Univerza *v Ljubljani*
Fakulteta *za elektrotehniko*

Computer Vision
03b – Lens distortion

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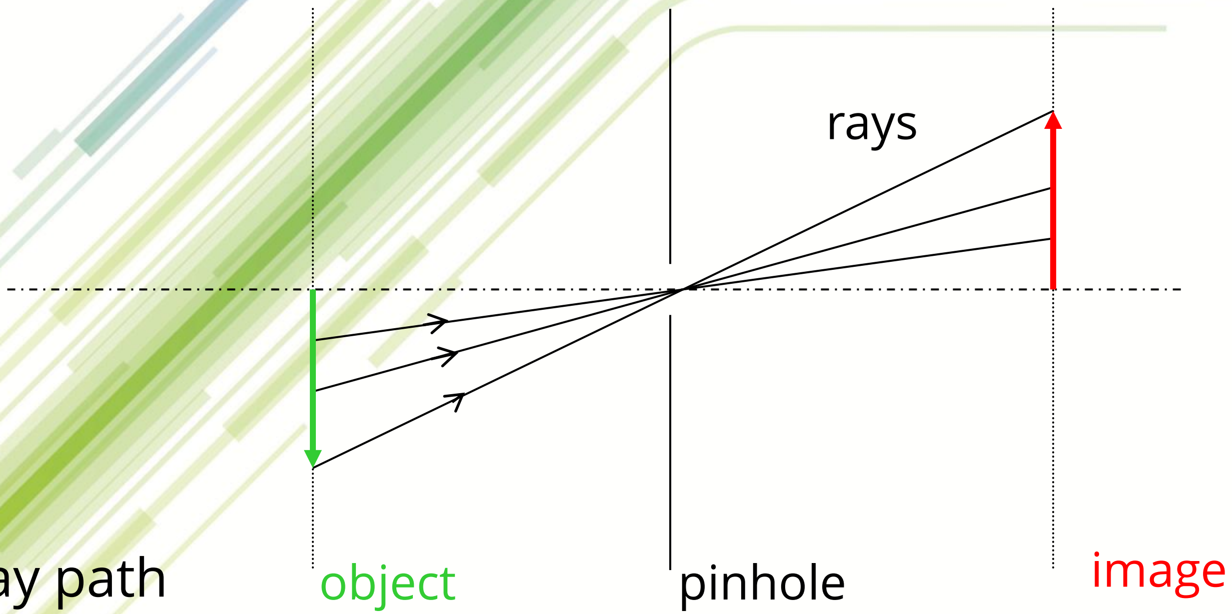
Introduction

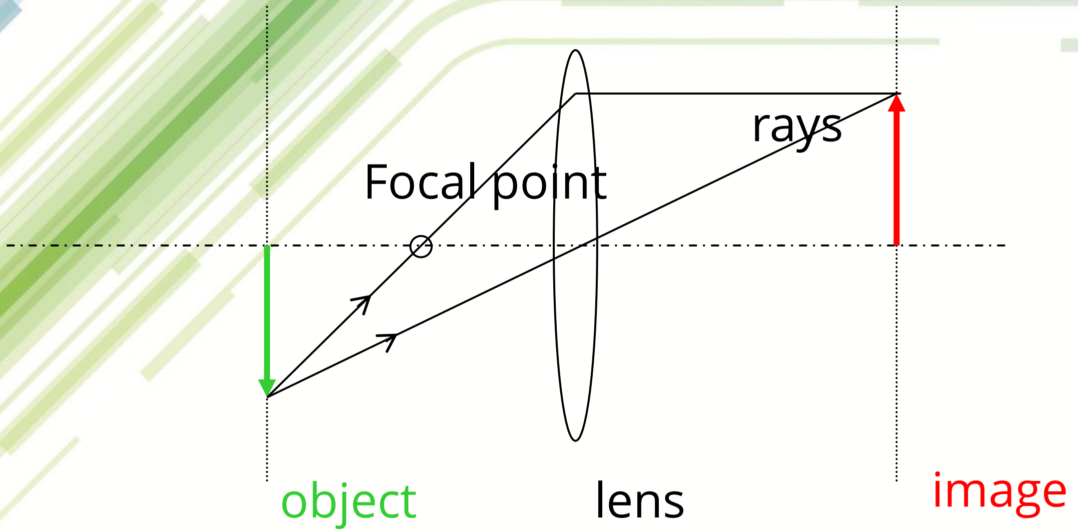
- Pinhole camera



  - Straight ray path
  - No distortions – a perfect camera
  - Impractical: at ISO film speed of 200, 1-3 sec exposure time, in the sun!

