Complementarity and the Amplification of Financial Frictions

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This Version: October 15, 2025
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Abstract

Does improving the access to financing lead to more productive firms? We study the implications of financial frictions on technology adoption. We develop a model of heterogeneous producers facing collateral constraints and a technology adoption decision. Producers' choices are connected through the final good market, creating complementarity in adoption. We find that more complementarity between producers amplifies the effects of financial frictions, resulting in more sensitive adoption. Our results imply that economies with high complementarity benefit more from financial liberalizations, incentivizing the adoption of more productive technologies.

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1 Introduction

Financial frictions not only distort capital allocation but also technology adoption, which changes the firm-level productivity distribution. The aggregate effect of the distorted distribution could be sizable if a large share of producers give up adopting more productive technologies due to the borrowing constraints. In this case, policies to relax financial frictions bring large benefits to the economy. The recent development literature sheds light on what determines the size of aggregate effect from distortions, emphasizing the role of complementarity between firms' decisions in determining technology adoption, where the individual adoption decision depends on the share of adopters in the economy.

In this paper, we focus on financial frictions as the main source of distortions. How much does complementarity amplify the effect of financial frictions? How much can financial frictions explain the TFP differences across countries? We build a heterogeneous-agent model with financial frictions and technology adoption. We introduce intermediate goods to the production of final good and adoption good, which creates complementarity between producers. We calibrate the model to the US economy without distortions and conduct counterfactual analyses under different levels of substitutability.

We show that less elasticity of substitution between intermediate goods amplifies the effects of financial frictions. Specifically, when the differentiated goods are less substitutable (more complementary), adoption rates respond more to a relaxation of financial frictions. This results in larger aggregate effects on TFP. Our findings suggest economies with higher complementarity benefit more from better access to financing, helping to reconcile theoretical predictions with empirical observations.

We focus on two interaction channels: intermediate good substitutability and technology of adoption. When intermediate goods are more complementary, individual producers have more incentives to increase output through positive externalities. This externality is likewise reflected in technology adoption: when other producers adopt more productive technologies, the value of adoption increases generating an amplification effect. The effects are larger when the economy features more complementarity. However, the externality also has the opposite effect when producers are more constrained and their production level is negatively distorted by financial frictions.

Our model features a mass of intermediate good producers, who are heterogeneous in the quality of their output and their wealth, and a unit mass of workers. The mass of intermediate good producers and the efficiency of workers grow at an exogenous, constant rate. Producers have the option to adopt a more productive technology by paying a one-time adoption cost in units of intangible capital, increasing their optimal scale. Intangible capital is produced with a constant returns to scale technology that employs final good and labor. The final good producer aggregates intermediate goods using a constant elasticity of substitution technology, creating linkages between the producers' decisions.

Intermediate good producers employ capital and labor to produce according to a decreasing returns to scale production function and an idiosyncratic, stochastic quality. Their capital demand is subject to a collateral constraint, and they can rent against their assets and intangible capital. Producers' output therefore depends on both their quality and their wealth. Intermediate goods are perfectly substitutable within the same quality, so producers are price takers. Intermediate goods are then aggregated into the final good, that is used for investment and consumption.

Producers' are initially unproductive. They have the option to pay a one-time adoption cost to use a more productive technology. We interpret the adoption cost as investment in intangible capital. Adopting the productive technology allows to increase the optimal scale of production through a jump in productivity and allows to pledge intangible capital as collateral to rent physical capital.

Related Literature The macro-development literature has widely studied misallocation of inputs, with influential work by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009).[1] Our model is the most related to frameworks with financial frictions as the source of misallocation. Some examples with an entry or adoption margin include Midrigan and

[1] Gopinath et al. (2017) study the increased cost of misallocation in Europe and the decline in sectoral total factor productivity in response to a decline in the real interest rate. Inspired by the analysis in Bau and Matray (2023), we focus on the access to financial markets as changes in collateral constraints and distortions.

Xu (2014), Bento and Restuccia (2017), and Fattal-Jaef (2022). Buera, Kaboski and Shin (2011) has a sectoral choice that can also be interpreted as an entry margin. Our paper introduces differentiated intermediate inputs in a Melitz (2003) fashion. We also consider an adoption-specific good to study the amplification through prices. This specification allows for an expanded notion of sector and the analysis of amplification of distortions through complementarity in production.

The paper also relates to the literature on complementarity in technology adoption. Murphy et al. (1989) explores this idea in a model with aggregate demand spillovers.[2] Buera et al. (2021) analyze multiplicity and the amplification of distortions in a static model with heterogeneity. Our model focuses on the amplification of financial frictions and technology adoption through production complementarity, which introduces externalities. We study this problem in a heterogeneous-agent dynamic framework with financial frictions as the underlying distortion, meaning that distortions arise endogenously and have spill-over effects on other producers.

Our model helps explain the recent empirical evidence on misallocation in Bau and Matray (2023). They use a financial liberalization in India during the 2000s that expanded the international funding allowed for firms to study the effects of misallocation. While they find improvements in the distribution of capital and an overall reduction of misallocation, they get statistically insignificant results when analyzing changes in within-firm TFP. We rationalize this fact through the introduction of complementarity in production in a standard misallocation model. We find that low complementarity between producers can partially mute technological adoption when firms' financing is expanded.

