Descriptive statistics

Applied Data Analysis (ADA) - May 2025

Nomades Advanced Technologies
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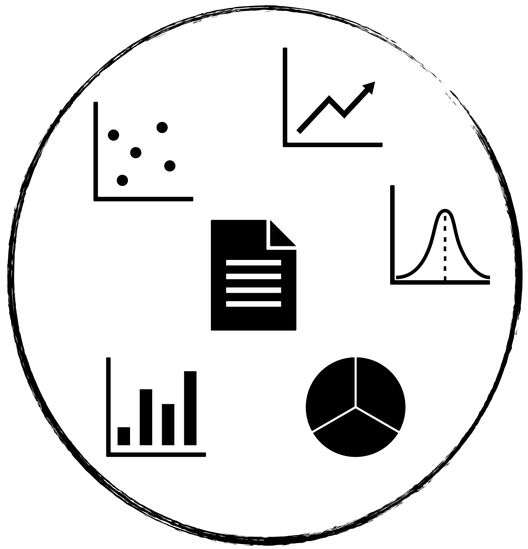
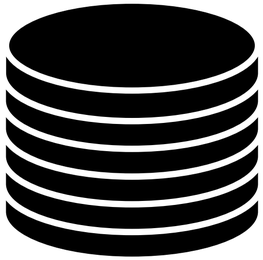
- Monday : Understand data structures
  - Population vs sampling
  - Central tendency measures
  - Dispersion measures
- Tuesday : Introduction to probability theory
- Wednesday : Central Limit Theorem confidence interval and test hypothesis
- Thursday : Feature selection and correlation matrix
- Friday : Statistics with scikit-learn

How a project is built ?



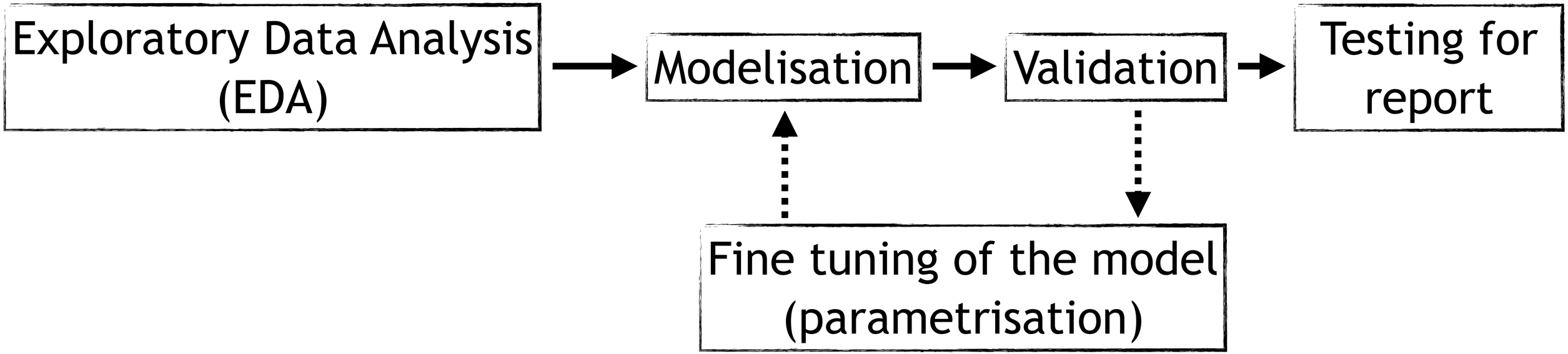
Raw data



Workable dataset

Modelisation, predictions, hidden informations, etc…

How a project is built ?



What’s descriptive statistics ?

Definition : descriptive statistics is about exploring and understanding a data set before going further into the modelisation.

Remark : Not the same as inferential statistics where we use a sample data set to make predictions on a larger population.

Different types of data

Unstructured

- Images
- Text
- Videos
- Time Series
- …

Structured

- Numerical values
  - Continuous
  - Categorical

Review on mean and median

1 - *Mean :* $\mu_X = \bar{X} = \frac{1}{n}\sum_{k=1}^{n} x_i$

`np.mean(x)`

2 - *Weighted mean :* $\bar{X} = \frac{1}{n}\sum_{k=1}^{n} w_i x_i$

`np.average(x, weights = w)`

3 - *Median :* $x_{\left[\frac{n}{2}\right]}$

`np.median(x)`

Review on variability measures

1 - *Variance :* $\text{Var}[X] = \sigma_X^2 = \frac{1}{n}\sum_{k=1}^{n}\left(x_i - \mu_X\right)^2$

`np.var(x)`

2 - *Standard deviation :* $\sigma_X = \sqrt{\text{Var}[X]}$

`np.std(x)`

3 - *Covariance :* $\text{Cov}(X, Y) = \mathbb{E}\left[(X - \mu_X)(Y - \mu_Y)\right]$

`(Exercice)`

4 - *Correlation :* $\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

`(Exercice)`

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`np.cov(X)`

4 - *Correlation :* $\rho_{X,Y} = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$

`np.corrcoef(X)`

Key for analysis is Visualisation
- See you data -

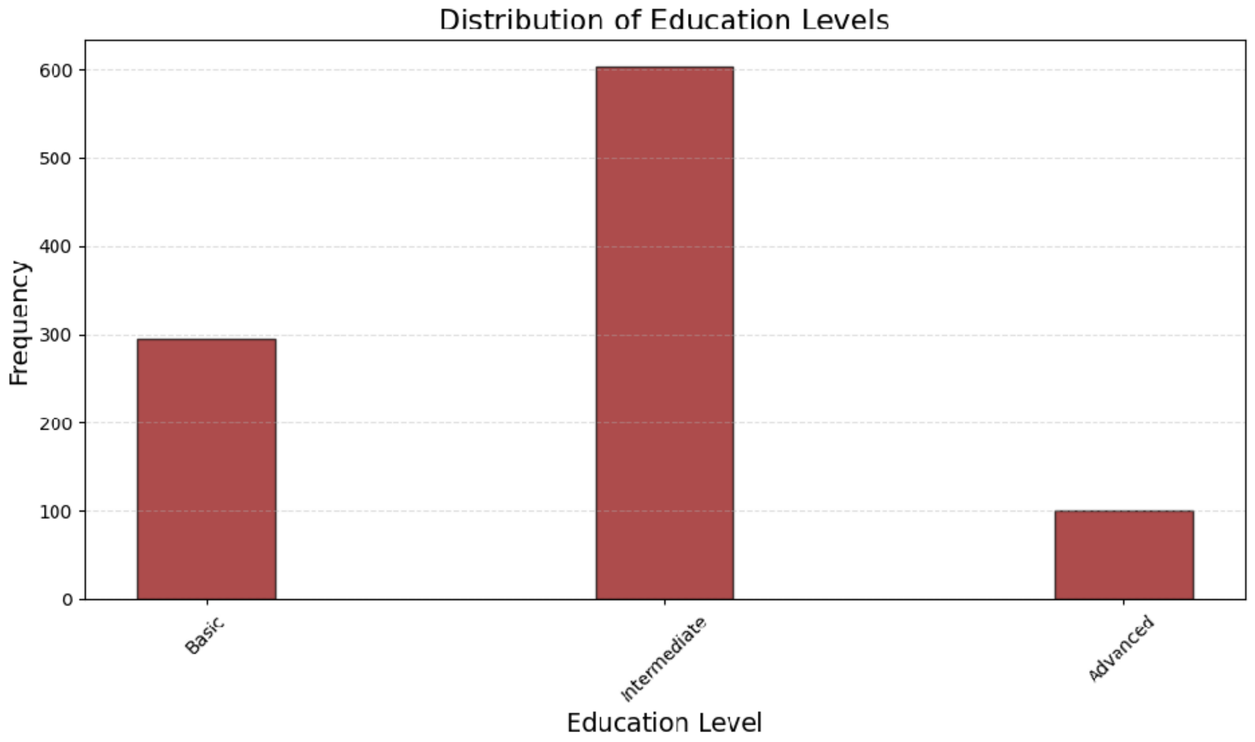
df.describe() is your friend
when you first see a data frame

ID of the person	Age	Family size	Education level	Annual revenue [CHF]
0	21	“No family“	Intermediate	141 475
1	22	“No family“	Intermediate	68 479
2	48	Large	Basic	129 630
3	52	“No family“	Intermediate	159 280
4	62	Small	Basic	83 903
5	78	“No family“	Basic	39 281
6	25	“No family“	Intermediate	77 452
7	12	Small	Intermediate	358 865
8	53	Medium	Advanced	95 682