Introduction

- Optical system with lens



  - Rays are *deflected* on their way through the lens
  - Flexible – rays are *focused*, forming brighter image
  - Introduction of *imperfections* and tradeoffs
  - Result: optical *aberrations*

In practice: compound lens

- Optical system with lens
  - Several (or many) optical elements to achieve better performance
  - Exact (theoretical) analysis is very difficult
  - Or impossible due to trade secrets of lens manufacturers
  - Example of compound lens: AF Nikkor 20mm f/2.8D:

Fundamental literature

- If (!) you need to dive deeper into the optical systems:
  - Lens imperfections poorly covered in computer vision literature (except radial distortion)
  - Well studied in the field of *photogrammetry* (*photogrammetry*): the science of making reliable measurements from photographs
  - "The bible":
    Chester C. Slama (Ed.), *Manual of Photogrammetry*, 4th Ed. American Society of Photogrammetry, Falls Church, VA, 1980.
  - Difficult to get (in Slovenia: library @ Institute of Forestry)
  - Last edition: 2005 (5th edition) – sold out.

Lens aberrations

- Aberrations degrade image quality, some affect geometry as well
  - Astigmatism
  - Coma
  - Curvature of field
  - Color aberrations

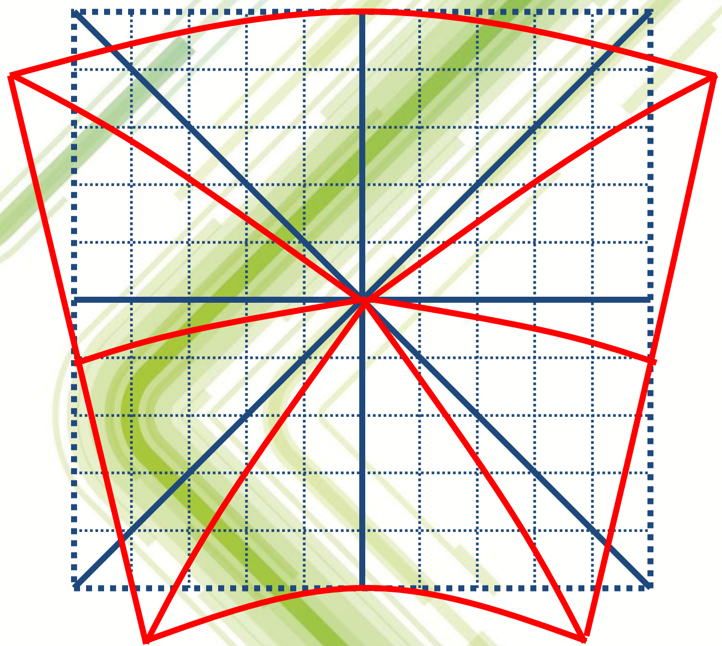
- Degradation of quality is visible -> manufacturers try hard to reduce them even in consumer-grade equipment

Lens distortions

- Distortions
  - Radial
  - tangential
- Distortions affect image geometry
- Present, but not very visible to consumers
- Consequence: linear camera model is not sufficient in practice

Lens distortions

- Tangential distortion
  - Misalignment in elements of the optical system
  - Image points are displaced in the *tangential* direction

