

Bias Estimation of Biological Reference Points Under Two-Parameter SRRs

Nick Grunloh



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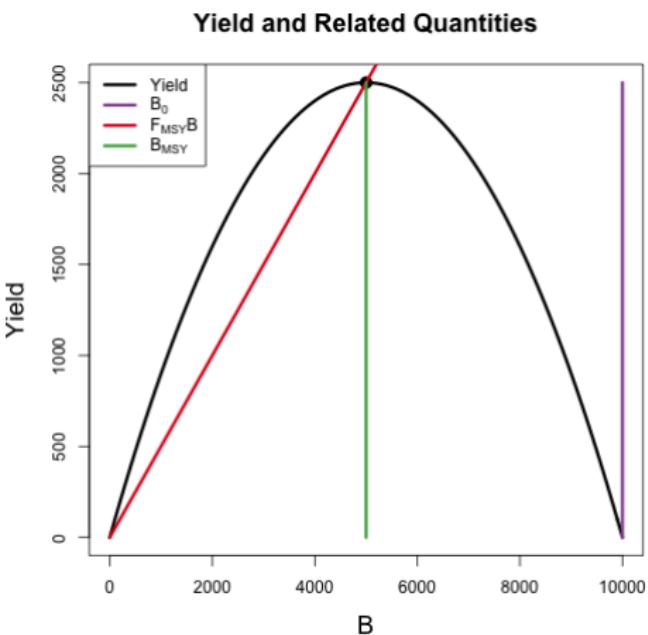


General Modeling Structures

$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

$$\frac{dB(t)}{dt} = P(B(t); \theta) - Z(t)B(t)$$

Reference Points : MSY, $\frac{F_{MSY}}{M}$, $\frac{B_{MSY}}{B_0}$



Conceptually:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B_0} \in (0, 1)$$

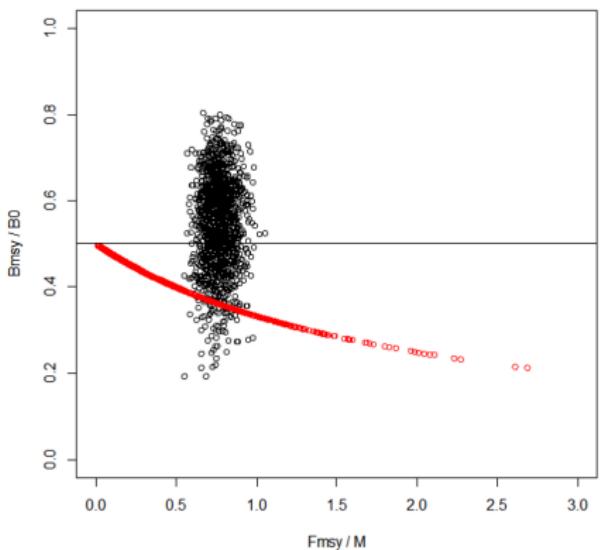
Two Parameter:

■ Shaefer Model:

$$F_{MSY} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{2}$$

■ BH Model:

$$\frac{F_{MSY}}{M} \in \mathbb{R}^+ \quad \frac{B_{MSY}}{B(0)} = \frac{1}{F_{MSY}/M + 2}$$



$$I_t = qB_t e^\epsilon \quad \epsilon \sim N(0, \sigma^2)$$

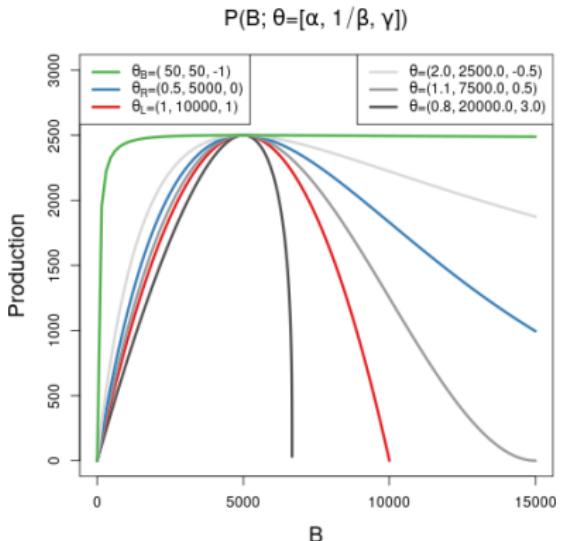
$$\frac{dB}{dt} = P(B; \theta) - (M + F)B$$

$$P(B; [\alpha, \beta, \gamma]) = \alpha B (1 - \beta \gamma B)^{\frac{1}{\gamma}}$$

