Inferential statistics

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- Monday : Metrics and training process
  - Supervised & Unsupervised learning
  - Evaluation metrics
  - Training process

- Tuesday : - content -

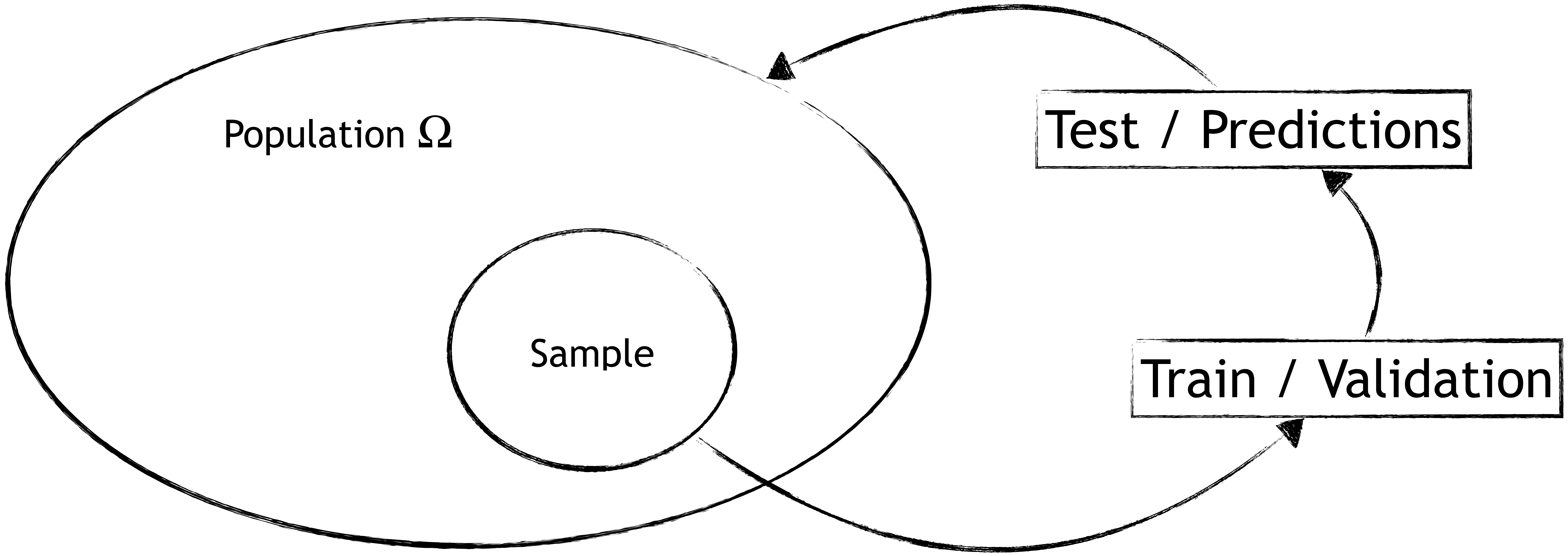
- Wednesday : - content -

- Thursday : - content -

- Friday : - content -

Inferential statistics

Definition : Inferential statistics is the idea of drawing conclusions about a population base on data from a sample.



Supervised learning

Definition : Supervised learning is a type of machine learning where the model is trained on a labeled dataset, meaning each input data point is paired with the correct output (target).

=> The goal is for the model to learn the mapping from inputs to outputs and make predictions on new unseen data.

Supervised learning
- Example -

- Spam detection in mails => Input : email text / Label : spam or not spam

- Prediction of house prices => Input : number of rooms, area, location / Label : price.

- Medical diagnosis => Input : patient symptoms and test results / Label : disease present or not.

- Image classification => Input : image pixels / Label : “cat”, “dog”, etc…

Unsupervised learning

Definition : Unsupervised learningn is a type of machine learning where the model is given unlabeled data.

=> The goal is to find patterns, groupings, or structures within the data without knowing the “correct” answers.

Unsupervised learning
- Example -

- ✤ Customer segmentation => <u>Task :</u> grouping customers by purchasing behaviour.

- ✤ Anomaly detection => <u>Task :</u> Finding fraudulent transactions in banking data.

- ✤ Topic modelling => <u>Task :</u> discovering themes in a collection of new articles.

- ✤ Dimensionality reduction => <u>Task :</u> visualizing high-dimensional data (e.g., genetics, images) using PCA.

Supervised learning
- Regression methods -

Definition : It models the relationship between one or more independent variables (features) and a continuous dependent variable (target).

=> It’s a supervised learning method.

