Univerza *v Ljubljani*
Fakulteta *za elektrotehniko*

Computer Vision
05-Color

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Quick recap of the previous lectures

- Geometric aspects of image formation
  - WHERE does the image appear on the sensor/film/screen
  - Camera geometry
  - Camera calibration
  - Camera geometry revisited with homogenous coordinates
- Photometric aspects of image formation
  - HOW bright is the image that appears on the sensor
  - Photometric lens equation
  - Lighting as important part of image formation process

- Obvious next topic: Color

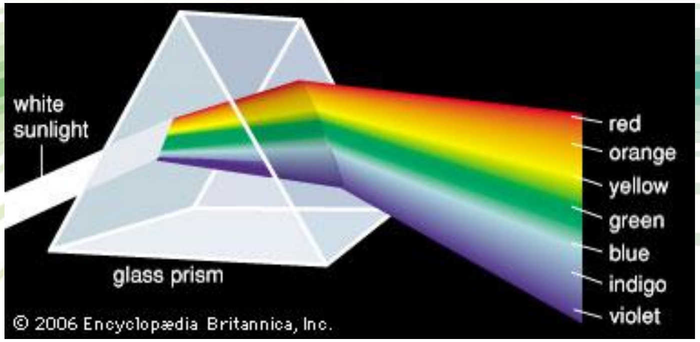
Outline

- Last 5 slides from the last presentation
  - Topic: structured light
  - Due to the computer problems the lecture was not finished
- Light spectrum
- Human eye
- Color, color perception and color spaces

Light

Light is electromagnetic radiation

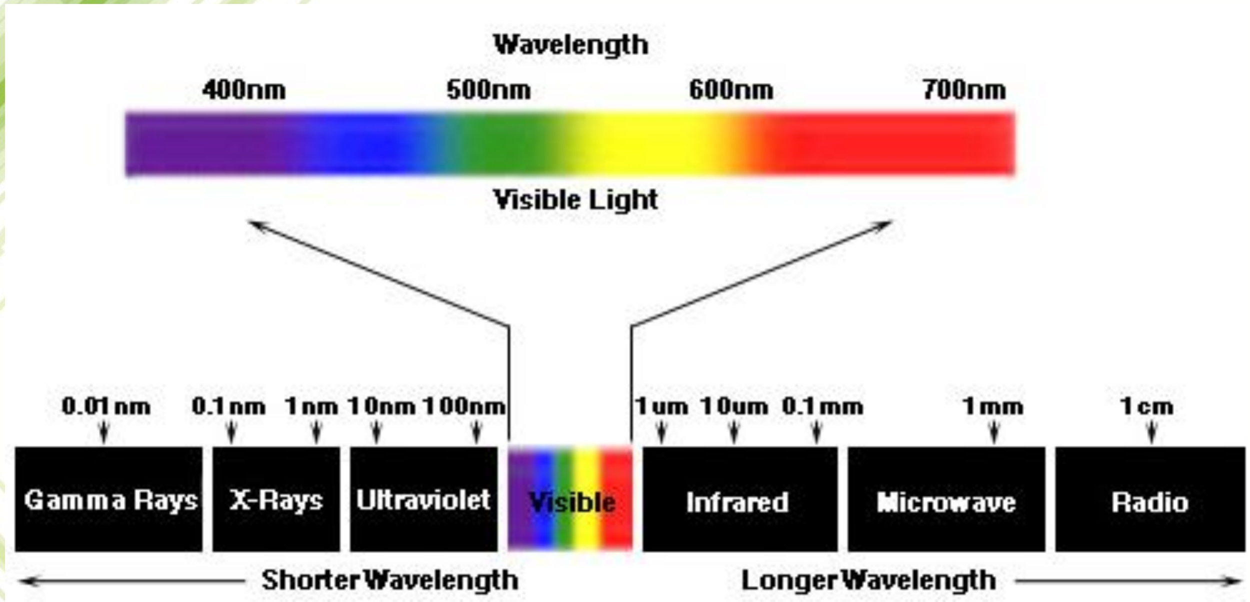
How does the prism work?



But colors only exist due to the human visual system!

Visible light

Vijolična	380-420	Violet
Modra	440-490	Blue
Zelena	490-560	Green
Rumena	560-590	Yellow
Oranžna	590-630	Orange
Rdeča	630-760	Red



Sources of light

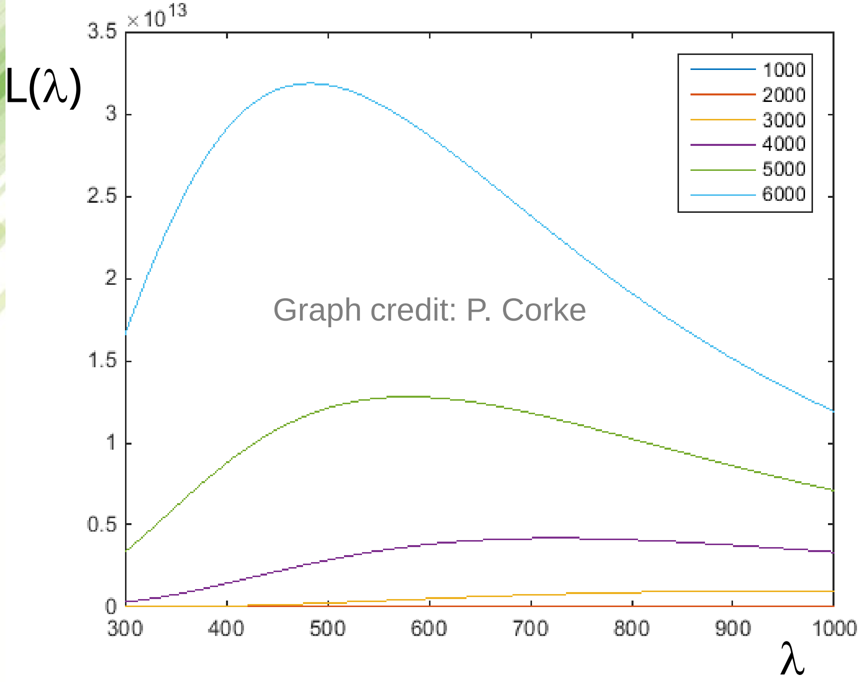
- Incandescence
  - Material emits light due to its temperature (tungsten lamp!)
- Electroluminescence
  - Semiconductor emits light under the influence of electrical field (LEDs)
- Fluorescence
  - Electrons or light excite material, the energy is finally released as visible light (could have multiple stages) – ("neon lamp")
- Chemiluminescence , Phosphorescence, Bioluminescence, etc.

Planck's law

Black body radiation!

$$L(\lambda,T)=\frac{2hc^2}{\lambda^5 e^{\frac{hc}{k\lambda T}}-1}\quad [\mathrm{W}\cdot\mathrm{sr}^{-1}\cdot\mathrm{m}^{-3}]$$

c = 2.998×10^{8} m/s
h = 6.626×10^{-34} Js
k = 1.381×10^{-23} J/K



λ - wavelenght
h – Planck constant
c – speed of light
k – Boltzmann constant
T – absolute temperature
L – power density in a given direction at given wavelenght, i.e., "spectral radiance"

Stefan-Boltzman law

Total energy radiated from a black body per unit of surface area:

$$L(T) = \sigma T^4 \quad [\frac{\mathrm{W}}{\mathrm{m}^2}]$$

$\sigma = 5.670373 * 10^{-8}$ W m^2 K^4

(σ = Stefan-Boltzmann constant)

T – absolute temperature

L – total power density

Example:

T (sun surface) ≈ 5700 K

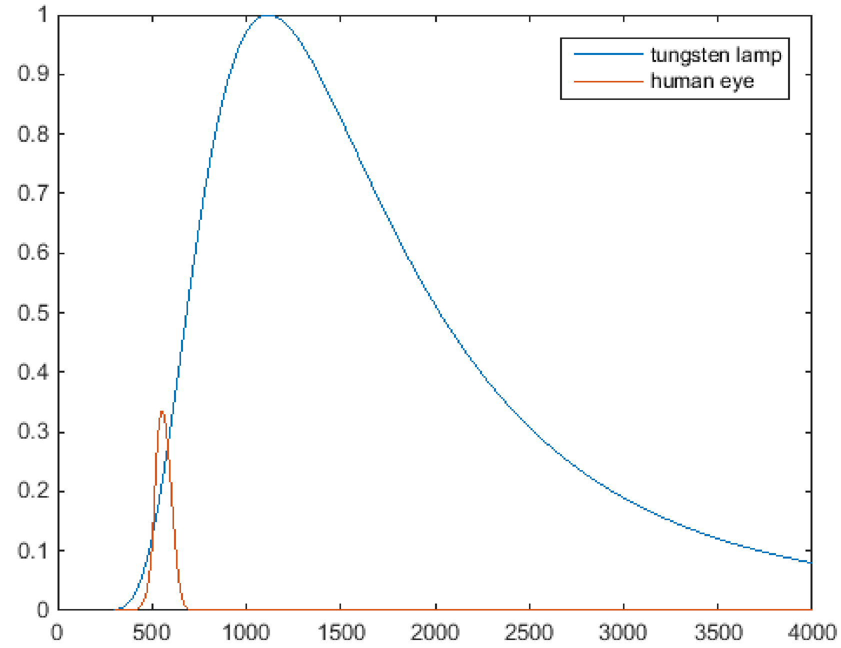
(Stefan‘s estimate ≈ 5400 K)

Wien's displacement law

- Peak of the Planck's curve for a blackbody
  - At which wavelength the radiation is the strongest?

$$\lambda_{\max} = \frac{2.8978 \cdot 10^{-3}}{T} \quad [\mathrm{m}]$$

On the right side the graph shows the curve for tungsten lamp and the sensitivity of the human eye. What do you see?



Graph credit: P. Corke

- Example:
  - Tungsten lamp, T=2800K, λ_{max}=1µm

Some fun with Planck & Wien

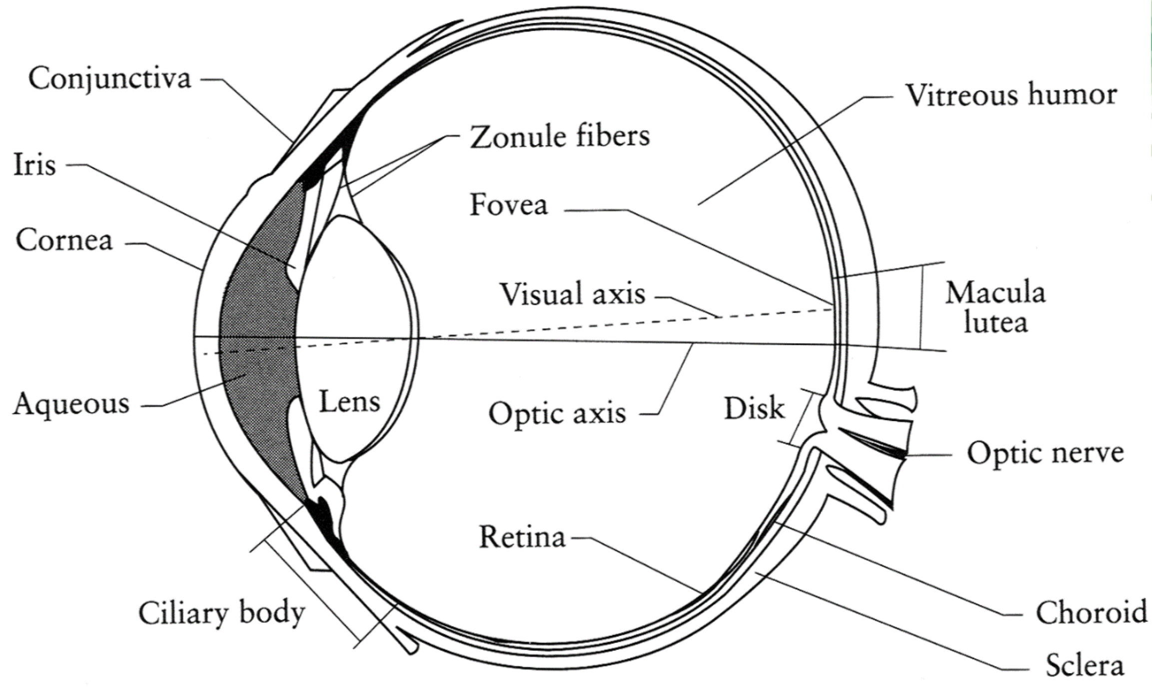
- What happens if we plug the temperature of human body into the Wien's equation?

$$\lambda_{\max} = \frac{2.8978 \cdot 10^{-3}}{T} \quad [\mathrm{m}]$$

  – We get, T=308K, λ_{max}=9-10 μm

- Do we really *glow in the dark?*

Human eye – human “camera sensor”

