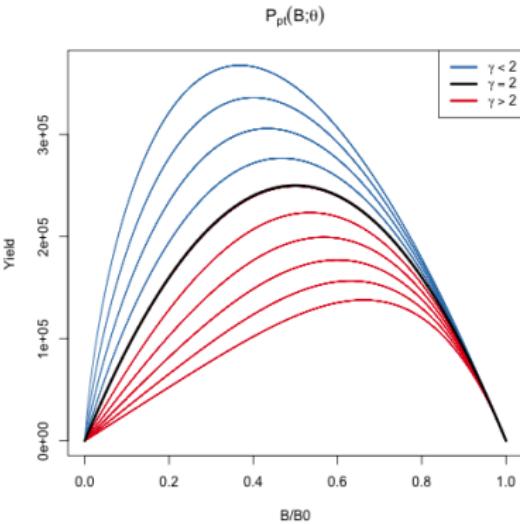
$$P_{pt}(B; [r, K, \gamma]) = \frac{rB}{\gamma - 1} \left( 1 - \left( \frac{B}{K} \right)^{(\gamma-1)} \right)$$

### Reference Points:

$$F^* = \frac{r}{\gamma}$$

$$B^* = K \left( \frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}} \quad B_0 = K$$

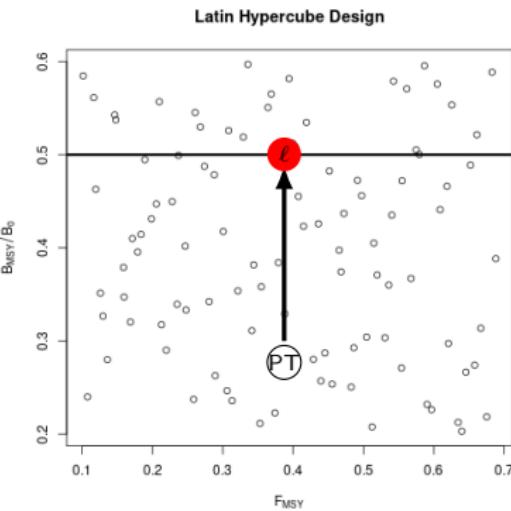
$$\frac{B^*}{B_0} = \left( \frac{1}{\gamma} \right)^{\frac{1}{\gamma-1}}$$



$$F^* = \frac{r}{\gamma} \quad \frac{B^*}{\bar{B}(0)} = \left(\frac{1}{\gamma}\right)^{\frac{1}{\gamma-1}}$$

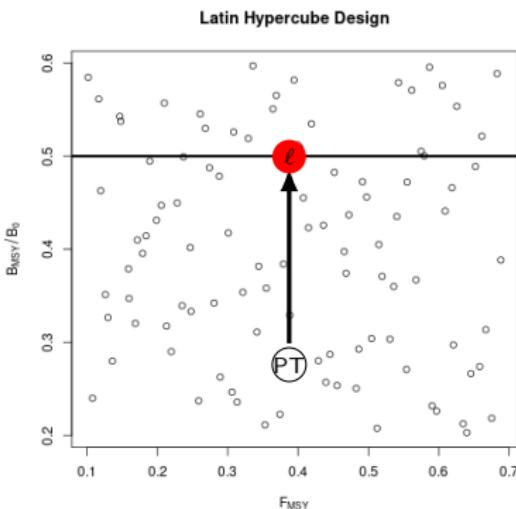
Closed-Form Inversion

$$r = \gamma F^* \quad \gamma = \frac{W\left(\frac{B^*}{\bar{B}(0)} \log\left(\frac{B^*}{\bar{B}(0)}\right)\right)}{\log\left(\frac{B^*}{\bar{B}(0)}\right)}$$

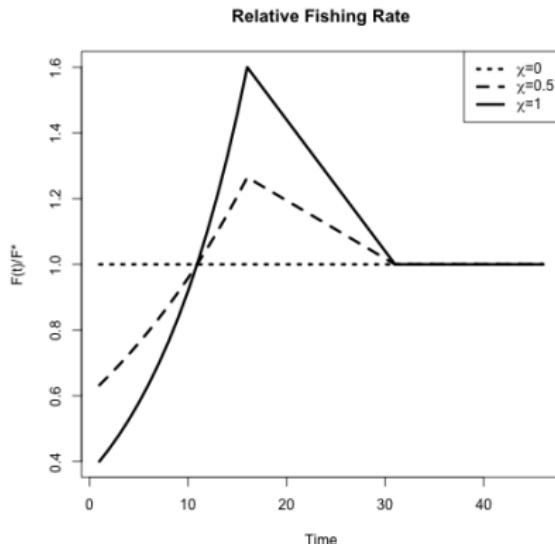


\* Lambert W function inverts  $xe^x$  s.t.  $W(xe^x) = x$

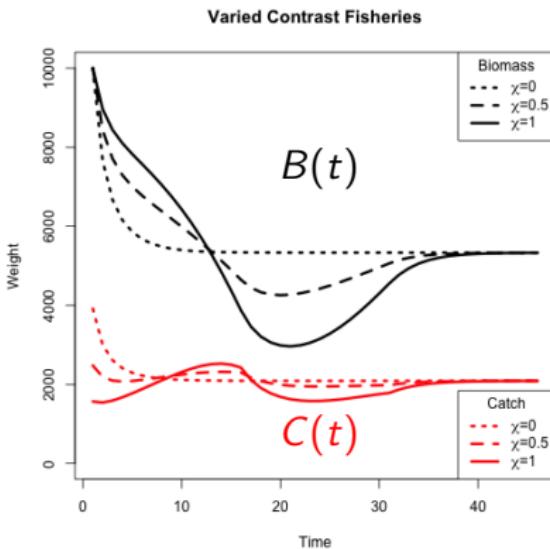
$$\underbrace{\left( F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{PT Truth}} \xrightarrow{\text{GP}} \underbrace{\left( \hat{F}_{MSY}, \frac{1}{2} \right)}_{\text{Schaefer Estimate}}$$

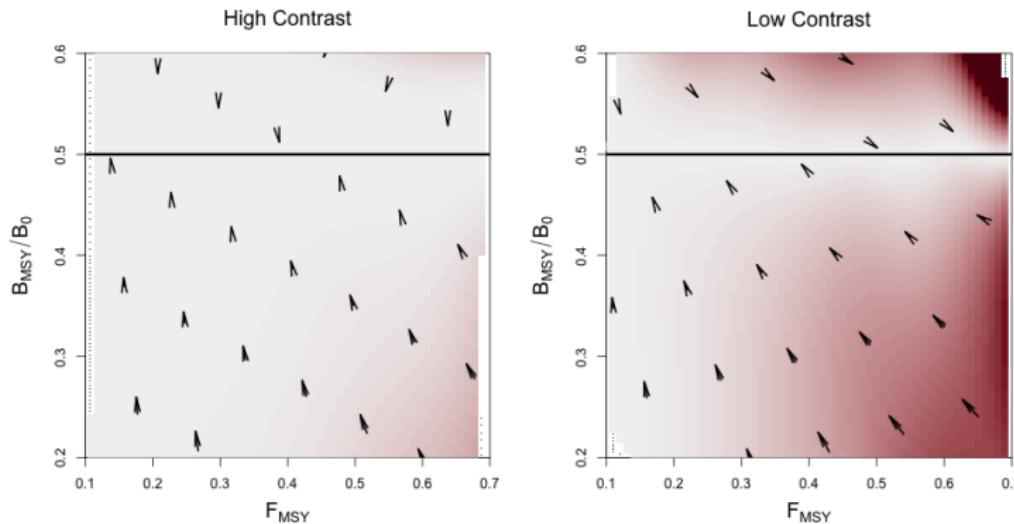


$$\begin{aligned}\frac{dB}{dt} &= P(B(t); \theta) - F(t)B(t) \\ &= P(B(t); \theta) - F_\theta^* \underbrace{\frac{F(t)}{F_\theta^*}}_{\text{Relative Fishing}} B(t)\end{aligned}$$