$\gamma = -1 \Rightarrow$ **Beverton-Holt**

$\gamma \rightarrow 0 \Rightarrow$ **Ricker**

$\gamma = 1 \Rightarrow$ **Logistic**

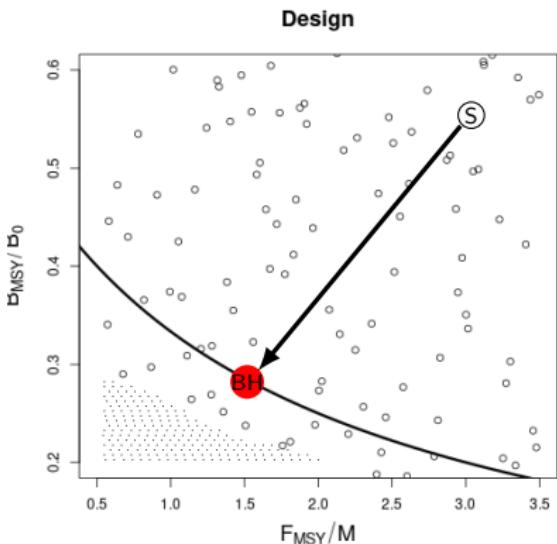


Simulation Design

- LHS design based on analytical results similar to Schnute and Richards 1998, CJFAS
- GP Metamodeling of RP bias

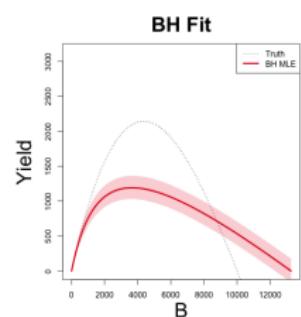
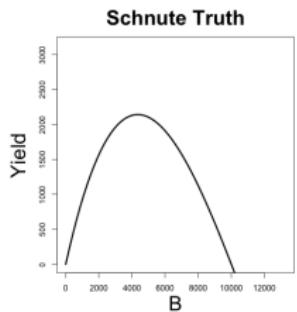
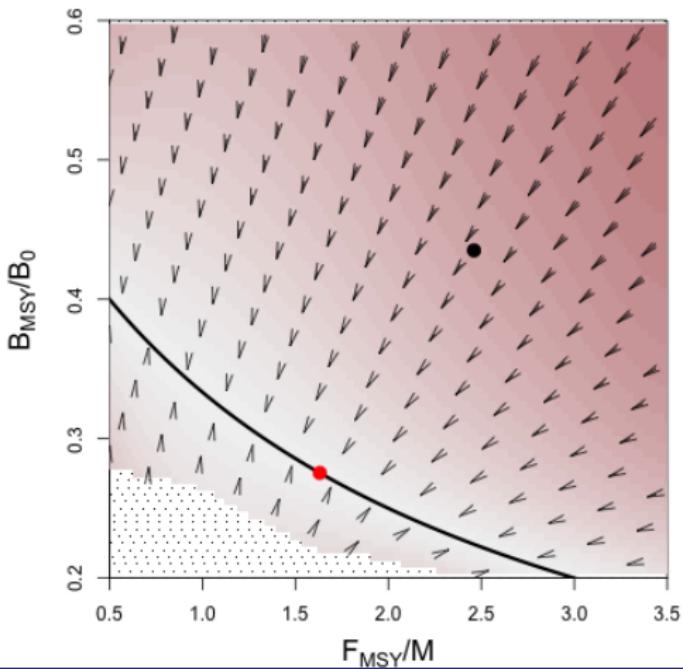
$$\left(\frac{F_{MSY}}{M}, \frac{B_{MSY}}{\bar{B}(0)} \right) \xrightarrow{\text{GP}} \left(\frac{\hat{F}_{MSY}}{M}, \frac{\hat{B}_{MSY}}{\bar{B}(0)} \right)$$