2 Model

Our model builds on Midrigan and Xu (2014). The economy is populated by producers and workers. The efficiency of labor and the measure of producers grow at the same constant

[2] In open-economy settings, Okuno-Fujiwara (1988), Rodrik (1996), and Rodríguez-Clare (1996) study coordination failures and industrialization policies. We do counterfactual analysis in an open economy setting but abstract from coordination failures. Our model instead features externalities in adoption.

rate g. Specifically, producers are born exogenously and initially operate an *unproductive* technology. Producers transform labor and capital into a differentiated good according to their type z. Within a type z, producers engage in perfect competition. A competitive firm aggregates differentiated goods into the final good.

2.1 Sectoral Problems

2.1.1 Final Good Sector

A competitive firm uses intermediate goods to produce the final good. Intermediate goods are indexed by $z \in \mathbb{R}_{++}$. The technology of production for the final good is

$$Y = \left[\int Y(z)^{1-\frac{1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \tag{2.1}$$

where $\sigma > 1$. Therefore, the final good producer demands intermediates from each type z according to the inverse demand

$$p(z) = [Y(z)/Y]^{-1/\sigma}. \tag{2.2}$$

The final good is the numeraire in our model, so $P = \left[\int p(z)^{1-\sigma} dz\right]^{\frac{1}{1-\sigma}} = 1$. By assumption, intermediate good producers with quality z take $p(z)$ as given and their output is perfectly substitutable among producers with the same quality. For simplicity, we refer to the mass of producers with quality z as *sector* z. We now describe their problem.

2.1.2 Intermediate Good Sector

Intermediate good producers are characterized by their exogenous, stochastic quality $z \in Z = \mathbb{R}_{++}$, their asset holdings $a \in A = [\underline{a}, \bar{a}]$, and their technology of production. They are born exogenously at rate g and live indefinitely. The idiosyncratic quality z follows the stochastic process

$$\log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t \tag{2.3}$$

with $\rho \in (0,1)$ and $\varepsilon_t \sim N(0, \sigma_z)$.

There are two types of technologies (or producers): productive and unproductive. All producers can employ the unproductive technology. Upon payment of a one-time adoption cost, producers earn the right to use the productive technology and we refer to them as productive producers. Producers maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to their technology-specific budget and collateral constraints.

Productive Producers The production function is

$$y^p(k,\ell) = [z \exp(\phi)]^{1-\eta_p} \left(k^{\alpha_p} \ell^{1-\alpha_p}\right)^{\eta_p}, \tag{2.4}$$

where $\eta_p \in (0,1)$ denotes the returns to scale, and $\phi > 0$ is the productivity advantage of the productive technology. Given their quality z, market price $p(z)$, and wealth a, producers maximize their profits. They employ labor and capital at market prices W and $r + \delta$, respectively. Producers face a collateral constraint to rent capital: they can borrow capital depending on their wealth a and their intangible capital $P_\kappa \kappa$ (in units of the numeraire). The constrained profit maximization problem is

$$\Pi^p(z,a) = \max_{k,\ell} \; p(z) \left[z \exp(\phi)\right]^{1-\eta_p} k^{\alpha_p \eta_p} \ell^{(1-\alpha_p)\eta_p} - W\ell - (r+\delta)k, \tag{2.5}$$

subject to the collateral constraint

$$k \leq \frac{1}{1-\theta} a + \frac{\theta}{1-\theta} P_\kappa \kappa,$$

$P_\kappa \kappa$ corresponds to the producer's fixed stock of intangible capital. We assume they can pledge intangible capital as collateral to rent capital up to a fraction $\theta/(1-\theta)$. In the self-financing world ($\theta = 0$), intangible capital is not pledgeable and producers can only rent up to their wealth level.

The demand for inputs is given by

$$r + \delta + \mu^p(z,a) = \alpha_p \eta_p \frac{p(z) y^p(z,a)}{k^p(z,a)},$$

and

$$W = (1-\alpha_p)\eta_p \frac{p(z) y^p(z,a)}{\ell^p(z,a)},$$

where $\mu^p(z,a) \geq 0$ is the multiplier on the collateral constraint. When $\mu^p(z,a) = 0$, producers are not constrained in their capital demand and can achieve their optimal scale. When they are constrained, $r + \delta + \mu^p(\cdot)$ will reflect the shadow return on capital the producer is facing. Optimal profits are[3]

$$\begin{aligned} \Pi^p(z,a) &= p(z) y^p(z,a) - W \ell^p(z,a) - [r+\delta]\, k^p(z,a) \\ &= z \exp(\phi) p(z)^{\frac{1}{1-\eta_p}} \underbrace{\left(\frac{\alpha_p \eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p \eta_p}{1-\eta_p}}}_{1} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}} \\ &\quad \times \underbrace{\left[1 - \eta_p + \frac{\mu^p(z,a)}{r+\delta+\mu^p(z,a)} \alpha_p \eta_p\right]}_{2}. \end{aligned} \tag{2.6}$$

The excess return on capital $\mu^p(\cdot)$ has two distinct effects on profits. First, since the capital to output ratio within a firm is distorted when $\mu^p(\cdot) > 0$, producers hire less capital than they would like to and thus output is relatively low compared to the optimal scale.[4] Second, provided that the labor share $(1-\alpha)\eta$ is fixed, a higher capital return distorts the profit