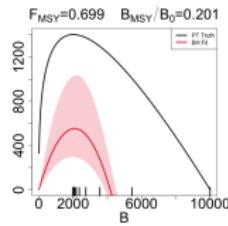
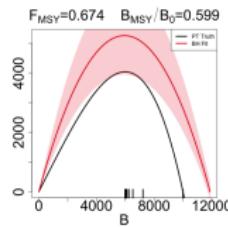
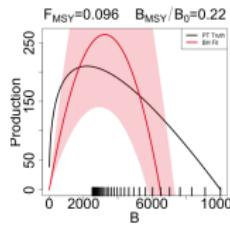
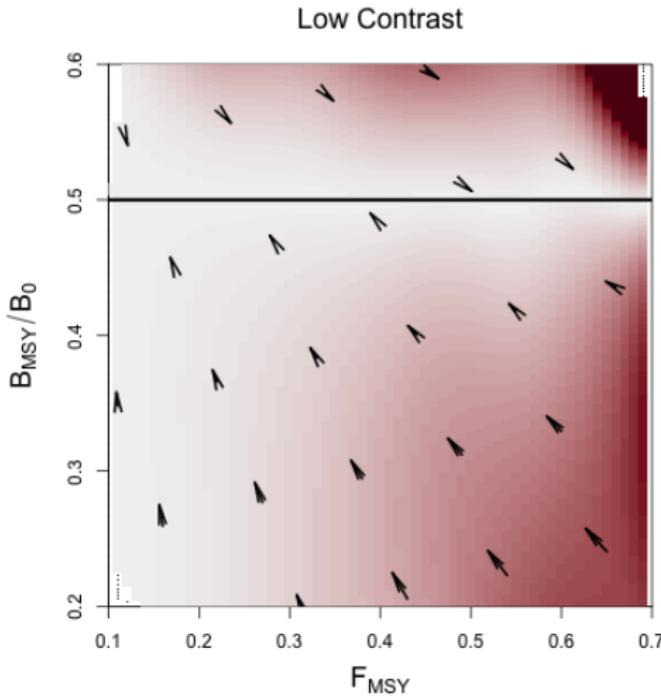
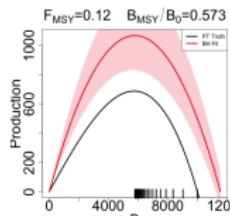


$$\begin{aligned}\frac{dB}{dt} &= P(B(t); \theta) - F(t)B(t) \\ &= P(B(t); \theta) - F_\theta^* \underbrace{\frac{F(t)}{F_\theta^*} B(t)}_{C(t)}\end{aligned}$$



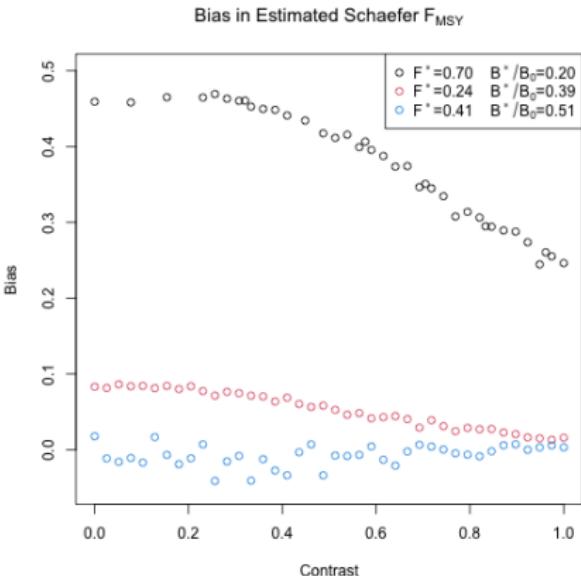


# Mechanism for Bias in $F_{MSY}$ via Contrast



# Mechanism for Bias in $F_{MSY}$ via Contrast

- Low contrast limits the observation window on production.
- As contrast increases  $F^*$  bias diminishes.
- Bias in  $\frac{B^*}{B_0}$  remains, but these biases are independent of  $F^*$ .



# Summary

- PT three-parameter generalization allows fully analytical RP designs.
- Novel simulation framework explicitly controlling for RP misspecification.
- The useful notion of contrast developed here together with the simplified geometry of the Schaefer model exposes a mechanism for RP bias
- What to do when the simulation design is not analytical?

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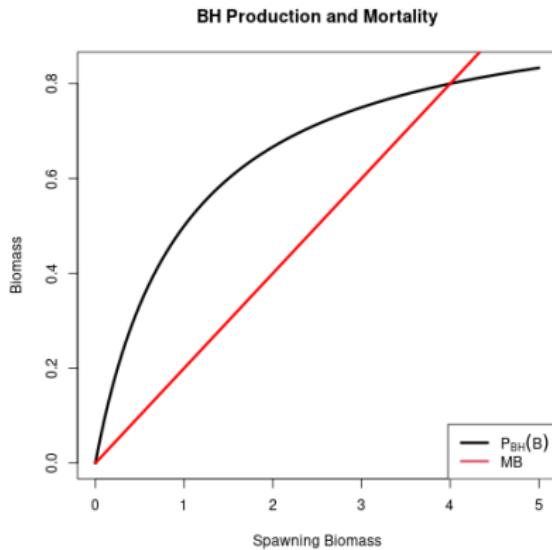
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$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = P_{BH}(B; [\alpha, \beta]) - (M + F)B$$

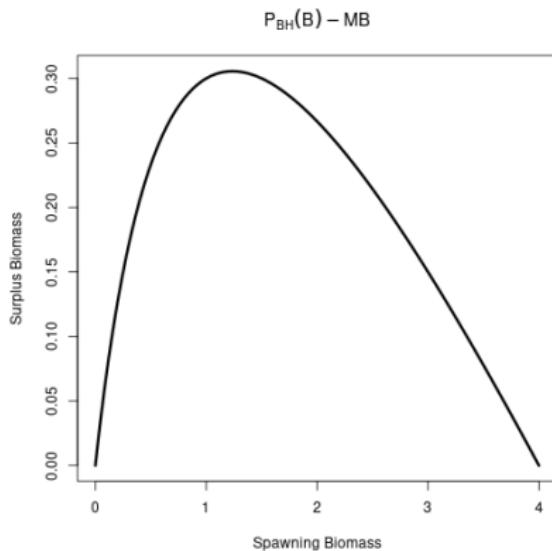
$$P_{BH}(B; [\alpha, \beta]) = \frac{\alpha B}{1 + \beta B}$$



$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \underbrace{P_{BH}(B; [\alpha, \beta]) - MB}_{\text{Surplus Production}} - FB$$

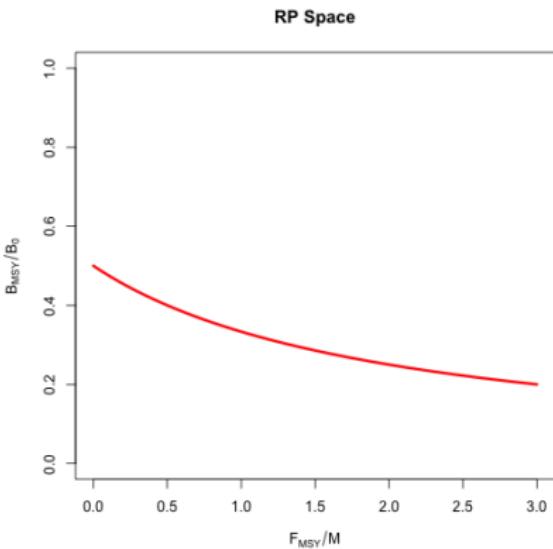
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$$\frac{dB}{dt} = \underbrace{P_{BH}(B; [\alpha, \beta]) - MB}_{\text{Surplus Production}} - FB$$

$$P_{BH}(B; [\alpha, \beta]) = \frac{\alpha B}{1 + \beta B}$$



# Three Parameter Schnute Generalization

$$P_s(B; [\alpha, \beta, \gamma]) = \alpha B (1 - \beta \gamma B)^{\frac{1}{\gamma}}$$

Logistic

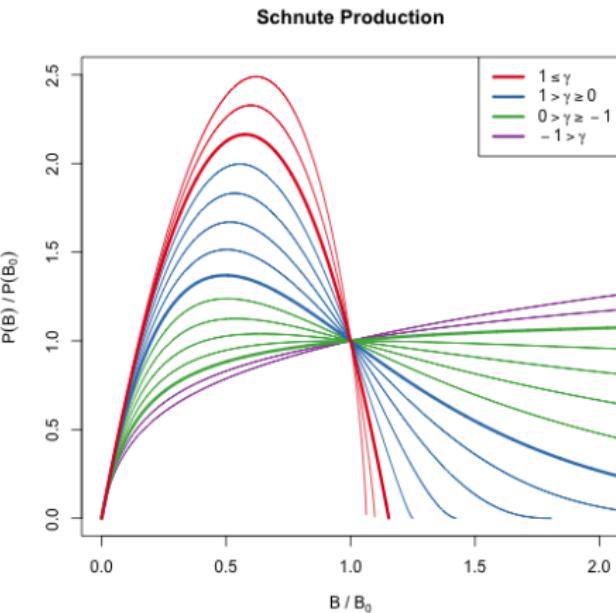
$$\gamma = 1$$

Ricker

$$\gamma \rightarrow 0$$

Beverton-Holt

$$\gamma = -1$$



# Schnute RPs Challenges

$$\begin{array}{c} \text{RPs} \mapsto \theta \\ \text{Incomplete Inversion} \\ \theta \mapsto \text{RPs} \end{array}$$

$$\alpha = (M + F^*) \left(1 + \frac{\gamma F^*}{M + F^*}\right)^{1/\gamma}$$
$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^\gamma\right)$$
$$\frac{B^*}{B_0} = \frac{1 - \left(\frac{M+F^*}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}.$$

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Schnute & Richards (1998). CJFAS.