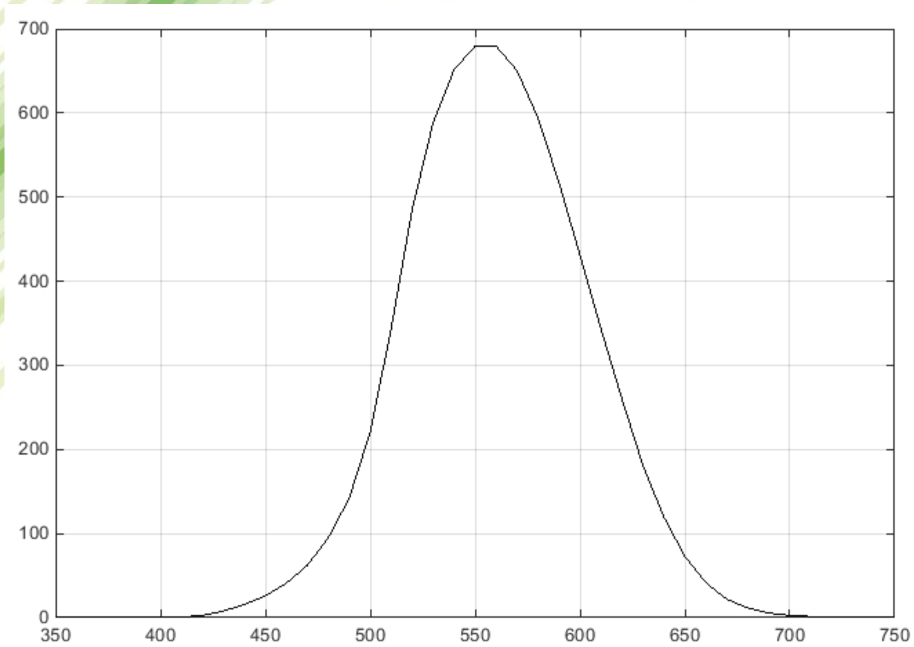


- Iris: pupil 2-8mm diameter
- Fovea: most sensitive part of retina, ~ 1,5 deg., ~ 0,5 mm in diameter

Sensitivity of the human eye

– 683 lm/W at 555 nm

Luminosity curve of a „*standard photometric observer*“, CIE 1931



Human eye is most sensitive in the middle of the visible spectrum, at 555 nm, *photopic vision* (daylight condition)

For *scotopic vision* (in darkness) the sensitivity peaks at 510 nm

Retina structure

- Light receptors: rods & cones
  - 0,5 – 4 μ in diameter
- Cones: mostly in fovea,

~ 6M cells

  - Wavelength selective
  - Thus, we perceive “colors”
- Rods: Distributed over retina, except in fovea, ~ 100 M,
  - More sensitive than cones, but not wavelength selective

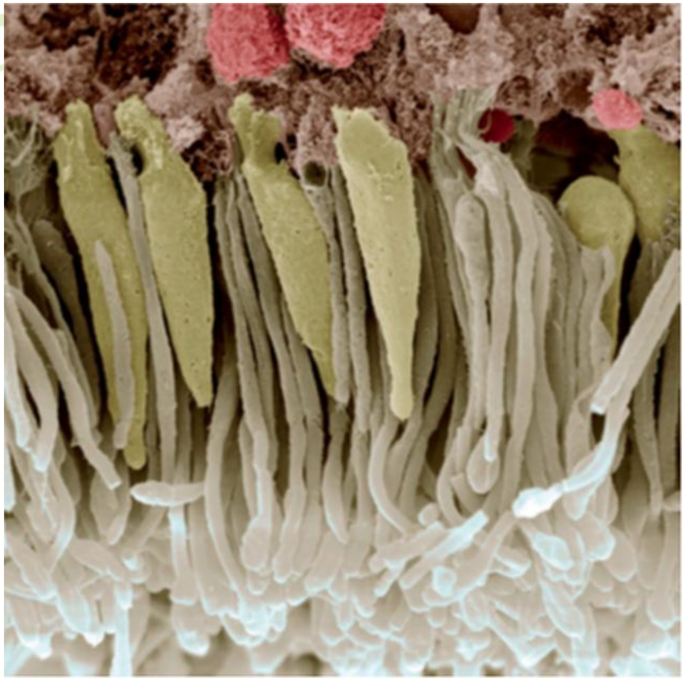
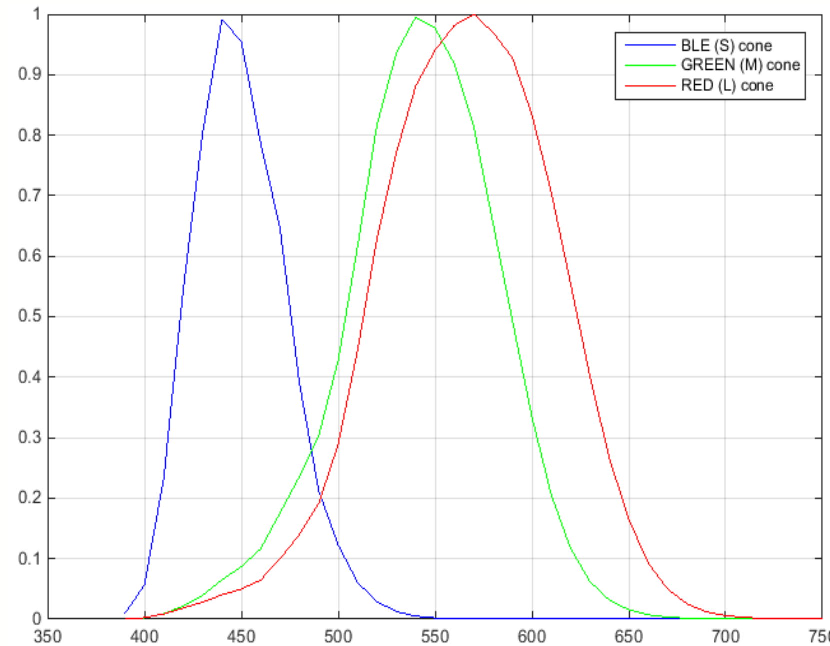


Image credit: P. Corke

Spectral sensitivity of the eye

- Three types of cones:
  - Usually named (R, G, B), but better (L(ong), M(edium), S(hort))
  - This means humans are trichromats
  - Peak B: 440 nm, Peak G: 540 nm, Peak R: 580 nm
  - Spectral responses of cones:
    (not to scale!)



- Ratio of cones

  Red : green : blue

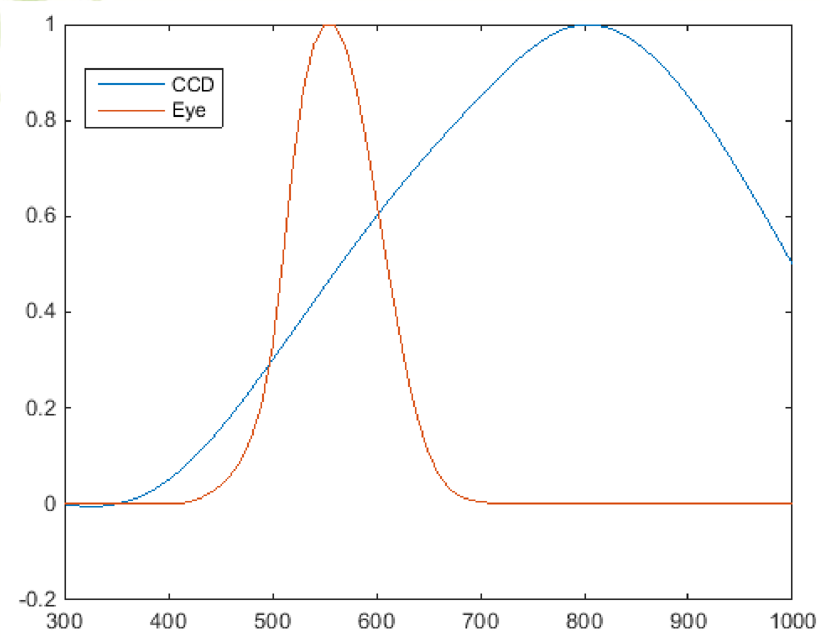
  L(λ) : M(λ) : S(λ)

  65%: 33%: 2%

  40 : 20 : 1

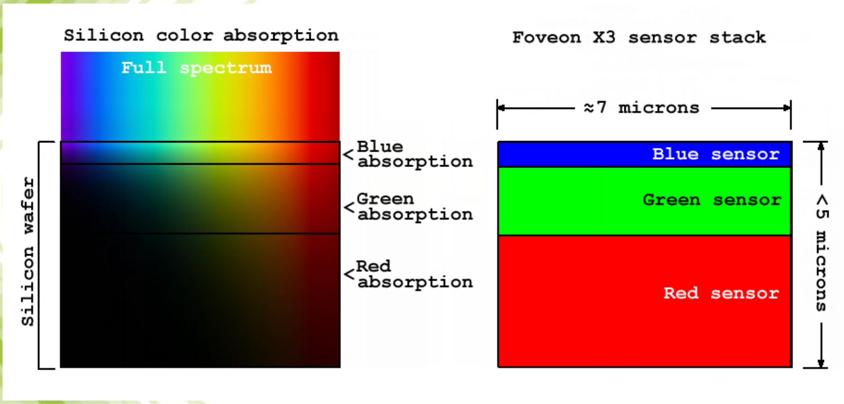
Camera sensor vs. eye sensitivity

- CCD and CMOS image sensors are most sensitive in NIR spectrum (Near IR)
  - Some security cameras provide infrared scene illumination for covert night time monitoring.
  - Some cameras are fitted with Infrared (cut-off) filters to prevent the sensor becoming saturated by ambient infrared illumination.

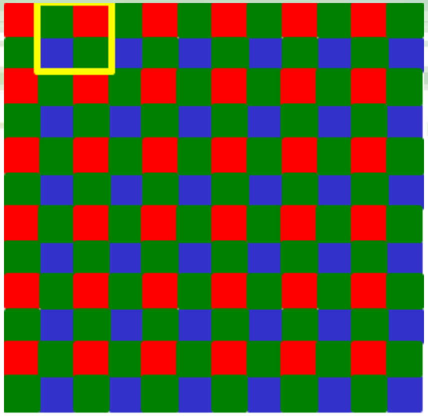


Sensors in color cameras

- There are special arrangements that allow us to get R, G and B component of *each pixel*
  - But most inexpensive cameras are not built this way
- 3 CCD cameras contain 3 separate sensors, each covered by R, G or B filter
- Layered image sensor, eg. Foveon X3



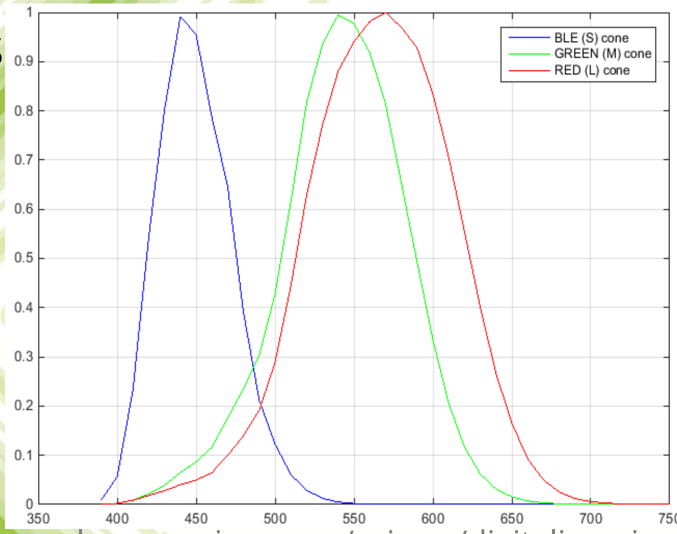
Sensors in *most of the* color cameras



- Bayer pattern of (micro)filters
  - covers the whole sensor, each pixel has either R, G or B filter in front of it
  - Human eye sensitivity vs. Bayer filter transmission spectral profiles:

Peak B: 440 nm, Peak G: 540 nm, Peak R: 580 nm

"real" RGB image is obtained only by post-processing (on-camera or in the PC)



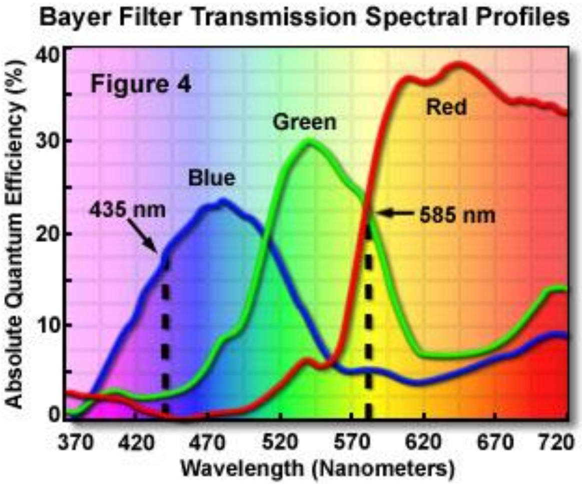
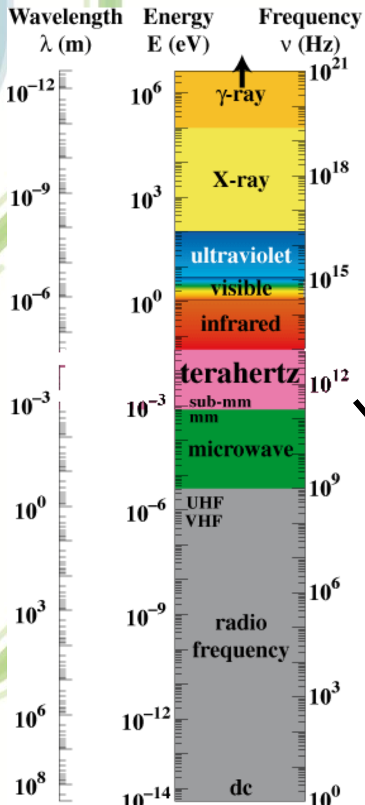


Image credit: http://www.olympusmicro.com/primer/digitalimaging/cmosimagesensors.html

Visible spectrum and beyond




Visible light


Near Infra-red


Far infra-red (FLIR)


THz (Teraherz) imaging

Slide credit: http://userweb.elec.gla.ac.uk/d/dpaul/terahertz.html

Surface appearance, reflection

- The spectrum of light reflected from a surface, its luminance (L) largely depends:
- on the spectrum of the light illuminating the surface (E)
- and the reflective properties of the surface R, (material, structure).