[3]Input demands are

$$\begin{aligned} k^p(z,a) &= [z \exp(\phi)]\, p(z)^{\frac{1}{1-\eta_p}} \left(\frac{\alpha_p \eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p \eta_p}{1-\eta_p}+1} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}}, \\ \ell^p(z,a) &= [z \exp(\phi)]\, p(z)^{\frac{1}{1-\eta_p}} \left(\frac{\alpha_p \eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p \eta_p}{1-\eta_p}} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}+1}. \end{aligned}$$

Output is

$$y^p(z,a) = [z \exp(\phi)]\, p(z)^{\frac{\eta_p}{1-\eta_p}} \left(\frac{\alpha_p \eta_p}{r+\delta+\mu^p(z,a)}\right)^{\frac{\alpha_p \eta_p}{1-\eta_p}} \left(\frac{(1-\alpha_p)\eta_p}{W}\right)^{\frac{(1-\alpha_p)\eta_p}{1-\eta_p}}.$$

[4]From the first order condition with respect to capital, $\frac{\alpha_p \eta_p}{r+\delta+\mu^p(\cdot)} = \frac{k^p(\cdot)}{p(z) y^p(\cdot)}$.

share, which equals $1 - \eta$ in the unconstrained case.

Given profits and interest payments from their asset holdings, they choose their consumption level c and future asset holdings a' in order to maximize their (recursive) value

$$V^p(z,a) = \max_{c,a'} \log(c) + \beta \mathbb{E}_{z'} \left[V^p(z',a') \middle| z \right],$$

subject to the budget constraint

$$c + a' = \Pi^p(z,a) + (1+r)a.$$

Unproductive Producers Upon birth, producers can only operate a less productive or traditional technology that we call *unproductive*. They are born with quality drawn from the stationary distribution of z and wealth $a_0 = \min\{a \in A \cap \mathbb{R}_{++}\}$. The production function is

$$y^u(k,\ell) = z^{1-\eta_u} \left(k^{\alpha_u} \ell^{1-\alpha_u} \right)^{\eta_u}, \tag{2.7}$$

with $\eta_u \leq \eta_p$ and $\alpha_u \leq \alpha_p$. This parameterization implies that the unproductive technology is less intensive in capital, less profitable, and has a smaller optimal scale. Therefore, unproductive producers will have a smaller operation for the same (z,a) than productive producers. Given their quality z, market price $p(z)$, and wealth a, producers maximize their profits

$$\Pi^u(z,a) = \max_{k,\ell} \; p(z) z^{1-\eta_u} k^{\alpha_u \eta_u} \ell^{(1-\alpha_u)\eta_u} - W\ell - (r+\delta)k, \tag{2.8}$$

subject to the collateral constraint

$$k \leq \frac{1}{1-\theta} a.$$

Similar to productive producers, the unproductive producers' profits are

$$\begin{aligned}\Pi^u(z,a) &= p(z)y^u(z,a) - W\ell^u(z,a) - [r+\delta]\,k^u(z,a)\\ &= zp(z)^{\frac{1}{1-\eta_u}}\left(\frac{\alpha_u\eta_u}{r+\delta+\mu^u(z,a)}\right)^{\frac{\alpha_u\eta_u}{1-\eta_u}}\left(\frac{(1-\alpha_u)\eta_u}{W}\right)^{\frac{(1-\alpha_u)\eta_u}{1-\eta_u}}\\ &\quad\times\left[1-\eta_u+\frac{\mu^u(z,a)}{r+\delta+\mu^u(z,a)}\alpha_u\eta_u\right]. \qquad (2.9)\end{aligned}$$

where $\mu^u(z,a) \geq 0$ is the multiplier on the collateral constraint.

In addition to the consumption-saving decision, unproductive producers have the option to adopt the productive technology. Adoption occurs upon payment of a one-time cost κ in units of the adoption good. Therefore, their current value is

$$V^u(z,a) = \max_{c,a'} \log(c) + \beta \max\left\{\mathbb{E}_{z'}\left[V^u(z',a')\,|\, z\right], \mathbb{E}_{z'}\left[V^p(z',a'-P_\kappa\kappa)\,|\, z\right]\right\},$$

subject to the budget constraint

$$c + a' - \xi(z,a)P_\kappa\kappa = \Pi^u(z,a) + (1+r)a.$$

The indicator function $\xi(z,a)$ equals 1 when an unproductive producer decides to adopt the productive technology next period, and 0 otherwise. Since the quality process is independent, the distribution of z' conditional on z does not depend on the adoption decision.