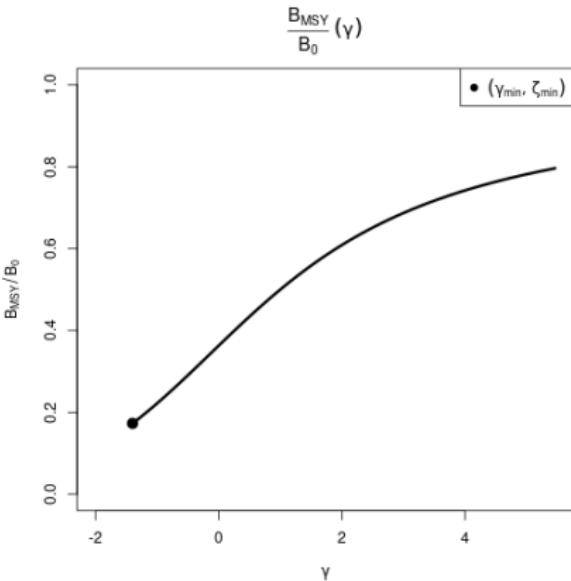
# Schnute RPs Challenges



Schnute & Richards (1998). CJFAS.

# Sudo Inverse Inverse-CDF Sampling

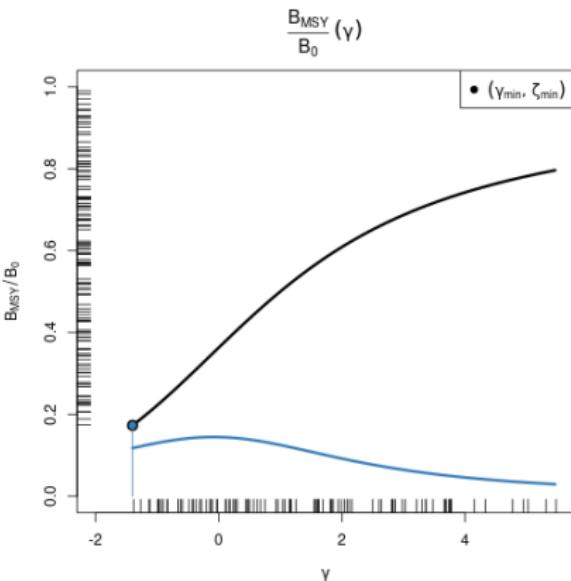
$$\frac{B^*}{B_0}(\gamma) = \frac{1 - \left( \frac{M+F^*}{\alpha(\gamma)} \right)^\gamma}{1 - \left( \frac{M}{\alpha(\gamma)} \right)^\gamma}$$



# Sudo Inverse Inverse-CDF Sampling

$$\frac{B^*}{B_0}(\gamma) = \frac{1 - \left(\frac{M+F^*}{\alpha(\gamma)}\right)^\gamma}{1 - \left(\frac{M}{\alpha(\gamma)}\right)^\gamma}$$

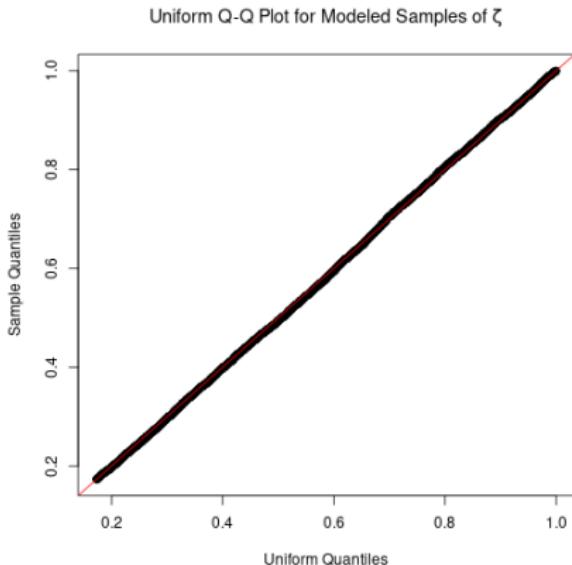
$$\gamma' \sim \zeta_{min}\delta(\gamma_{min}) + (1 - \zeta_{min})t(\mu, \sigma, \nu)\mathbf{1}_{\gamma > \gamma_{min}}$$



## Sudo Inverse Inverse-CDF Sampling

$$\frac{B^*}{B_0}(\gamma) = \frac{1 - \left(\frac{M+F^*}{\alpha(\gamma)}\right)^\gamma}{1 - \left(\frac{M}{\alpha(\gamma)}\right)^\gamma}$$