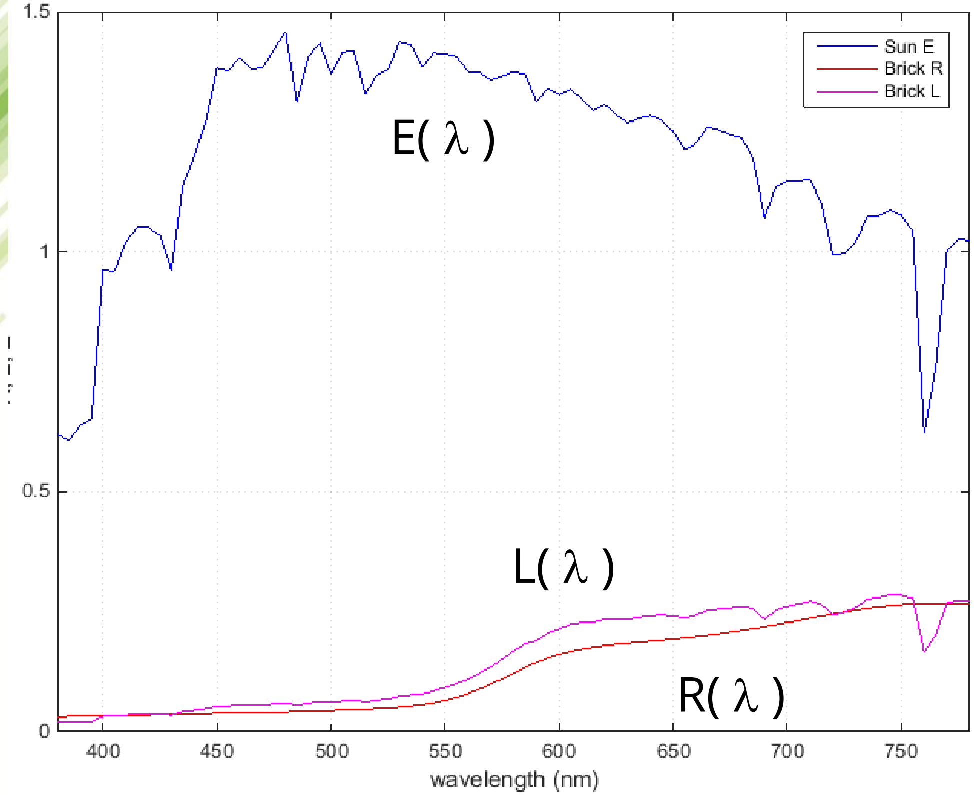
$$L(\lambda) = E(\lambda)\, R(\lambda)\ [W/m2]$$

Surface appearance, reflection

- Example: what is reflected from the red brick?

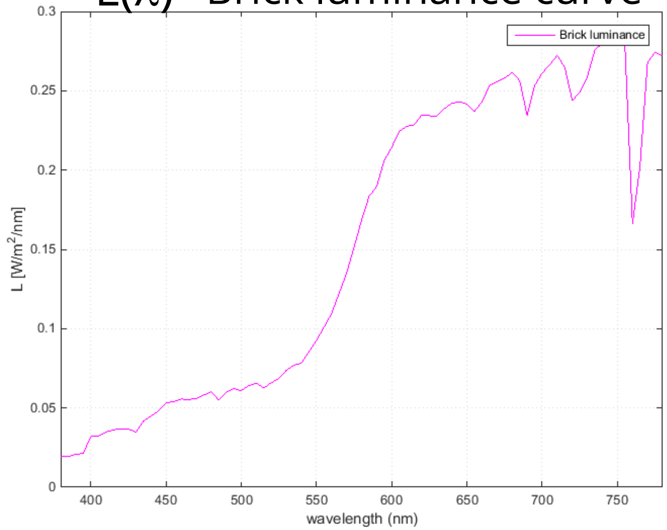
$$L(\lambda) = E(\lambda)\, R(\lambda) \; [W/m2]$$

- – $E(\lambda)$ is the sun light at the surface level
- – $R(\lambda)$ is the reflectance of the brick
- – $L(\lambda)$ is the sun light reflected from the brick

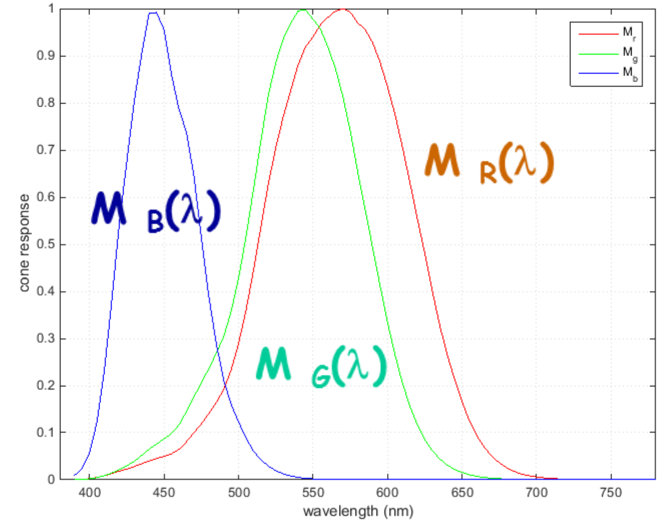


How do WE see the red brick?

$L(\lambda)$ Brick luminance curve



Eye color sensitivity curves

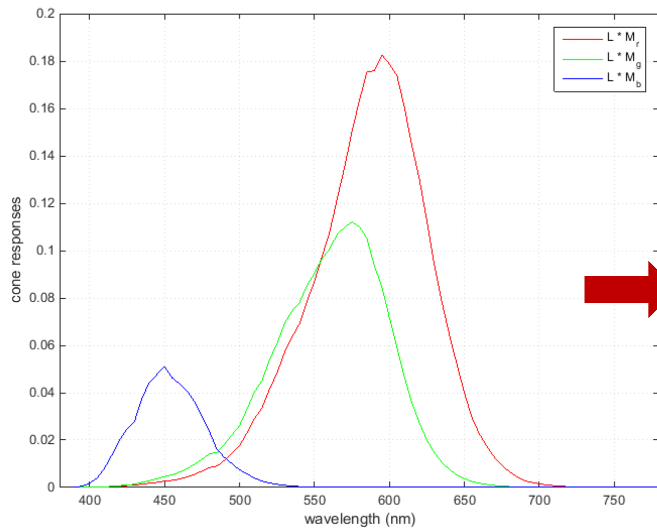


Light receptors integrate over the spectrum, the net result are three "color" stimuli – *tristimulus.*

ρ = 1.6323

γ = 1.0054

β = 0.2843



$$\rho = \int L(\lambda) M_R(\lambda) d\lambda$$

$$\gamma = \int L(\lambda) M_G(\lambda) d\lambda$$

$$\beta = \int L(\lambda) M_B(\lambda) d\lambda$$

Tristimulus & metamerism

- Light receptors integrate over the spectrum!
  - the net result are three color stimuli – tristimulus.
- We humans are *trichromats*.

$$\rho = \int L(\lambda) M_R(\lambda) d\lambda$$
$$\gamma = \int L(\lambda) M_G(\lambda) d\lambda$$
$$\beta = \int L(\lambda) M_B(\lambda) d\lambda$$

- Consequence: surfaces causing *equal ρ, γ, β* stimuli (tristimulus)
  - *appear of the same color*, regardless their actual spectra.
  - the phenomenon is known as metamerism.

Reproducing color

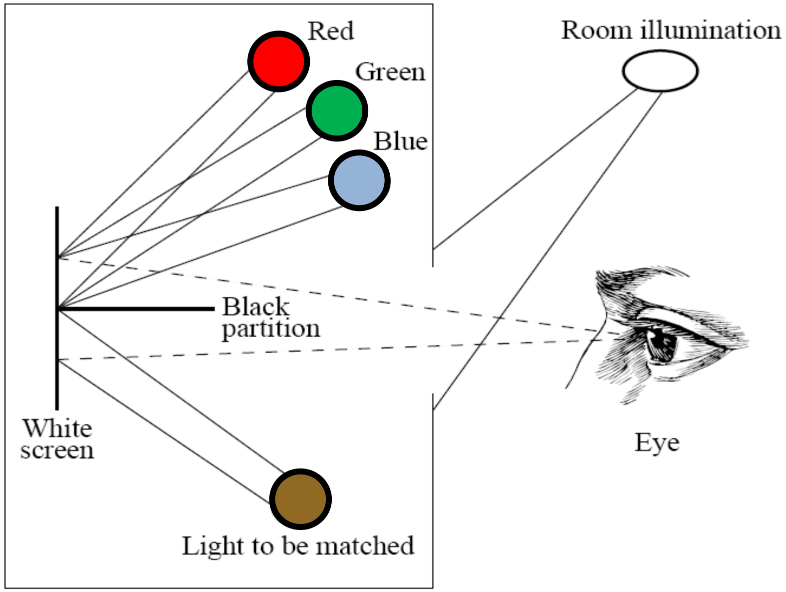
- But more importantly
  - An arbitrary visual stimulus can be generated by just three monochromatic stimuli, the primary colors.
- Note that a set of primaries is not unique.
  - There is a number of different possible primaries, with the only constraint that the third color cannot be matched by the combination of the other two.
- One set of primaries:
  - CIE 1976 primaries [nm]: red = 700; green=546.1; blue=435.8;
  - (those are the emission lines of mercury lamp)

Grassman's law of additivity

Matching the chromatic sensation

$$\boxed{T = t_R R + t_G G + t_B B}$$

Think of R,G and B as unit vectors spanning 3D RGB space



$$T_1 = t_{1,R} R + t_{1,G} G + t_{1,B} B$$

$$T_2 = t_{2,R} R + t_{2,G} G + t_{2,B} B$$

$$T_1 + T_2 = (t_{1,R} + t_{2,R}) R + (t_{1,G} + t_{2,G}) G + (t_{1,B} + t_{2,B}) B$$

$$T_1 = T_2 \Leftrightarrow \begin{bmatrix} t_{1,R} & t_{1,G} & t_{1,B} \end{bmatrix} = \begin{bmatrix} t_{2,R} & t_{2,G} & t_{2,B} \end{bmatrix}$$

$$kT = kt_R R + kt_G G + kt_B B$$

- chromatic sensation can be described *in terms of an effective stimulus* consisting of *linear combinations* of different light colors

Reproducing color

- Let us start with the monochromatic light stimulus

$$L(\lambda) = \begin{cases} L_\lambda & \text{if} \quad \lambda = \lambda_s \\ & 0 \end{cases}$$

- The responses of the cones are (integrals disappear):

$$\rho = L_\lambda M_R(\lambda_s)$$
$$\gamma = L_\lambda M_G(\lambda_s)$$
$$\beta = L_\lambda M_B(\lambda_s)$$

Reproducing color

- Next, let's take three primary light sources
  - Rlight, Glight, Blight, of wavelengths $\lambda_R, \lambda_G, \lambda_B$
  - and intensities *r,g,b*

$$\rho = r \cdot M_R(\lambda_R) + g \cdot M_R(\lambda_G) + b \cdot M_R(\lambda_B)$$
$$\gamma = r \cdot M_G(\lambda_R) + g \cdot M_G(\lambda_G) + b \cdot M_G(\lambda_B)$$
$$\beta = r \cdot M_B(\lambda_R) + g \cdot M_B(\lambda_G) + b \cdot M_B(\lambda_B)$$