Key for analysis is Visualisation
- Univariate analysis -

1 - Univariate analysis for categorical variables

Bar plots

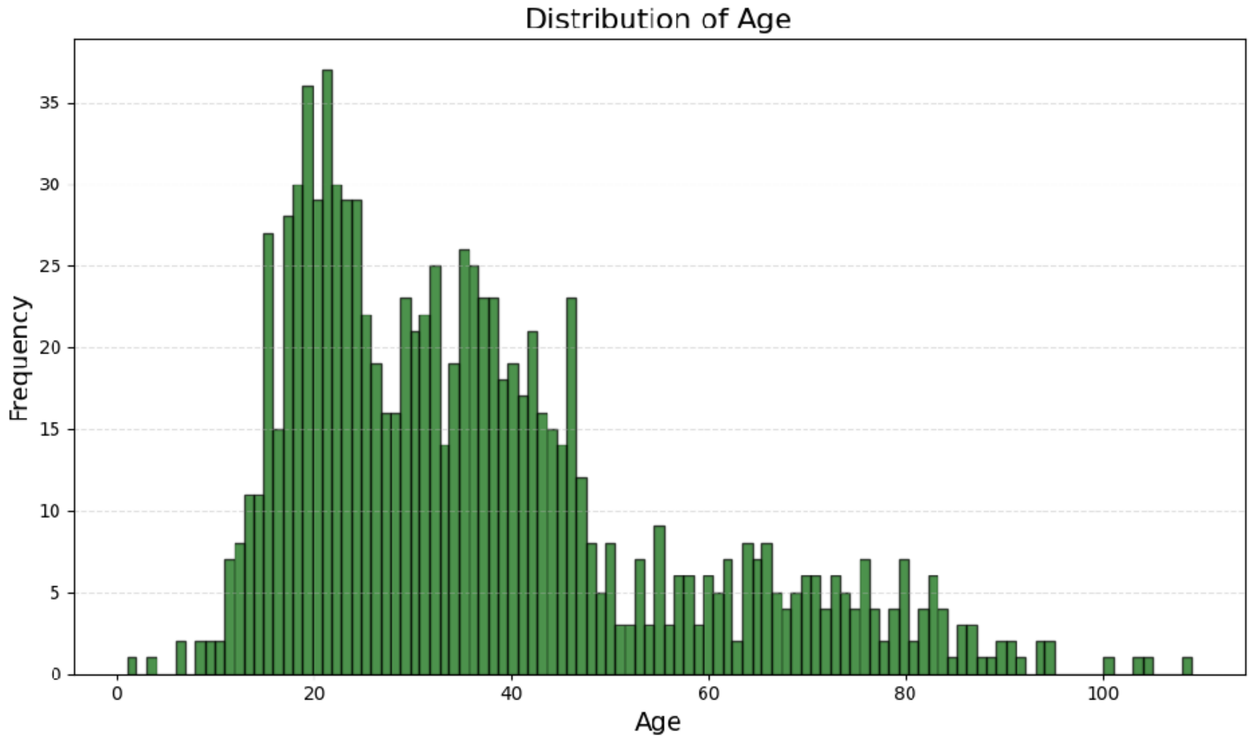


References

Key for analysis is Visualisation
- Univariate analysis -

2 - Univariate analysis for continuous variables

Histograms



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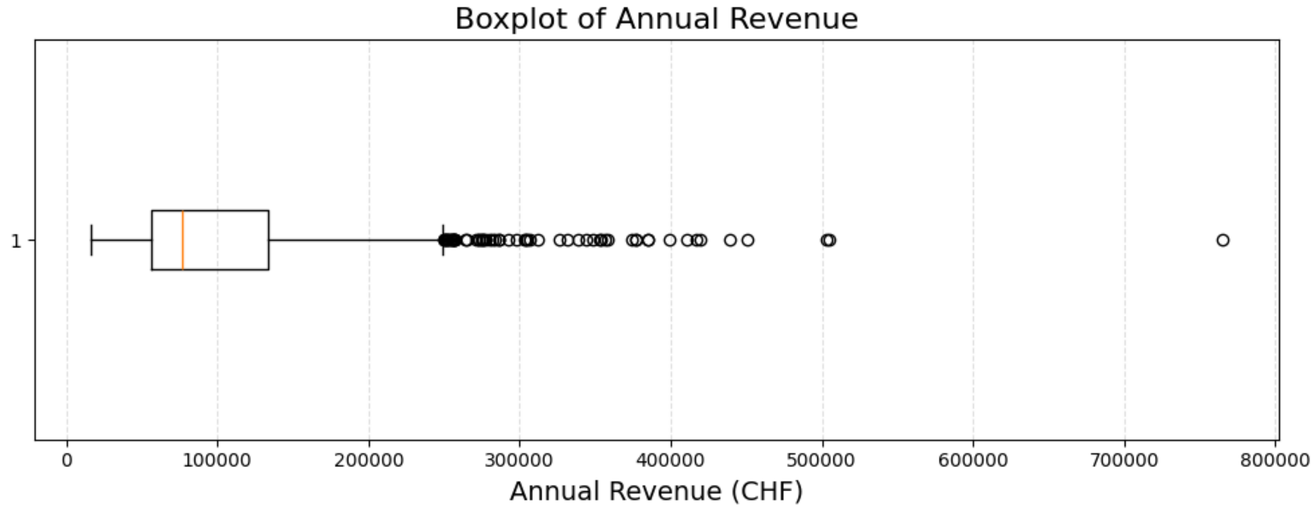
Boxplots



References

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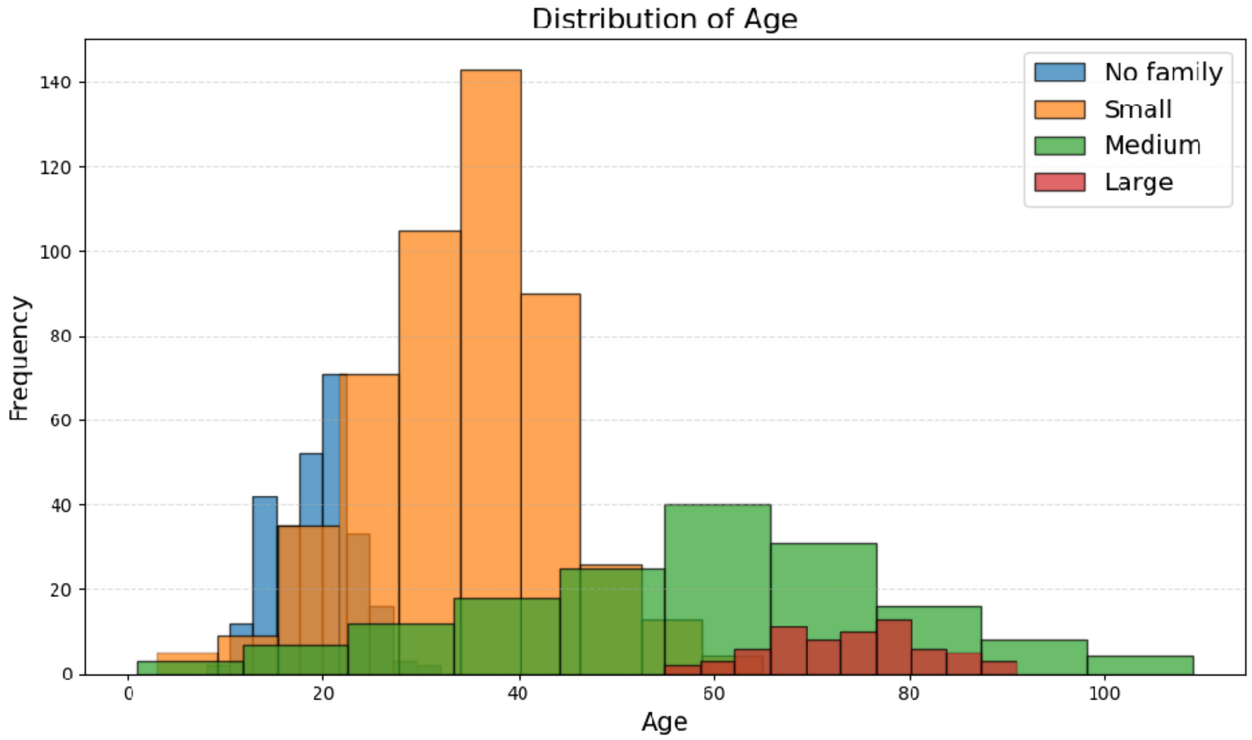
- Multivariate analysis -

4 - Multivariate analysis for continuous and/or categorical variables

Multivariate



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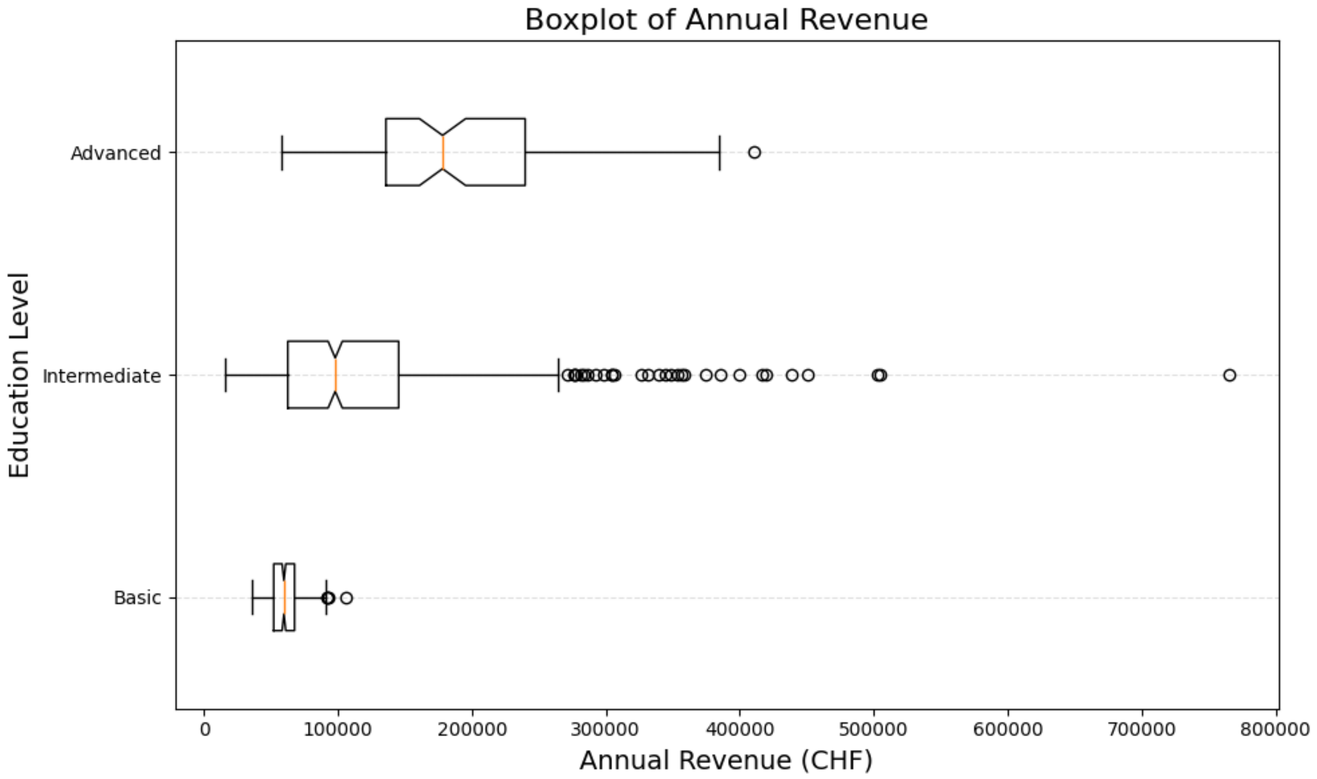
Bivariate boxplots



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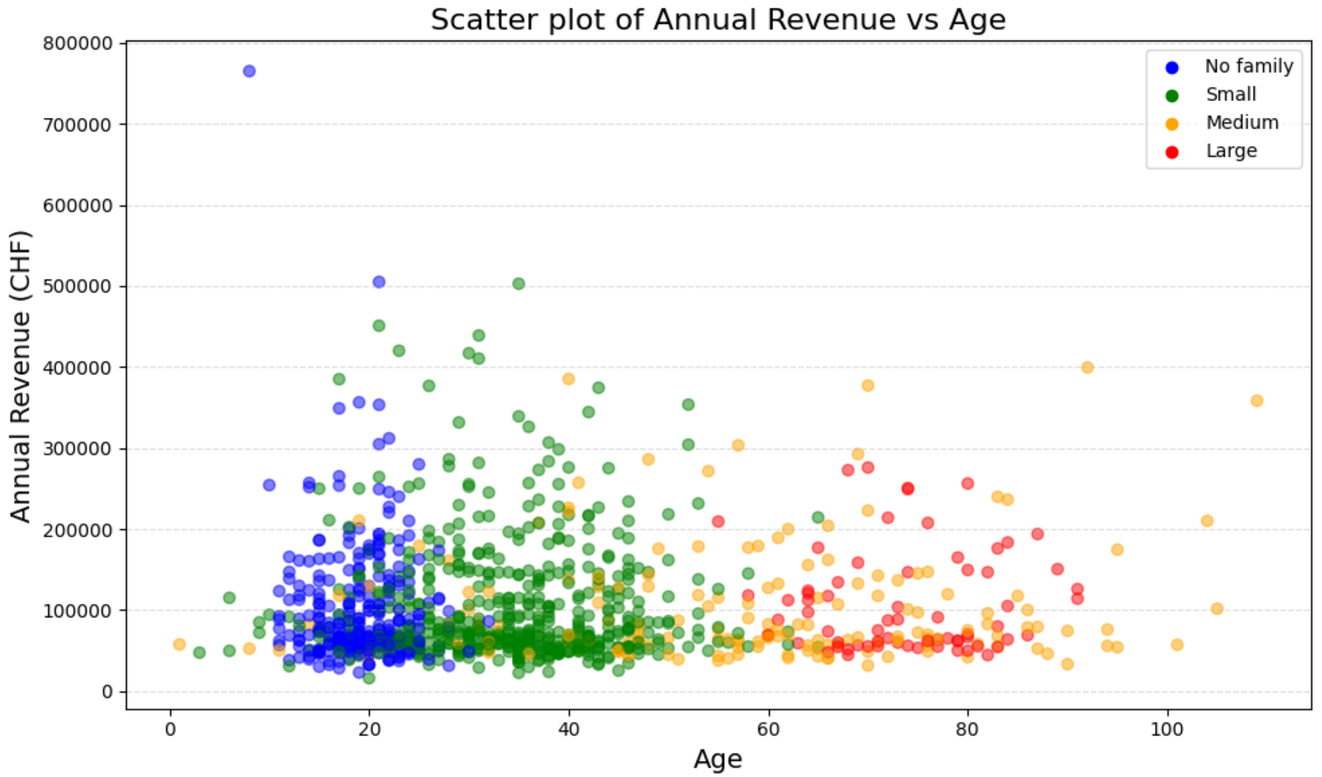
Colored scatter plots