2.1.3 Adoption Good Sector

Producers need to acquire κ units of the adoption good in order to utilize the productive technology. We interpret this adoption cost as investment in intangible capital, such as intellectual property or blueprints. The adoption good is produced by a competitive firm that combines final goods and labor according to

$$y_\kappa(x,\ell) = \frac{1}{\gamma^\gamma(1-\gamma)^{1-\gamma}}x^\gamma\ell^{1-\gamma}.$$

Therefore, the price of the adoption good is equal to the marginal cost $P_\kappa = P^\gamma W^{1-\gamma}$. The elasticity γ indicates the intensity of the adoption good in final goods. When γ is lower, the production of adoption good is more intensive in labor and thus more time is needed to produce the same output. The demand for inputs conditional on total demand for adoption good κM, where M is the mass of adopters, is

$$X^\kappa = \gamma P^{-(1-\gamma)} W^{1-\gamma} \kappa M \quad \text{and} \quad L^\kappa = (1-\gamma) P^\gamma W^{-\gamma} \kappa M.$$

2.1.4 Workers

There's a unit measure of workers in the economy. Each of them is subject to an idiosyncratic shock ν_t to labor efficiency that follows a finite-state Markov process. Their efficiency grows at rate g, hence their labor supply equals $g^t \nu_t$, which they supply inelastically. They consume and save in order to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(c_t),$$

subject to the budget constraint

$$c_t + a_{t+1} = W g^t \nu_t + (1+r) a_t.$$

2.2 Aggregation

To get expressions for aggregate output and productivity, we assume $\alpha_u = \alpha_p \equiv \alpha$ and $\eta_u = \eta_p \equiv \eta$. For compactness, denote capital return as $R^i(z,a) = r + \delta + \mu^i(z,a)$.

2.2.1 Aggregation Within Quality

Given sectoral price $p(z)$, the quantity produced in sector z is

$$Y(z) = p(z)^{\frac{\eta}{1-\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} (\alpha\eta)^{\frac{\alpha\eta}{1-\eta}}$$
$$\times \underbrace{\left[[z\exp(\phi)] \int_A R^p(z,a)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^p(z,a) + z\int_A R^u(z,a)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^u(z,a) \right]}_{\equiv \Lambda_y(z)}.$$

The aggregator $\Lambda_y(z)$ is a weighted average of productivities. For clarity, we can rewrite it as

$$\Lambda_y(z) \propto \exp(\phi) \int_A \left(\frac{k^p(z,a)}{y^p(z,a)}\right)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^p(z,a) + \int_A \left(\frac{k^u(z,a)}{y^u(z,a)}\right)^{-\frac{\alpha\eta}{1-\eta}} d\Omega^u(z,a),$$

where the weights depend on the capital to output ratio and reflect the level of distortion from the optimal scale. Substituting the sectoral price $p(z) = [Y(z)/Y]^{-1/\sigma}$, we get

$$Y(z) = Y^{\frac{\eta}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta\sigma}{(1-\eta)\sigma+\eta}} (\alpha\eta)^{\frac{\alpha\eta\sigma}{(1-\eta)\sigma+\eta}} \Lambda_y(z)^{\frac{(1-\eta)\sigma}{(1-\eta)\sigma+\eta}}.$$

When σ is lower, varieties are less substitutable and the quantity produced $Y(z)$ depends more on the aggregate quantity Y. In particular, $d\log Y(z)/[d\log Y] \to \eta$ as $\sigma \to 1$ (more complementarity), and $d\log Y(z)/[d\log Y] \to 0$ as $\sigma \to \infty$ (no complementarity). The elasticity of substitution therefore governs the interconnections between sectors, implying that complementarity (low σ) creates positive externalities between varieties. Similarly, input demands in the sector are

$$K(z) = \int_A k^p(z,a) d\Omega^p(z,a) + \int_A k^u(z,a) d\Omega^u(z,a)$$
$$= Y^{\frac{1}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta(\sigma-1)}{(1-\eta)\sigma+\eta}} (\alpha\eta)^{\frac{\alpha\eta(\sigma-1)}{(1-\eta)\sigma+\eta}+1} \Lambda_y(z)^{-\frac{1}{(1-\eta)\sigma+\eta}}$$
$$\times \underbrace{\left[[z\exp(\phi)] \int_A R^p(z,a)^{-\frac{\alpha\eta}{1-\eta}-1} d\Omega^p(z,a) + z\int_A R^u(z,a)^{-\frac{\alpha\eta}{1-\eta}-1} d\Omega^u(z,a) \right]}_{\equiv \Lambda_k(z)},$$

and

$$
\begin{aligned}
L(z) &= \int_A \ell^p(z,a)d\Omega^p(z,a) + \int_A \ell^u(z,a)d\Omega^u(z,a) \\
&= Y^{\frac{1}{(1-\eta)\sigma+\eta}} \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta(\sigma-1)}{(1-\eta)\sigma+\eta}+1} (\alpha\eta)^{\frac{\alpha\eta(\sigma-1)}{(1-\eta)\sigma+\eta}} \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}}.
\end{aligned}
$$

Let $\mathcal{A}(z) \equiv Y(z)\left[K(z)^\alpha L(z)^{1-\alpha}\right]^{-\eta}$ denote TFP in sector z. Then,

$$
\mathcal{A}(z) = \frac{\Lambda_y(z)^{1-(1-\alpha)\eta}}{\Lambda_k(z)^{\alpha\eta}}.
$$

This expression for TFP is similar to Midrigan and Xu (2014), where outputs are perfect substitutes. Therefore, variety substitutability does not directly affect productivity within a sector as producers face the same price and z is exogenous. Despite no direct effect, substitutability has an indirect effect on sectoral TFP $\mathcal{A}(z)$ through capital returns and the marginal distribution over wealth. This fact highlights the role of the interaction between collateral constraints and variety substitutability, resulting in more amplification of financial frictions.