Schnute TruthBH Estimate



High Contrast

Bias Direction for $(F_{MSY}/M, B_{MSY}/B_0)$ Jointly

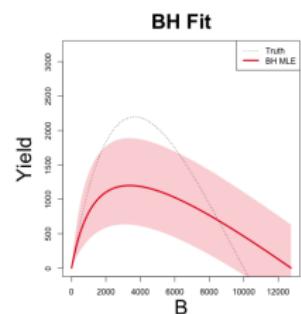
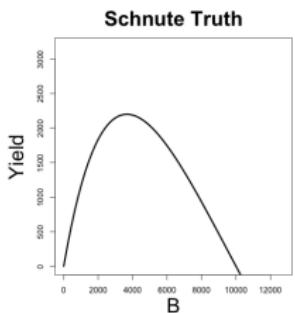
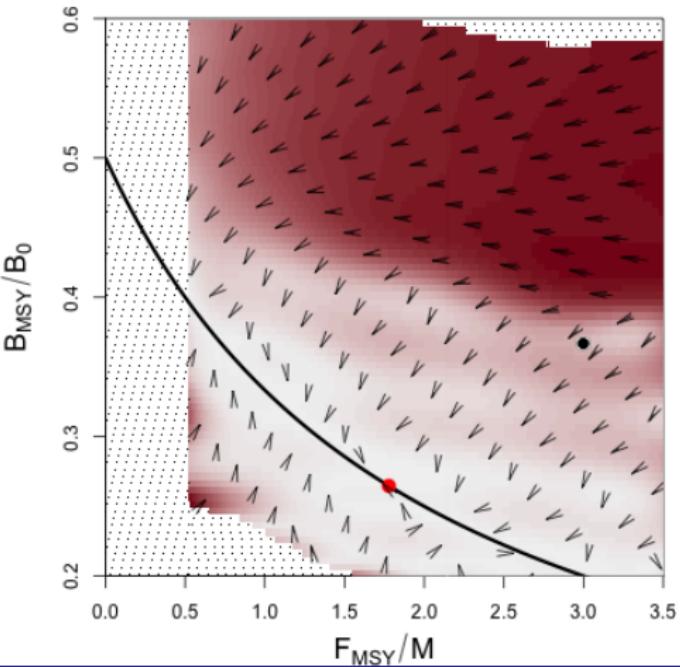


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Low Contrast

Bias Direction for $(F_{MSY}/M, B_{MSY}/B_0)$ Jointly

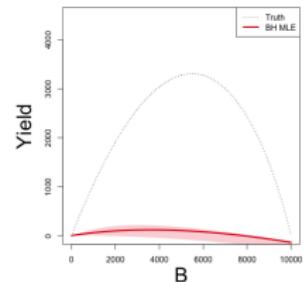


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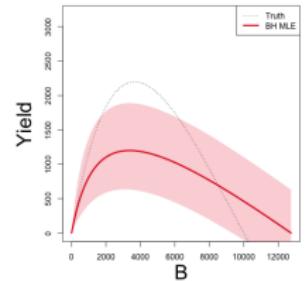
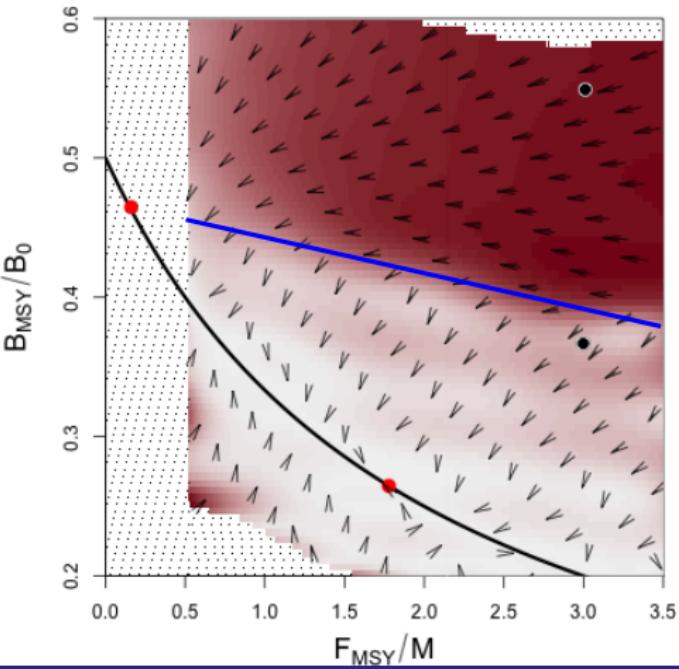
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Low Contrast

