Univerza *v Ljubljani*
Fakulteta *za elektrotehniko*

Computer Vision 10 – Image processing and analysis 2b

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Quick recap of the previous lectures

- Image formation
- Color
- Image processing
- Edges, Hough transform
- Segmentation, morphological operations

Outline

- Image analysis, continued

- Morphological operations on grayscale images
- Distance transform
- Region boundaries & active contours
- Region descriptors, moments

Morphological operations on grayscale images

- Until now, we always assumed thresholding *before* applying
  - Erosion
  - Dilation
  - Opening
  - Closing

- Can those operations be performed on grayscale images?
  - Yes, we can expand the definition of erosion and dilation to fit grayscale images as well

Erosion and dilation on grayscale images

- Now the image is not constrained to 0 and 1
  - Instead of forcing the central element to 0 or 1, we substitute it with the min or max pixelwise operation!
- Example:
  - SE – structuring element of ones in 5x5 window
  - Slide SE through the image
- Erode: min(Image, SE)
  - therefore, if at least one of pixels within SE belongs to background (0), the central pixel will become 0
- Dilate: max(Image, SE)
  - therefore, if at least one of pixels within SE belongs to foreground (255), the central pixel will become 255

Distance transform

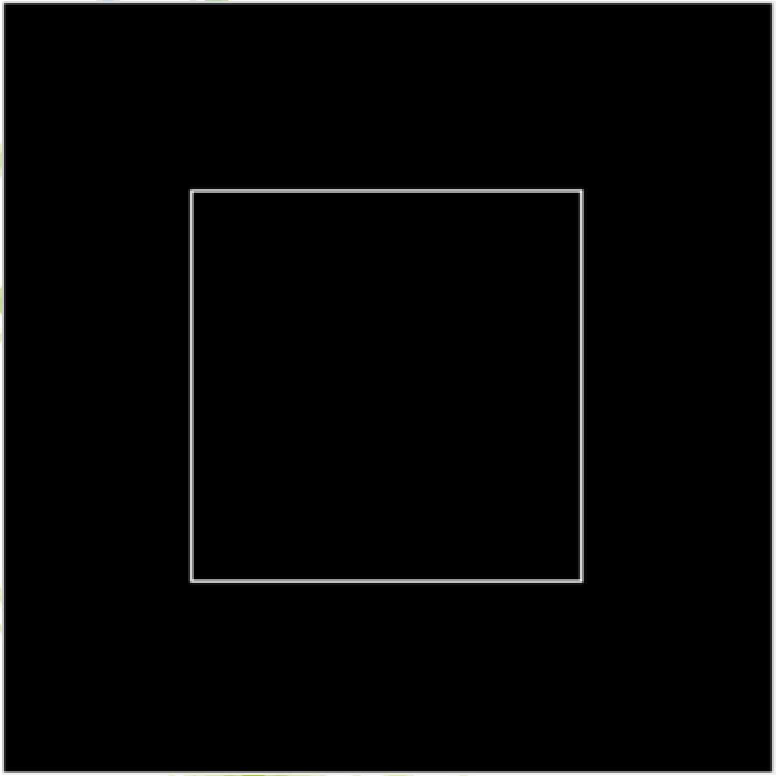
- Mostly used on binary images
  - Pixel values are replaced with the distance to important structures in the image, such as background.

```
. . . . X . . .
. . X X X X X .
. . X X X X X .
. . X X X X X .
. . . . X . . .
```