2.2.2 Aggregation Between Qualities

To compute aggregate productivity, let $K = \int_Z K(z)dz$ and $L = \int_Z L(z)dz$. Aggregate output is

$$
Y = \left(\frac{(1-\alpha)\eta}{W}\right)^{\frac{(1-\alpha)\eta}{1-\eta}} (\alpha\eta)^{\frac{\alpha\eta}{1-\eta}} \left[\int_Z \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{(1-\eta)\sigma+\eta}{(1-\eta)(\sigma-1)}}.
$$

We can then get an expression for the sectoral price:

$$
p(z) = \left[\int_Z \left(\frac{\Lambda_y(z)}{\Lambda_y(\tilde{z})}\right)^{-\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} d\tilde{z}\right]^{\frac{1}{\sigma-1}}
$$

The sectoral price $p(z)$ relates sectoral efficiency: when a sector is relatively more efficient than the others (lower $\Lambda_y(z)$), its price is lower.

Define $\mathcal{A} \equiv Y\left[K^{\alpha}L^{1-\alpha}\right]^{-\eta}$, then

$$\mathcal{A} = \frac{\left[\int_Z \Lambda_y(z)^{\frac{(1-\eta)(\sigma-1)}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{\sigma}{\sigma-1}-(1-\alpha)\eta}}{\left[\int_Z \Lambda_y(z)^{-\frac{1}{(1-\eta)\sigma+\eta}} \Lambda_k(z) dz\right]^{\alpha\eta}}.$$

Since we want to measure the effect of distortions on aggregate TFP, define average productivity in sector z as

$$\Lambda^e(z) = z\left[\exp(\phi)\int_A d\Omega^p(z,a) + \int_A d\Omega^u(z,a)\right].$$

Average productivity is naturally increasing in the number of adopters within z: when all producers adopt, $\Lambda^e(z) = z\exp(\phi)$ times the mass of producers in z; when none does, $\Lambda^e(z) = z$ times the mass of producers in z. TFP is increasing in the number of producers given decreasing returns to scale, hence a "love for variety" effect. Under no distortions ($\mu^i(a,z) = 0$), $\Lambda_k(z) = (r+\delta)^{-1}\Lambda_y(z) = (r+\delta)^{-\frac{\alpha\eta}{1-\eta}-1}\Lambda^e(z)$. Similarly, $\mathcal{A}^e(z) = \Lambda^e(z)^{1-\eta}$. The undistorted aggregate productivity is

$$\mathcal{A}^e = \left[\int_Z \mathcal{A}^e(z)^{\frac{\sigma-1}{(1-\eta)\sigma+\eta}} dz\right]^{\frac{(1-\eta)\sigma+\eta}{\sigma-1}} \tag{2.10}$$

We interpret $\log\mathcal{A}^e - \log\mathcal{A}$ as the loss from misallocation. That is, for a fixed measure $\Omega^i(z,a)$, $i = \{u,p\}$, the productivity gain of reallocating capital across producers.

2.3 Equilibrium Conditions

Let $\Omega_t^i(z,a)$ denote the measure of producers with quality z and wealth a using technology i at time t. Denote $f(z'|z)$ the conditional distribution of quality and $\bar{f}(z)$ the stationary distribution over Z. The measure of productive producers evolves over time according to

$$\Omega_{t+1}^p(z',A) = \int_A\int_Z f(z'|z)d\Omega_t^p(z,a) + \int_A\int_Z f(z'|z)\xi(z,a)d\Omega_t^u(z,a).$$

The measure accounts for producers that were productive in the previous period and those who adopted after t. The measure of unproductive producers is

$$\Omega^u_{t+1}(z', A) = \int_A \int_Z f(z'|z)\,[1 - \xi(z, a)]\, d\Omega^u_t(z, a) + (g - 1)\bar{f}(z) N_t,$$

where $N_t = g^t$ is the total number of producers at t. We normalize the mean of ν_t, the idiosyncratic efficiency of workers, so that $L_t = g^t$. Each period, the mass of adopters is

$$M_t = \int_{Z\times A} \xi(z, a) d\Omega^u_t(z, a).$$

The quantity produced in sector z is

$$Y_t(z) = \sum_{i=u,p} \int_A y^i(z, a) d\Omega^i_t(z, a).$$

Output of final good is

$$\begin{aligned} Y_t &= \left[\int_Z Y_t(z)^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[\int_Z \left(\int_A y^p(z, a) d\Omega^p_t(z, a) + \int_A y^u(z, a) d\Omega^u_t(z, a)\right)^{1-\frac{1}{\sigma}} dz\right]^{\frac{\sigma}{\sigma-1}}, \end{aligned}$$

The market clearing conditions for labor and assets are

$$\begin{aligned} L_t &= L^p_t + L^u_t + L^\kappa_t \\ &= \sum_{i=u,p} \int_{Z\times A} \ell^i(z, a) d\Omega^i_t(a, z) + (1 - \gamma) P^\gamma W^{-\gamma} \kappa M_t, \end{aligned} \tag{2.11}$$

$$\begin{aligned} 0 &= A^w_t + A^p_t + A^u_t - (K^p_t + K^u_t) \\ &= A^w_t + \sum_{i=u,p} \int_{Z\times A} a^i(z, a) d\Omega^i_t(a, z) - \sum_{i=u,p} \int_{Z\times A} k^i(z, a) d\Omega^i_t(a, z). \end{aligned} \tag{2.12}$$

A competitive equilibrium in this economy consists of prices $\{\{p(z) : z \in Z\}, W, r\}$ and measures $\Omega^i_t(z, a)$ such that all agents maximize their problems given prices and all markets clear, i.e., $Y_t(z) = p(z)^{-\sigma} Y_t$ for all $z \in Z$, and equations 2.11 and 2.12 hold with equality.