- For the perceived colors to be equal, tristimuli must match, thus

$$L_\lambda M_R(\lambda_S) = rM_R(\lambda_r) + gM_R(\lambda_g) + bM_R(\lambda_b)$$
$$L_\lambda M_G(\lambda_S) = rM_G(\lambda_r) + gM_G(\lambda_g) + bM_G(\lambda_b)$$
$$L_\lambda M_B(\lambda_S) = rM_B(\lambda_r) + gM_B(\lambda_g) + bM_B(\lambda_b)$$

Reproducing color

$$L_\lambda \begin{bmatrix} M_R(\lambda_S) \\ M_G(\lambda_S) \\ M_B(\lambda_S) \end{bmatrix} = \begin{bmatrix} M_R(\lambda_r) & M_R(\lambda_g) & M_R(\lambda_b) \\ M_G(\lambda_r) & M_G(\lambda_g) & M_G(\lambda_b) \\ M_B(\lambda_r) & M_B(\lambda_g) & M_B(\lambda_b) \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix}$$

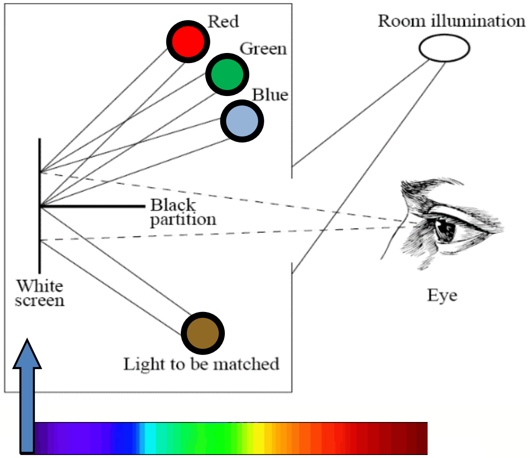
$$\begin{bmatrix} r \\ g \\ b \end{bmatrix} = L_\lambda \begin{bmatrix} M_R(\lambda_r) & M_R(\lambda_g) & M_R(\lambda_b) \\ M_G(\lambda_r) & M_G(\lambda_g) & M_G(\lambda_b) \\ M_B(\lambda_r) & M_B(\lambda_g) & M_B(\lambda_b) \end{bmatrix}^{-1} \begin{bmatrix} M_R(\lambda_S) \\ M_G(\lambda_S) \\ M_B(\lambda_S) \end{bmatrix}$$

- Transformation is linear!
  - the transformation matrix depends on the responses of the cones to the chosen primaries

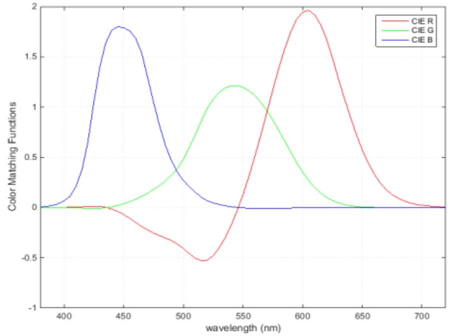
Color matching functions

- Once again, let's assume
  - the light to be matched to be monochromatic (i.e. a very narrow band (5nm) around a central wavelength λ
  - and of unit strength $U(\lambda)$
- By matching it
  - we get coefficients for R,G,B primaries

$$\boxed{U(\lambda_i) = r(\lambda_i)R + g(\lambda_i)G + b(\lambda_i)B}$$

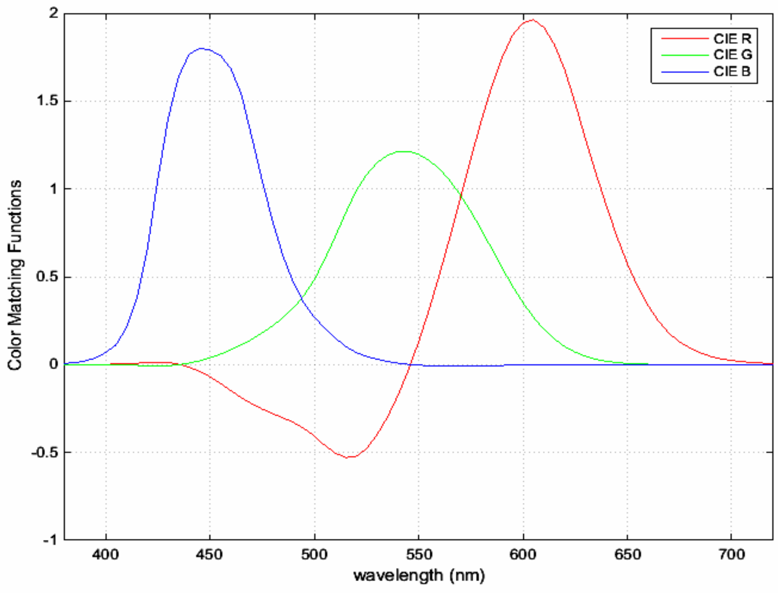


We repeat the experiment for all wavelenght bands, say from 400 – 700 nm. (i=0,1,2,...,N-1). N = (700-400)/5 + 1= 61



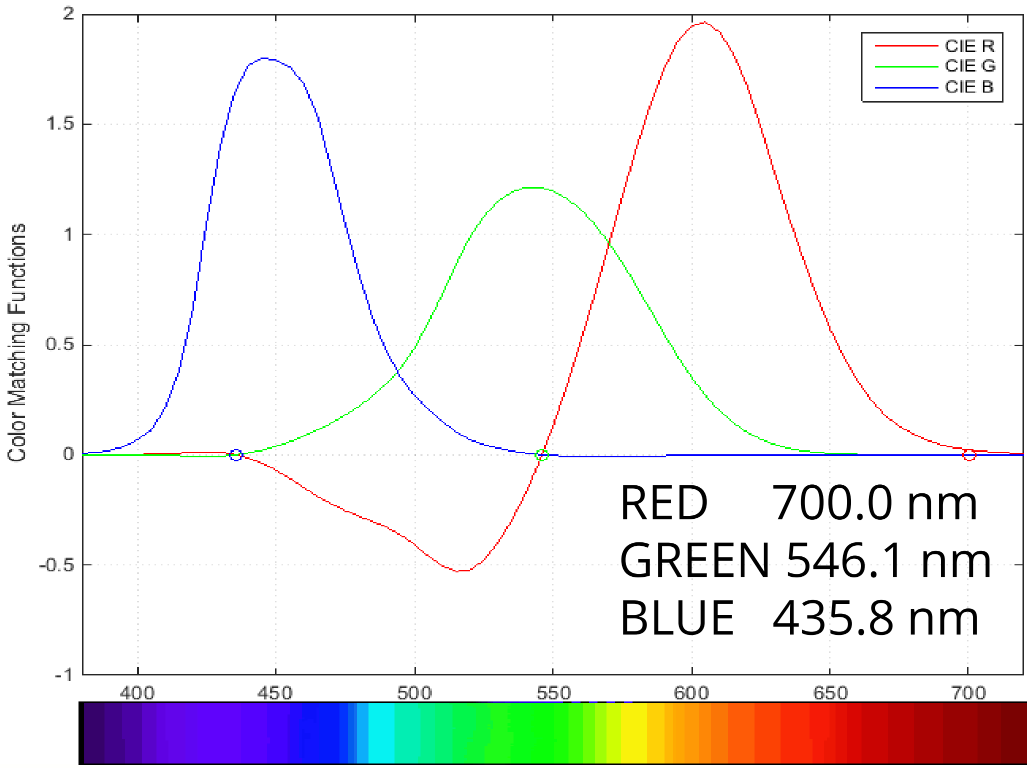
Color matching functions

- The final result are three functions,
  - the “color matching functions” for the “standard” 2 deg. observer.
  - Each curve shows how much of the corresponding primary is required to match the monochromatic light of wavelength λ
  - For example to create the sensation of light at 600 nm (orange) we would need 2*red source + 0.5* green source



Reproducing color

- The original CMF based on CIE RGB primaries, based on Guild (1931) and Wright (1928)
  - Created by using human test subjects!
  - Can you explain the meaning of negative values?!

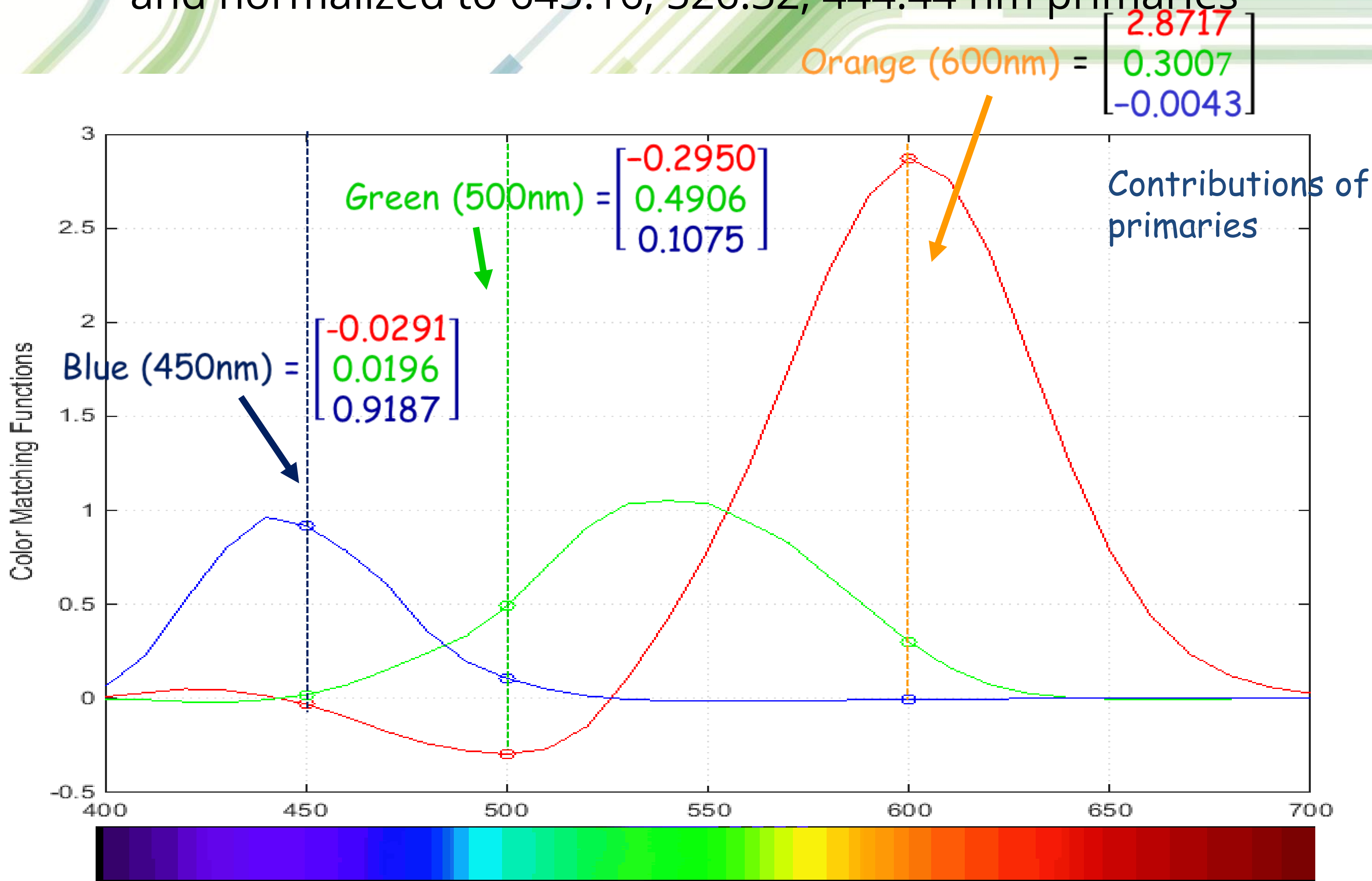


Lets state our motive again

- Perhaps you lost the thread – WHY are we doing this?
  - We have real world, that emits continuous spectrum of light.
  - People have only 3 kinds of color receptors, for red, green and blue color
  - So we can reproduce any digital image by mixing together proper amount of red, green and blue for each pixel.
  - That's how your computer or phone screen works.
  - **But the for example source of red light in your monitor can be different from the source in your phone screen** (different wavelength but still red!)
  - So the Color Matching Functions tell us how to mix different kinds of sources to get *perceptually* the same effect!