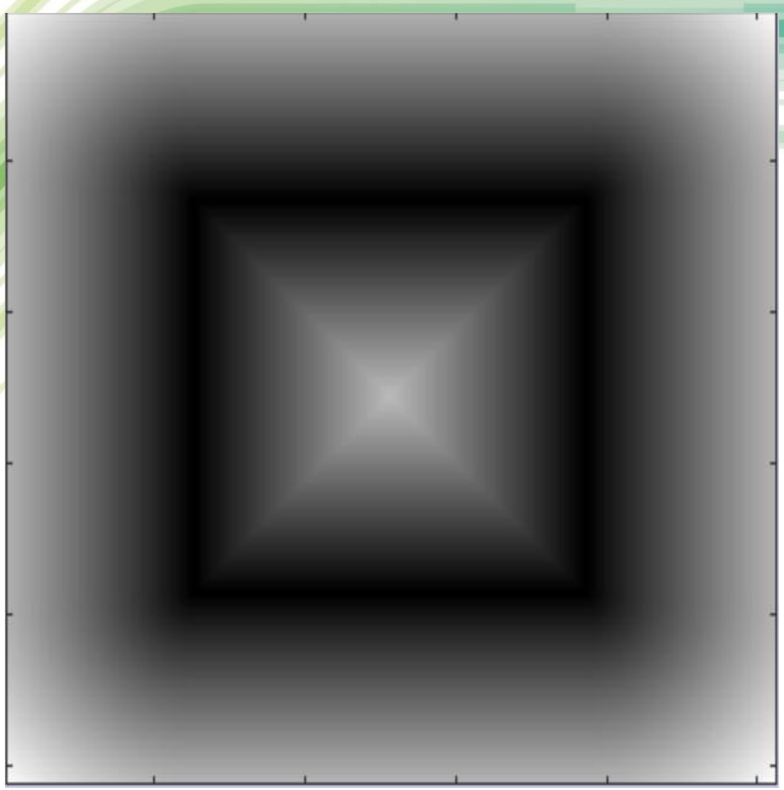
```
. . . . 1 . . .
. . 1 1 2 1 1 .
. . 1 2 3 2 1 .
. . 1 1 2 2 1 .
. . . . 1 . . .
```

- But typical “important structures” are edges, or silhouettes
- Note: dots could be labeled as well.

Distance transform - illustration

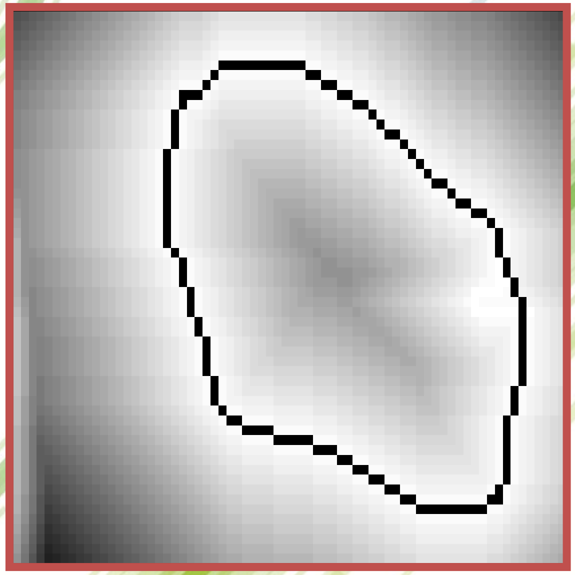


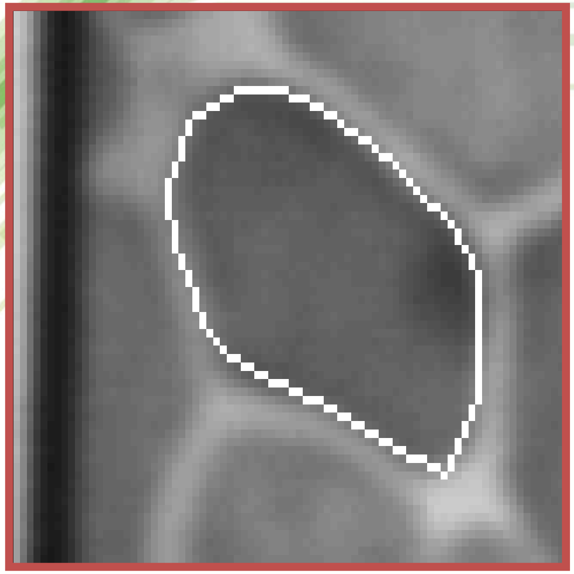
Input image: edge pixels (e.g. after edge detection)



Distance transform. Pixel values are proportional to the pixel distance from the edge

Distance transform - illustration





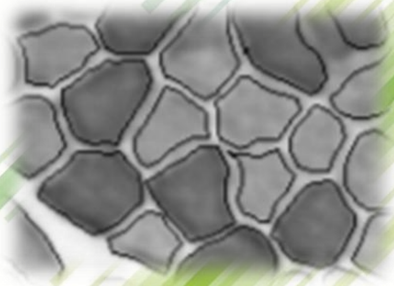
- Many use cases – e.g. template matching
  - How does it work in that case?