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# Schnute LHS Design

Logistic

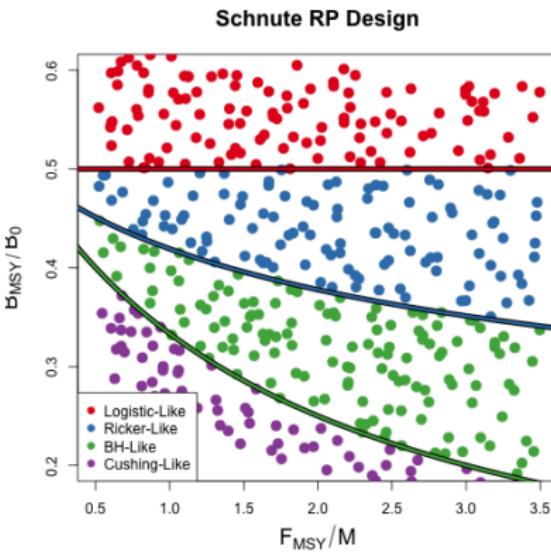
$$\gamma = 1$$

Ricker

$$\gamma \rightarrow 0$$

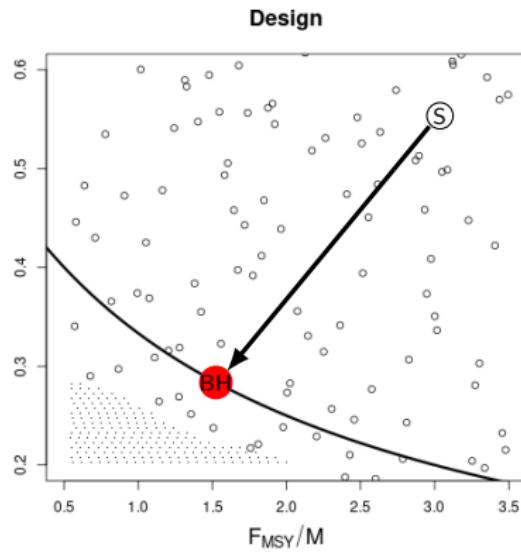
Beverton-Holt

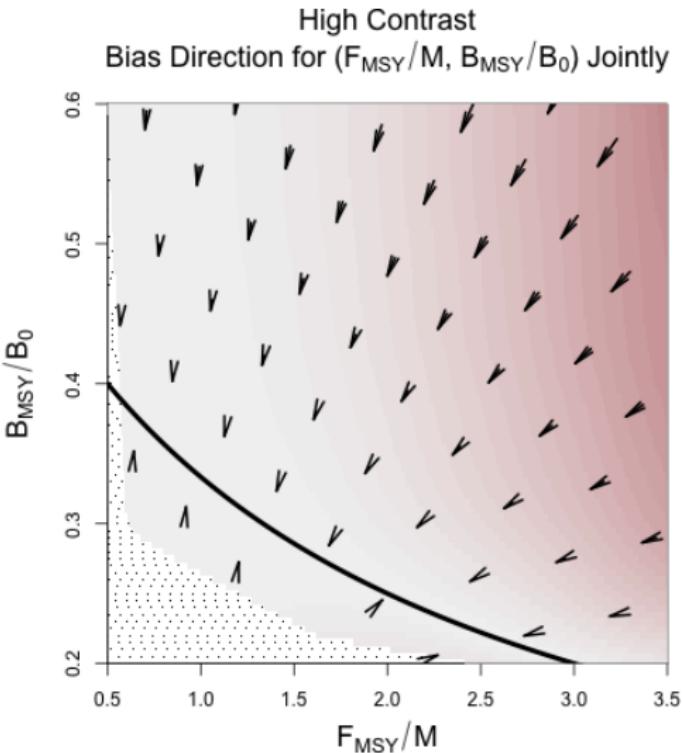
$$\gamma = -1$$

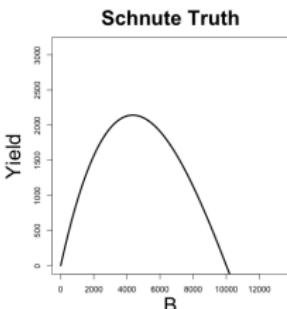
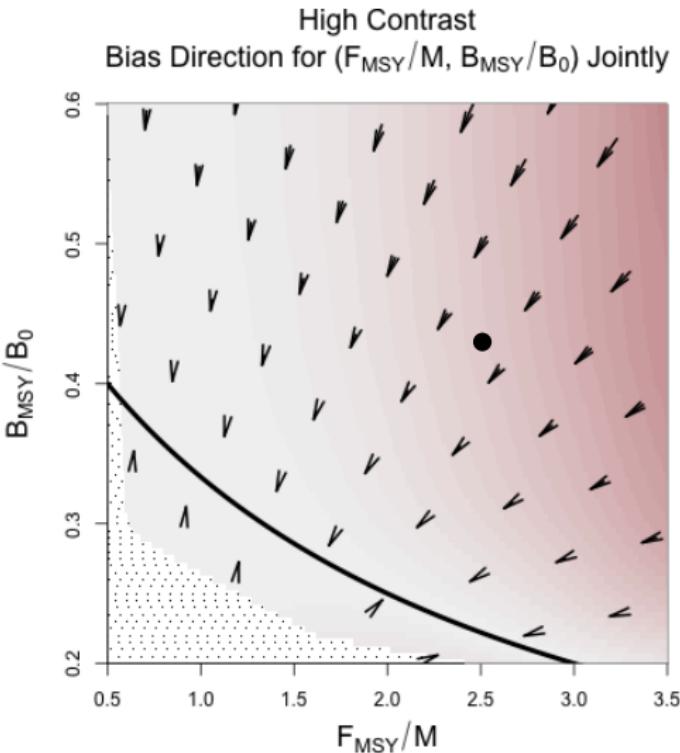


Schnute LHS Design

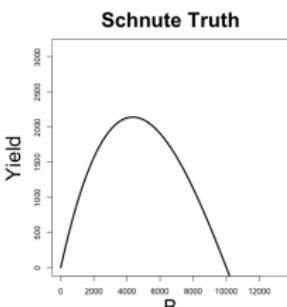
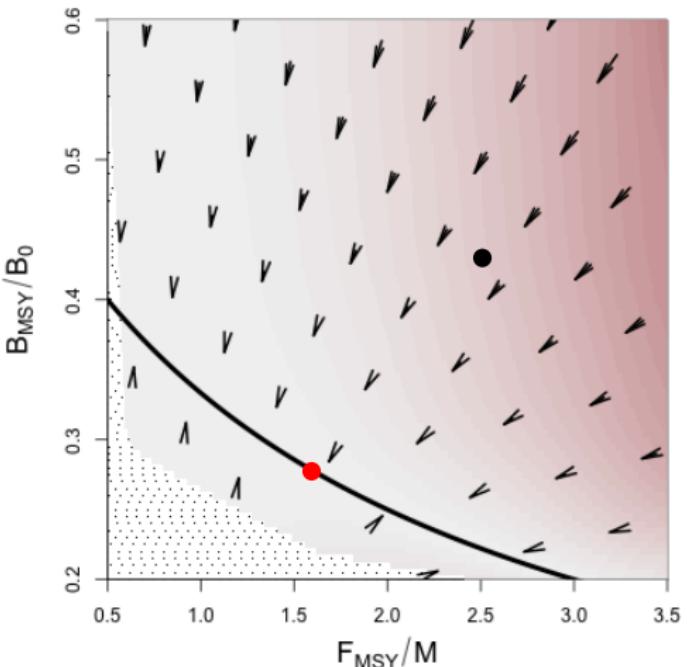
$$\underbrace{\left( \frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right)}_{\text{Schnute Truth}} \xrightarrow{\text{GP}} \underbrace{\left( \frac{\hat{F}_{MSY}}{M}, \frac{1}{\hat{F}_{MSY}/M + 2} \right)}_{\text{BH Estimate}}$$



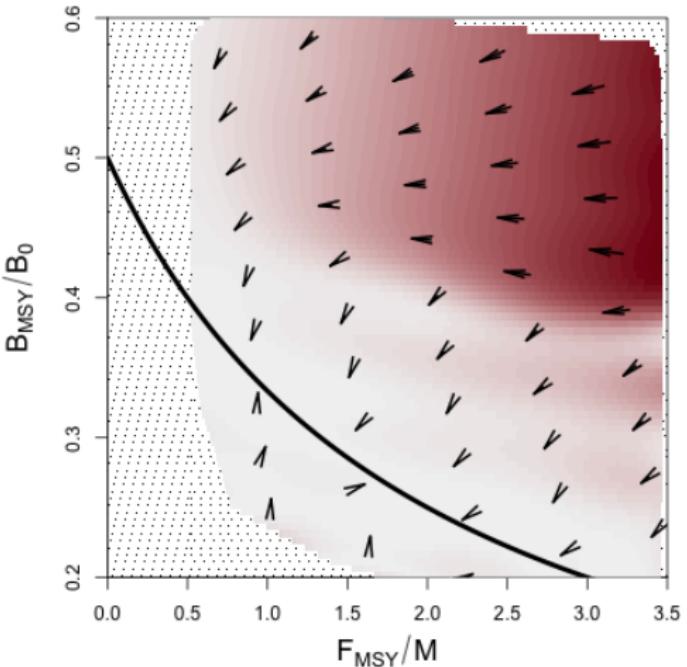




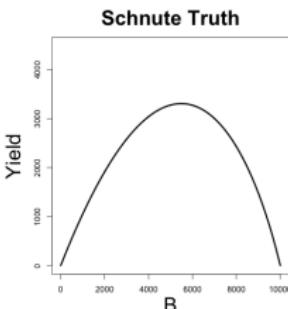
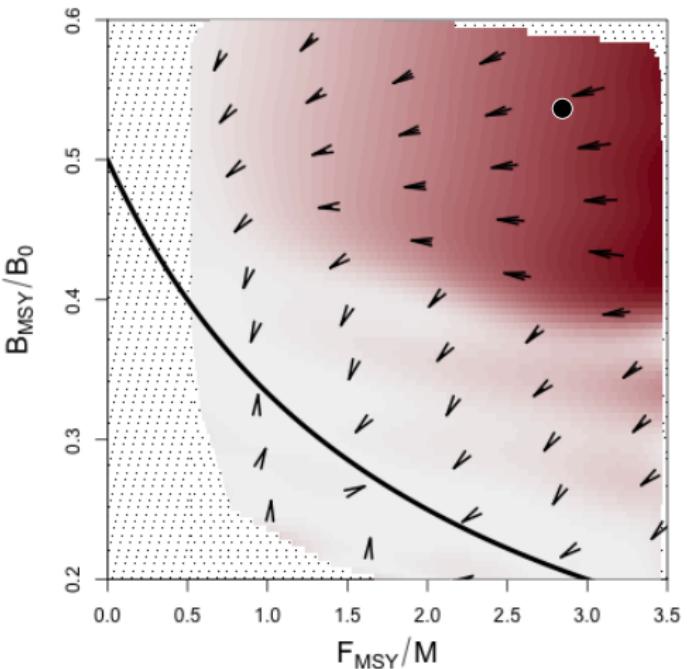
## High Contrast Bias Direction for $(F_{MSY}/M, B_{MSY}/B_0)$ Jointly