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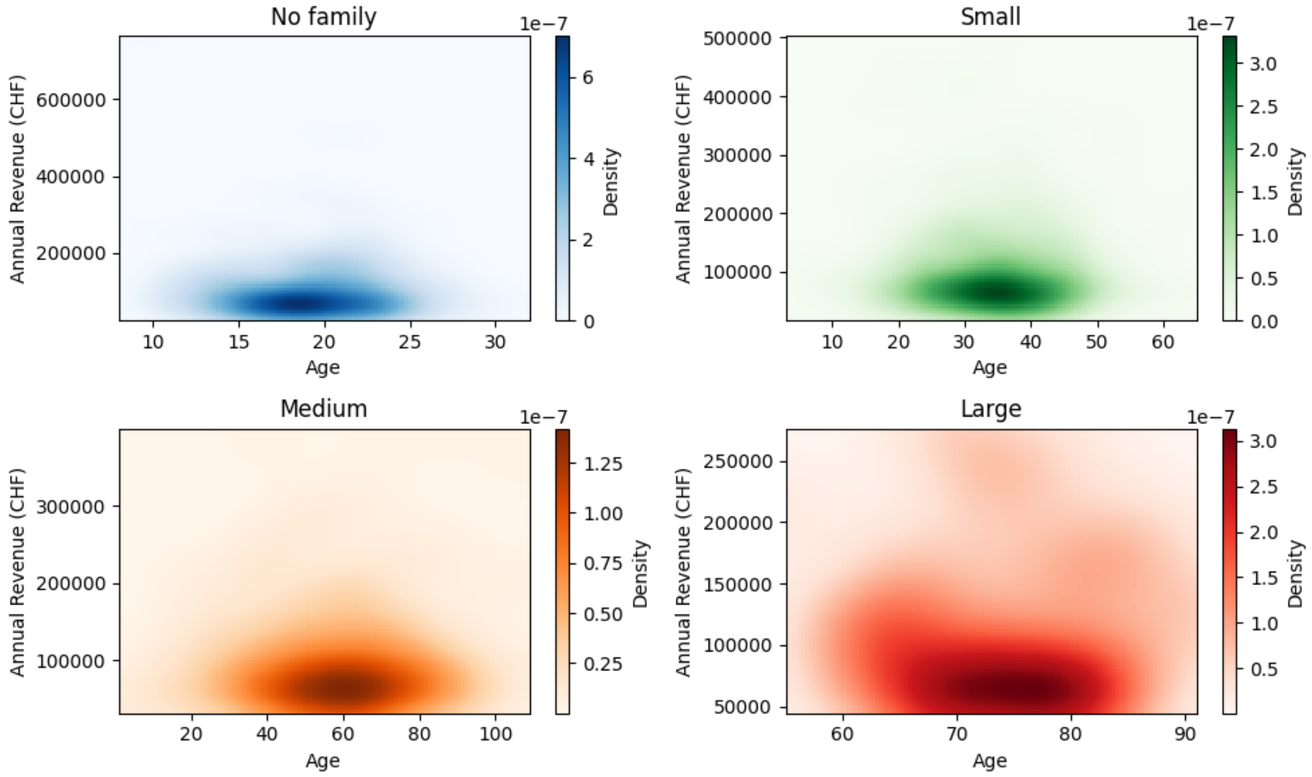


References

Data cleaning

Real-world datasets are generally never perfectly ready for use. Therefore, you often need to modify them to make them usable. Here are some steps:

- Remove duplicates / absurd data
- One-hot encoding
- Normalisation of continuous data
- Synchronisation of time series
- Manage outliers
- Missing data (be careful of bias)

Remove duplicates

When working with multiple datasets from various sources, you might encounter duplicated data. Here are some key steps :

- Clearly identify each sample in the dataset
- Remove the samples that contain the least information or have the lowest quality assurance
- Warning : Samples can be very similar but still different! Be careful not to remove too many

One-hot encoding

For the feature / attribute “Family size”, how would you tell your program whether you have no family or a large family ?

One-hot encoding

For the feature / attribute “Family size”, how would you tell your program whether you have no family or a large family ?

Idea 1 : Assign a class value, e.g., No family = 0, Small = 1, Medium = 2, Large = 3.

Idea 2 : Use one-hot encoding! Add a column for each class and set the value to 1 if the sample belongs to that class, and 0 otherwise.

One-hot encoding

ID of the person	Age	Family size	Education level	Annual revenue [CHF]
0	21	“No family“	Intermediate	141 475
1	22	“No family“	Intermediate	68 479
2	48	Large	Basic	129 630

ID of the person	Age	Family No Family	Family Small	Family Medium	Family Large	Education Basic	Education Intermediate	Education Advanced	Annual revenue [CHF]
0	21	1	0	0	0	0	0	1	141 475
1	22	0	1	0	0	0	1	0	68 479
2	48	0	0	0	1	1	0	0	129 630

Normalisation of continuous data set

Without normalisation, we may encounter the following issues:

- ✳ Large values of some features may have more importance during the modelisation
- ✳ Convergence and reliability of the models may be impacted during the process

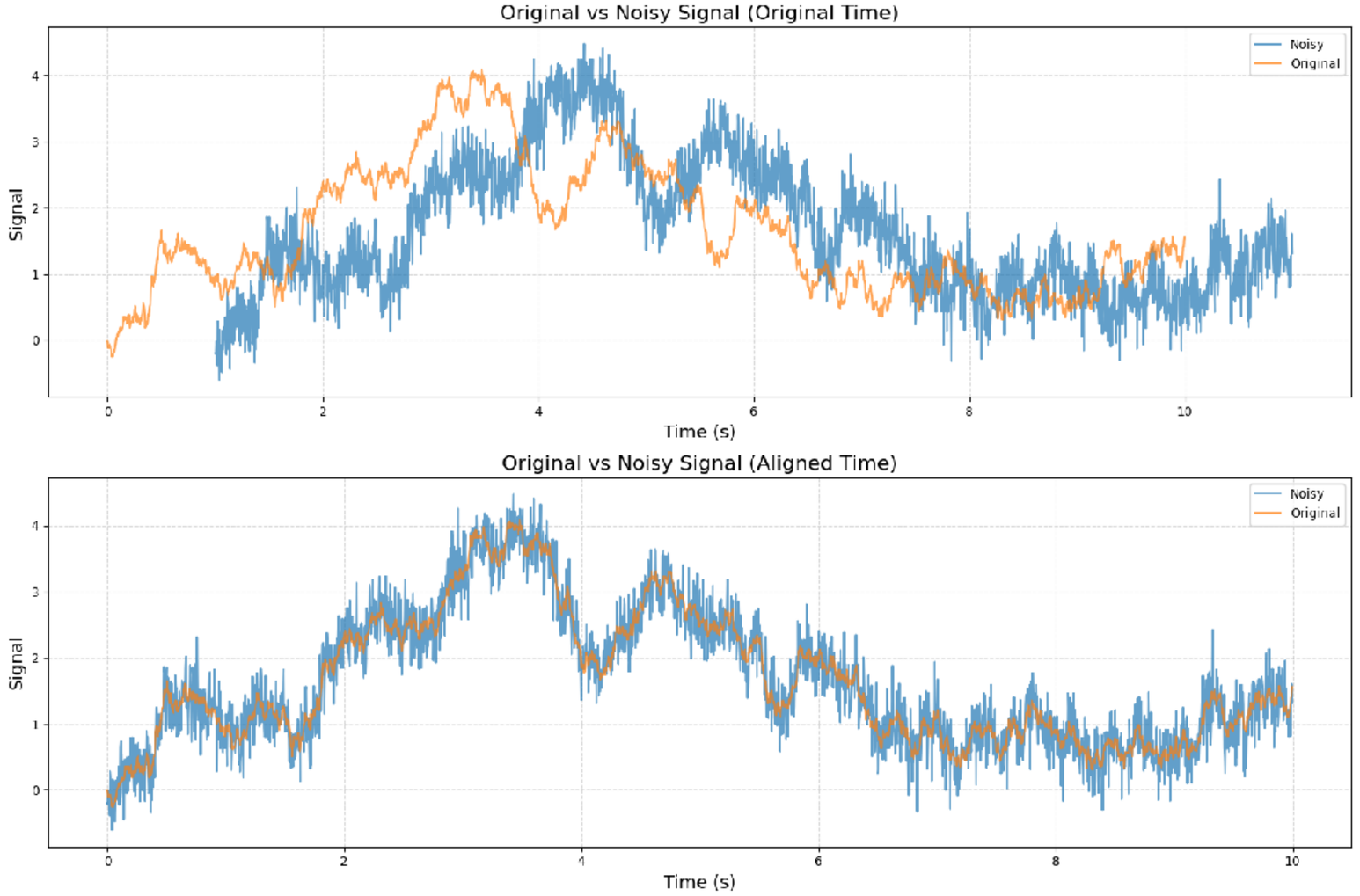
Some normalisation formulas

1 - *Min-max normalisation :* $\tilde{x}_i = \dfrac{x_i - \min(X)}{\max(X) - \min(X)}$

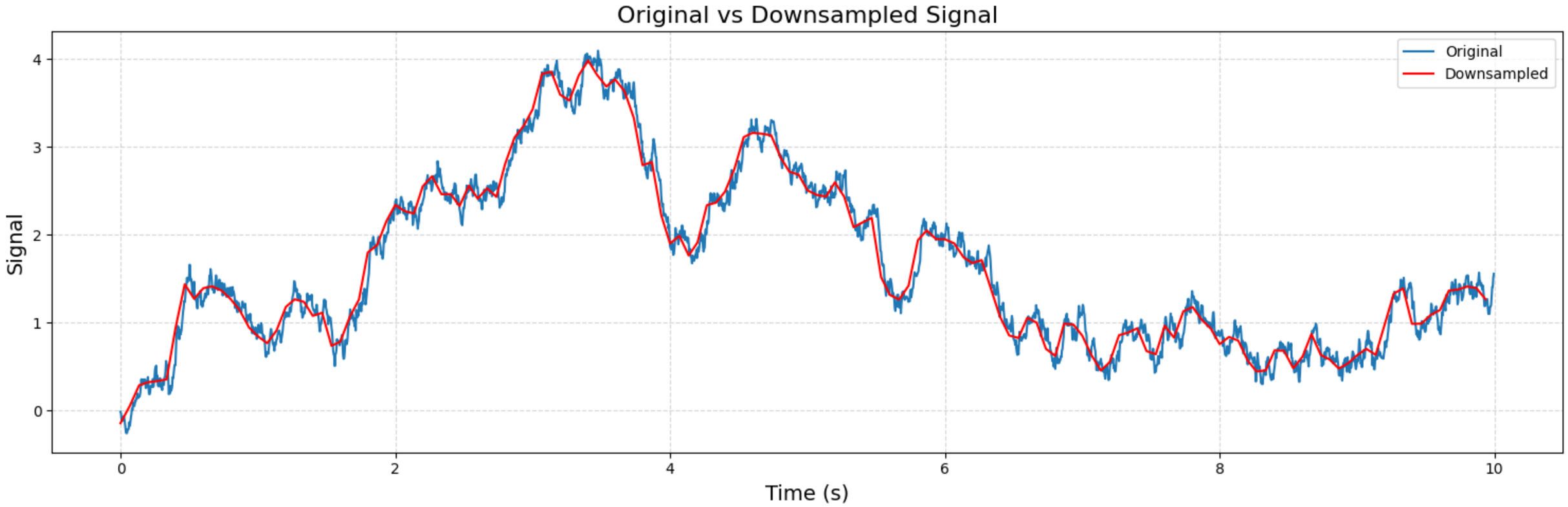
2 - *Standardisation :* $\tilde{x}_i = \dfrac{x_i - \mu}{\sigma}$