CMF, a few monochromatic examples

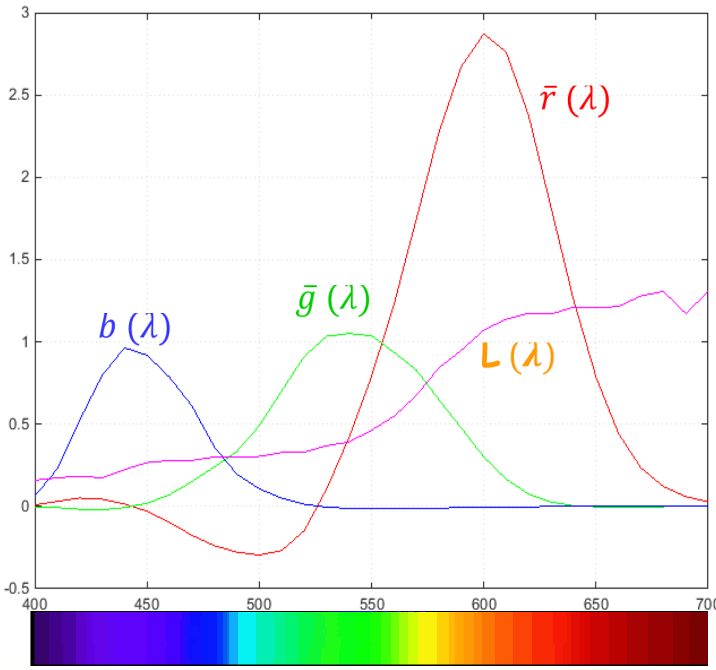
– These CMFs were derived by Stiles & Burch (1955), corrected and normalized to 645.16, 526.32, 444.44 nm primaries



Reproducing color

Due to the principle of linearity one can expect that based on these three color matching functions it should be possible to produce proportions of primaries for any light to be matched with an arbitrarily richer spectral content.

- These CMFs were derived by Stiles & Burch, corrected and normalized to 645.16, 526.32, 444.44 nm primaries.
- Various CMFs are accessible at http://cvrl.ioo.ucl.ac.uk/



Sensing and measuring color

– Let‘s take an arbitrary light source of richer spectral content S

$$S=[S(\lambda_0),S(\lambda_1),\cdots,S(\lambda_i),\cdots,S(\lambda_{N-1})]$$

$$S=S(\lambda_0)U(\lambda_0)+S(\lambda_1)U(\lambda_1)+\cdots+S(\lambda_i)U(\lambda_i)+\cdots+S(\lambda_{N-1})U(\lambda_{N-1})=\sum_{i=0}^{i=N-1}S(\lambda_i)U(\lambda_i)$$

$$S=\sum_{i=0}^{i=N-1}S(\lambda_i)U(\lambda_i)=\sum_{i=0}^{i=N-1}S(\lambda_i)[r(\lambda_i)R+g(\lambda_i)G+b(\lambda_i)B]$$

$$S=\sum_{i=0}^{i=N-1}[S(\lambda_i)r(\lambda_i)]R+\sum_{i=0}^{i=N-1}[S(\lambda_i)g(\lambda_i)]G+\sum_{i=0}^{i=N-1}[S(\lambda_i)b(\lambda_i)]B$$

$$S=s_R R+s_G G+s_B B$$

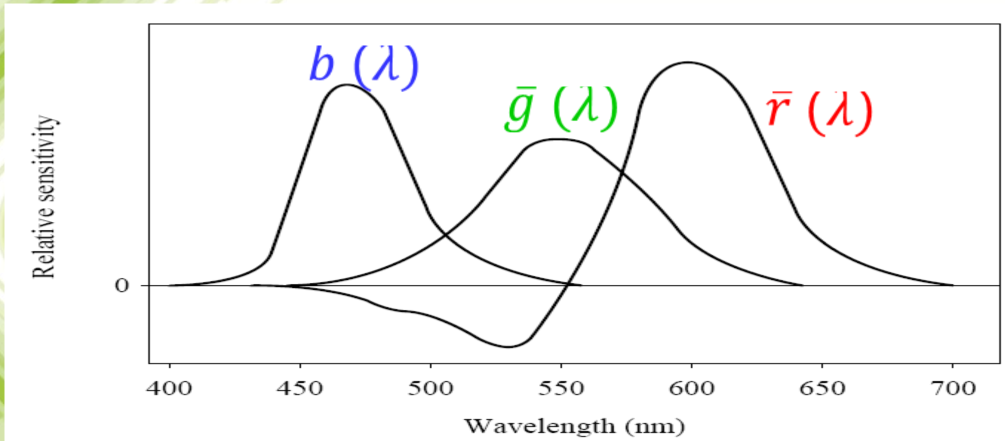
– It is possible to derive the components of S (primaries) by projection‘ of the light source onto the color matching functions, or by matching the light source to the primaries.

– Note: in principle we should deal with integrals. $s_R=\int S(\lambda)r(\lambda)d\lambda$

Sensing and measuring color

– Once the primaries are chosen, and the color matching functions for these primaries are known, we can derive the proportions (coefficients, or weights) for any color, S

$$\begin{bmatrix} s_R \\ s_G \\ s_B \end{bmatrix} = \begin{bmatrix} r(\lambda_0) & r(\lambda_1) & \cdots & r(\lambda_i) & \cdots & r(\lambda_{N-1}) \\ g(\lambda_0) & g(\lambda_1) & \cdots & g(\lambda_i) & \cdots & g(\lambda_{N-1}) \\ b(\lambda_0) & b(\lambda_1) & \cdots & b(\lambda_i) & \cdots & b(\lambda_{N-1}) \end{bmatrix} \begin{bmatrix} S(\lambda_0) \\ S(\lambda_1) \\ \vdots \\ S(\lambda_i) \\ \vdots \\ S(\lambda_{N-1}) \end{bmatrix}$$



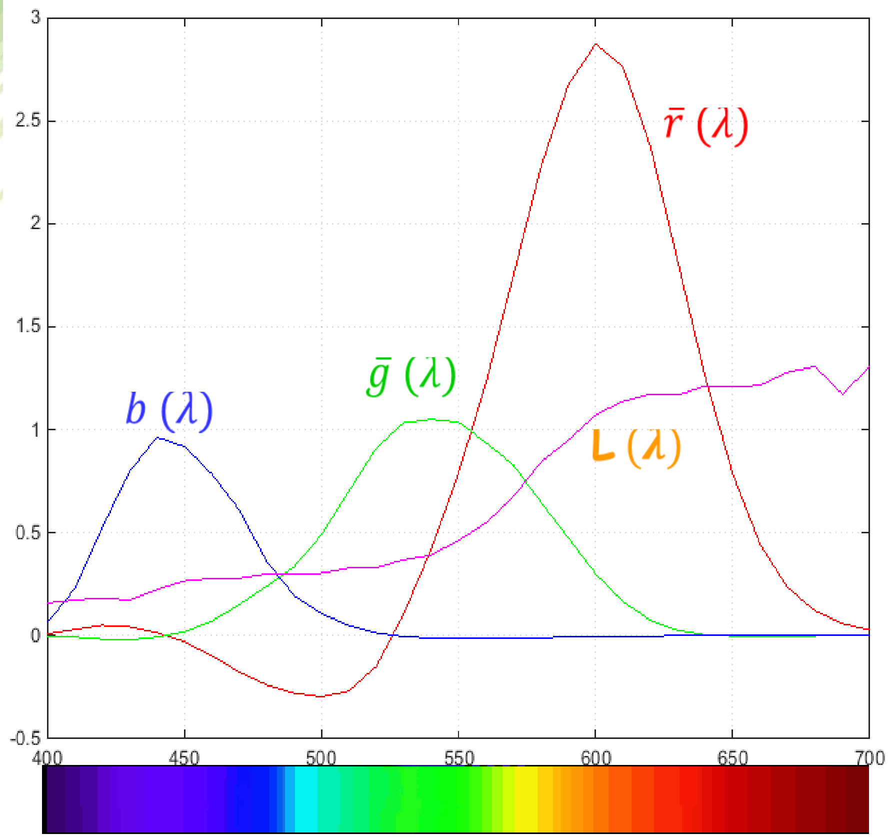
CMF example, red brick

- CMFs derived by Stiles & Burch (1955). Primaries: 645.16 nm, 526.32 nm, 444.44 nm

This is what we get for a red brick wall example

RGB_for_brick =
- 0.613
- 0.141
- 0.038

Do not forget – these representations are equivalent only for human observers!



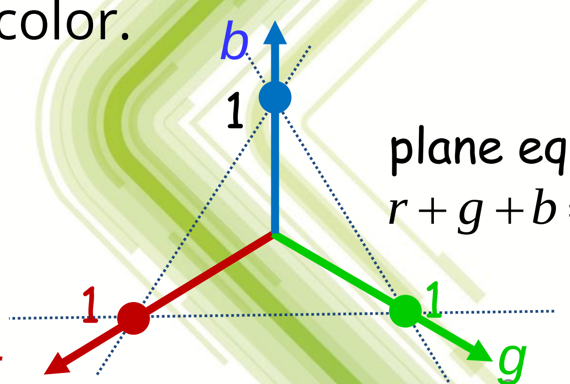
Sensing color

– The tristimulus values encode the color and lightness (brightness, luminance). To achieve brightness invariance we can normalize the values such that they sum to one.

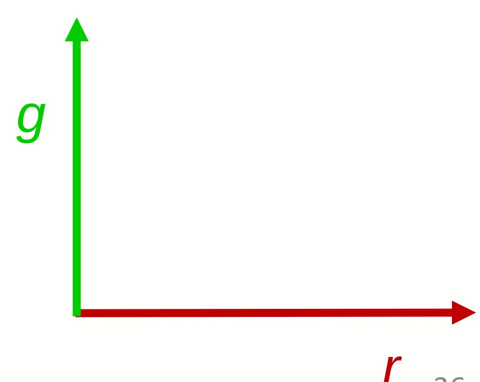
$$r = \frac{R}{R+G+B} \qquad g = \frac{G}{R+G+B} \qquad b = \frac{B}{R+G+B} \qquad r+g+b=1$$

– Using the constraint r + g + b = 1 one component becomes redundant. With intensity eliminated two components are sufficient to represent color.

– Therefore, by convention, only r and g are used to describe color.



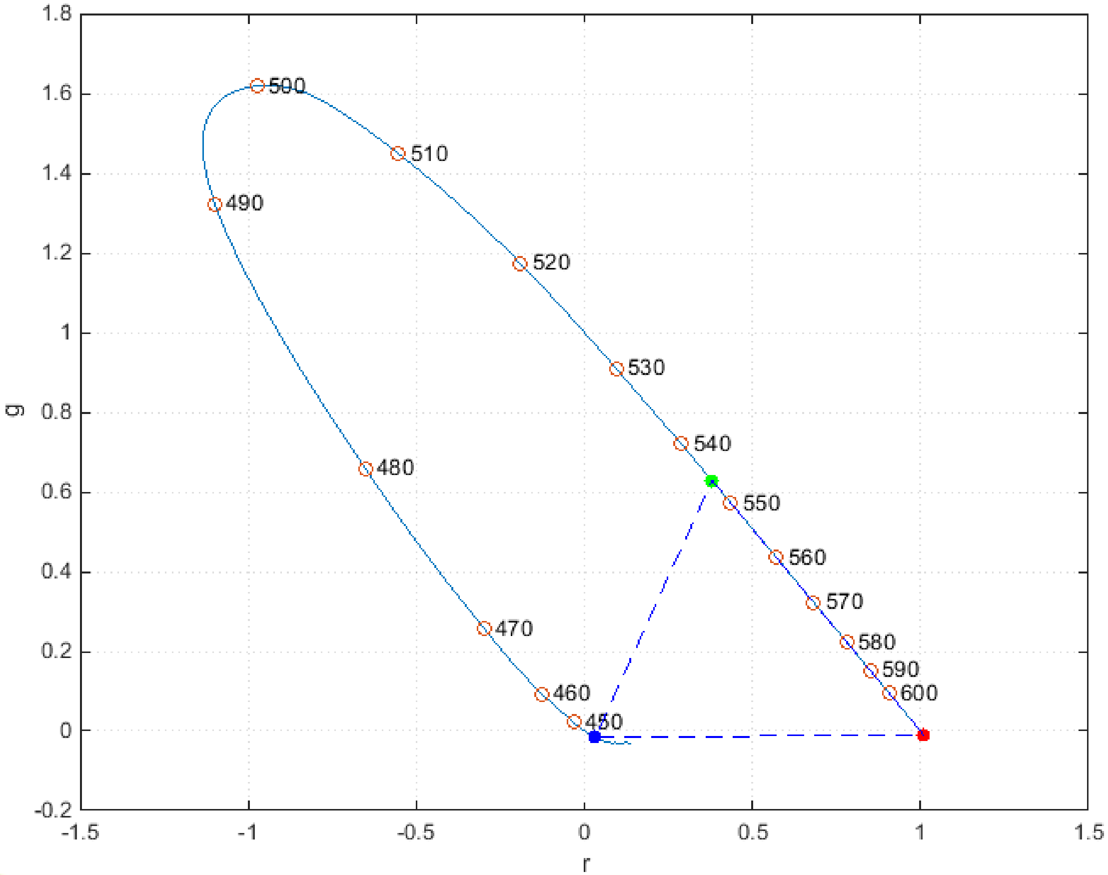
plane eq.
$r+g+b=1$



rg chromacitiy diagram

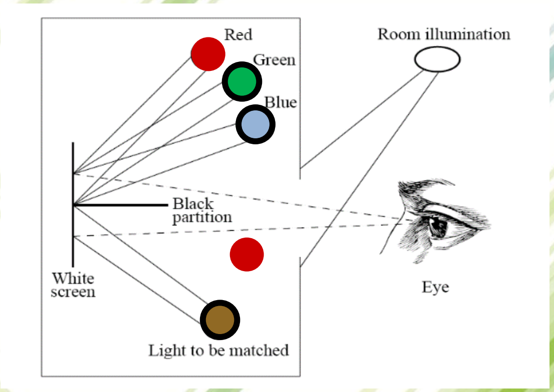
- The *inner part of horseshoe-shaped region* represents colors that can be sensed by the human eye