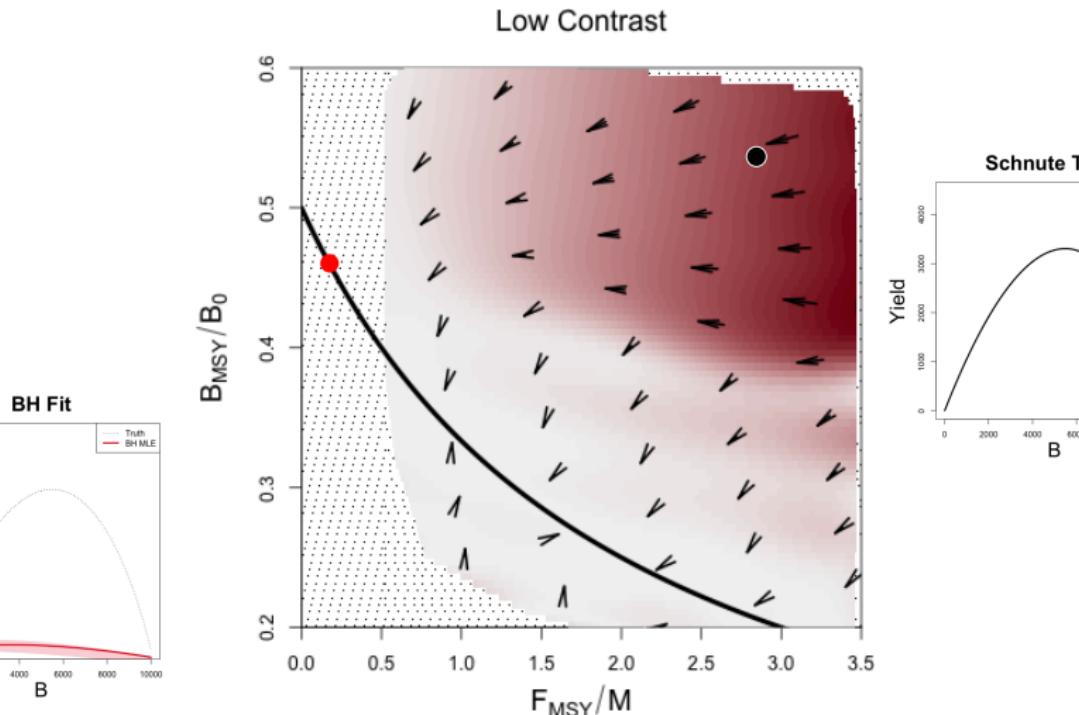


## Low Contrast

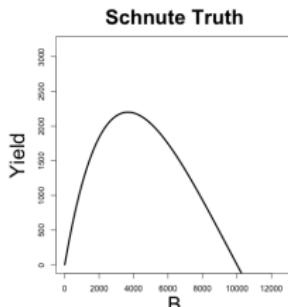
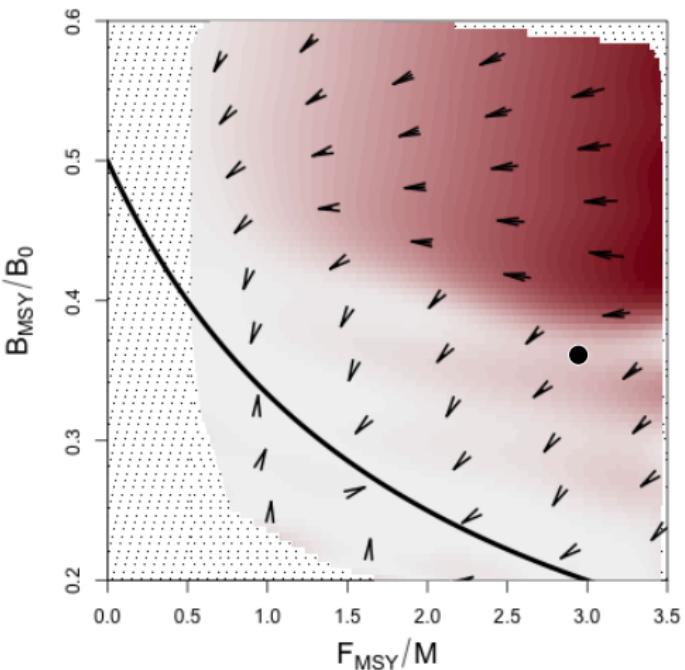


## Low Contrast

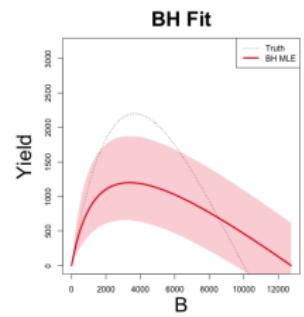
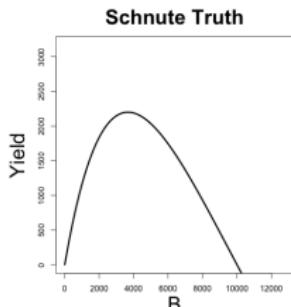
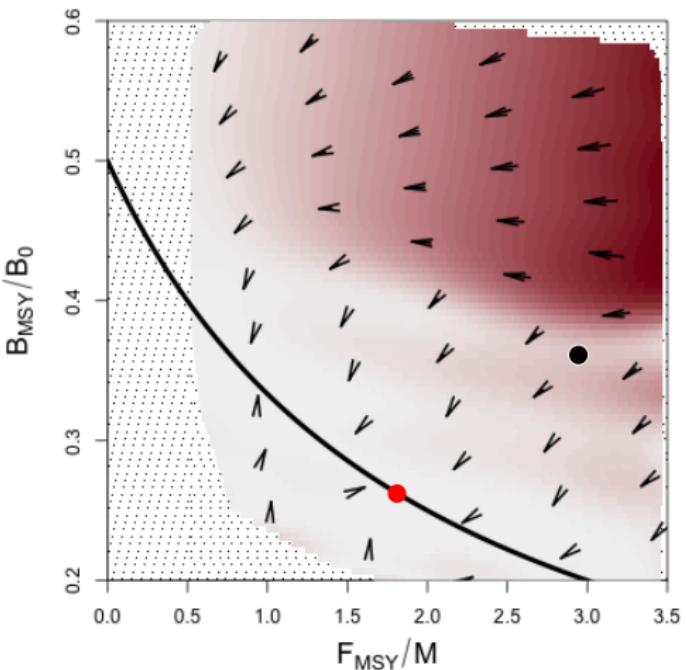




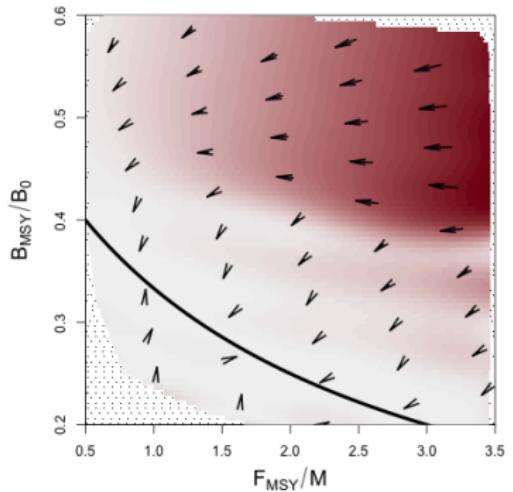
## Low Contrast



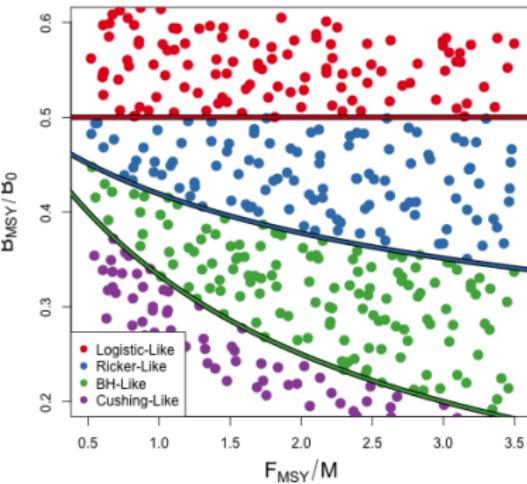
## Low Contrast



## Low Contrast



Schnute RP Design



# Summary

- Schnute three-parameter generalizing model is very attractive since it interpolates the most common models of productivity.
  - Logistic, Ricker, Beverton-Holt
- A stable method of generating simulation designs despite non-analytical and numerically treacherous  $RPs \leftrightarrow \theta$ .
- GP metamodel demonstrates that misspecified BH models enjoy some sense of optimality in RP estimation.
  - Nearly shortest distance RP mapping as mediated by contrast.
  - but misspecified BH models induce a risk structure in RPs.
- Can more complex biological dynamics help RP estimation?

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# General Modeling Structure

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB}{dt} = \underbrace{w(a_s)R(B; \theta)}_{\text{Recruitment Biomass}} + \underbrace{\kappa [w_\infty N - B]}_{\text{Net Growth}} - \underbrace{(M + F)B}_{\text{Mortality}}$$
$$\frac{dN}{dt} = R(B; \theta) - (M + F)N$$

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Walters, C. J. (2020). The continuous time Schnute-Deriso delay difference model for age-structured population dynamics, with example application to the Peru anchoveta stock. University of British Columbia.