Other morphological operations

- Top hat (transform):
  - Result = original image – opening
- Bottom hat (transform):
  - Result = original image – closing
- Watershed transform:
  - Think of an image as a landscape of mountains (light regions) and valleys (dark regions)
  - then flood the valleys.
  - Ridges define solution.

Contour based approaches

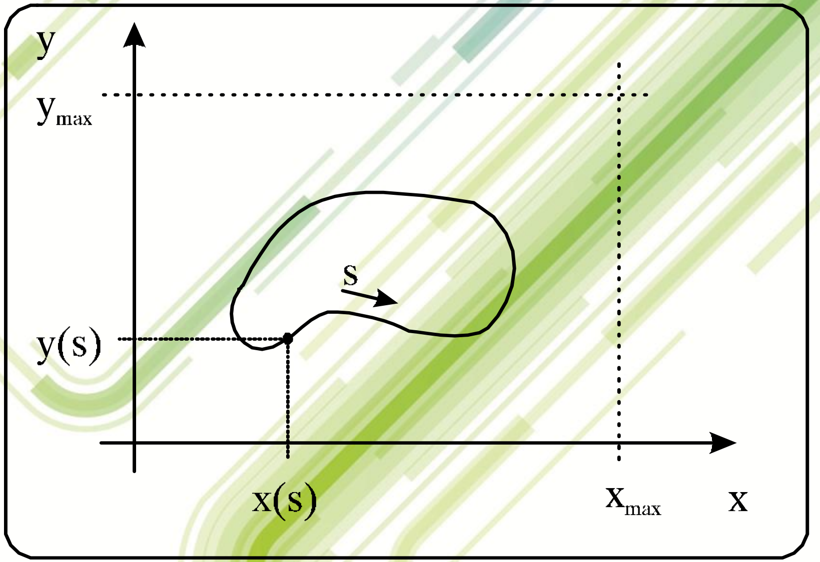
- Often, the best way do describe a region is by specifying the boundary



- Boundary = contour = contour based approaches.

Region description using boundary

- Object region is described by pixels on the boundary



Input: 2D image

Output: parametric curve

I(x, y) → (x(s), y(s))

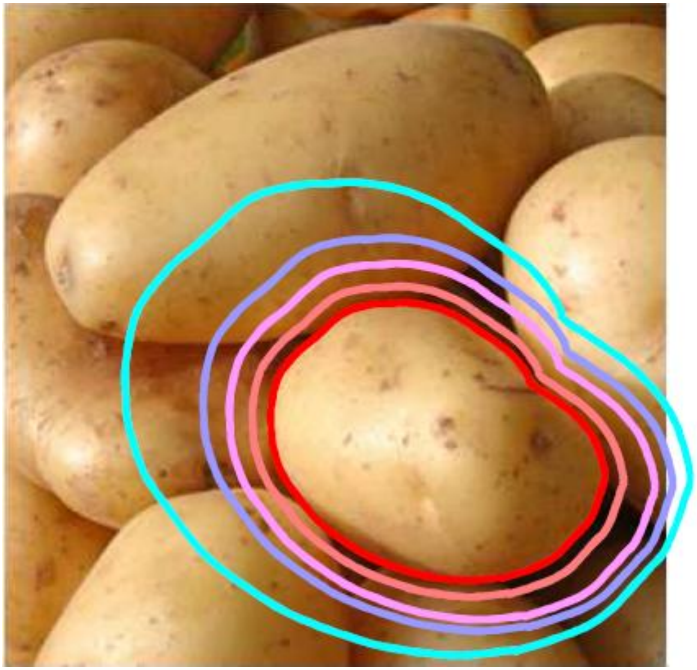
- Various algorithms for contour following (tracing) exist. We will not discuss those.