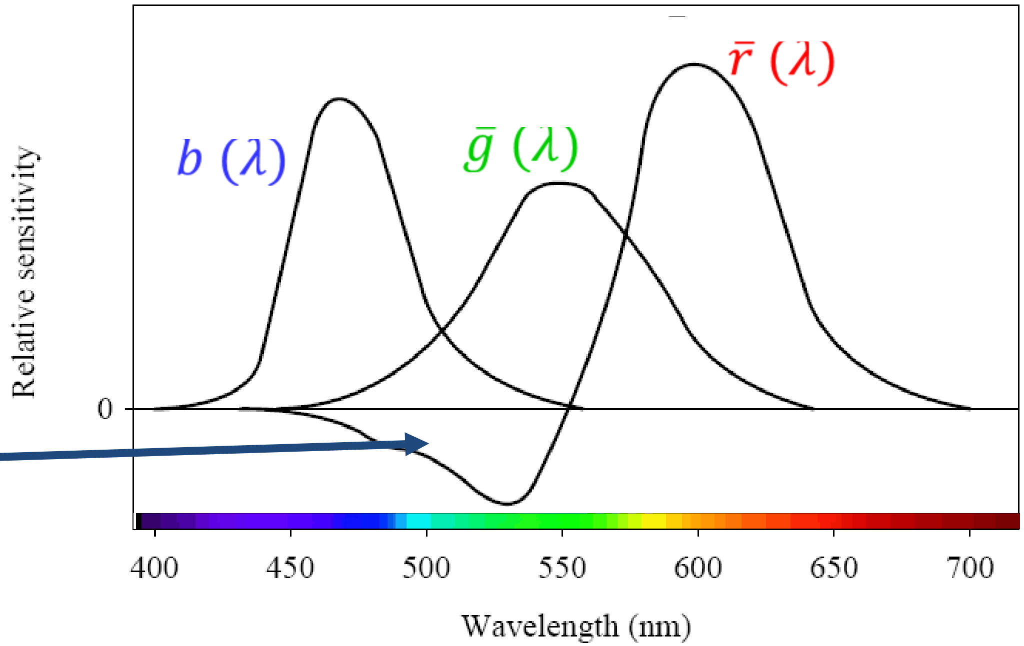
- Colors within dotted triangle can be reproduced using primaries defined by vertices.



Sensing color

– Back to color matching functions!





Negative weigths?

$$T + t_B B = t_R R + t_G G$$

$$T = t_R R + t_G G - t_B B$$

Conclusion: with the chosen set of primaries it is not possible to reproduce all the colors sensed by the human eye.

Sensing color

Based on experimentally derived curves (RGB) CIE (1931) standardized color matching functions for *imaginary primaries*, X, Y (intensity) and Z that totally enclose the spectral locus.

Standard “color space” CIE XYZ (Color space)

$$X = \int L(\lambda)\bar{x}(\lambda)d\lambda \qquad Y = \int L(\lambda)\bar{y}(\lambda)d\lambda \qquad Z = \int L(\lambda)\bar{z}(\lambda)d\lambda$$

- CIE criteria for selecting primaries
  - CMF are non-negative everywhere
  - Equal amounts of primaries should produce white light
  - CMF for Y should follow the luminous-efficiency curve of the human eye.

RGB vs. XYZ

– Derivation of CIE CMF (CIE Color Matching Functions)

$$\begin{aligned}
\bar{x}(\lambda) &= 0.49\bar{r}(\lambda) + 0.31\bar{g}(\lambda) + 0.20\bar{b}(\lambda) \\
\bar{y}(\lambda) &= 0.17697\bar{r}(\lambda) + 0.81240\bar{g}(\lambda) + 0.01063\bar{b}(\lambda) \\
\bar{z}(\lambda) &= 0.00\bar{r}(\lambda) + 0.01\bar{g}(\lambda) + 0.99\bar{b}(\lambda)
\end{aligned}$$

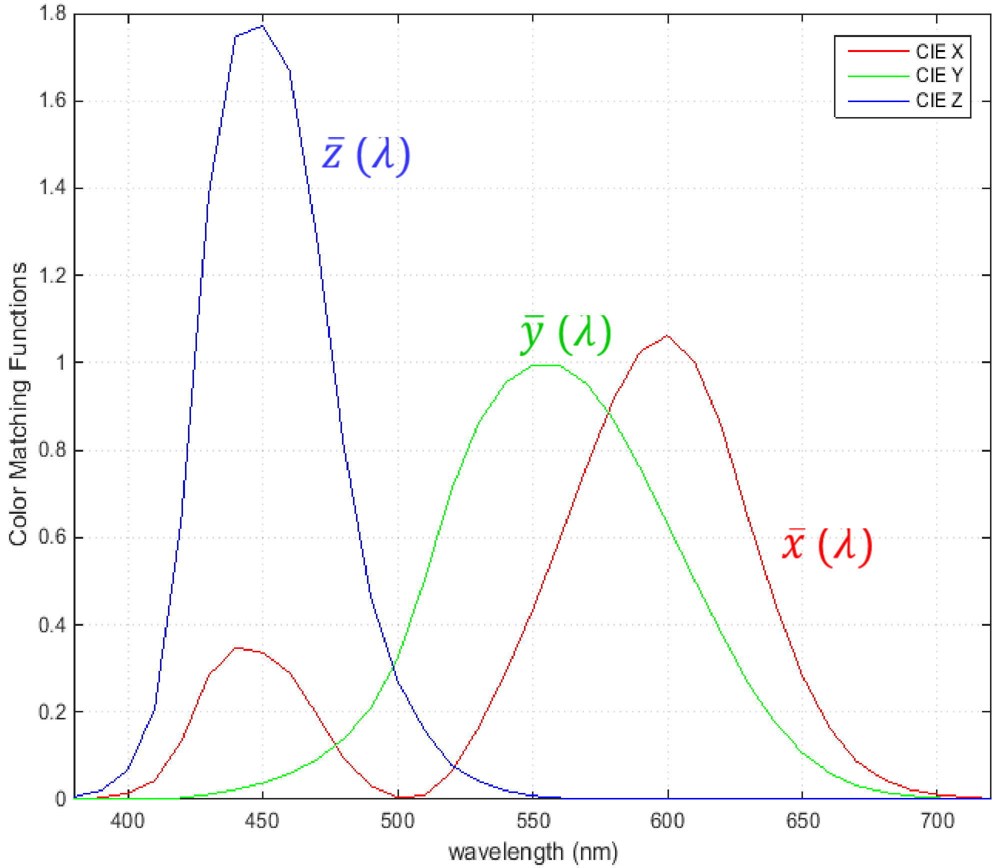
– Transformation in matrix form

$$\begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} = \begin{bmatrix} 0.49000 & 0.31000 & 0.20000 \\ 0.17697 & 0.81240 & 0.010630 \\ 0.0000 & 0.01000 & 0.99000 \end{bmatrix} \begin{bmatrix} \bar{r}(\lambda) \\ \bar{g}(\lambda) \\ \bar{b}(\lambda) \end{bmatrix}$$

– CIE 1931 standard observer (for 2 deg vision – fovea)

– Note: often, the matrix is normalized differently, i.e. divided by 0.17697).

CIE Color matching functions



CIE 1931 standard observer (for 2 deg vision – fovea)

xy chromaticity diagram

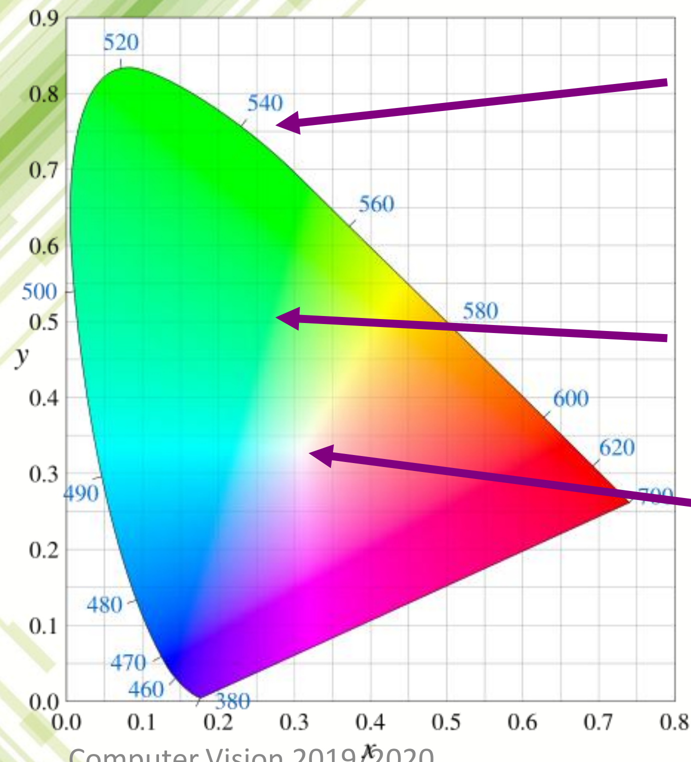
– It is difficult to visualize 3D (X,Y,Z) color space in 2D plane.
– Two normalized coordinates are sufficient to describe color
– Intensity change does not substantially influence color perception

$$x = \frac{X}{X+Y+Z}$$

$$y = \frac{Y}{X+Y+Z}$$

$$z = \frac{Z}{X+Y+Z}$$

$$x + y + z = 1$$



Saturated (pure) colors are on the edge

Less saturated with more white added.

White (point of equal energy, x = y = z = 1/3)