# Individual Growth

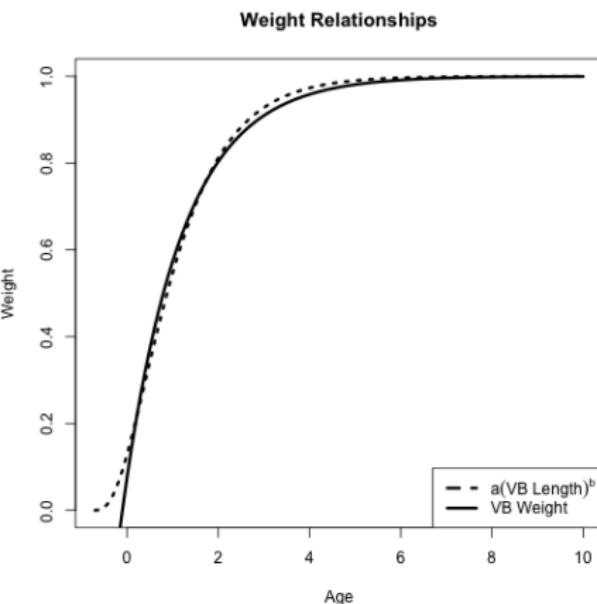
$$w(a) = w_\infty(1 - e^{-\kappa(a-a_0)})$$

$a_s$  : Lagged Maturity &  
Knife-Edge Selectivity

$\kappa$  : Individual Growth

- Instant Growth:  
(Production Model)

$$a_s \rightarrow 0 \quad \kappa \rightarrow \infty$$



# Individual Growth

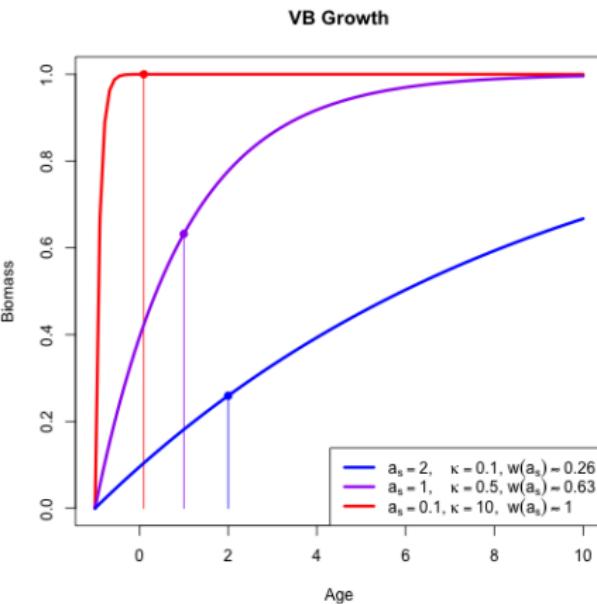
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$\kappa$  : Individual Growth

- Instant Growth:  
(Production Model)

$$a_s \rightarrow 0 \quad \kappa \rightarrow \infty$$



# Schnute Recruitment

$$R(B; \alpha, \beta, \gamma) = \alpha B_{t-a_s} (1 - \beta \gamma B_{t-a_s})^{\frac{1}{\gamma}}$$

Logistic

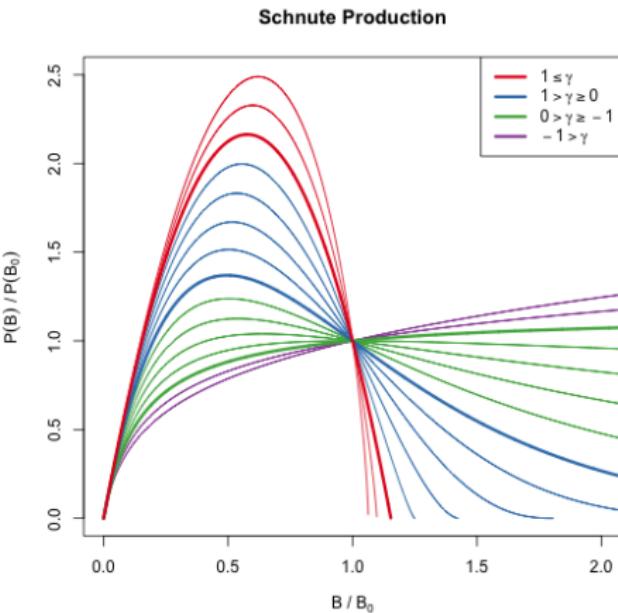
$$\gamma = 1$$

Ricker

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Beverton-Holt

$$\gamma = -1$$

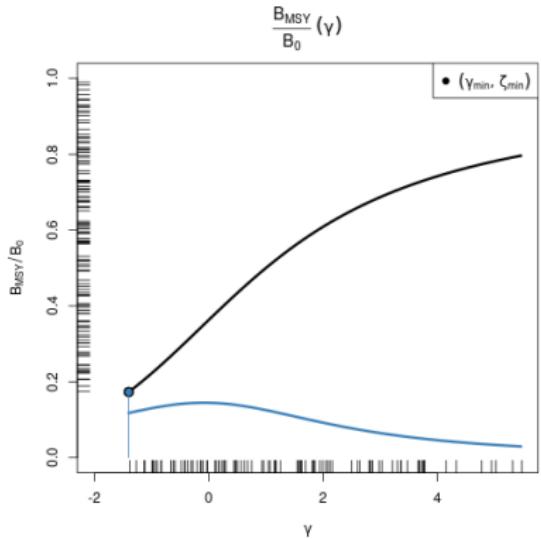


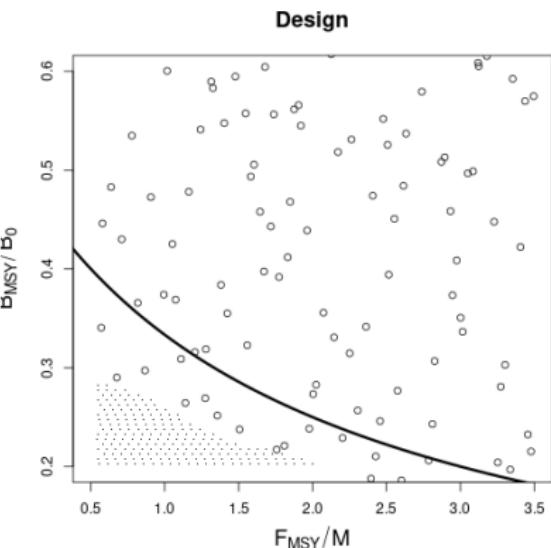
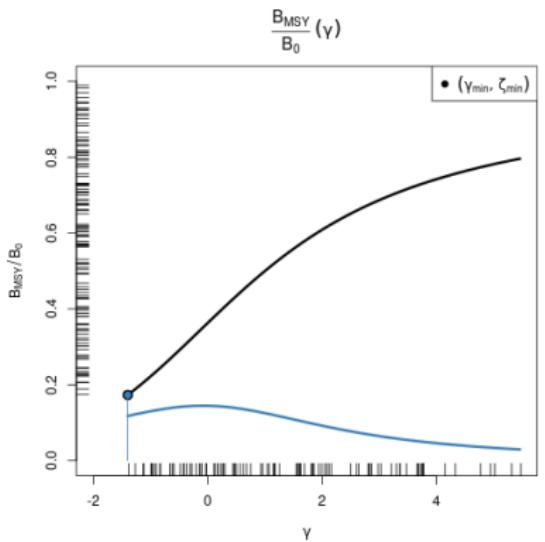
## Growth Further Complicates RPs

$$\alpha(\gamma) : \alpha = \left[ \left( \frac{Z^*(Z^* + \kappa)}{w(a_S)(Z^* + \frac{\kappa w_\infty}{w(a_S)})} \right)^\gamma + \left( \frac{\gamma F^*}{w(a_S)} \right) \left( \frac{Z^*(Z^* + \kappa)}{w(a_S)(Z^* + \frac{\kappa w_\infty}{w(a_S)})} \right)^{\gamma-1} \left( 1 + \frac{\left( \frac{\kappa w_\infty}{w(a_S)} \right) \left( \kappa - \frac{\kappa w_\infty}{w(a_S)} \right)}{(Z^* + \frac{\kappa w_\infty}{w(a_S)})^2} \right) \right]^{\frac{1}{\gamma}}$$

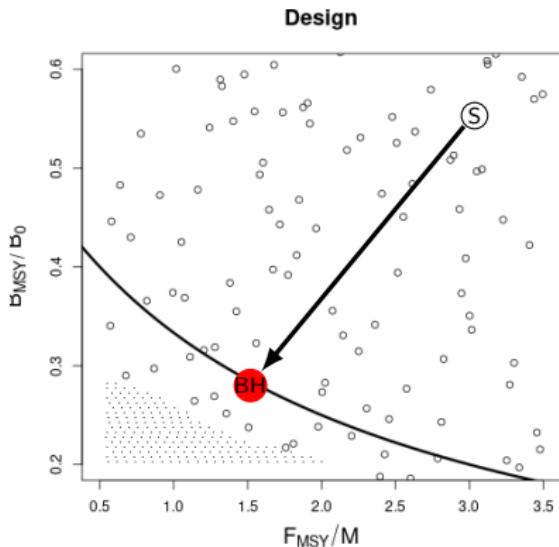
$$\beta(\alpha(\gamma), \gamma) : \quad \quad \quad \beta = \frac{1}{\gamma B_0} \left( 1 - \left( \frac{M(M + \kappa)}{\alpha w(a_s)(M + \frac{\kappa w_\infty}{w(a_s)})} \right)^\gamma \right)$$

$$\frac{B^*}{B_0}(\alpha(\gamma), \gamma) := \frac{\frac{1 - \left( \frac{(F^* + M)(F^* + M + \kappa)}{\alpha w(a_S)(F^* + M + \frac{\kappa w_\infty}{w(a_S)})} \right)^\gamma}{1 - \left( \frac{M(M + \kappa)}{\alpha w(a_S)(M + \frac{\kappa w_\infty}{w(a_S)})} \right)^\gamma}}{B^*}$$