Active contour models

- Active contour models – “snakes”
  - (Kass, Witkin, Terzopoulos, 1988):
  - Contour is placed onto the image.
  - Contour acts as an elastic string that is moving, therefore “snake”.
- The contour

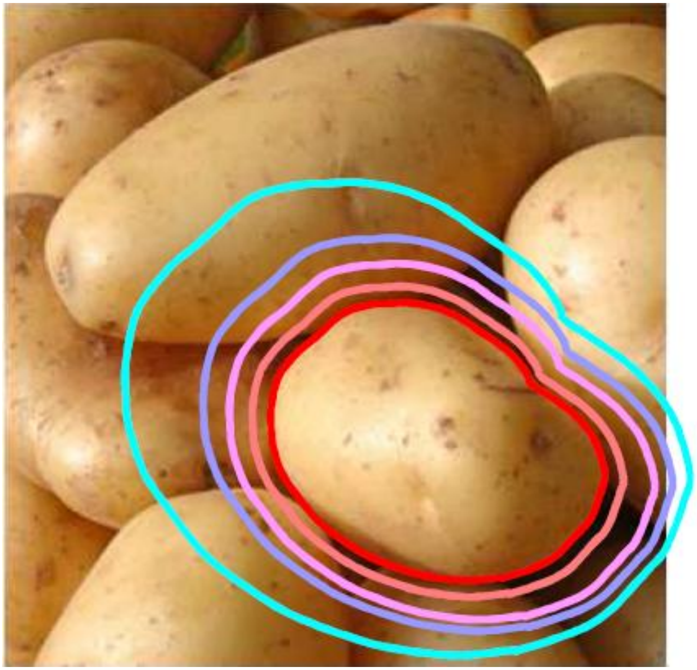
  $v(s)=(x(s),y(s)),\ s \in [0,1]$

  is fitted to the boundary of image region



Active contour models

- $v(s)=(x(s),y(s))$, $s \in [0,1]$
- Two types of forces influence the contour
  - *Image forces* attract snake to distinctive image structures, like edges,
  - *internal snake forces* resist tearing or folding. That is, internal forces keep contour continuous and smooth.



Active contour models

- The contour v(s)=(x(s),y(s)), s $\in$ [0,1] has an energy functional

$$E(\mathbf{v}) = \int_0^1 \left(E_{int}(\mathbf{v}(s)) + E_{ext}(\mathbf{v}(s))\right) ds$$

Internal (snake) energy
Increases if snake is
forced to deform

External (image) energy
Decreases if snake fits the
Image feature (contour) well

- Task: minimize E(v),
  - That is: produce best fit with minimum deformation

Active contour models

- Internal energy: smoothness constraint

$$E_{int}(\mathbf{v}(s)) = \frac{1}{2}\left(\alpha(s)\left|\frac{\partial \mathbf{v}(s)}{\partial s}\right|^2 + \beta(s)\left|\frac{\partial^2 \mathbf{v}(s)}{\partial s^2}\right|^2 \right)$$

Makes snake to behave like elastic band, prevents tearing

Makes snake to behave like a string, prevents bending

Active contour models

- Various implementation approaches exist:
  - direct discretization of energy functional
  - solution of Euler differential equation

$$-\alpha(s)\frac{\partial^2 \mathbf{v}(s)}{\partial s^2}+\beta(s)\frac{\partial^4 \mathbf{v}(s)}{\partial s^4}+\nabla P(\mathbf{v}(s))=0$$

Internal energy terms

External – image energy (that attracts the snake towards image edges)

Snake in the discrete form

- Using the finite differences method to solve the differential equation

$$\frac{1}{h}\left(\frac{\alpha_i}{h}(\mathbf{v}_i-\mathbf{v}_{i-1})-\frac{\alpha_{i+1}}{h}(\mathbf{v}_{i+1}-\mathbf{v}_i)\right)+$$

$$+\frac{1}{h^2}\left(\frac{\beta_{i-1}}{h^2}(\mathbf{v}_{i-2}-2\mathbf{v}_{i-1}+\mathbf{v}_i)-\frac{2\beta_i}{h^2}(\mathbf{v}_{i-1}-2\mathbf{v}_i+\mathbf{v}_{i+1})+\frac{\beta_{i+1}}{h^2}(\mathbf{v}_i-2\mathbf{v}_{i+1}+\mathbf{v}_{i+2})\right)$$

$$+\nabla P(\mathbf{v}_i)=0.$$

$$\mathbf{Av}+\nabla P(\mathbf{v})=0$$

A – stiffness matrix

v – vector of discrete contour points

$\nabla P(\mathbf{v})$ - image “potential field”

Snake in the discrete form

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$$\mathbf{Av}+\nabla P(\mathbf{v})=0$$

