Contents

I **Part One**

1 **Newtonian particle mechanics** 17
1.1 Inertial frames and the Galilean transformation 17
1.2 Newton's laws of motion 19
1.3 One-dimensional motion: Drag forces 22
1.4 Oscillation in one-dimensional motion 26
1.5 Resonance 30
1.6 Motion in two or three dimensions 35
1.7 Systems of particles 38
1.8 Conservation laws 40
1.9 Collisions 56
1.10 Forces of Nature 58
1.11 Summary 60

2 **Relativity** 73
2.1 Foundations 73
2.2 Relativistic kinematics 80
2.3 Relativistic dynamics 87
2.4 Summary 113

3 **The Variational Principle** 125
3.1 Fermat's principle 125

3.2 The calculus of variations . . . 126
3.3 Geodesics . . . 133
3.4 Brachistochrone . . . 136
3.5 Several Dependent Variables . . . 141
3.6 Mechanics from a variational principle . . . 142
3.7 Motion in a uniform gravitational field . . . 143
3.8 Arbitrary potential energies . . . 149
3.9 Summary . . . 150

4 Lagrangian mechanics . . . 161
4.1 Nonconservative forces . . . 161
4.2 Forces of constraint and generalized coordinates . . . 162
4.3 Hamilton's principle . . . 163
4.4 Generalized momenta and cyclic coordinates . . . 169
4.5 When is $H \neq E$? . . . 176
4.6 Systems of particles . . . 177
4.7 The Hamiltonian . . . 185
4.8 When is $H \neq E$? . . . 189
4.9 The moral of constraints . . . 190
4.10 Small oscillations about equilibrium . . . 192
4.11 Recap . . . 194
4.12 Summary . . . 195

5 From classical to quantum and back . . . 205
5.1 Classical waves . . . 205
5.2 Two-slit experiments and quantum mechanics . . . 213
5.3 Feynman sum-over-paths . . . 215
5.4 Helium atoms and the two slits, revisited . . . 219
5.5 The emergence of the classical trajectory . . . 224
5.6 Why Hamilton's principle? . . . 229
5.7 The Jacobi action . . . 230
5.8 Summary . . . 233

II Part Two

6 Constraints and Symmetries . . . 239
6.1 Contact forces . . . 239
6.2 Symmetries and conservation laws: a preview . . . 251
6.3 Cyclic coordinates and generalized momenta . . . 251
6.4 A less straightforward example . . . 254
6.5 Infinitesimal transformations . . . 254

6.6 Symmetry . . . 258
6.7 Noether's theorem . . . 259
6.8 Some comments on symmetries . . . 270
6.9 Summary . . . 271

7 Gravitation . . . 279
7.1 Central forces . . . 279
7.2 The two-body problem . . . 281
7.3 The effective potential energy . . . 284
7.4 The shape of central-force orbits . . . 287
7.5 Bertrand's Theorem . . . 295
7.6 Orbital dynamics . . . 295
7.7 Summary . . . 303

8 Electromagnetism . . . 311
8.1 Gravitation revisited . . . 311
8.2 The Lorentz force law . . . 315
8.3 The Lagrangian for electromagnetism . . . 319
8.4 The two-body problem, once again . . . 321
8.5 Coulomb scattering . . . 324
8.6 Motion in a uniform magnetic field . . . 327
8.7 Relativistic effects and the electromagnetic force . . . 342
8.8 Summary . . . 345

9 Accelerating frames . . . 349
9.1 Linearly accelerating frames . . . 349
9.2 Rotating frames . . . 352
9.3 Pseudoforces in rotating frames . . . 355
9.4 Pseudoforces on Earth . . . 360
9.5 Spacecraft rendezvous and docking . . . 367
9.6 Summary . . . 375

10 From black holes to random forces . . . 383
10.1 Beyond Newtonian gravity . . . 383
10.2 The Schwarzschild geometry . . . 388
10.3 Geodesics in the Schwarzschild spacetime . . . 392
10.4 The Event Horizon and Black Holes . . . 400
10.5 Magnetic gravity . . . 402
10.6 Gauge symmetry . . . 405
10.7 Stochastic forces . . . 407
10.8 Summary . . . 413

III Part Three

11 Hamiltonian formulation 419

11.1 Legendre transformations 419
11.2 Hamilton's equations 423
11.3 Phase Space 426
11.4 Canonical transformations 432
11.5 Poisson brackets 438
11.6 Liouville's theorem 444
11.7 Summary 445

12 Rigid Body Dynamics 455

12.1 Rigid Bodies 455
12.2 Rotations 457
12.3 Infinitesimal Rotations 459
12.4 The Euler Angles 463
12.5 Rotational kinetic energy 468
12.6 Potential Energy 475
12.7 Angular Momentum 477
12.8 Torque 480
12.9 Principal Axes 480
12.10 The parallel-axis theorem 484
12.11 Torque-Free Dynamics 485
12.12 Gyroscopes 491
12.13 Summary 495

13 Coupled oscillators 499

13.1 Linear systems of masses and springs 499
13.2 More realistic bound systems 511
13.3 Vibrational degrees of freedom 519
13.4 The continuum limit 521
13.5 Summary 536

14 Complex systems 547

14.1 Integrability 548
14.2 Conservative chaos 556
14.3 Dissipative chaos 560
14.4 Perturbation techniques 565
14.5 Numerical techniques 565
14.6 Summary 573

15 Seeds of Quantization . . . 575
15.1 Hamilton-Jacobi theory . . . 575
15.2 Hamilton's characteristic function . . . 578
15.3 Action angle variables . . . 581
15.4 Adiabatic invariants . . . 583
15.5 Early quantum theory . . . 587
15.6 Optics: from waves to rays . . . 589
15.7 Schrödinger's wave mechanics . . . 593
15.8 Quantum operators and the bracket . . . 597
15.9 Summary . . . 600

IV Appendices

Appendices . . . 607

A Appendix Coordinate systems . . . 609
A.1 Cartesian coordinates . . . 609
A.2 Cylindrical coordinates . . . 610
A.3 Spherical coordinates . . . 612

B Appendix Integral theorems . . . 615
B.1 Green's theorem . . . 615
B.2 Stocke's theorem . . . 617
B.3 Divergence theorem . . . 620

C Appendix Dimensional reasoning . . . 623

D Appendix Runge-Kutta . . . 627

E Appendix Fractal dimension . . . 631

F Appendix The logistic map . . . 633