Regression methods
- Illustration of some models -

- *Simple linear regression :* single feature vs target.
- *Multiple linear regression :* multiples features to predict the target.
- *Polynomial regression :* Useful for modelling non-linear trends.
- *Regularized regression :* Penalizes weights to prevent overfitting and manage multicollinearity => Improves generalisation.

Supervised learning
- Classification methods -

Definition : It models the relationship between one or more independent variables (features) and a categorical dependent variable (target).

=> It's also a supervised learning method.

Classification methods
- Example -

- *Logistic regression :* Probability model for binary tasks.
- *Decicision tree :* Tree based model, interpretable model.
- *Random forest :* multiple random trees.
- *Support Vector Machines (SVM) :* Find optimal separating hyperplane.
- *Neural Networks :* Flexible and powerful (especially for high-dimensional inputs).

Evaluation of models
- Metrics -

How do you know you model is working better than the others ?

➡ You define metrics to evaluate the model’s performance.

➡ But can you use the same metrics for regression and classification tasks ?
➡ How do you chose your metric ?

Evaluation of models
- Metrics for regression -

Goal : We need to have a metric that evaluate how close predictions of continuous variables are to the actual target variables.

1 - Error terms : Let y_i be the true value and $\hat{y}_i$ the predicted value:

$$\epsilon_i = y_i - \hat{y}_i$$

$$\Rightarrow \text{ Error} = \sum_{i=1}^{n} \epsilon_i$$

Evaluation of models
- Metrics for regression -

2 - Mean Absolute Error (MAE) : It measures average absolute difference.

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i|$$

Remark : Easy to interpret but doesn't penalise large errors.

Evaluation of models
- Metrics for regression -

3 - Mean Squared Error (MSE) : It measures the square of errors.

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$$

Remark : Commonly used in optimisation!

Evaluation of models
- Metrics for regression -

<u>4 - Root Mean Squared Error (RMSE) :</u> Its measure has the same units as the target variable.

$$\text{RMSE} = \sqrt{\text{MSE}}$$

<u>Remark :</u> More sensitive to outliers that MAE.

Evaluation of models
- Metrics for regression -

5 - R-Squared (R^2) : It measures a proportion of variance explained.

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (y_i - \hat{y}_i)^2}{\sum_{i=1}^{n} (y_i - \bar{y})^2}$$

Remark : 0 means that the prediction is not better than the mean as a naïve prediction.

Evaluation of models
- Metrics for regression -

6 - Mean Average Percentage Error (MAPE) : It measures a prediction accuracy of the models.

$$\text{MAPE} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right|$$

Remark : Very intuitive error interpretation in terms of relative error.

Evaluation of models
- Metrics for classification -

Goal : We need to have a metric that evaluate the quality of the prediction given by a specific model.

0 - Basic concepts:

- *True Postive (TP) :* true value = 1, predicted value = 1
- *True Negative (TN) :* true value = 0, predicted value = 0
- *False Postive (FP) :* true value = 0, predicted value = 1
- *False Negative (FN) :* true value = 1, predicted value = 0

Evaluation of models
- Metrics for classification -

1 - Accuracy: It measures the proportion of correct predictions.

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN}$$

Remark : Issues when facing imbalanced classes.

Evaluation of models
- Metrics for classification -

2 - Precision : It measures how many predicted positives were correct.

$$\text{Precision} = \frac{???}{???}$$

Remark : Useful for imbalanced classes.

Evaluation of models
- Metrics for classification -

<u>2 - Precision :</u> It measures how many predicted positives were correct.

$$\text{Precision} = \frac{TP}{TP + FP}$$

<u>Remark :</u> Useful for imbalanced classes.

Evaluation of models
- Metrics for classification -

3 - Recall : It measures how many actual positives were captured.

$$\text{Recall} = \frac{???}{???}$$

Remark : Useful for imbalanced classes.

Evaluation of models
- Metrics for classification -

<u>3 - Recall :</u> It measures how many actual positives were captured.

$$\text{Recall} = \frac{TP}{TP + FN}$$

<u>Remark :</u> Useful for imbalanced classes.

Evaluation of models
- Metrics for classification -

<u>4 - F1-score :</u> It measures harmonic mean of precision and recall.

$$\text{F1-score} = 2\frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

<u>Remark :</u> Useful for imbalanced classes.

Evaluation of models
- Metrics for classification -

<u>5 - Confusion matrix :</u> It shows all 4 outcomes (TP, TN, FP, FN) in a 2x2 table.