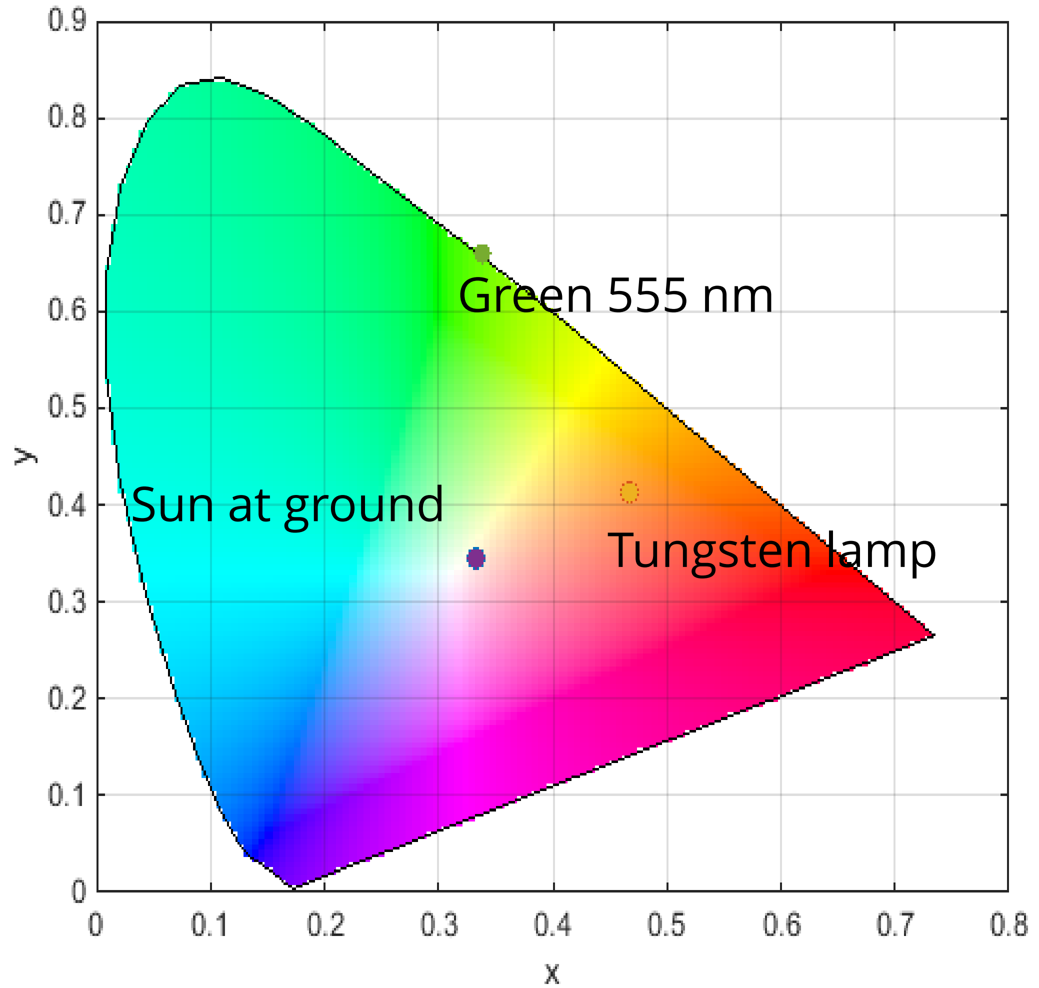
xy chromaticity diagram, examples

Examples: CIE x y

Green 555 nm
0.3016 0.6923

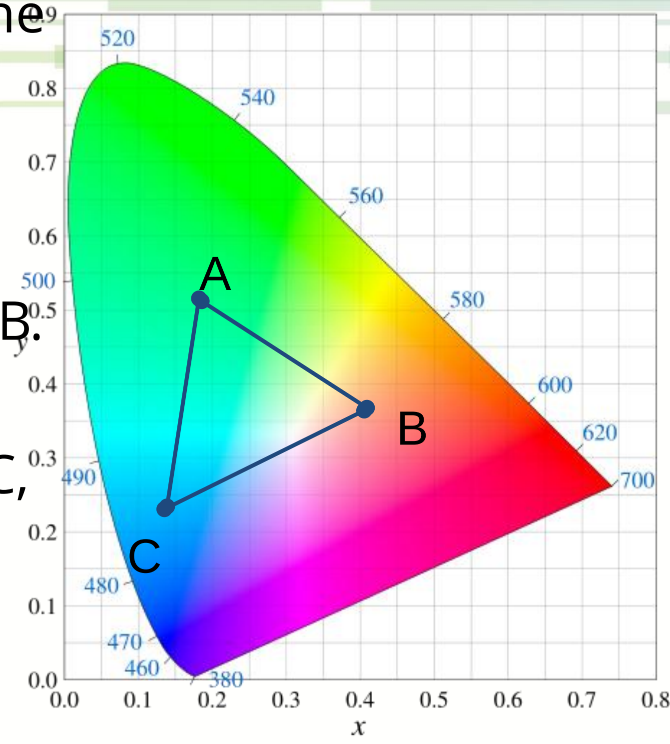
Tungsten bulb
at 2600K
0.4679 0.4126

Sunshine on the ground
0.3323 0.3454



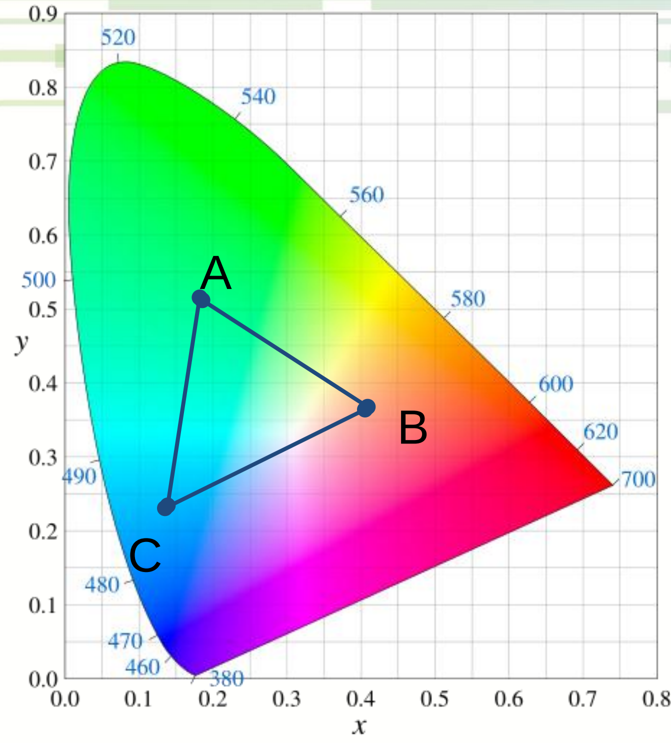
xy chromaticity diagram

Horseshoe shaped region represents all the colors that human eye can sense.
(*eye gamut*)
Mixture of two colors, such as A and B, can produce all the colors along the line AB.
(*gamut is a line*)
Mixture of three colors, such as A, B, and C, can produce all the colors within triangle, but not others.
(*gamut is a triangle*)
A single triangle cannot cover the whole gamut of the eye.



xy chromaticity diagram

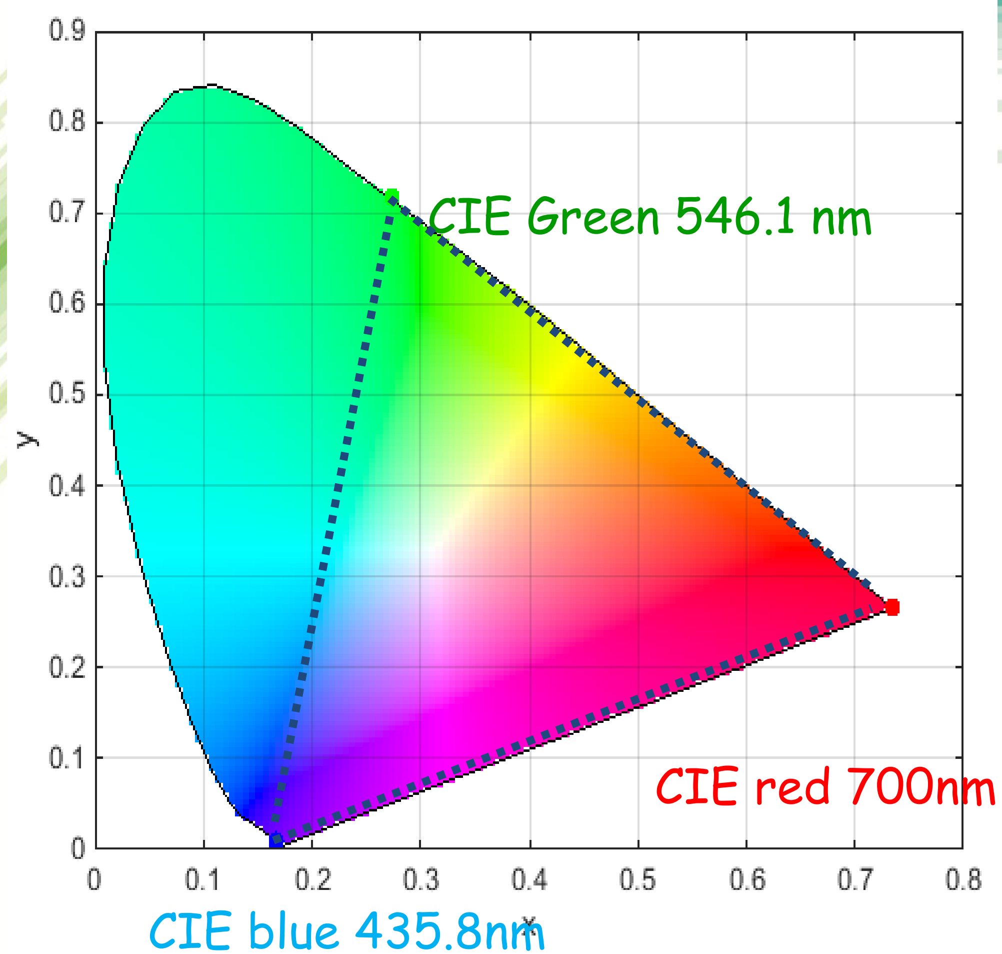
- Important:
  - Equal proportions of A and B do not produce color that lies on the middle of line between them.
  - Thus, color space is not perceptually homogeneous, or “uniform”



CIE RGB in xy chromaticity diagram

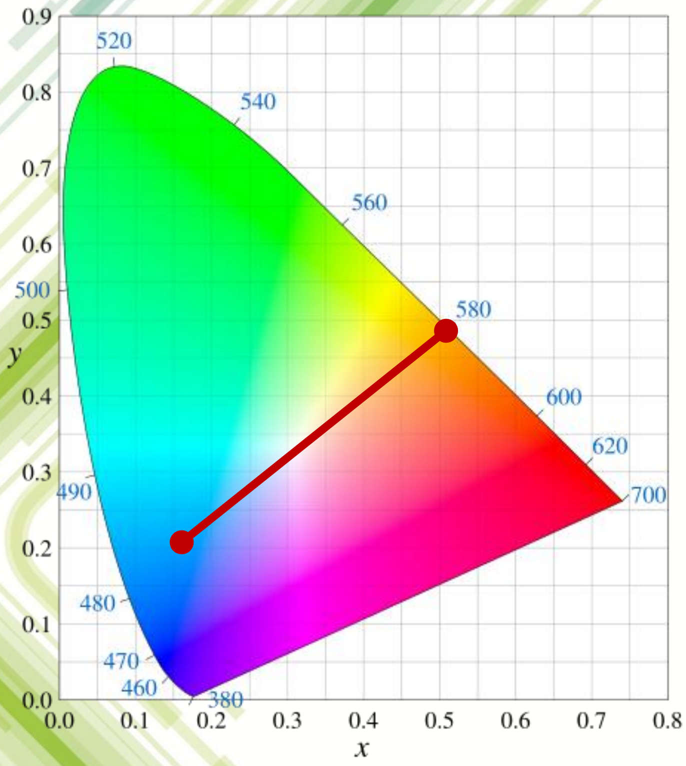
Example: CIE xy with CIE RGB primaries

RED 700 nm:
0.7347, 0.2653
GREEN 546.1:
0.2737, 0.7174
BLUE 435.8:
0.1665, 0.0089
Only colors within triangle, defined by the primaries can be produced



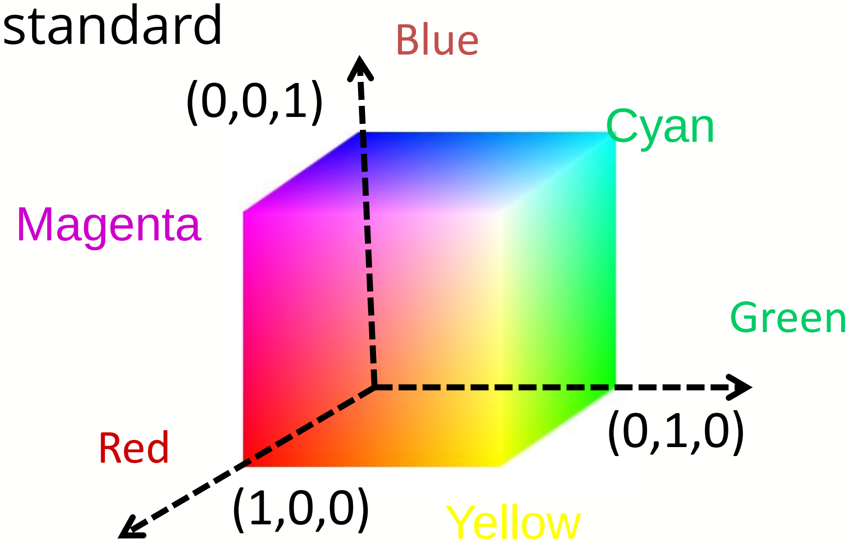
xy chromaticity diagram

- Complementary colors (wavelengts
  - if mixed together produce white



RGB color space

- Primaries are not unique.
  - Therefore, mostly for historical reasons, and technology, various “color spaces” exist today.
  - CIE RGB 1976
    - R = 700 nm, G = 546,1 nm, B = 435.8
    - Primaries used in Wright and Guild experiment (1931)
  - CIE RGB, CIE XYZ (1964) 10 deg standard
    - R = 645,16 nm, G = 526,32 nm, B = 444,44 nm
    - Primaries used in Stiles & Burch experiment (10 deg observer)



Transformations between color spaces

- Tristimuli must match

$$S = \begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} S'_1 & S'_2 & S'_3 \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix}$$

- Transformation between primaries

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix}$$

$$\begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix} = \begin{bmatrix} S'_1 & S'_2 & S'_3 \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix}$$

Transformations between color spaces

$$\begin{bmatrix} S_1 & S_2 & S_3 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{32} & c_{33} \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix} = \begin{bmatrix} S'_1 & S'_2 & S'_3 \end{bmatrix} \begin{bmatrix} P'_1 \\ P'_2 \\ P'_3 \end{bmatrix}$$

$$\begin{bmatrix} S'_1 \\ S'_2 \\ S'_3 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{32} & c_{33} \end{bmatrix}^T \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}$$

- Different color spaces require different transformation matrices.
- But, for particular transformation, the matrix is constant.