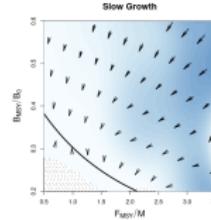
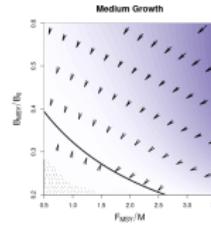
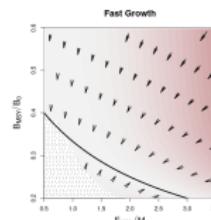
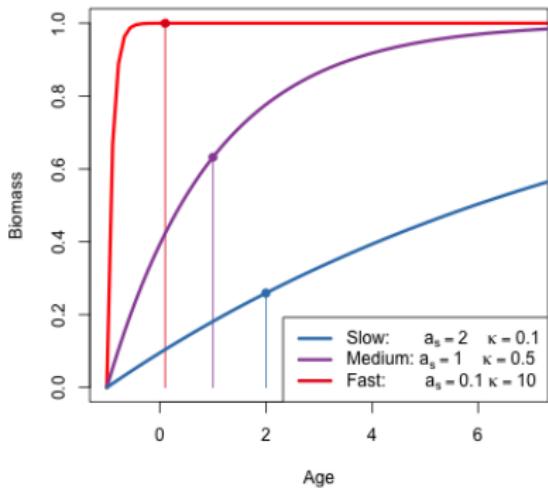


$$\underbrace{\left( \frac{F^*}{M}, \frac{B^*}{\bar{B}(0)} \right)}_{\text{Schnute Truth}} \xrightarrow{\text{GP}} \underbrace{\left( \frac{\hat{F}^*}{M}, \frac{B^*}{B_0}(-1; \hat{F}^*) \right)}_{\text{BH Estimate}}$$



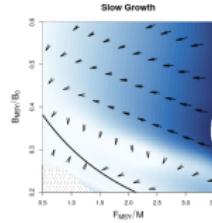
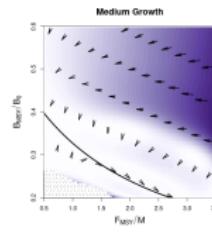
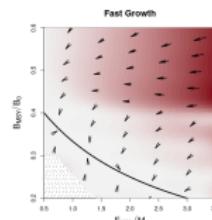
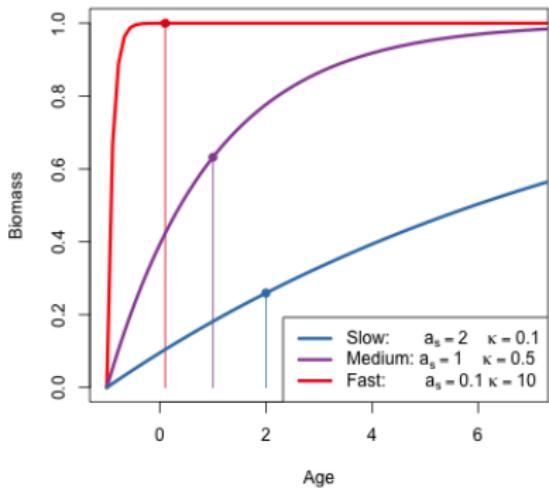
# High Contrast

$$w(a) = w_\infty(1 - e^{-\kappa(a+1)})$$

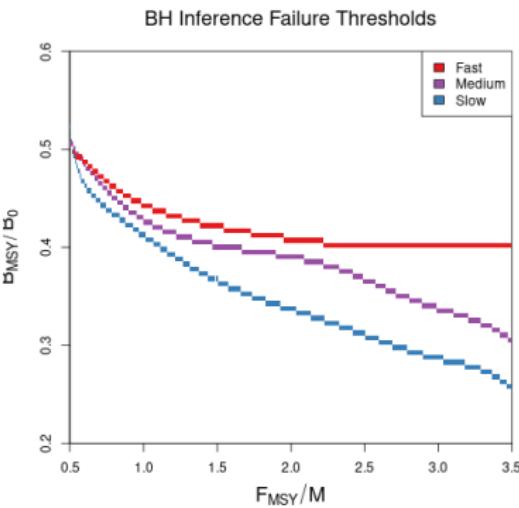
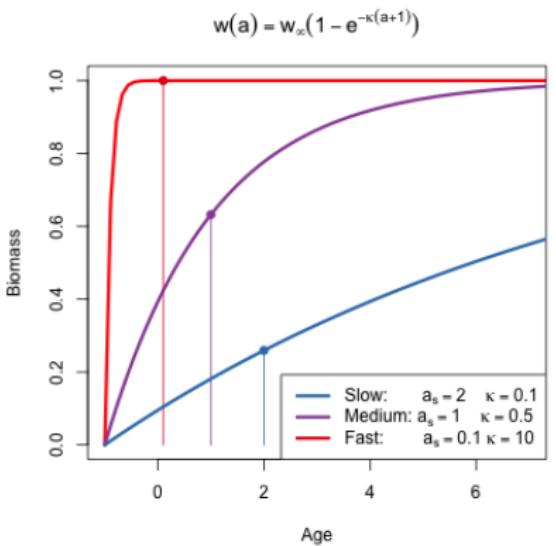


# Low Contrast

$$w(a) = w_\infty(1 - e^{-\kappa(a+1)})$$

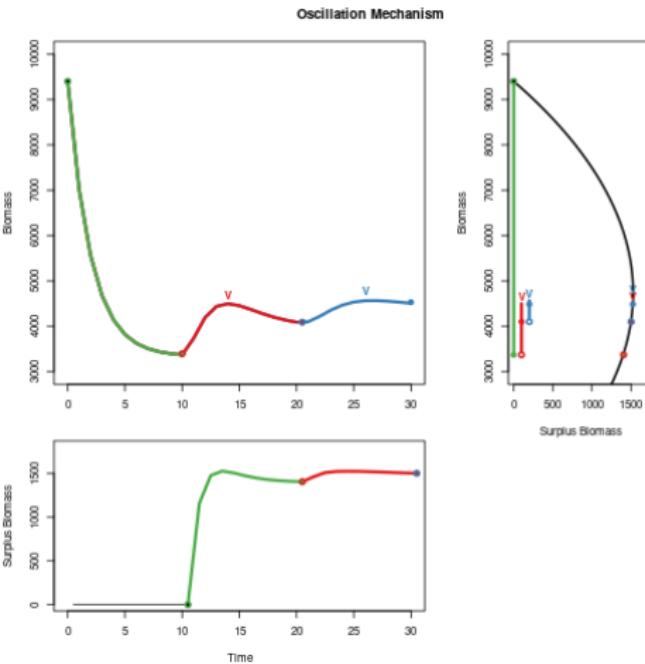


# Low Contrast



# Oscillation

- Large  $a_s$  window.
- Fishing shocks within  $a_s$  window.
- Repeated shocks can lead to chaos.



# Conclusions

- Similar RP mapping under BH-DDM as the BH-SPM.
- As growth dynamics are emphasized, the BH model becomes more brittle.
- Emphasizes the need to use more flexible SRR models.

## Future Work:

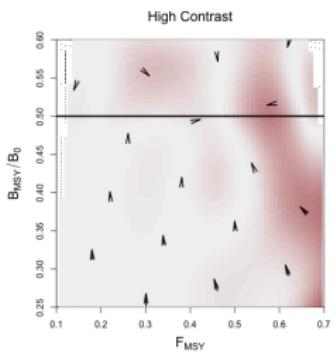
- Recruitment Deviations
- Further metamodeling techniques can expedite the simulation into more challenging simulation applications.
  - Targeted acquisition functions
  - Non-stationarity metamodels

Many Thanks:

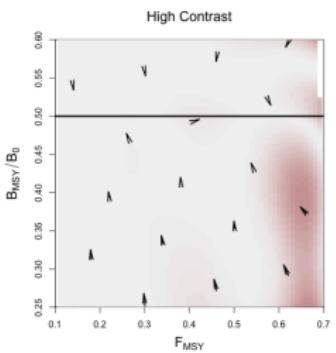
- UCSC Advisors
- Collaborators at NOAA
- NMFS Sea Grant



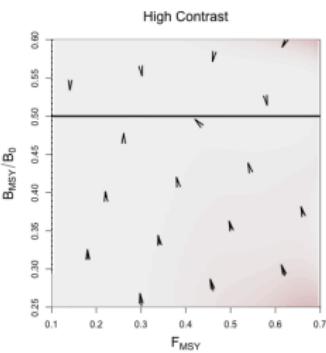
## High Contrast PT $\sigma = 0.12$ Data