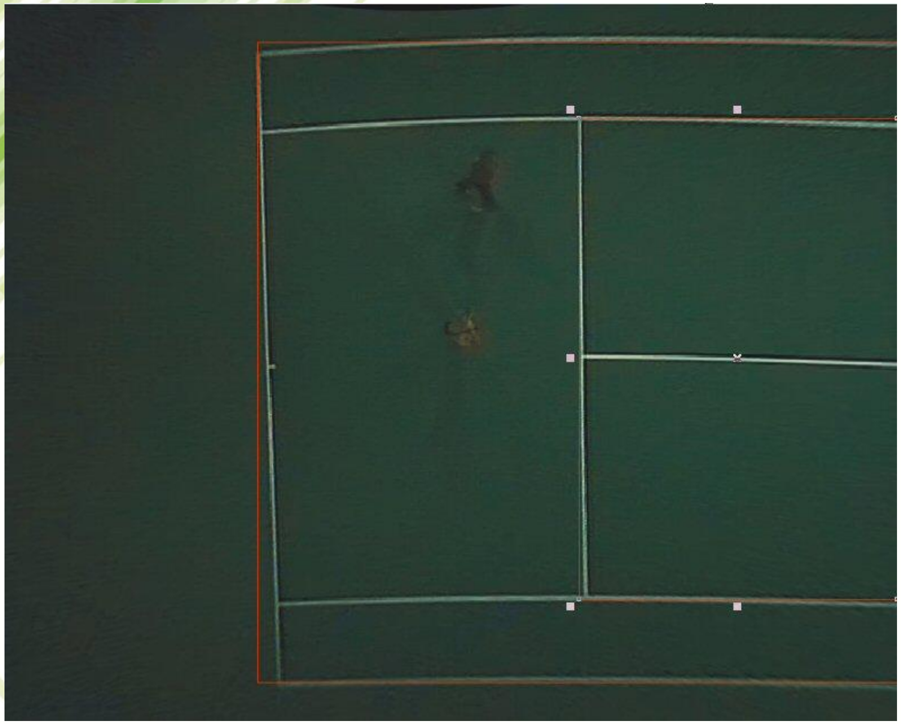


Ideal image

Image with tangential distortion

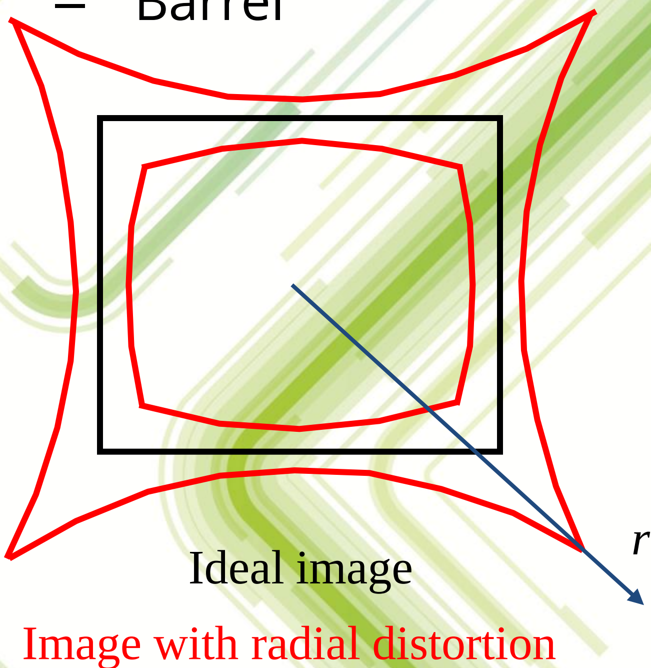
Tangential distortion – an example

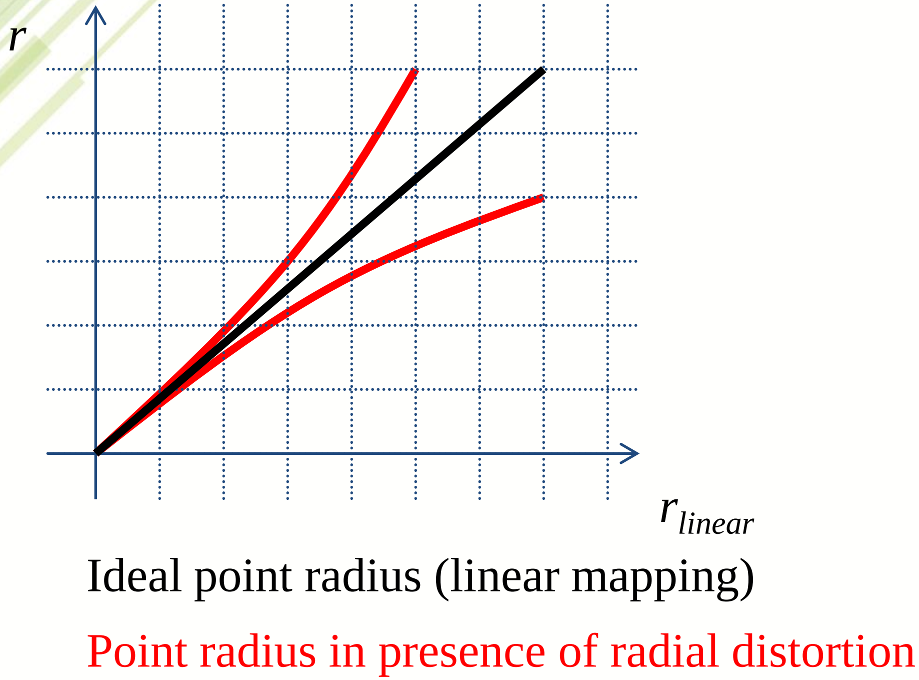
- Rarely one can see an obvious case
  - In this case: lens was not properly attached to the camera (misalignment!)



Lens distortions

- Radial distortion
  - Image points are displaced in radial direction - 2 types
  - “Pincushion”
  - “Barrel”