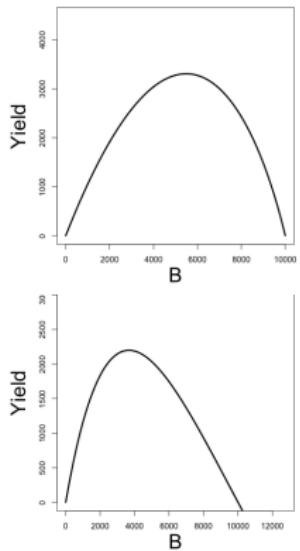
BH Fit



BH Fit

Bias Direction for $(F_{MSY}/M, B_{MSY}/B_0)$ Jointly

Schnute Truth



Individual Growth Extension

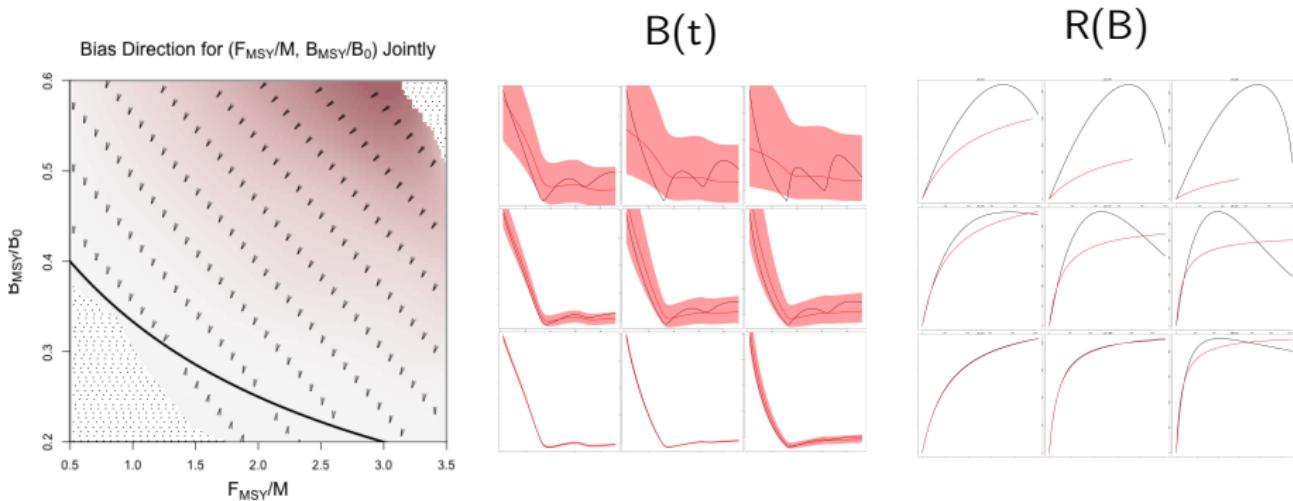
$$\begin{aligned}\frac{dB}{dt} &= \overbrace{w(a_0)R(B; \theta)}^{\text{Recruitment Biomass}} + \overbrace{\kappa [w_\infty N - B]}^{\text{Net Growth}} - \overbrace{(M + F)B}^{\text{Mortality}} \\ \frac{dN}{dt} &= R(B; \theta) - (M + F)N\end{aligned}$$

$$R(B; [\alpha, \beta, \gamma]) = \alpha B(t - a_0)(1 - \beta \gamma B(t - a_0))^{\frac{1}{\gamma}}$$

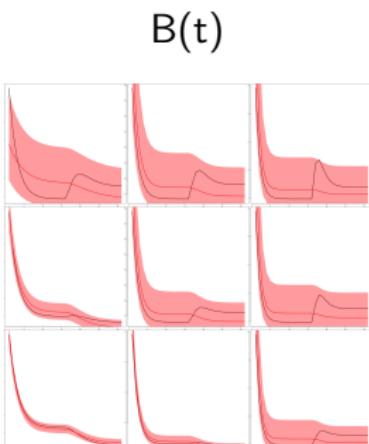
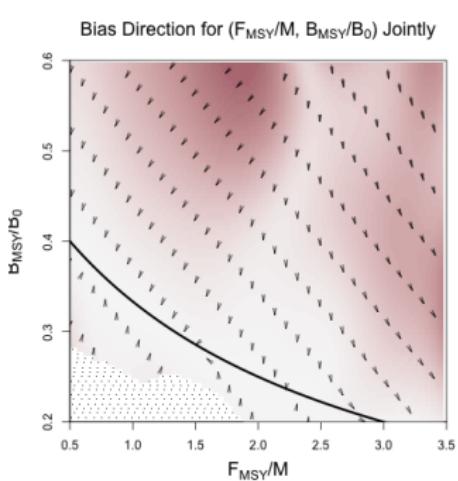
$$w(a) = w_\infty(1 - e^{-\kappa a})$$

$$\theta' = [\alpha, \beta, \gamma] \quad \text{Species Properties: } a_0, \kappa, w_\infty, M$$