2.4 Adoption Gain

To better understand the adoption decision by producers, consider the profit gap for a given state (z, a):

$$\begin{aligned}
\log\left[\frac{\Pi^p(z,a)}{\Pi^u(z,a)}\right] = \quad & \phi - \frac{\alpha\eta}{1-\eta}\log\left(\frac{r+\delta+\mu^p(z,a)}{r+\delta+\mu^u(z,a)}\right) \\
& + \log\left(1-\eta+\frac{\mu^p(z,a)}{r+\delta+\mu^p(z,a)}\alpha\eta\right) \\
& - \log\left(1-\eta+\frac{\mu^u(z,a)}{r+\delta+\mu^u(z,a)}\alpha\eta\right).
\end{aligned}$$

In the perfect-borrowing world, the log profit gap reduces to the productivity gain ϕ. However, when the producers are constrained, they also have to account for two additional effects. First, a distorted capital to output ratio reduces revenue overall and is reflected by the shadow return on capital $r+\delta+\mu^i(\cdot)$. By adopting the productive technology, producers get access to more financing by pledging their intangible capital as collateral and simultaneously increase their optimal scale. Second, a high return on capital increases the profit share of producers: since the labor share is constant, any return in excess of the market price of capital is absorbed by the producer. Since producers are price takers and output is perfectly substitutable within quality z, the elasticity of substitution σ does not directly affect the profit gap.

3 Quantitative Analysis

In this section, we conduct counterfactual analysis to understand the interaction between technology adoption and financial frictions. We focus on the role of intermediate good substitutability σ, which governs how connected individual producers' problems are. We first calibrate our model to the US economy under perfect borrowing, yielding a benchmark without distortions. We then evaluate the model featuring low and high complementarity in production, for different degrees of financial frictions.

3.1 Calibration

We calibrate the model to the US economy as an undistorted benchmark ($\theta = 1$) in a closed economy setting. We assign standard values to several parameters, following mostly Midrigan and Xu (2014). For the production function, we choose capital elasticity $\alpha = 0.33$ and span of control $\eta = 0.85$, both common across technologies. Capital depreciation is 0.06. The subjective discount factor β is $0.92/g$, consistent with Buera, Kaboski and Shin (2011) after adjusting for the growth rate of consumption along the balanced growth path in our model. We fix the growth rate g to 3 percent. In our model, g equals the growth rate of per-capita GDP and the entry rate of establishments. While real GDP per capita grows at about 2 percent annually, the entry rate of establishments in the US Census' Business Dynamics Statistics is significantly larger at around 8 percent for the smallest size reported.

For the idiosyncratic shock of workers, we use a two-state Markov process with $\nu_i \in \{0, 1\}$. We can interpret these values as unemployment and employment from the perspective of the worker. We set the probability of staying in the low state to 0.5, and pick the probability of staying in the high state such that employment equals 60% of population as reported by the Bureau of Labor Statistics. The mean of ν is normalized so that the total number of efficiency units of labor supplied equals the total number of producers. For the quality process, we pick values in line with those in Asker et al. (2014) for the productivity process in the US. Specifically, we choose $\rho = 0.9$ and $\sigma_z = 0.75$, making the process fairly persistent.

We choose the elasticity of substitution between varieties $\sigma = 3$ for the benchmark, which is comparable to Hsieh and Klenow (2009) and Buera et al. (2021). In our benchmark calibration, we use $\gamma = 1$, meaning that intangible capital is only produced with final good. We then compare to the case of $\gamma = 0$, where intangible capital is produced only using labor. For the productivity gain $(1 - \eta)\phi$, we rely on the estimations by Buera et al. (2021) and set it to 0.55.[5] We finally calibrate the adoption cost κ to match the ratio of investment in

[5]In particular, we interpret their productivity gap as labor productivity gap. We therefore scale it with

intangible capital to GDP, which equals around 4 percent according to the BEA's National Income and Product Accounts between 2000 and 2019. We specifically interpret private investment in intellectual property as investment in intangible capital. The resulting adoption cost κ equals 9.39.

Table 1: Parameter Values

Description	Parameter	Value	Target / Source
Collateral constraint	θ	1.00	Undistorted
Elasticity of substitution	σ	3.00	Benchmark
Goods elasticity in adoption	γ	1.00	Benchmark
Relative efficiency modern	$(1-\eta)\phi$	0.55	Buera et al. (2021)
Cost of entering modern sector	κ	9.39	Intangible investment to GDP
Span of control	η	0.85	Standard
Capital elasticity	α	0.33	Standard
Capital depreciation	δ	0.06	Standard
Discount factor	β/g	0.92	Midrigan and Xu (2014)
Growth rate	g	1.03	GDP growth & Entry
Quality Persistence	ρ	0.90	Asker et al. (2014)
Quality SD	σ_z	0.75	Asker et al. (2014)
Prob. remaining unemployed	–	0.50	Standard
Prob. remaining employed	–	0.67	Employment to population ratio