3 - *log-normalisation :* $\tilde{x}_i = \log(x_i + 1)$ or $\tilde{x}_i = \log(x_i)$

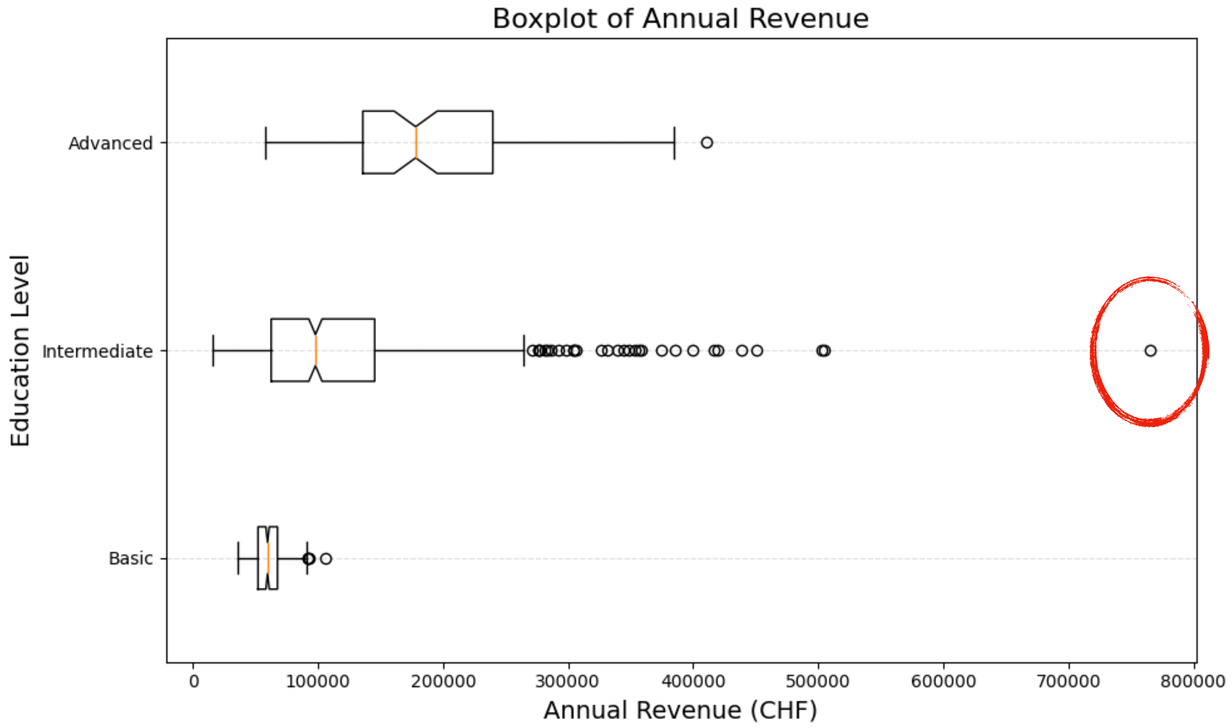
Synchronisation of time series



Re-sampling of time series



Manage outliers



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Missing data

ID of the person	Age	Family size	Education level	Annual revenue [CHF]
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2	48	Large	Basic	- None -
3	52	“No family“	Intermediate	159 280
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6	25	- None -	Intermediate	- None -
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Exercice for tomorrow

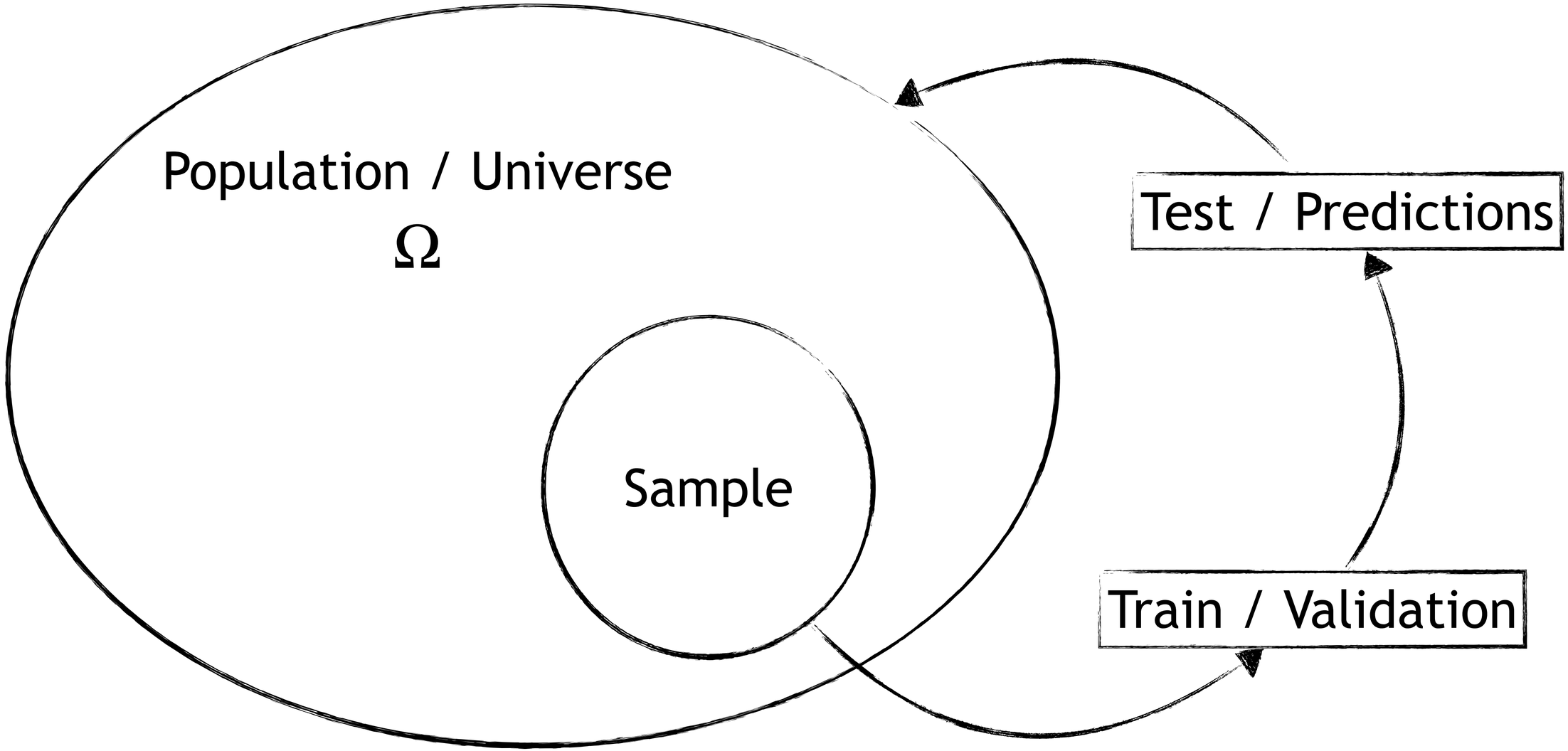
The idea is that you get a new data set of the houses there is in Boston. Multiple features are extracted in this data set such as:

- Garden size
- Distance from downtown
- Surface area
- Number of floors
- Type of walls [concrete, bricks, wood]
- Presence of a pool
- Estimated price

Exercice : Explore the features with the tools presented in the slides.

- Monday : Understand data structures
- Tuesday : Introduction to probability theory
  - Fundamental definitions
  - Probability law / distribution (discrete)
  - Discrete vs continuous probability
- Wednesday : Central Limit Theorem confidence interval and test hypothesis
- Thursday : Feature selection and correlation matrix
- Friday : Statistics with scikit-learn

Introduction to probability theory



Some definitions

- **Universe :** denoted as Ω, it represents the complete set of all possible elements or outcomes you are studying.

- **Sample :** it represents a subset of the universe, selected for analysis.

- **Event :** it represents a set of outcomes from an experiment.

Probability of an event

Definition : Let Ω be a finite sample space (set of all possible outcomes) and $A \in \Omega$ an event, then the probability $\mathbb{P}[A]$ is defined as follow :

$$\mathbb{P}[A] = \frac{\text{Number of favorable outcomes for A}}{\text{Total number of outcomes in } \Omega} = \frac{|A|}{|\Omega|}$$

Some practice

Excercice : You have a shuffled deck of 52 cards and you randomly pick one card. What is the probability of drawing a card with an even number (excluding face cards) ?

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Excercice : You have a shuffled deck of 52 cards and you randomly pick one card. What is the probability of drawing a card with an even number (excluding face cards) ?

Solution : You have to pick either 2, 4, 6, 8 and 10 (4 colours each), meaning you have 20 possible cards:

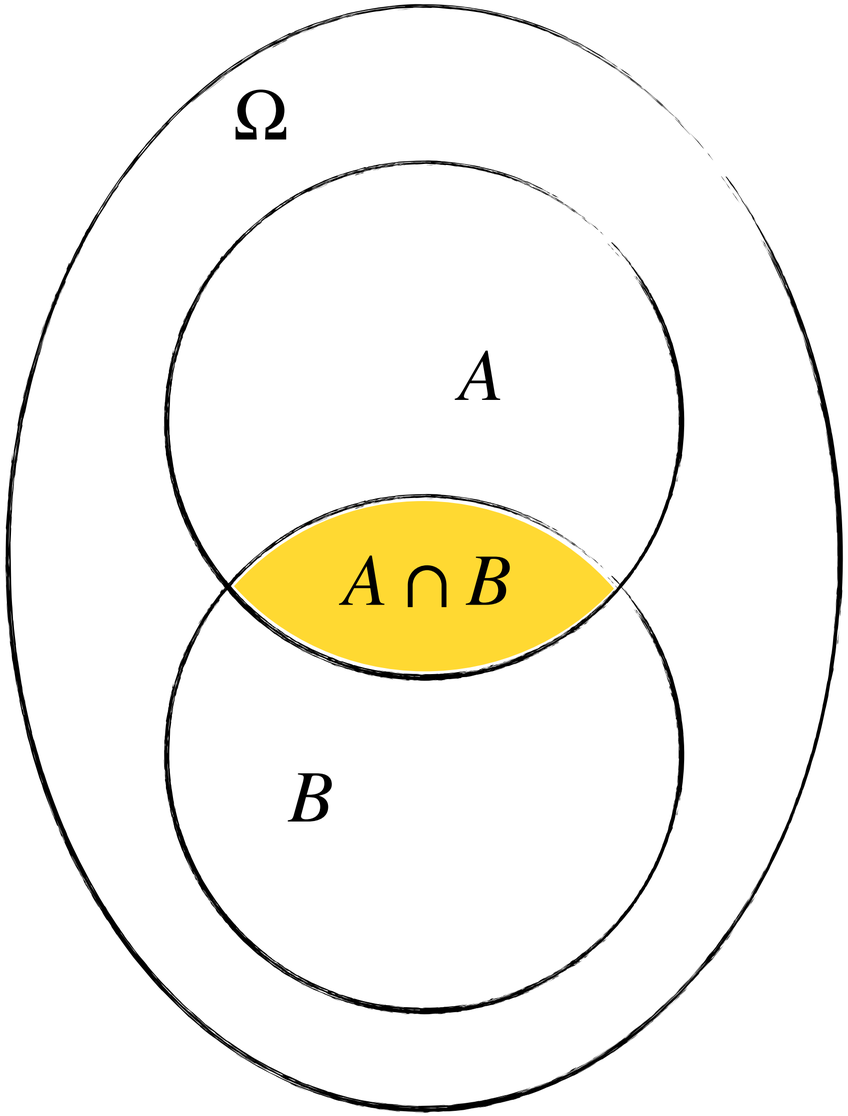
$$\mathbb{P}\left[\text{Pick even card}\right] = \frac{20}{52} \approx 0.38$$