High Contrast



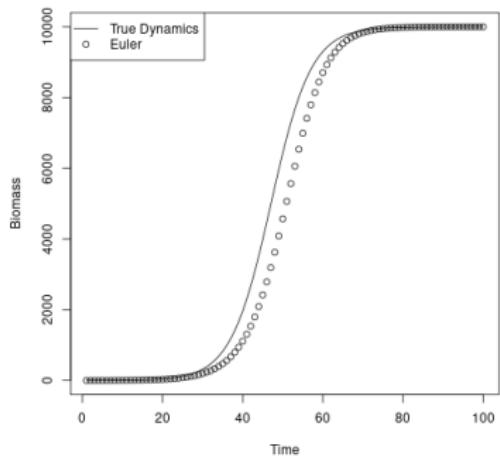
Low Contrast



Common Discretization

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$



Common Discretization

Instantaneous

$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

Yearly

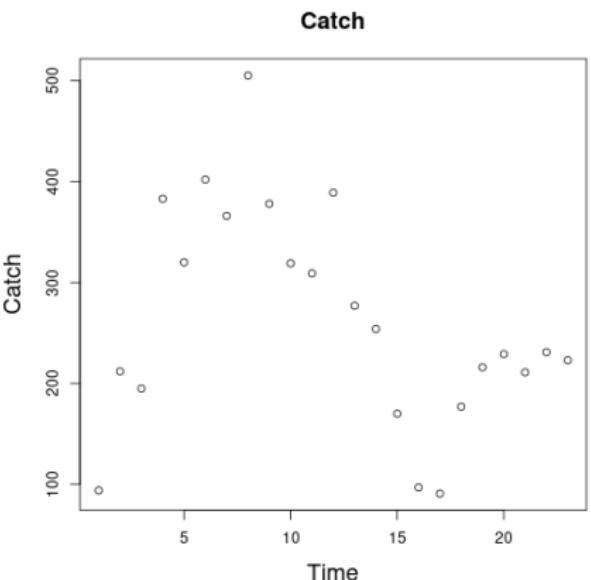
$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$
$$c(\tau) = \int_{\tau}^{\tau+1} C(t) dt$$

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graph TD; A["differential eqn"] --> B["discrete eqn"]; A --> C["integral eqn"]; C --> B;
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$$t \in \mathbb{R}^+ \quad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[c(t)] = \int_{\tau}^t C(t^*) dt^*$$

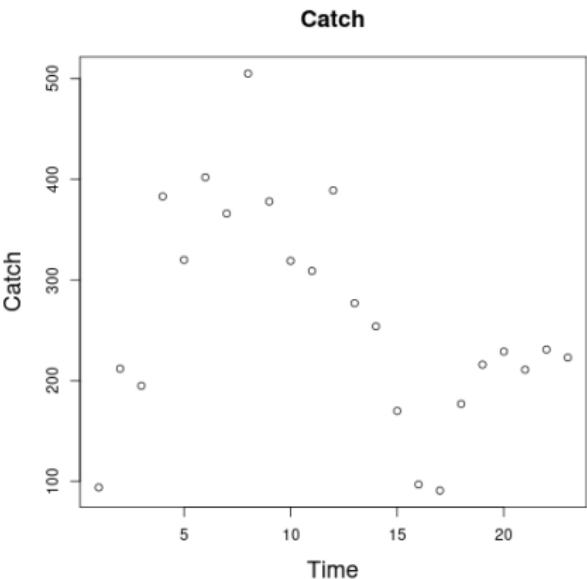
$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



$$t \in \mathbb{R}^+ \quad \tau = \lceil t \rceil - 1$$

$$\mathbb{E}[c(t)] = \int_{\tau}^t C(t^*) dt^*$$

$$C(t) = \beta_0 + \sum_{j=1}^{T-1} \beta_j (t - \tau_j) \mathbb{1}_{t > \tau_j}$$



$$c(\tau_i) = \beta_0 + \sum_{j=1}^{i-1} \beta_j \left[\left(\frac{\tau_i^2}{2} - \tau_j \tau_i \right) \mathbb{1}_{\tau_i > \tau_j} - \left(\frac{\tau_{i-1}^2}{2} - \tau_j \tau_{i-1} \right) \mathbb{1}_{\tau_{i-1} > \tau_j} \right] + \epsilon_i$$