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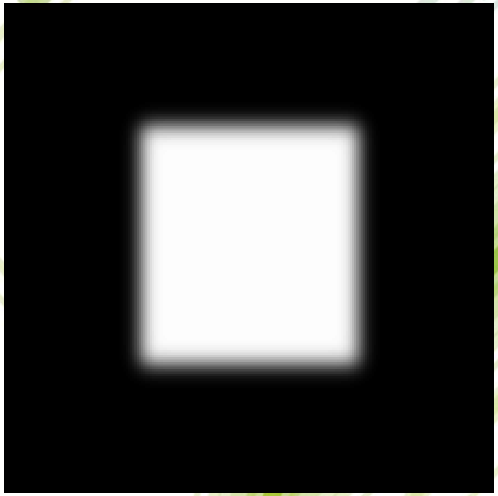
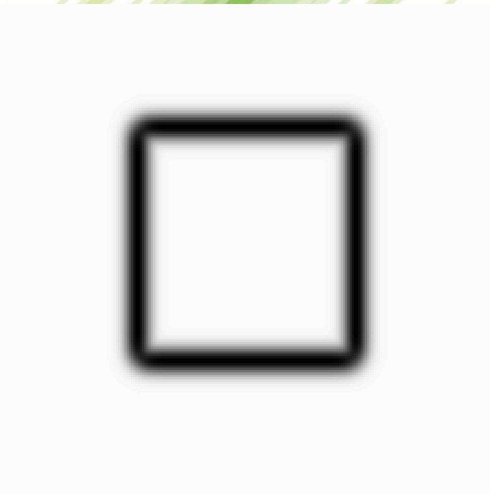
External energy – an example

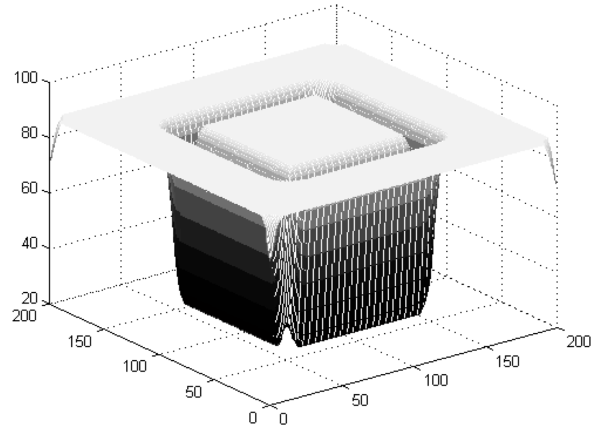
- Snake is attracted to the shape in the first image