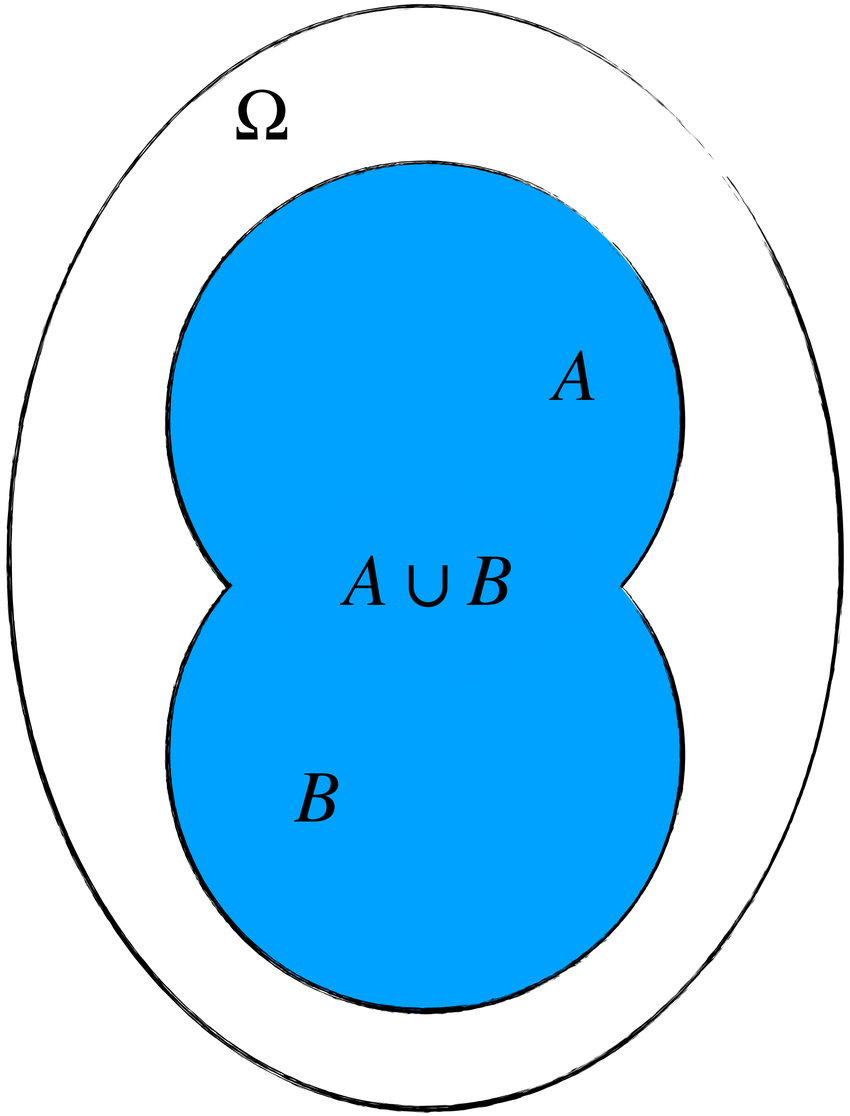
Some properties

Property : Let Ω be a finite sample space, then we have the following properties:

- $\mathbb{P}[\Omega] = 1$ and $\mathbb{P}[\varnothing] = 0$
- for all event $A \in \Omega$, $0 \leq \mathbb{P}[A] \leq 1$
- $\sum_{i=1}^{n} \mathbb{P}[A_i] = 1$ where $\Omega = \left\{ A_i \,|\, i = 1, \ldots, n \text{ and } A_i \cap A_j = \varnothing \text{ for } i \neq j \right\}$

Visualisation of set theory





And more properties

Property : Let Ω be a finite sample space and $A, B \in \Omega$ two events such that $A \cap B = \varnothing$, then we have the following :

$$\mathbb{P}[A \cup B] = \mathbb{P}[A] + \mathbb{P}[B]$$

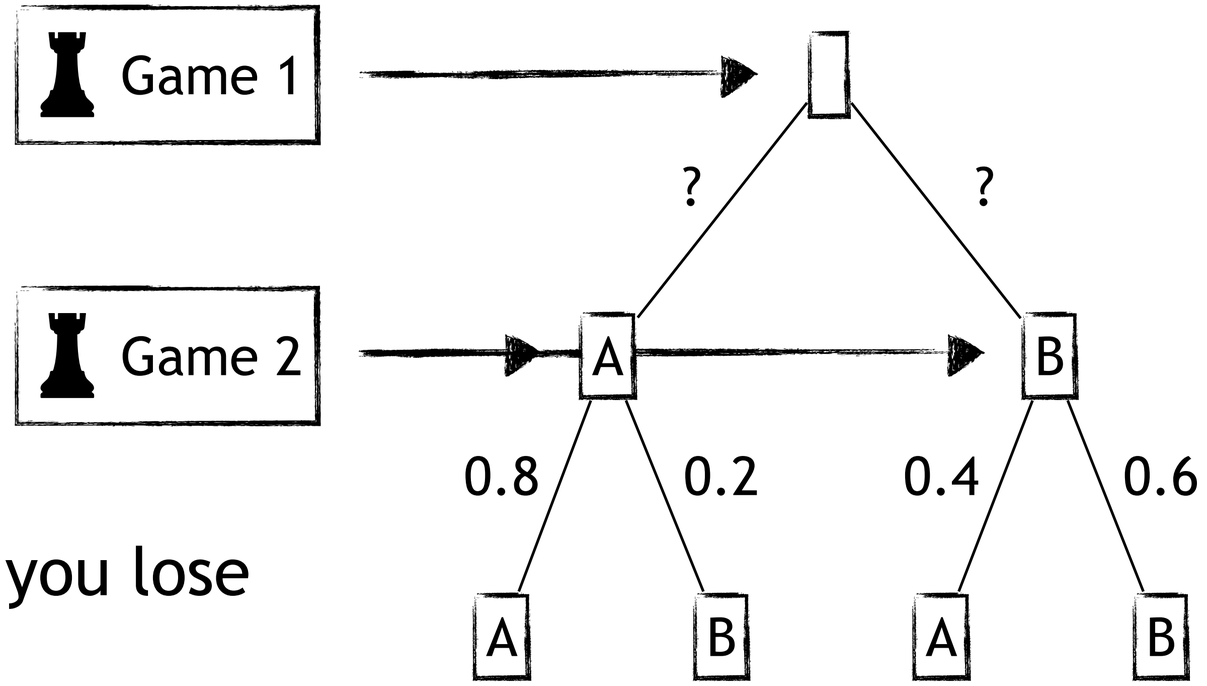
Bayes' theorem : Let Ω be a finite sample space and $A, B \in \Omega$ two events such that $\mathbb{P}[B] > 0$, then we have the following :

$$\mathbb{P}[A \mid B] = \frac{\mathbb{P}[A \cap B]}{\mathbb{P}[B]}$$

Let’s play chess !

You play a chess game against me and let’s define the following events :

- Event A : you win the game.
- Event B : I win the game.



Exo 1 - What is the probability for you to win a game after a lose ?

Exo 2 - Your probability to win both games is 0.56, what is the probability you lose the first game ?

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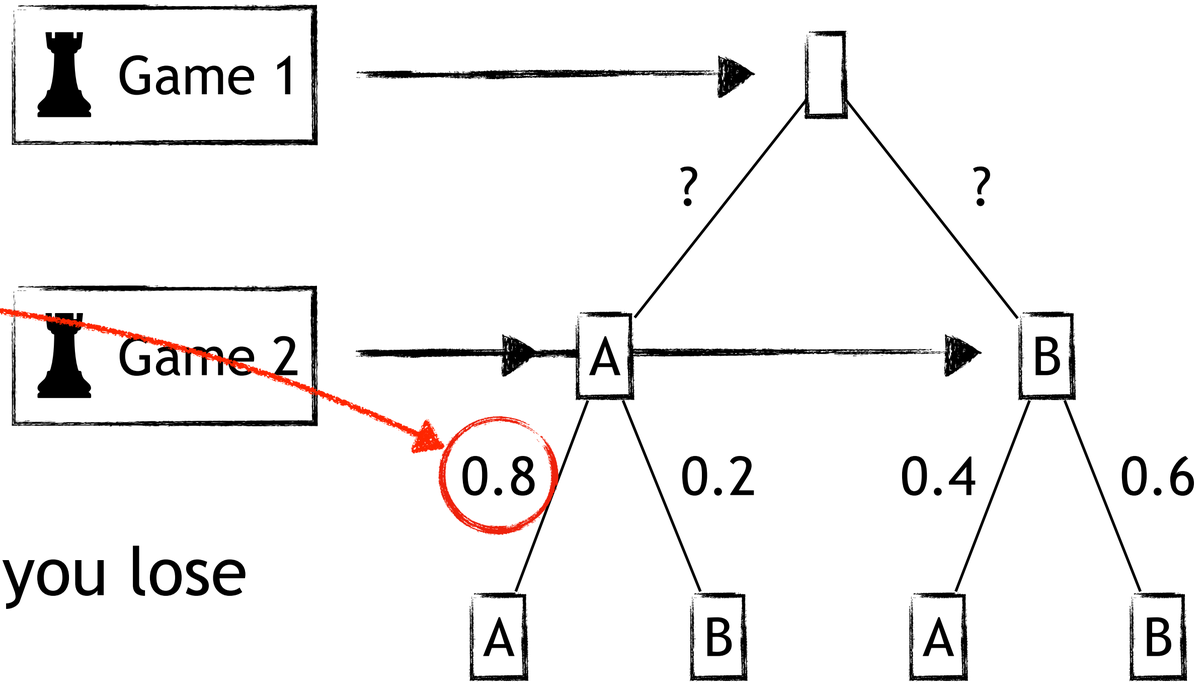