1x Samples



2x Samples



## 4x Samples

## Metamodel Details

$$\mathbf{y} = \widehat{\log(F_{MSY})} \quad -\text{or}- \quad \mathbf{y} = \widehat{\log(B_0)}$$

$$X = \left( \frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\mathbf{y} = \beta_0 + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\nu} + \epsilon$$

$$\boldsymbol{v} \sim N_n(\mathbf{0}, \tau^2 \boldsymbol{R}_\ell)$$

$$\epsilon \sim N_n(\mathbf{0}, \omega' \mathbf{I})$$

$$R(\mathbf{x}, \mathbf{x}') = \exp \left( \sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2} \right)$$

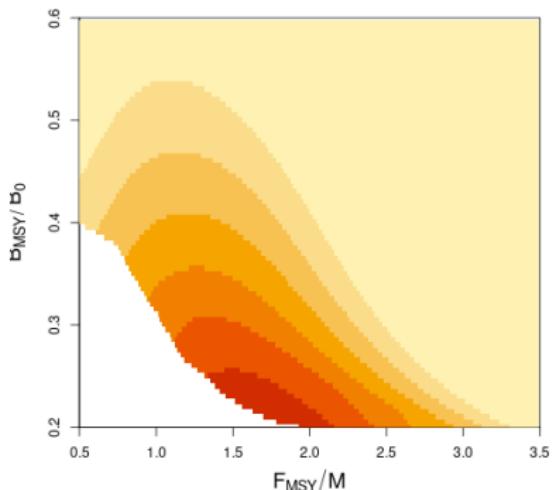
# Prediction

$$\hat{y}(\mathbf{x}^*) = \beta_0 + \mathbf{x}^* \boldsymbol{\beta} + \mathbf{r}(\mathbf{x}^*)' \mathbf{R}_\ell^{-1} \left( \mathbf{y} - (\beta_0 + \mathbf{X} \boldsymbol{\beta}) \right)$$

$$\hat{\sigma}^2(\mathbf{x}^*) = \mathbf{R}(\mathbf{x}^*, \mathbf{x}^*) - \mathbf{r}(\mathbf{x}^*)' R_\ell^{-1} \mathbf{r}(\mathbf{x}^*)$$

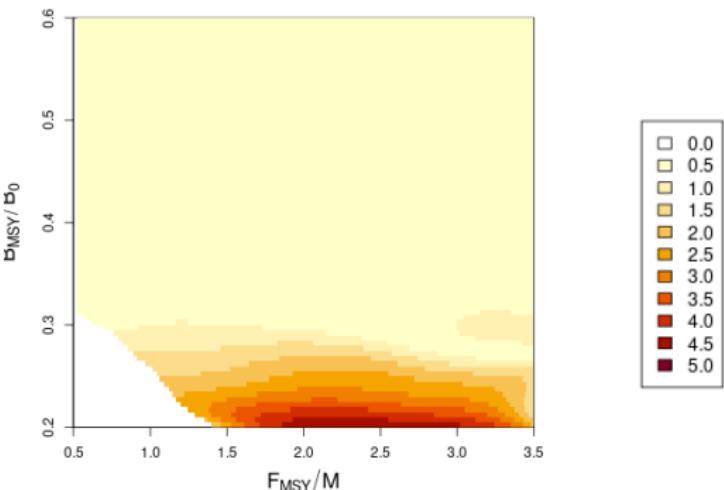
## Contrast

### Size of 1 SD Interval in RP Bias



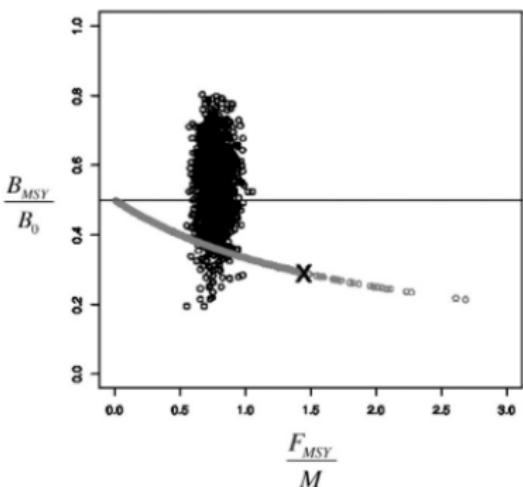
## No Contrast

### Size of 1 SD Interval in RP Bias

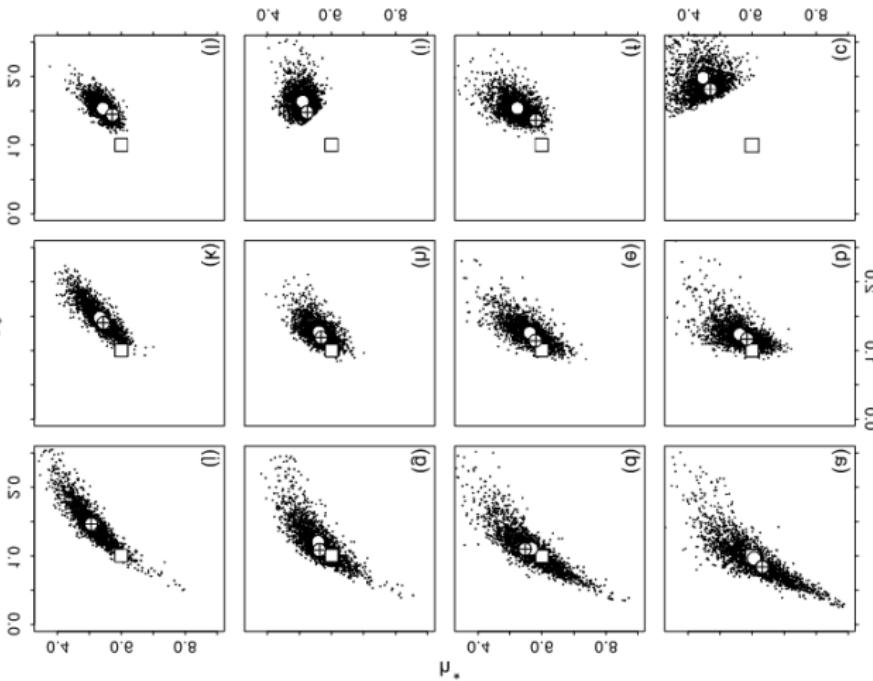


Mangel et al.

**Fig. 4.** DeYoreo et al. (2012) used both a BH-SRR and three-parameter SRR, similar to the S-SRR in a stock assessment of cowcod (*Sebastodes levis*). We show samples from posterior distributions arising from different values of steepness. Unlike most stock assessments, we plot  $B_{MSY}/B_0$  versus  $F_{MSY}/M$ . The grey circles show the results for the BH-SRR. This curve is another way of representing the constraint placed on a stock assessment by using a BH-SRR and specifying steepness — results must lie along this curve. The black circles represent the outcome of the three-parameter SRR. The black X represents the result when steepness is asserted to be 0.6.

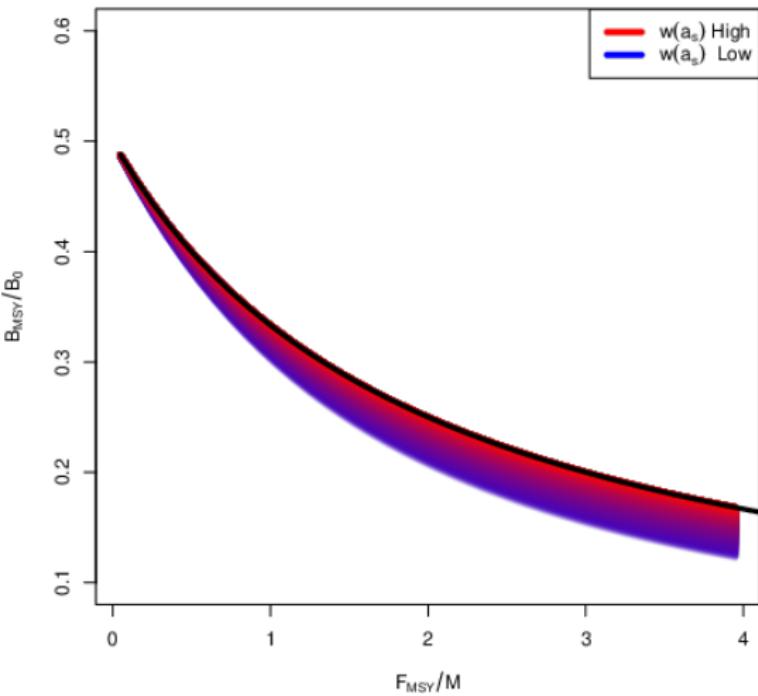


## Logistic



Schnute, J. T., & Kronlund, A. R. (2002). Estimating salmon stock recruitment relationships from catch and escape-  
ment data. Canadian Journal of Fisheries and Aquatic Sciences, 59(3), 433–449.

## Space of BH Reference Points



## Space of Reference Points