image

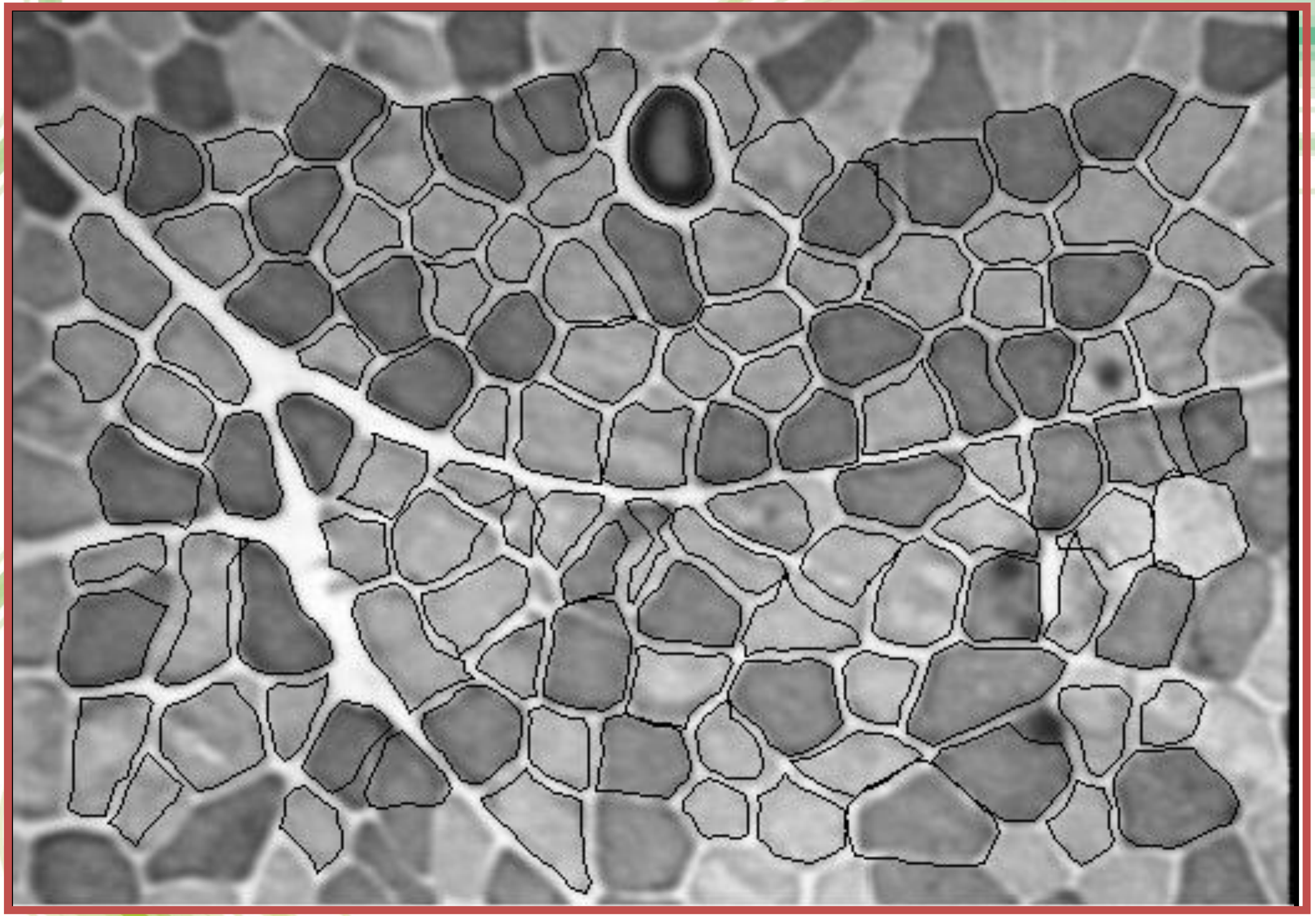


gradient



ext. energy

Snakes – segmentation example



- Initialization has to be provided (e.g. manual)

Snakes - problems

- Snakes are very sensitive to local minima
- Initialization of snake is a serious problem by itself
- All that snake does is smoothing the solution, no shape preferences
- Thus, many improvements are possible, and have been developed:
  - various snake derivatives,
  - and later point distribution models (PDM),
  - active shape models (ASM), ...

Snakes in tracking

Kalman snakes (J. Denzler)

Condensation snakes
Particle filtering approach
(Isard & Blake)

(video!)

Region (2D shape) description

- Once the regions are detected and different regions labeled, we can do many things to describe regions, i.e. compute
  - position,
  - orientation,
  - bounding box,
  - shape factors,
  - area,
  - perimeter,
  - texture, appearance...
- Matlab: regionprops

- But before that, we usually apply various

Region (2D shape) description

- But before calculating region descriptors, we usually apply various "morphological operations", i.e., perform *morphological filtering* as required.

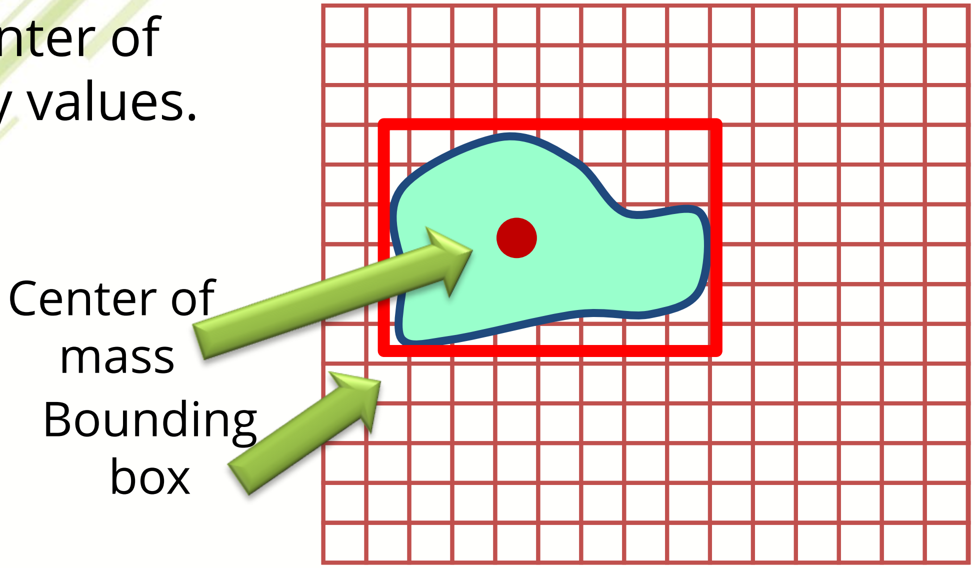
- Usually the whole procedure is as follows:
  - Image filtering/preprocessing
  - Image thresholding
  - Morphological operations (filtering)
  - Region analysis (region descriptors) as high level features