$$\beta_j \sim N(0, \phi)$$

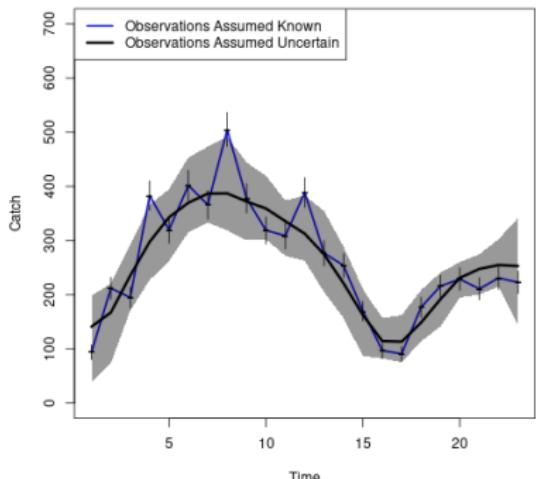
$$\phi \sim \text{Half-Cauchy}(0, 1)$$

$$\epsilon_i \sim N(0, \sigma_i^2)$$

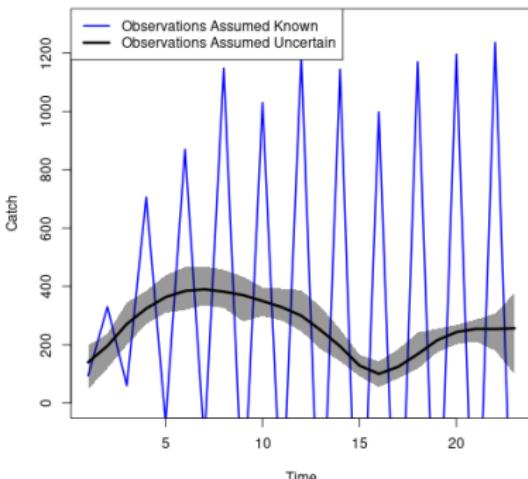
$$\frac{dB}{dt} = P_\theta(B(t)) - C(t)$$

$$B(\tau + 1) \approx B(\tau) + P_\theta(B(\tau)) - c(\tau)$$

Observed Catch with Predictive Interpolations



Interpolated Instantaneous Catch



Many Thanks:

- UCSC Advisors
- SWFSC Groundfish
- NMFS Sea Grant



Metamodel Details

$$\mathbf{x} = \left(F_{MSY}, \frac{B_{MSY}}{\bar{B}(0)} \right)$$

$$\begin{aligned}\hat{\mu} &= \beta_0 + \boldsymbol{\beta}' \mathbf{x} + f(\mathbf{x}) + \epsilon \\ f(\mathbf{x}) &\sim \text{GP}(0, \tau^2 R(\mathbf{x}, \mathbf{x}')) \\ \epsilon_i &\sim \mathcal{N}(0, \hat{\omega}_i).\end{aligned}$$

$$R(\mathbf{x}, \mathbf{x}') = \exp \left(\sum_{j=1}^2 \frac{-(x_j - x'_j)^2}{2\ell_j^2} \right)$$

Schnute RP-Parameter System of Equations

$$\frac{B_{MSY}}{B_0} = \frac{1 - \left(\frac{M+F_{MSY}}{\alpha}\right)^\gamma}{1 - \left(\frac{M}{\alpha}\right)^\gamma}$$

$$\alpha = (M + F_{MSY}) \left(1 + \frac{\gamma F_{MSY}}{M + F_{MSY}}\right)^{1/\gamma}$$

$$\beta = \frac{1}{\gamma B_0} \left(1 - \left(\frac{M}{\alpha}\right)^\gamma\right)$$