		Prediction	
		0	**1**
Truth	**0**	TN	FP
	1	FN	TP

Evaluation of models
- Metrics for classification -

<u>6 - Log-loss (cross-entropy loss) :</u> It measures probabilistic confidence in predictions.

$$\text{Log-loss} = -\frac{1}{n}\sum_{i=1}^{n}\left[y_i \log(p_i) + (1 - y_i)\log(1 - p_i)\right]$$

<u>Remark :</u> Commonly used in optimisation!

Underfitting

<u>Definition :</u> The model is too simple to capture underlying patterns. It can be visualised by a poor performance both on training and validation sets.

=> Make the model more complex by modifying its parameters or completely change the model.

Overfitting

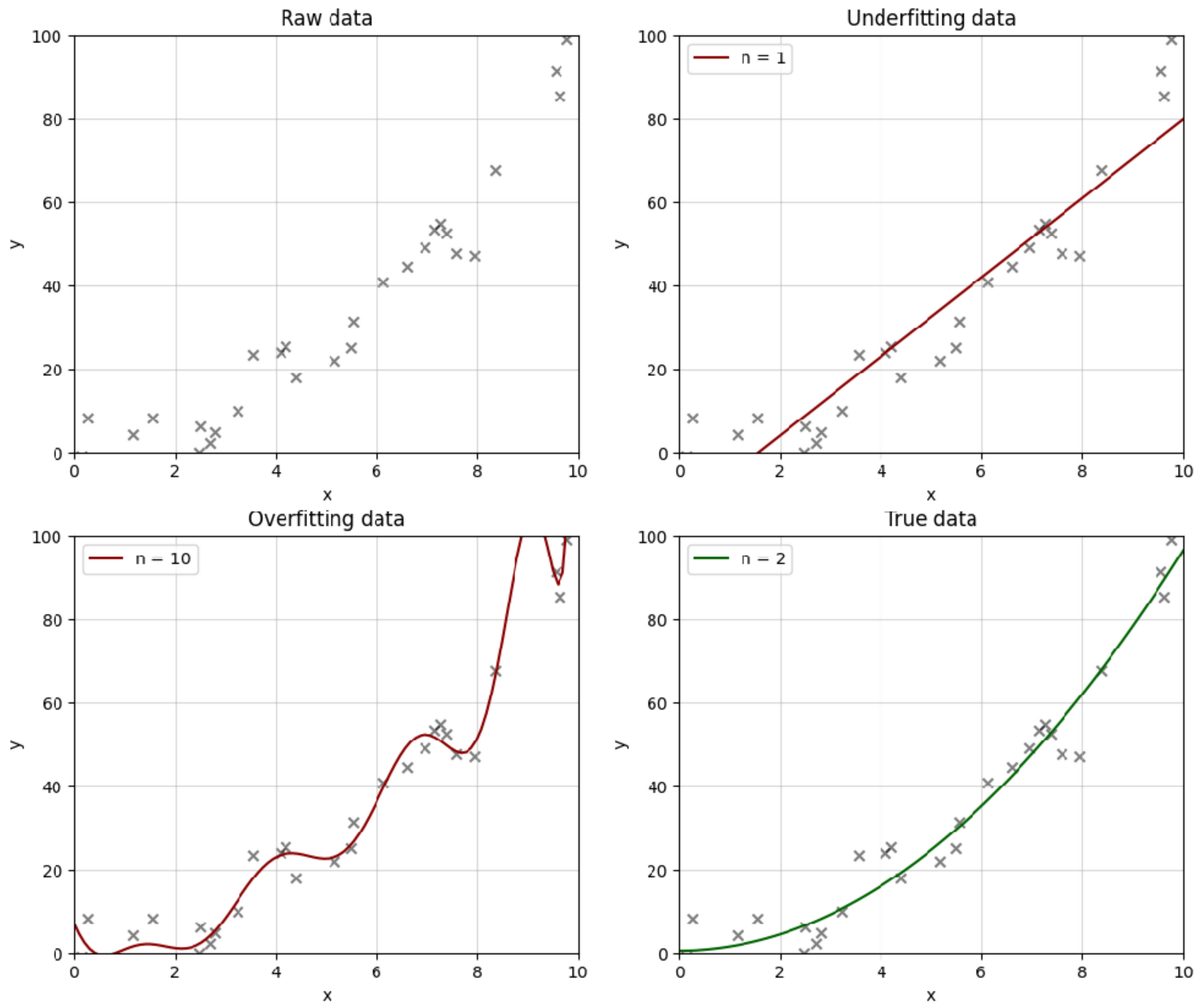
<u>Definition :</u> The model learns noise and details from the training data. It can be visualised by a good performance on training set but poorly on validation set.