Region (2D shape) descriptors

- Size, area and / or perimeter, bounding box
- simply count pixels that belong to region / boundary
- Position
  - (e.g. x and y of the center of *mass*) = mean x and y values.

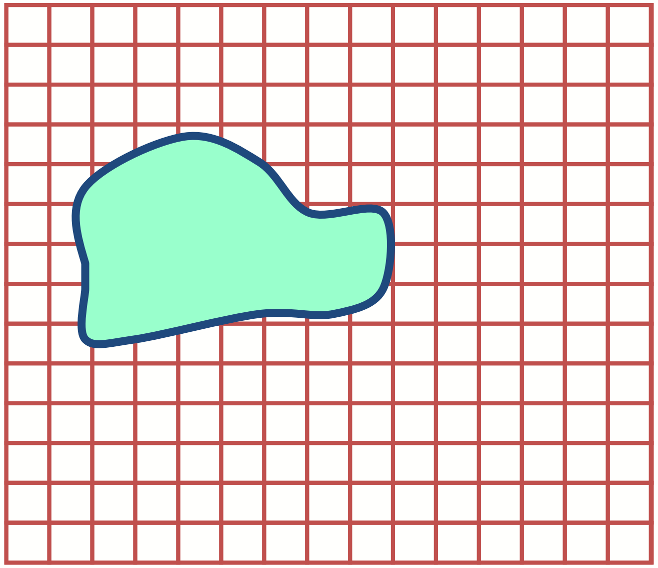
$$x_c = \sum_{x,y} x f(x,y) / \sum_{x,y} f(x,y)$$

$$y_c = \sum_{x,y} y f(x,y) / \sum_{x,y} f(x,y)$$



Region (2D shape) descriptors

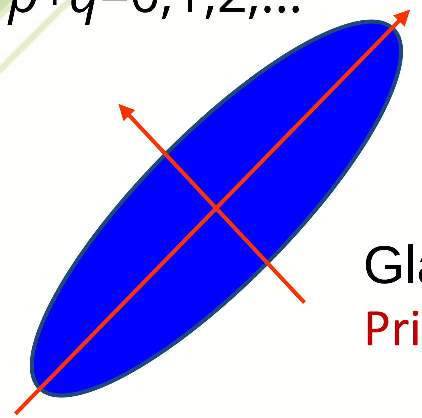
- Orientation
  - (e.g. principal axes, moments of inertia, covariance matrix)
- Shape factors
  - compactness, roundness, eccentricity, moments, …
- Fourier descriptors, bending energy, …



Region (2D shape) descriptors

- x and y are treated as random variables.
  - Raw (Statistical) moments of order *p*+*q*=0,1,2,…

$$m_{pq} = \sum_{x,y} x^p y^q f(x,y)$$



- Central moments

$$\mu_{pq} = \sum_{x,y} (x - x_c)^p (y - y_c)^q f(x,y)$$

  - Used for shape description, orientation estimation, normalization

Raw and central moments

- Image moments are essentially a generalization of “intuitive” descriptors.
  - For example, center of mass, as derived from raw moments:

$$x_c = \frac{m_{10}}{m_{00}}, \quad y_c = \frac{m_{01}}{m_{00}}$$

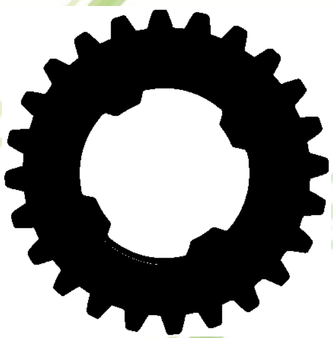
Shape/form factors

- Shape factors – global shape features
  - Compactness
  - Elongateness
  - Eccentricy
  - Moments