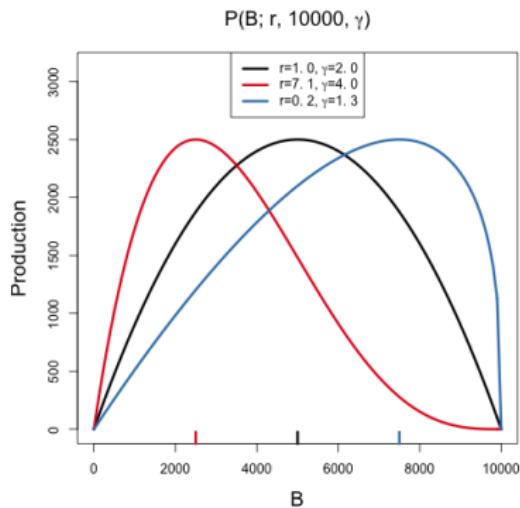
Pella-Tomlinson Production Model

$$I(t) \sim LN(qB(t), \sigma^2)$$

$$\frac{dB(t)}{dt} = P_\theta(B(t)) - F(t)B(t)$$

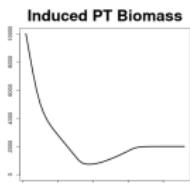
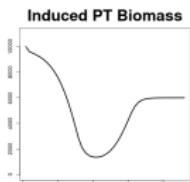
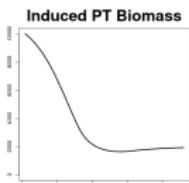
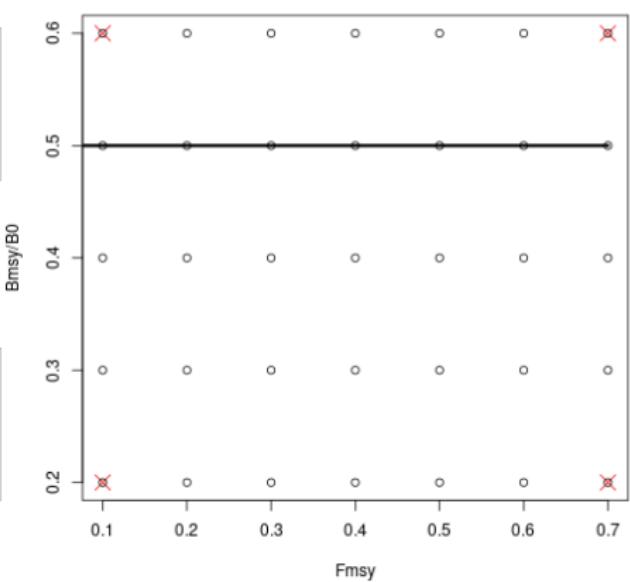
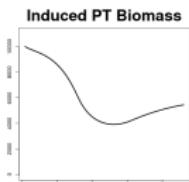
$$P(B; [r, K, \gamma]) = \frac{rB}{\gamma - 1} \left(1 - \frac{B}{K}\right)^{\gamma - 1}$$

$\gamma = 2 \Rightarrow$ Schaefer Model

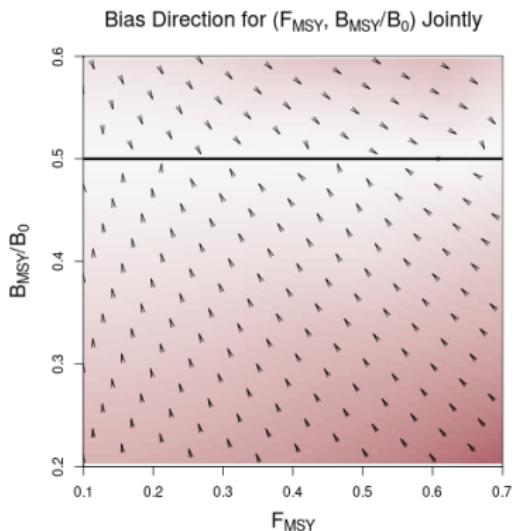


$$\theta = \left[r = F^* \left(\frac{1 - \frac{B^*}{\bar{B}(0)}}{\frac{B^*}{\bar{B}(0)}} \right) \left(1 - \frac{B^*}{\bar{B}(0)} \right)^{\left(\frac{\frac{B^*}{\bar{B}(0)} - 1}{\frac{B^*}{\bar{B}(0)}} \right)}, K = 10000, \gamma = \frac{1}{\frac{B^*}{\bar{B}(0)}} \right]$$