CIE RGB to CIE XYZ case #1

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{31} \\ c_{12} & c_{22} & c_{32} \\ c_{13} & c_{32} & c_{33} \end{bmatrix}^T \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

	CIE R	CIE G	CIE B
x	0.7347	0.2737	0.1665
y	0.2653	0.7174	0.0089
z	0	0.0089	0.8246

These are RGB primaries expressed in XYZ space

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0.7347 & 0.2737 & 0.1665 \\ 0.2653 & 0.7174 & 0.0089 \\ 0 & 0.0089 & 0.8246 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

CIE RGB to CIE XYZ case #1

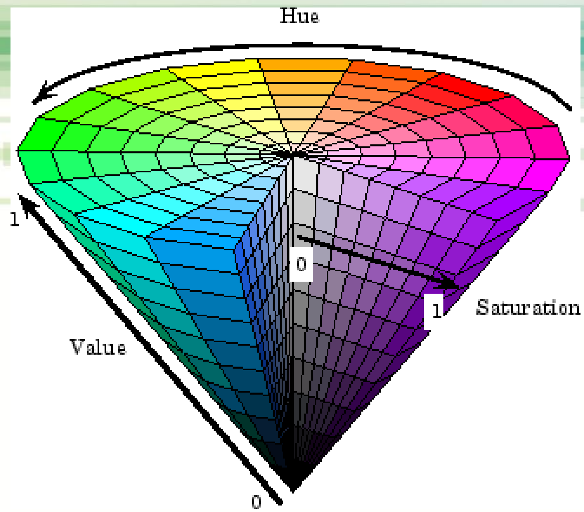
- In XYZ space luminance is encoded entirely in the Y component.
- Let us further demand that unity R, G, B result in unity Y,
- and that the color white follows the D65 illuminant.

$$\begin{bmatrix} x_{D65} \\ y_{D65} \\ z_{D65} \end{bmatrix} = \begin{bmatrix} 0.3127 \\ 0.3290 \\ 0.3582 \end{bmatrix} \Rightarrow \frac{1}{y_{D65}} \begin{bmatrix} x_{D65} \\ y_{D65} \\ z_{D65} \end{bmatrix} = C \begin{bmatrix} J_R & 0 & 0 \\ 0 & J_G & 0 \\ 0 & 0 & J_B \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \Rightarrow$$

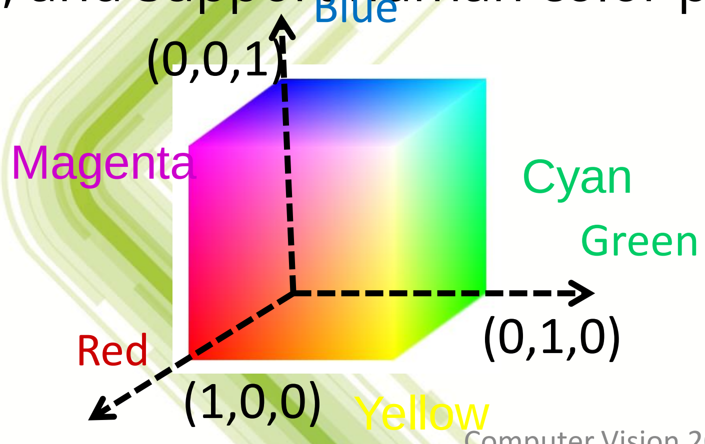
$$\underbrace{\begin{bmatrix} 0.7347 & 0.2737 & 0.1665 \\ 0.2653 & 0.7174 & 0.0089 \\ 0 & 0.0089 & 0.8246 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} 0.5614 & 0 & 0 \\ 0 & 1.1701 & 0 \\ 0 & 0 & 1.3077 \end{bmatrix}}_{\mathrm{diag}(J)} = \begin{bmatrix} 0.4124 & 0.3202 & 0.2178 \\ 0.1489 & 0.8394 & 0.0116 \\ 0 & 0.0104 & 1.0783 \end{bmatrix}$$

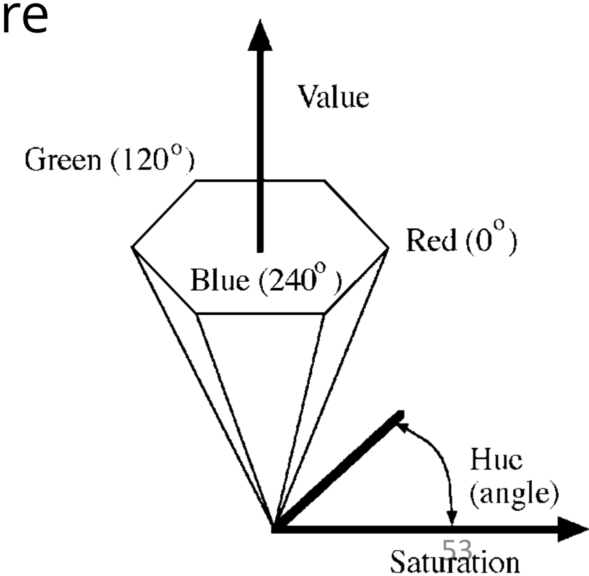
- Note that second row maps proportions of RGB color components to Y. Y = 0.1489 R + 0.8394 G + 0.0116 B

Other color spaces: HSV



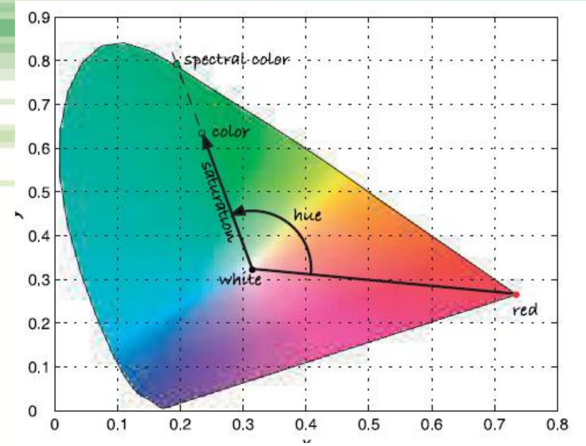
RGB, XYZ are Cartesian color spaces.
They describe luminance and chrominance (color). rg and xy ignore luminance, therefore these are chromaticity spaces.
Other options, besides Cartesian, are polar, spherical, cylindical, coordinate systems. One such space is HSV (HSL, HSI) space. These are more intuitive, and support human color perception.





Other color spaces: HSV

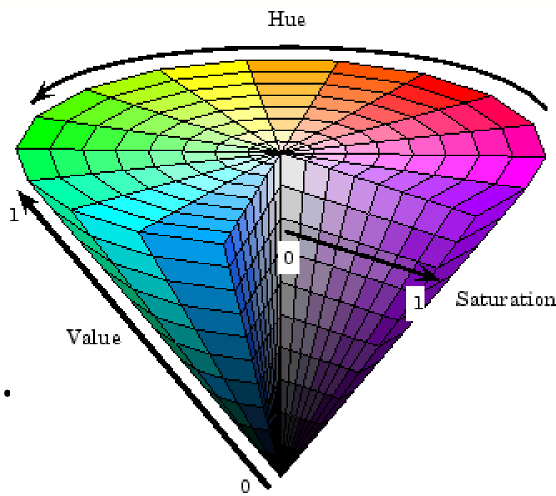
HSI, HSL, HSV, and also other notations are used. These are conceptually equivalent but not identical. Angle around axis encode HUE. Distance from the axis encodes SATURATION. Distance along the axis is proportional to VALUE. (Intensity, Lightness).



HSV is a cylindrical color system. Hue varies from 0 (red) to green(120) and Blue (240).

Saturation and Value are in the range [0..1].

In contrast to RGB spaces, HSV decouples lightness (VALUE) from color (HUE & SATURATION).



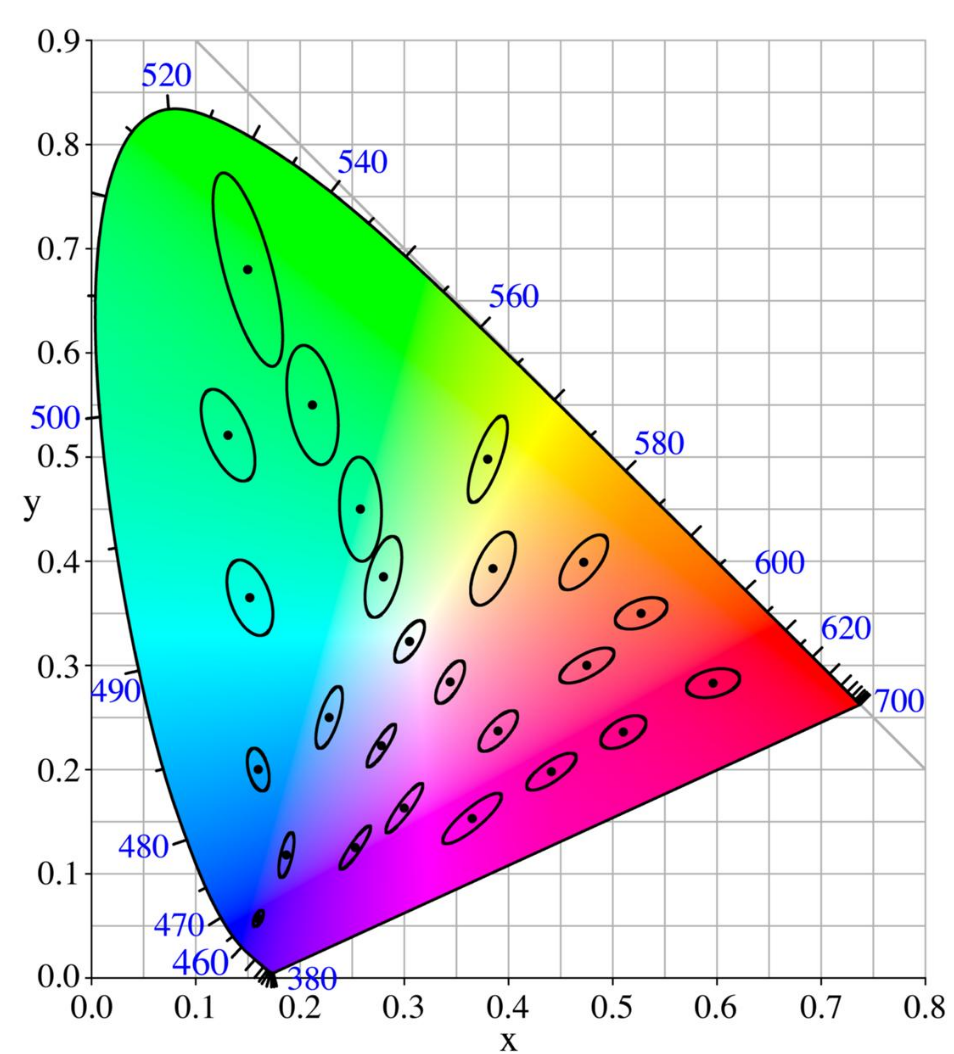
Left picture shows the relations between Hue, Saturation (chroma), and Value. Matlab: rgb2hsv, hsv2rgb

MacAdam ellipse in xy color space

Ellipses encircle regions of colors that are indistinguishable by the observer.

The size of ellipse differs from region to region.

Xy „color space“ is therefore perceptually Nonhomogeneous!

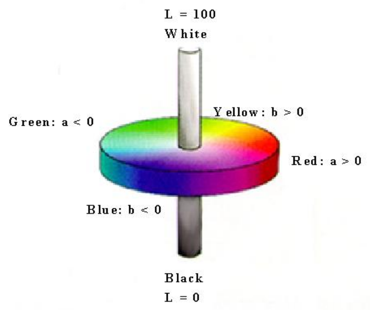


CIE L*a*b*

In this case colors are transformed so that they better correspond to the color sensitivity/discriminativity of the human eye.
If two colors perceptually appear close to each other,
they should be close to each other in the color space.
CIELAB and similarly CIELUV follow that goal. (developed byMacAdam, Hunter, and others)
CIELAB is called "perceptually homogeneous" color space)

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{(1/3)} - 16 \qquad \Delta E = \sqrt{(L*)^2 + (a*)^2 + (b*)^2}$$

$$a^* = 500\left[\left(\frac{X}{X_n}\right)^{(1/3)} - \left(\frac{Y}{Y_n}\right)^{(1/3)}\right] \qquad b^* = 200\left[\left(\frac{Y}{Y_n}\right)^{(1/3)} - \left(\frac{Z}{Z_n}\right)^{(1/3)}\right]$$



(X_n,Y_n,Z_n) - color white in CIE XYZ

Matlab: rgb2lab, lab2rgb

Further issues with color in CV

- People have strange ability:
  - We can distinguish colors that are very similar, but spatially close by
  - Never mix 2 batches of paint to paint the same wall!

- The question “What is white”
  - Related to the concept of white balance in photography
  - There are many kinds of white light

- Etc.

Sources

- Forsyth, Ponce: Computer vision, a modern approach
- Trucco, Verri: Introductory techniques for 3D Computer Vision.
- Corke, RVC, Springer 2012 (available online)
- Szeliski draft book (available online)
- Brown Uni CV course, and Wikipedia

Questions?