Table 2: Benchmark Results

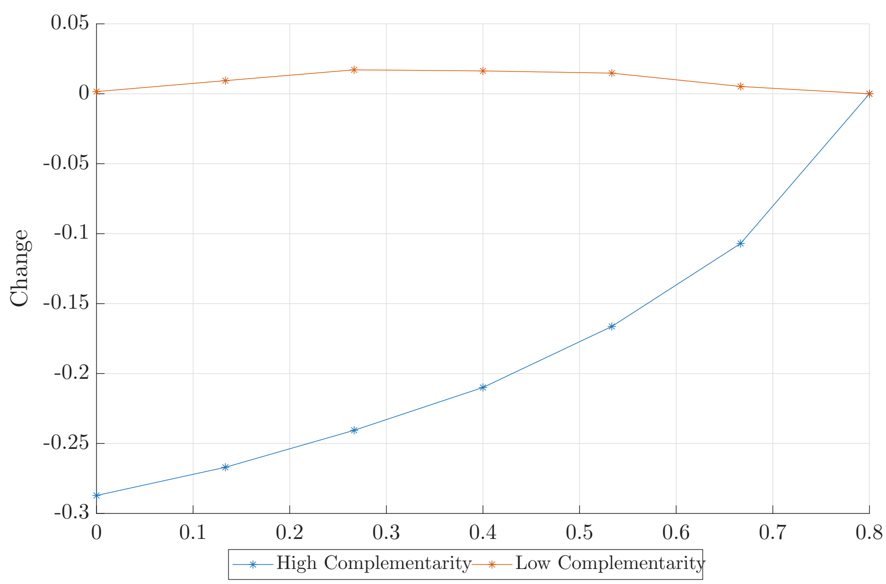
Variable	Value
Output	6.838
Capital to Output	2.228
Consumption to Output	0.759
Tangible Investment to Output	0.201
Intangible Investment to Output	0.040
Capital to Assets	10.06
Fraction Adopters	0.971
TFP	3.161

the output-labor elasticity, i.e., $(1-\eta)\phi = -(1-\alpha)\log(0.43)$.

3.2 Amplification Through Complementarity

We next analyze the interaction between complementarity (σ) and financial frictions (θ). In order to abstract from price changes that affect intertemporal choices and focus on the ability to borrow, the counterfactual economies take the interest rate r as given (small open economy). We set the interest rate equal to the benchmark. We compare the high complementarity case of $\sigma = 2$ and the low complementarity case of $\sigma \to \infty$ (perfect substitutes). When $\theta = 1$, producers are never constrained (perfect borrowing) and can always achieve their optimal scale. When $\theta = 0$, producers are in a self-financing economy and can demand capital up to their wealth level. Figure 1 and Figure 2 show the response of both economies to changes in the collateral constraint parameter θ.