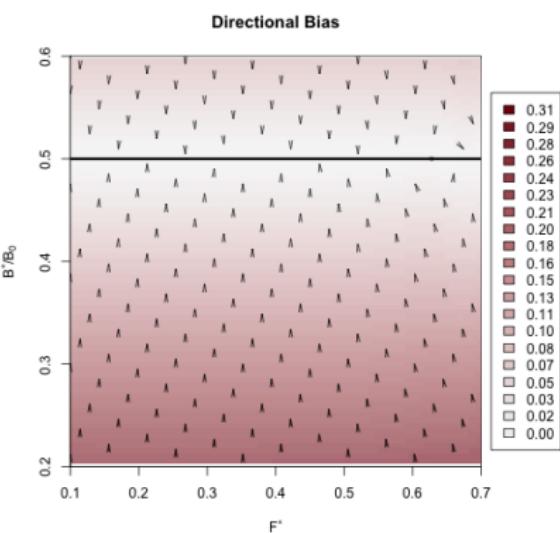
Reference Point Space



Low Contrast

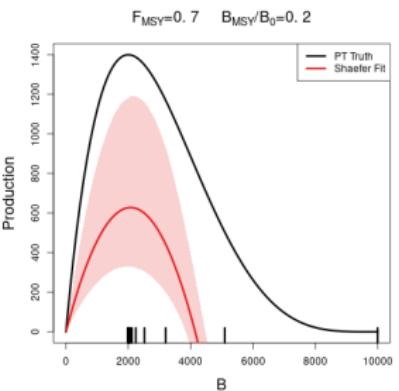
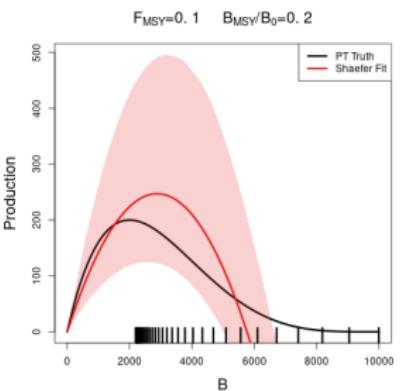
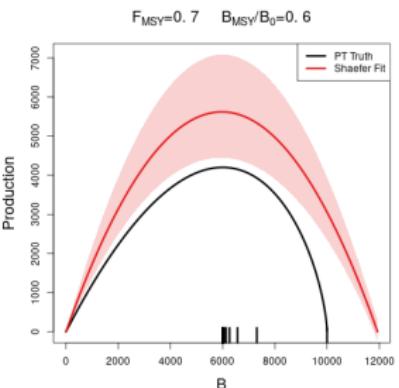
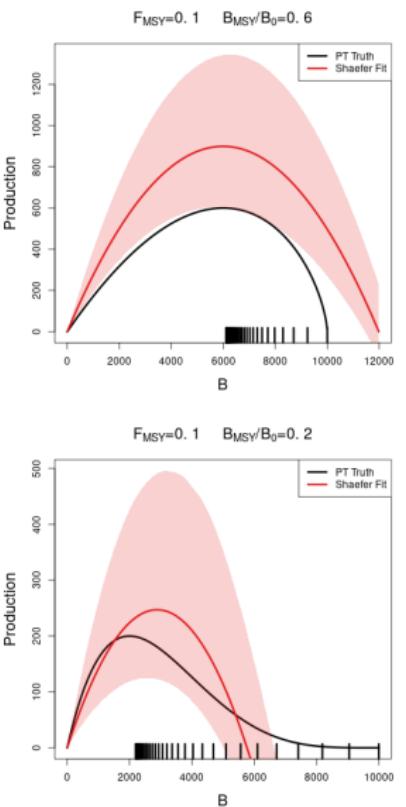
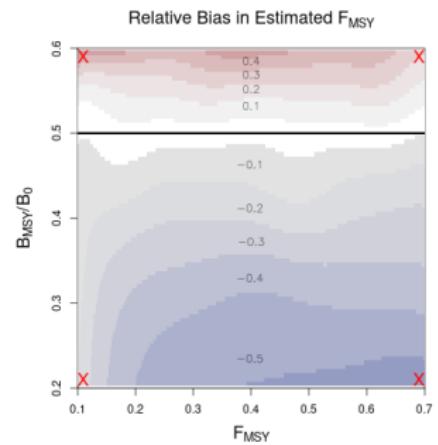


High Contrast

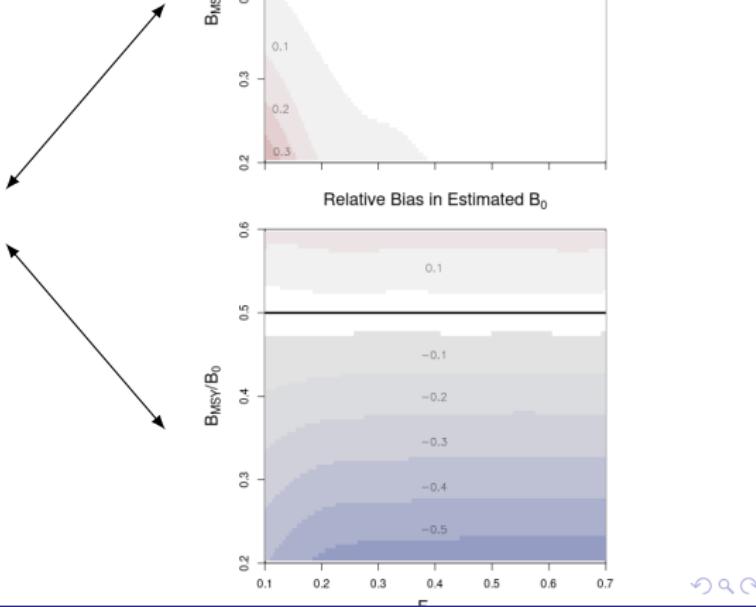
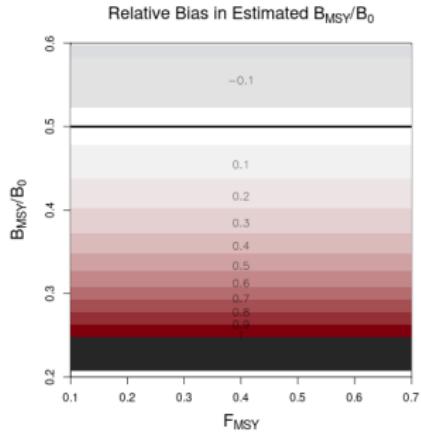


Low Contrast

F_{MSY}



Low Contrast Biomass



High Contrast