=> Simplify the model, regularisation or training data set not well defined regarding the validation set (split to redo).

Underfitting / Overfitting

Practice : Use the data set overfitting_data.csv, train a Polynomial Regression model on it and plot the results. Do it for $n = 1, 2$ and 10.

Underfitting / Overfitting



Practice
- Cross-validation using sklearn -

Training process
- General overview -

- ***Data preprocessing :*** Cleaning, normalisation, encoding, split, etc… (EDA)

- ***Model selection :*** Chose one or multiples model types.

- ***Loss function definition :*** Quantify how wrong the model is (not performance!)

- ***Optimization :*** Algorithms to minimize loss such as gradient descent.

- ***Evaluation :*** Measure performance on training and validation sets.

Practice
- KNN -

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Linear Models

Linear regression

Defintion : It’s a classic model for regression task, the idea is to predict a continuous target variable with a linear combination of the features (categorical or continuous).

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_n x_n + \epsilon$$

Assumptions : Linearity, independence, normality of errors.

Linear regression
- How it trained technically ? -

To compute the « right » parameters of the model, we are aiming to minimise a loss. This loss can be MSE like:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(y_i - \hat{y}_i\right)^2$$

How to minimise it ?

Linear regression
- Gradient descent -

We start from a random parameter β and we iterate from it until convergence of the method. More formally it gives a iteration n the following:

$$\begin{aligned}\beta_{n+1} &= \beta_n - \alpha \nabla_\beta \text{MSE} \\ &= \beta_n - \alpha \frac{2}{n} X^T (X\beta - y)\end{aligned}$$

Linear regression
- Practice -

<u>Exercice :</u> Load the data set `load_diabetes` from sklearn library, and train and test a linear regression model on this data set.

Polynomial and interaction terms regression

<u>Defintion :</u> We stick to a linear regression architecture but adding new feature to our data set. We add features combinations (e.g. x_i^2 or $x_i x_j$) to our set of features and do the same as before.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2 + \beta_4 x_2^2 + \epsilon$$

<u>Remark :</u> The main work here is done in EDA to determine which interaction should be included in your set of features

Ridge regression

<u>Defintion :</u> We face overfitting with linear regression and there is no inside parameters to play with to compensate this. Then, we use « regularizers ». Here we use L2-regularization, also called **Ridge regression :**

$$\text{loss} = \text{MSE} + \lambda \sum_i \beta_i^2$$

<u>Specificities :</u> Encourage small weights (but not zero) which helps keeping features when they are all important, keep the model smooth and reduce sensitivity to individual features.

Lasso regression

Defintion : Same situation as before but instead we use L1-regularization:

$$\text{loss} = \text{MSE} + \lambda \sum_i |\beta_i|$$

Specificities : Encourage weights to zero (sparsity), useful for feature selection, model robust when only few features matter.

Ridge and Lasso regressions
- Practice -

<u>Exercice :</u> Still on the data set `load_diabetes`, train and test Ridge and Lasso regression models and cross-validate the best parameter to use for λ.

Logistic regression

Definition : The logistic regression method is not for regression tasks but for classification tasks. The idea is to start from logistic function defined as:

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

And we define t as a linear combination of the exploratory variable such as $t = \beta_0 + \beta_1 x_1$ inducing (probability x_1 belongs to class 1):

$$p(x) = \sigma(\beta_0 + \beta_1 x_1) = \frac{1}{1 + e^{-\beta_0 - \beta_1 x_1}}$$

Logistic regression

Then, we use the log-loss (or cross-entropy loss) to train the logistic regression model. It can be defined as follow :

$$\mathscr{L}(\beta) = -\frac{1}{n}\sum_{i=1}^{n}\left[y_i \log(p_i) + (1 - y_i)\log(1 - p_i)\right]$$

Remark : Commonly used when face to binary classification.

Multinomial logistic regression

We follow the same idea as logistic regression but we are facing multi-class target instead of binary, i.e. $y \in \{0,1,\ldots,K\}$ and we note:

$$p_{ij} = \mathbb{P}\left[y_i = j | x\right]$$

for each class $j \in \{0,1,\ldots,K\}$ and element $i = 1,\ldots,n$.

Generalised Additive Models (GAM)

Definition : In the steps of polynomial and interaction terms regression models, we generalised this notion with GAM:

$$\mathscr{L}(t) = -\frac{1}{n}$$

where f_i are smooth functions (such as splines).