Figure 1: Amplification with Complementarity

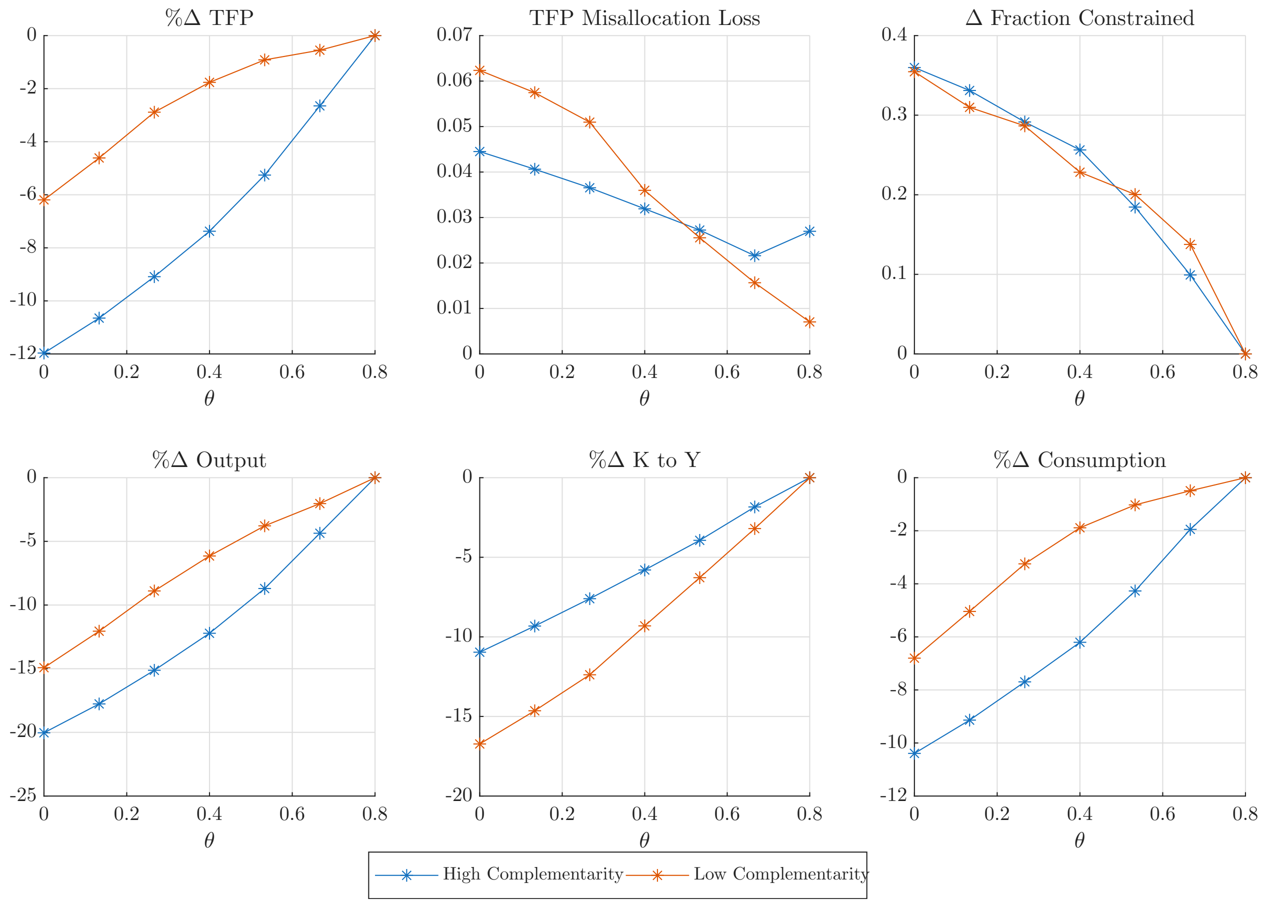


Note: Change in fraction of adopters relative to high θ case.

The economy with high complementarity experiences a larger decline in the fraction of productive producers when the financial frictions increase (lower θ). This reaction translates into a lower change in firm-level productivity when complementarity is low. Financial frictions are more costly in terms of aggregate consumption and output when complementarity is high, declining 3 and 5 percent more in the self-financing world relative to $\theta = 0.8$.

The quantitative results of our model help explain why financial liberalizations might

Figure 2: Additional Results



Note: Δ denotes change, %Δ denotes percent change. All changes relative to high θ case.

have small or no effects on within-firm TFP. When we consider the policy change in India during the 2000s, Bau and Matray (2023) find no significant change in TFP at the firm level and attribute most of the improvement in aggregate TFP to a reduction in capital misallocation. We interpret these results as occurring in a low complementarity environment, where weak production networks reduce the gains from adoption for producers.

3.3 Amplification Through Intangible Investment

We now consider the role of intangible investment as a driver of amplification. As opposed to the complementarity channel we analyzed in the previous section, the technology of production for intangibles affects individual decisions solely through prices as an equilibrium effect. We focus on the two boundary cases: intangibles are produced either with final good ($\gamma = 1$) or with labor ($\gamma = 0$). We compare for both cases an economy with $\theta = 0.8$ and its self-financing counterpart of $\theta = 0$.

When intangibles are produced with final good, more adoption increases both demand and supply for goods, and thus its partial effect on incumbents is not direct. To a certain extent, there are positive spillovers (amplification) in this case even when the overall effect is negative and there are no complementarities between producers. When intangibles are produced only with labor, more adoption increases the demand for labor from intermediate good producers and from the intangible capital producer. Given the fixed labor supply, an increase in the number of adopters unequivocally increases the wage in the economy, lowering the production of incumbents and therefore profits.

Table 3: Amplification through Intangible Investment

	Adoption in Goods, $\gamma = 1$		Adoption in Labor, $\gamma = 0$	
	Partial Financing	Self Financing	Partial Financing	Self Financing
Fraction Adopters	0.361	0.148	0.171	0.078
Fraction Constrained	0.450	0.823	0.437	0.803
K to Y	2.126	1.886	2.130	1.889
Consumption	1.000	0.879	0.852	0.786
Output	1.188	0.920	0.998	0.814
Intangible Investment to Y	0.019	0.010	0.029	0.013
TFP	1.000	0.861	0.907	0.798
Loss Misallocation %	1.5	4.4	1.1	4.2

Note: Consumption and Output are normalized to units of consumption in Column 1, the case with adoption in goods and partial financing. TFP is normalized to 1 for the same case.

Table 3 reports the main results of the exercise. Adoption is higher in the economy where intangibles are produced with final good and it's also more affected by financing: adoption is almost 60% lower in the self-financing case when $\gamma = 1$ compared to almost 55% lower when $\gamma = 0$. This result translates into a larger difference in output: 23% lower output in self-financing for adoption in goods versus 19% for adoption in labor. Aggregate TFP in the self-financing economy is 86% of the partial economy case when $\gamma = 1$ and 88% when $\gamma = 0$. The effects of the intangible production technology on all other aggregate variables are small. Therefore, the production of intangibles has larger positive spillovers when it's more intensive in goods rather than labor.

4 Concluding Remarks

We show that complementarity greatly affects how gains from adoption change with a relaxation of financial frictions. More complementarity amplifies the effects of financial frictions in the economy, implying that the gains from better access to financial markets increase with complementarity. Most importantly, relaxing financial constraints improves technology adoption relatively more when complementarity is higher between producers.

Our model features a heterogeneous mass of producers who have the option to adopt a more productive technology by paying a one-time adoption cost in units of intangible capital, increasing their optimal scale. Their capital demand is subject to a collateral constraint, and they can rent against their assets and intangible capital. Competition between producers is perfect within their quality and demand has a constant elasticity of substitution between qualities.

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Appendix

A Additional Figures

Figure 3: Amplification with Complementarity (Levels)