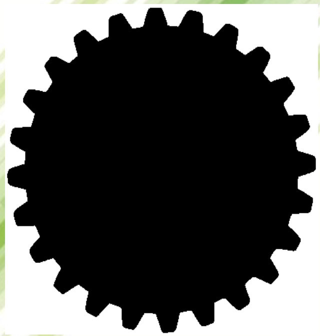
  …
  - Fourier descriptors
  - Bending energy (of an outline)

Compactness

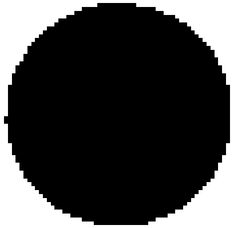
- Compactness = perimeter^2 / area



Compactness =7.4



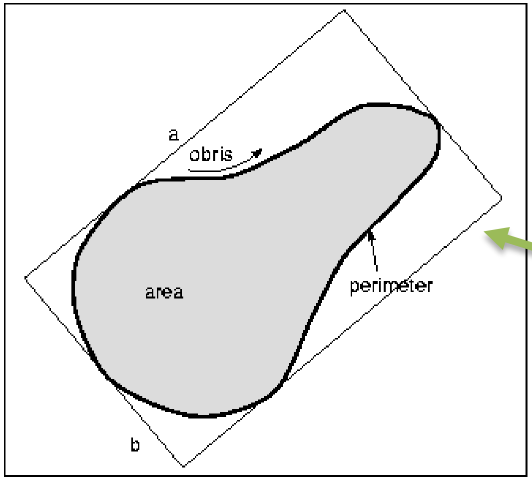
Compactness =2.8



Compactness =1.1
Almost a circle!

Eccentricity

- Eccentricity = major axis / perpendicular axis = (a / b)

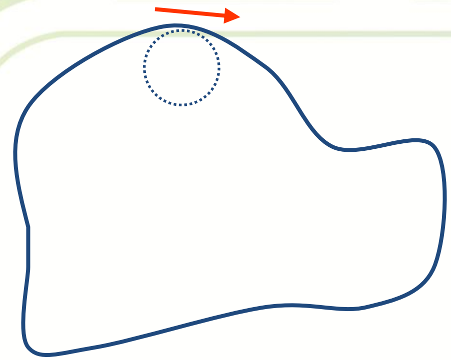


Bounding box

Shape from outline

Bending energy $BEN = \int_s k^2(s)ds$

$(x(s), y(s)), \quad (0 \leq s < L)$



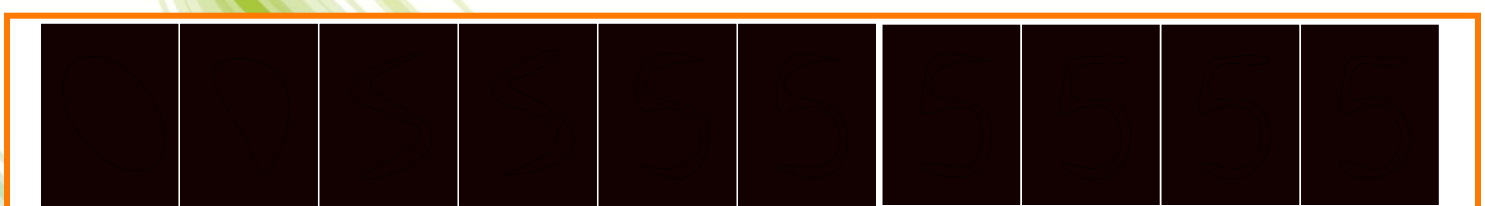
$$k = \frac{1}{r} = \left\| \frac{dT}{ds} \right\| = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\sqrt{(\dot{x}^2 + \dot{y}^2)^3}}$$

k : curvature

r : radius of osculating circle

dT : tangent vector

Fourier descriptors



Literature

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Questions?