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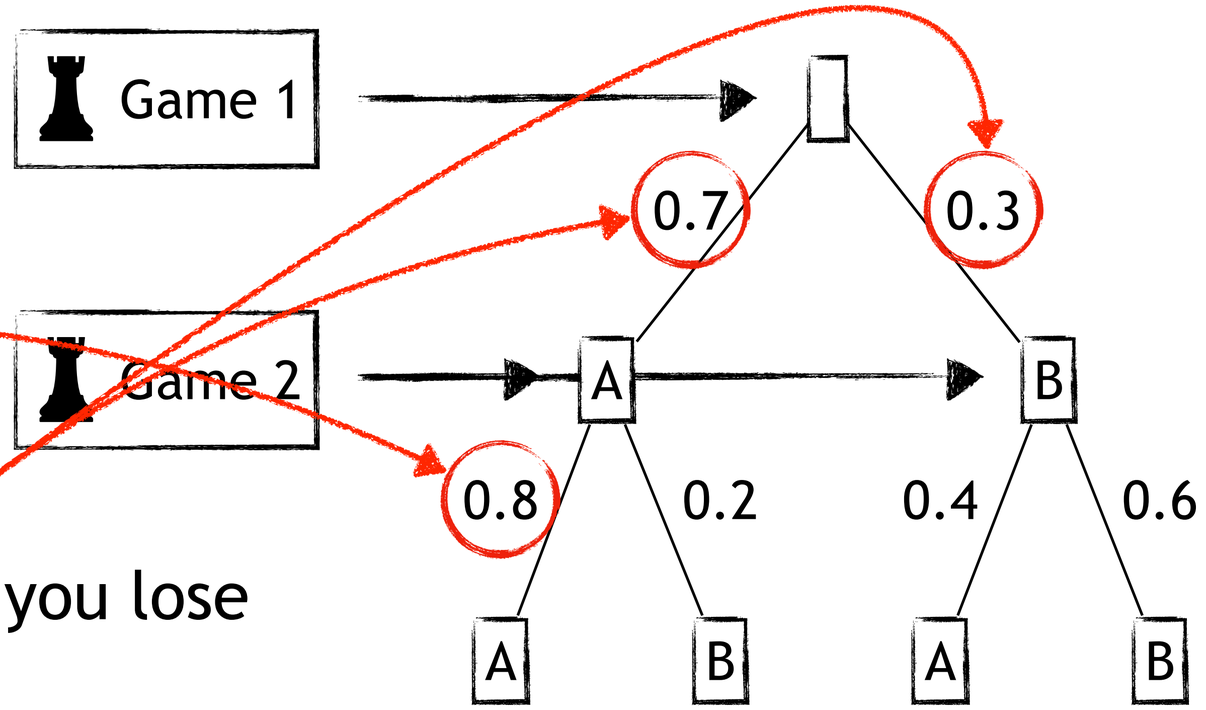
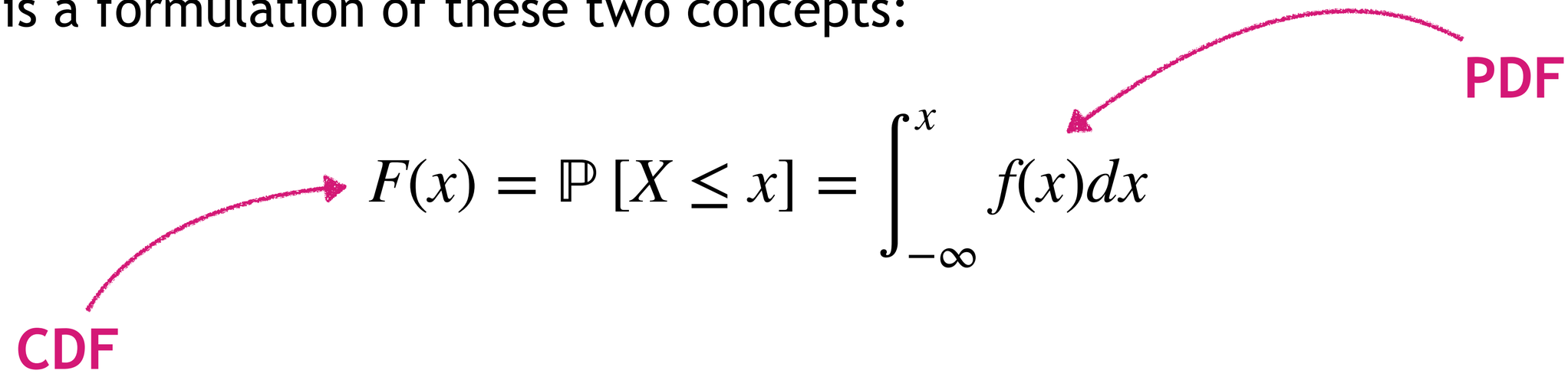


Last slide of theory !

In probability theory, there are two main concepts called the Cumulative Distribution Function (**CDF**) and the Probability Density Function (**PDF**), which describe how a random variable is theoretically defined.

Here is a formulation of these two concepts:



Bernoulli distribution

Let’s take again the chess example:

- Event A : You win a game with p = 0.7
- Event B : You lose a game with q = 1 - p = 0.3

$$F(x) = \begin{cases} \mathbb{P}[A] = p \\ \mathbb{P}[B] = q = 1 - p \end{cases}$$

References

Binomial distribution

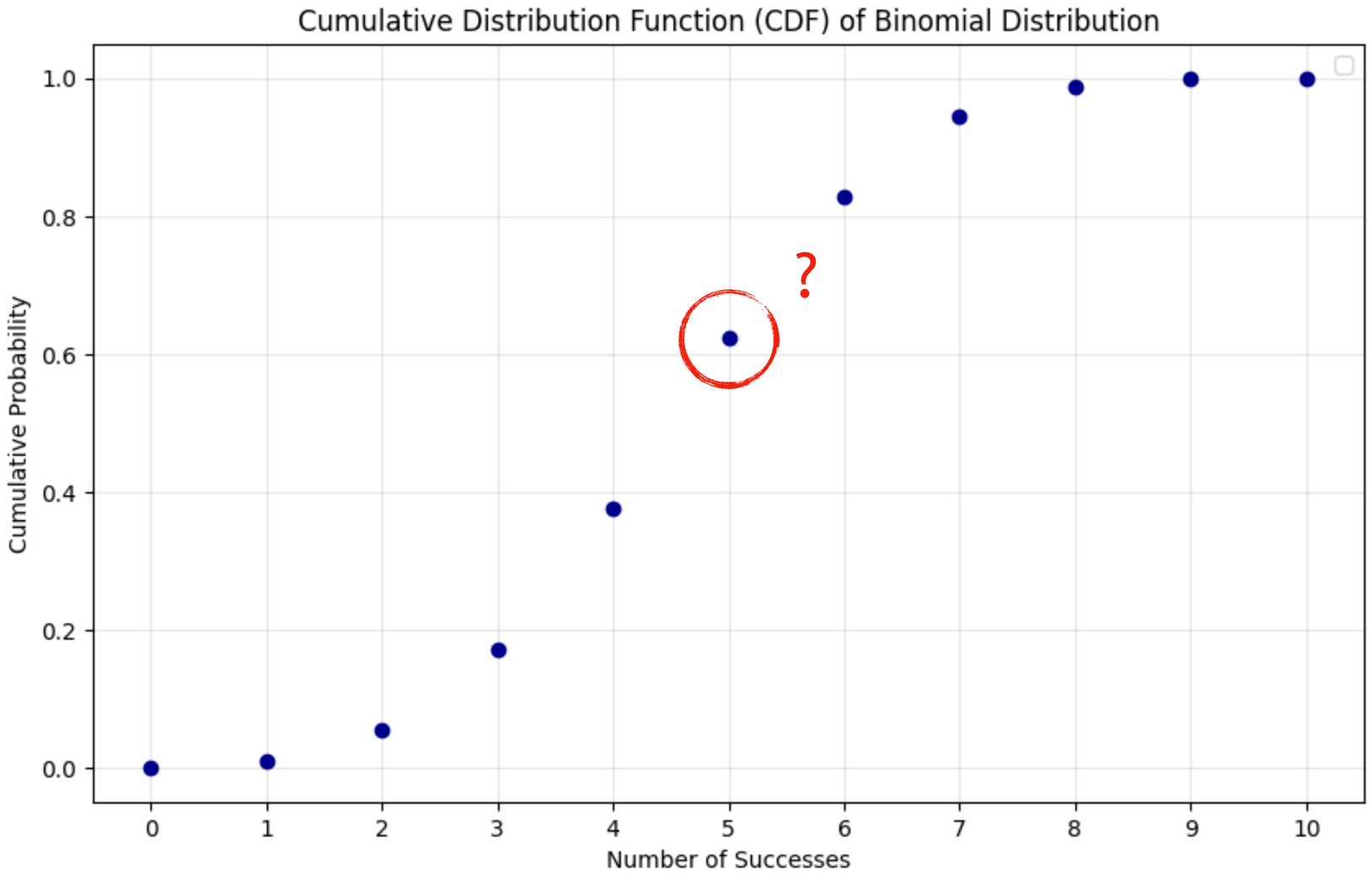
Let’s take again the chess example and you play n games against me with the same probability after each game. What’s the probability you win k times ?

Binomial distribution

Let’s take again the chess example and you play n games against me with the same probability after each game. What’s the probability you win k times ?

$$\mathbb{P}\left[\text{Win } k \text{ times}\right] = \binom{n}{k} p^k (1-p)^{k-1}$$

Binomial distribution



Geometric distribution

Let’s suppose you don’t have the context: what kind of event X could be represented by the following geometric distribution ?

$$\mathbb{P}\left[X = k\right] = (1-p)^{k-1}p$$

Uniform distribution

Let’s consider the size of a randomly selected person in meters. For now, we consider a minimum at 1.60m and a maximum at 2.00m and there is an even chance to be in this range.

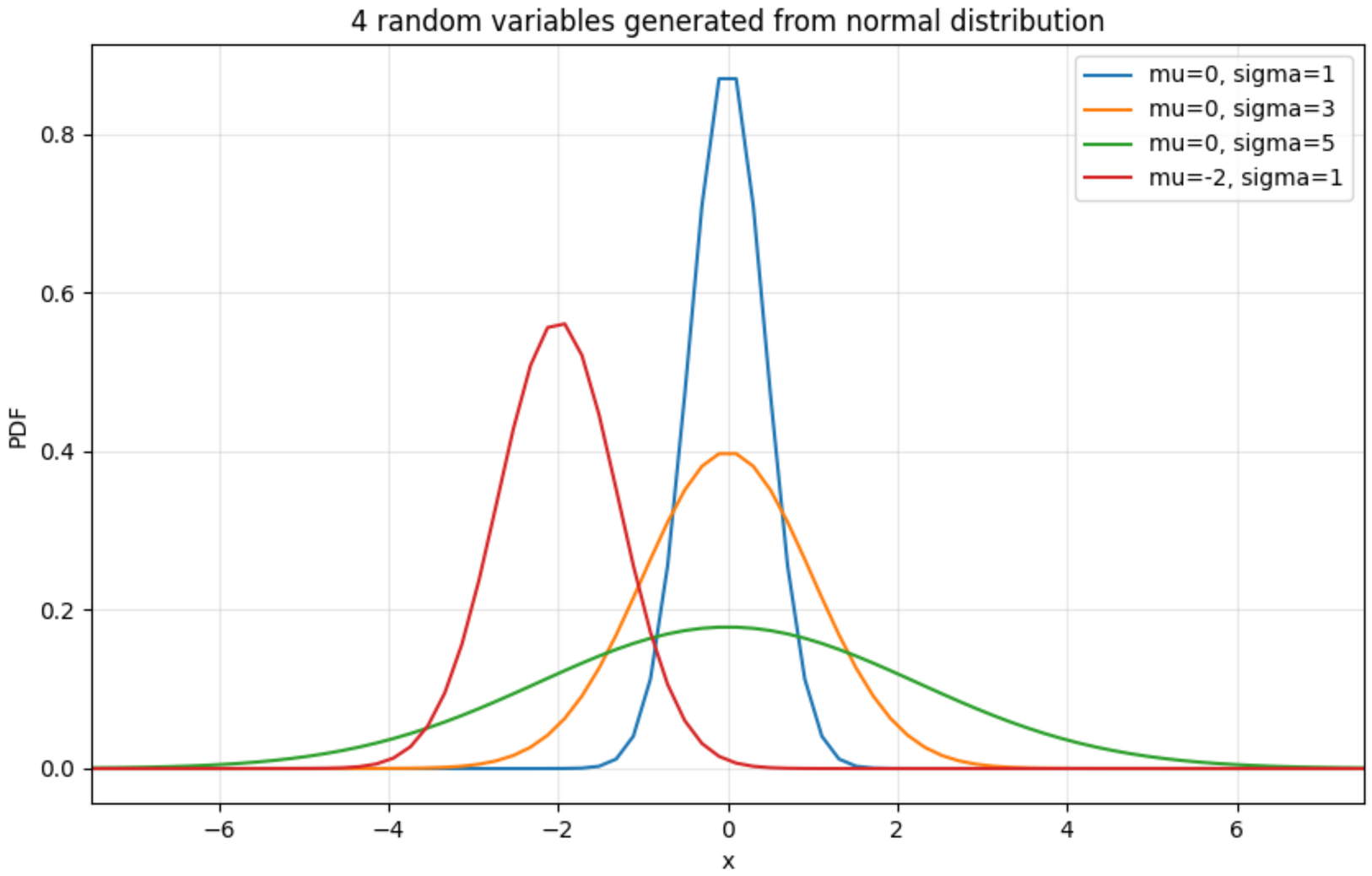
$$f(x) = \frac{1}{b-a}, \text{ for } x \in [a, b].$$

Normal distribution

The distance of employees' houses from the company's main office can be represented by a normal distribution.

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Normal distribution



Exponential distribution

You are waiting for a bus to arrive and in average you know that a bus pass every 10 minutes.

- ✦ What would be the CDF and the PDF ?
- ✦ What is the probability you wait less than 2 minutes before a bus pass ?

Exponential distribution

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- ✦ What would be the CDF and the PDF ?
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$$\mathbb{E}[X] = \frac{1}{\lambda}$$

$$F(x) = \mathbb{P}[X \leq x] = \begin{cases} 1 - e^{-\lambda x} & \text{, if } x \geq 0 \\ 0 & \text{, otherwise} \end{cases}$$

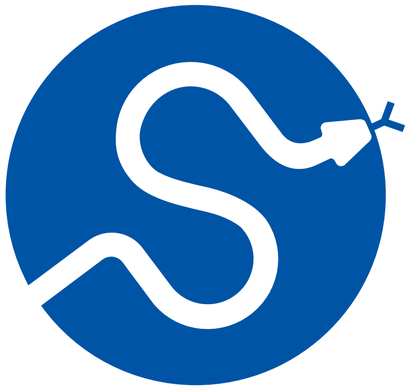
Introduction to SciPy

Name ? - SciPy stands for Scientific Python

What ? - Open source library built-on NumPy

Why ? - Gives access to powerful tools such as:

- Statistics
- Signal processing -> for time series processing
- Linear algebra
- Test hypothesis
- and more…



What are the main strengths of SciPy ?

- Easy-to-use scientific functions
- Optimised for performance and reliability
- Works perfectly with NumPy (and with pandas if you bridge your data through NumPy)
- Don’t waste time implementing functions yourself !
  - ✦ There’s a high probability that the function you want is already implemented in SciPy, NumPy or other libraries…

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  - Central Limit Theorem
  - Confidence interval
  - Test hypothesis
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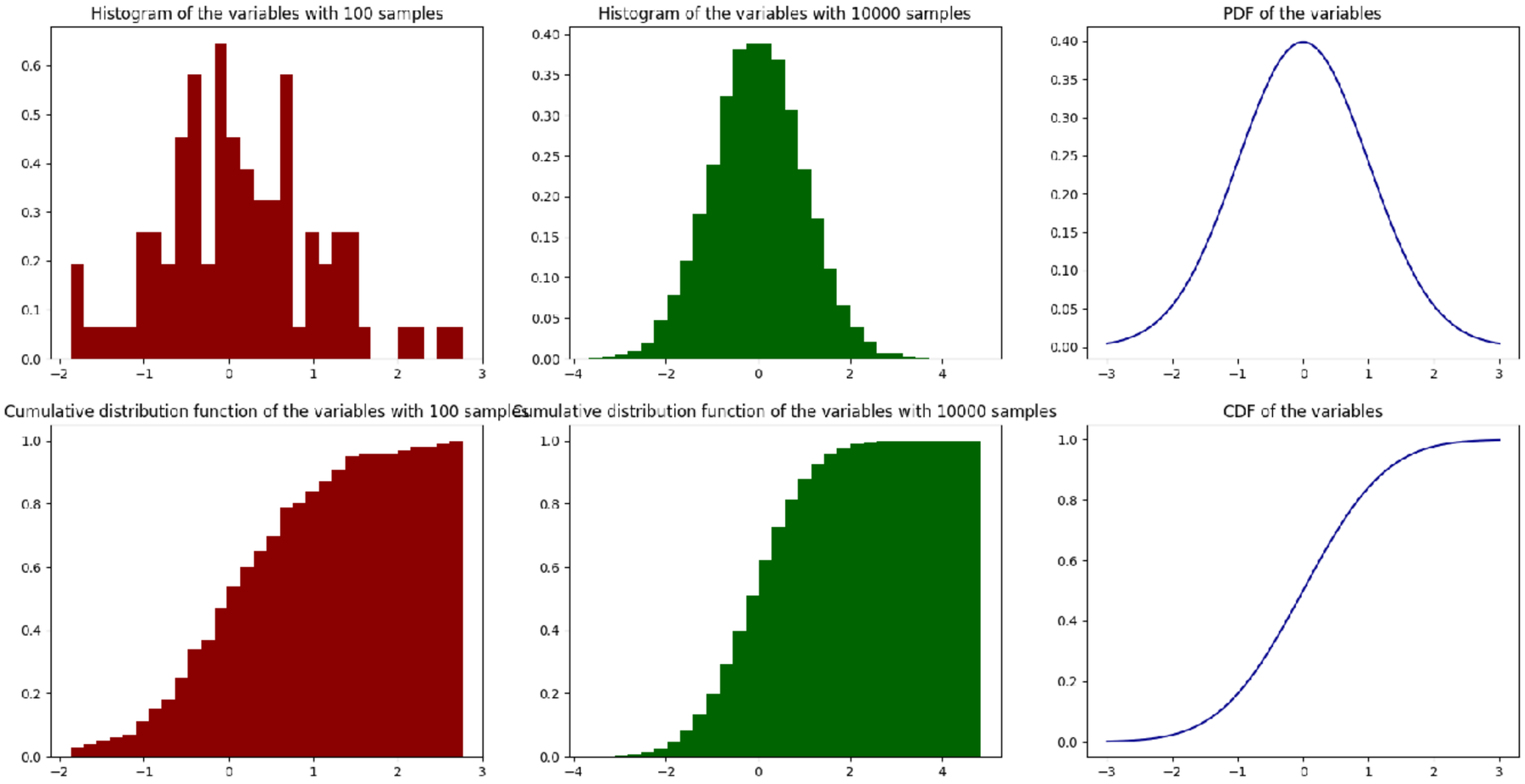
Small reminder on the difference between PDF and CDF

$$F(x) = \mathbb{P}\left[X \leq x\right] = \int_{-\infty}^{x} f(x)dx$$

Small reminder on the difference between PDF and CDF



Central Limit Theorem

Question 1: You have a fair 6-face dice and X represents the result after a roll of the dice as a random variable. What is the distribution of X ?

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Answer : a discrete uniform distribution ! There are two types of uniform distributions : continuous and discrete.

Central Limit Theorem

Still with the same fair dice, you roll it 5-times. So, you have 5 new random variables we note X_1, X_2, X_3, X_4 and X_5, which are all respectively discrete uniform distribution.

We note the average of the results as: $\bar{X} = \frac{1}{5}\sum_{i=1}^{5} X_i$.

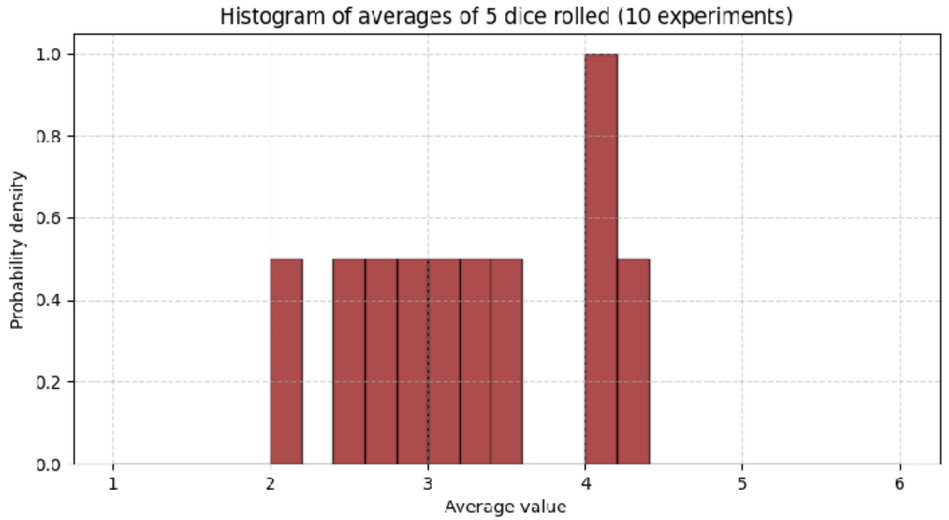
Question 2: Using the library SciPy, built a function that returns the average of the experiment describe above.

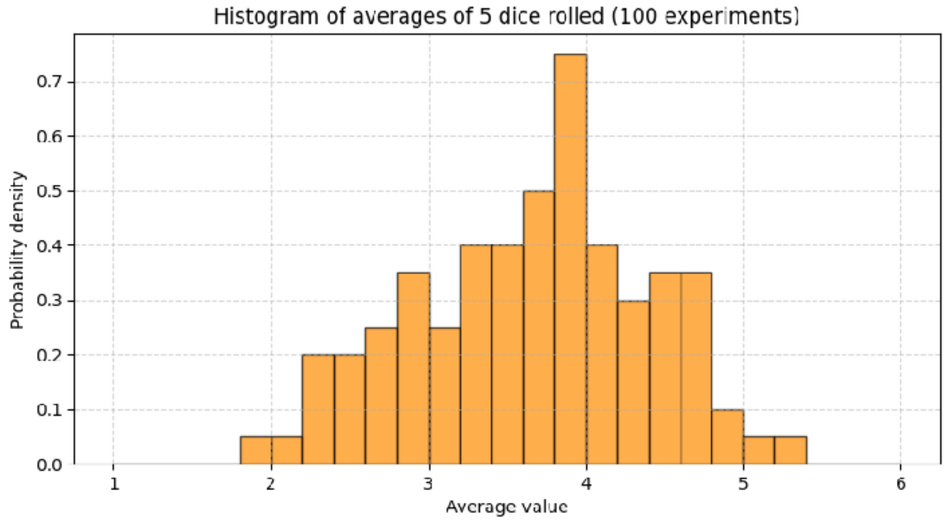
Central Limit Theorem

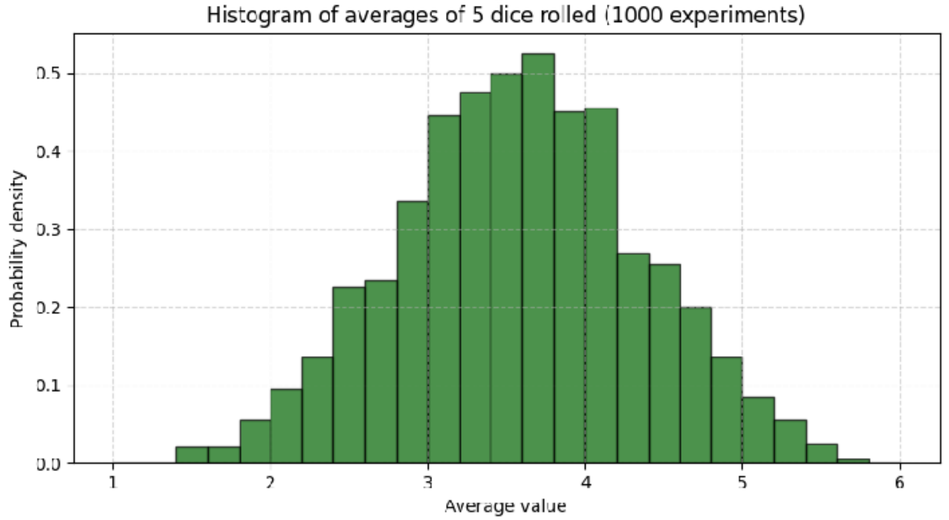
Now, you have a new random variable we note $\bar{X}$, corresponding to the average result of rolling a fair dice 5 time (uniform distribution).

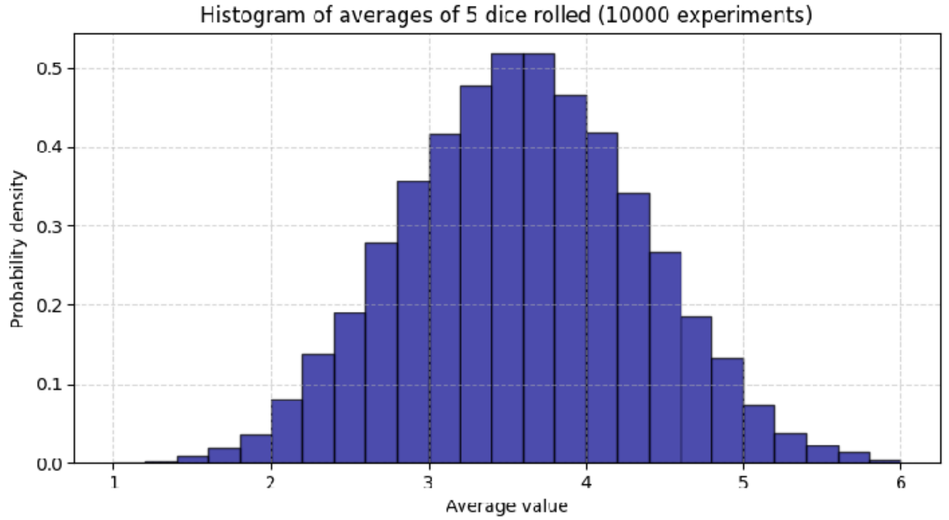
Question 3: What happens if we repeat this operation 10 times? 100 times? 1,000 times? 10,000 times?

Central Limit Theorem









Central Limit Theorem

Central Limit Theorem (CLT) : Let $X_1, X_2, \ldots, X_n$ be independent and identically distributed (i.i.d.) random variables, each with mean μ and finite variance σ^2. Then, as n becomes large, the following normalized random variable:

$$Z_n = \sqrt{n}\frac{\bar{X}_n - \mu}{\sigma}$$

converges in distribution to a standard normal distribution (i.e. centrée et réduite), where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$.

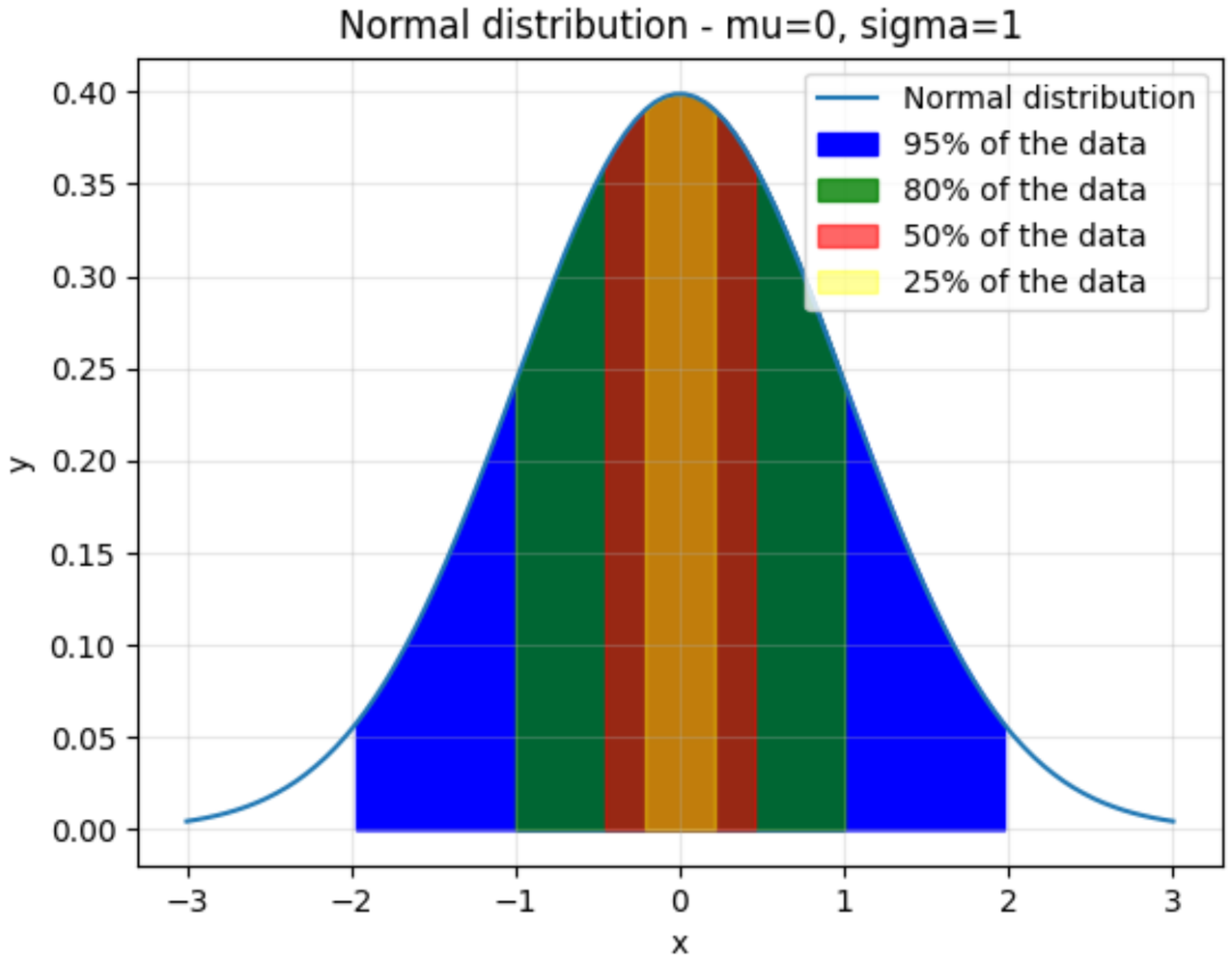
Confidence intervals

Let's recall the formula $Z_n = \sqrt{n}\dfrac{\bar{X}_n - \mu}{\sigma}$ and we know by the CLT that this distribution is standard normal.

If we note z_κ the value such that κ % of the possible values Z are less than z_κ, then we have:

$$\mathbb{P}\left[-z_{\alpha/2} \leq Z_n \leq z_{\alpha/2}\right] \approx 1 - \alpha$$

Confidence intervals



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Confidence intervals

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$$\mathbb{P}\left[-z_{\alpha/2} \leq \sqrt{n}\frac{\bar{X}_n - \mu}{\sigma} \leq z_{\alpha/2}\right] \approx 1-\alpha$$

$$\mathbb{P}\left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right] \approx 1-\alpha$$

Confidence intervals

Definition : Following the concepts introduced before, we define a $1-\alpha\,\%$ confidence interval $C_{1-\alpha}$ for an estimator of μ such as:

$$C_{1-\alpha} = \left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right],$$

assuming σ is known and n is large enough to let the CLT apply.

Confidence intervals

Interpretation : « We are at $1-\alpha\,\%$ confident that the true population mean μ lies within the interval $C_{1-\alpha}$. »

$$C_{1-\alpha} = \left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$$

Confidence intervals

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Question : How do you compute the z-score ?

Confidence intervals

Interpretation : « We are at $1 - \alpha\,\%$ confident that the true population mean μ lies within the interval $C_{1-\alpha}$. »

$$C_{1-\alpha} = \left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right]$$

Question : How do you compute the z-score ?

Answer : Use SciPy ! (clue: norm.ppf)

Confidence intervals

Question : Now you know how to compute your z-score, use the formula below to compute the $90\,\%$, $95\,\%$ and $99\,\%$ confidence intervals of the distribution `normal_ci.csv` knowing that the standard deviation is equal to 6.

$$C_{1-\alpha} = \left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right]$$

Confidence intervals

The formula presented before suppose we know exactly the parameter of the standard deviation σ, which is not case in practice.

What is the solution ?

Confidence intervals

The formula presented before suppose we know exactly the parameter of the standard deviation σ and n is large enough, which is not always the case in practice.

What is the solution ?

=> Use an estimator of the standard deviation with the sample standard deviation s you can compute directly from the data you have.

Confidence intervals

By using an approximation of the standard deviation, you can no longer use the normal distribution to determine the confidence interval.

You have to use the Student’s distribution.

Confidence intervals

You want the Student's distribution CDF and PDF ?

$$f(t) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{\nu\pi}\,\Gamma\left(\frac{\nu}{2}\right)}\left(1+\frac{t^2}{\nu}\right)^{-\frac{\nu+1}{2}}$$

$$F(t) = 1 - \frac{1}{2}I_{\frac{\nu}{\nu+t^2}}\left(\frac{\nu}{2},\frac{1}{2}\right) \quad \text{for } t \geq 0$$

Confidence intervals

Question : Now you know how to compute your z-score, use the formula below to compute the $90\,\%$, $95\,\%$ and $99\,\%$ confidence intervals of the distribution `exponential_ci.csv` using the student's distribution because you don't know the standard deviation σ.

$$C_{1-\alpha} = \left[\bar{X}_n - z_{\alpha/2}\frac{\sigma}{\sqrt{n}}; \bar{X}_n + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \right]$$

Test hypothesis

Null hypothesis : It's a baseline assumption, usually there is no effect or no difference.

$$H_0 : \mu = \mu_0$$

Alternative hypothesis : What we want to test for, typically that there is an effect or a difference.

One sided - $H_1 : \mu \neq \mu_0$

Tow sided - $H_1 : \mu > \mu_0$ *or* $\mu < \mu_0$

Test hypothesis

The test statistic presented before (normal and student's) measures how far the sample mean is from the hypothesized population mean under H_0, in units of standard error.

This test allows us to decide wether you reject or not the hypothesis H_0.

Now, how to compute the test statistic under the hypothesis H_0 ?

=> We need to talk about p-values.

p-values

Definition : The p-value is the probability, under the null hypothesis H_0, of observing a test statistic as extreme as or more extreme than the value actually obtained in the sample.

$$p\text{-value} = \mathbb{P}[\text{Test statistic} \geq \text{observed}] \qquad \text{(one-tailed)}$$

$$p\text{-value} = 2\mathbb{P}[\text{Test statistic} \geq |\text{observed}|] \qquad \text{(two-tailed)}$$

p-values

Example : In a z-test using the z-score presented before, the p-value is computed as follow:

$$p\text{-value} = 2\mathbb{P}\left[Z > |z_{\text{obs}}|\right]$$

We then compare this p-value to a significance level α we have chosen (typically 0.05).

p-values

For the comparison we have the following situations:

- p-value < α : reject H_0
- p-value > α : fails to reject H_0, not enough evidence to contradict

Interpretation : The smaller the p-value, the stronger the evidence against H_0.

p-values vs confidence intervals

- A **confidence level** $1 - \alpha$ (e.g., 95%) defines the range within which we expect the true parameter to lie.
- If the **null value** μ_0 lies **outside** the confidence interval, the **p-value will be less than** $\alpha \rightarrow$ reject H_0.
- Conversely, if μ_0 is **inside** the confidence interval, the p-value will be **greater than** $\alpha \rightarrow$ fail to reject H_0.

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  - Correlation matrix
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