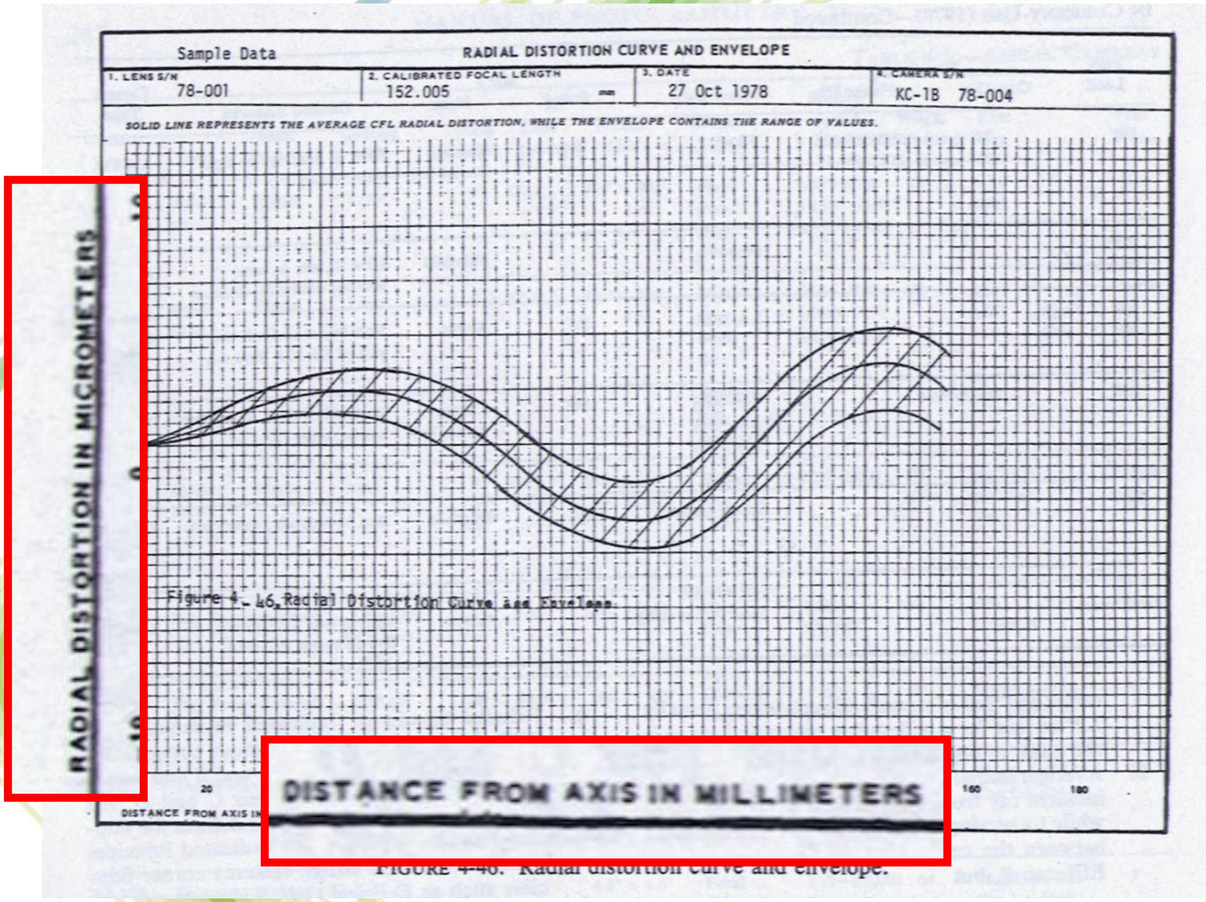


Radial distortion – an example

- Pincushion type
  - Rarely seen, not very obvious
  - Sometimes caused by overcompensated barrel distortion?
- Barrel type
  - ubiquitous in wide-angle, consumer-grade lens

Radial distortion – measured

- Source: Manual of Photogrammetry, 1980



Somewhat confusing treatment of radial distortion (1/2)

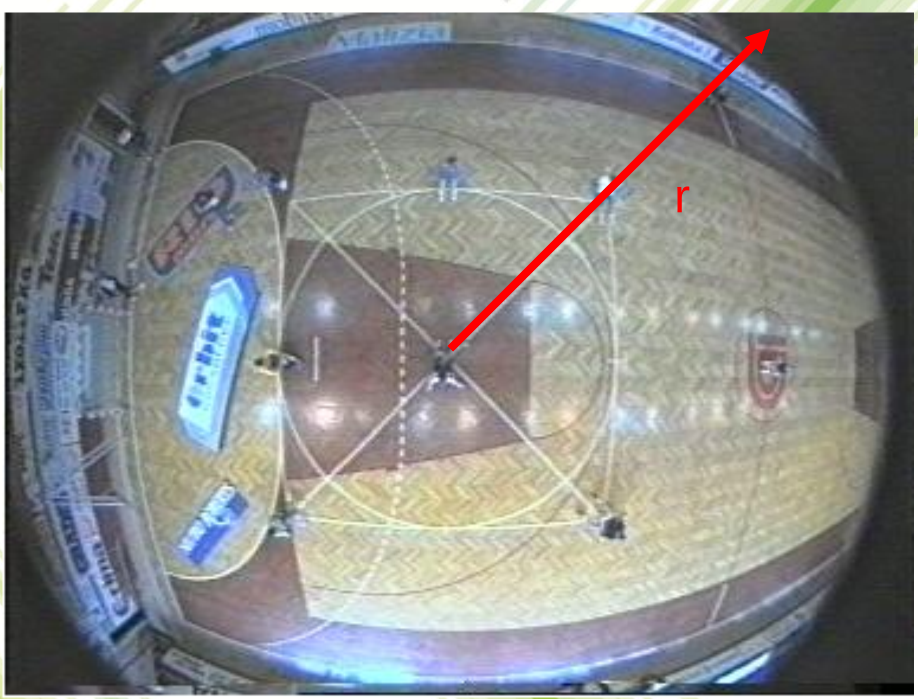
- In photogrammetry
  - Extremely small distortions (e.g. 100 µm at r=100 mm)
  - Complex, polynomial shape of distortion function
  - Difficult to spot even on megapixel digital images
- In camera calibration
  - "Significant distortion" = 3%-4% (R. Tsai, 1987)
  - Polynomial approximation methods, correcting any type of distortion
- In computer vision "in general"
  - "Large distortion" = up to 30%, wide angle, fisheye lenses
  - Almost always, barrel type distortion is addressed
  - Attempts to find alternative approximations to barrel distortion function

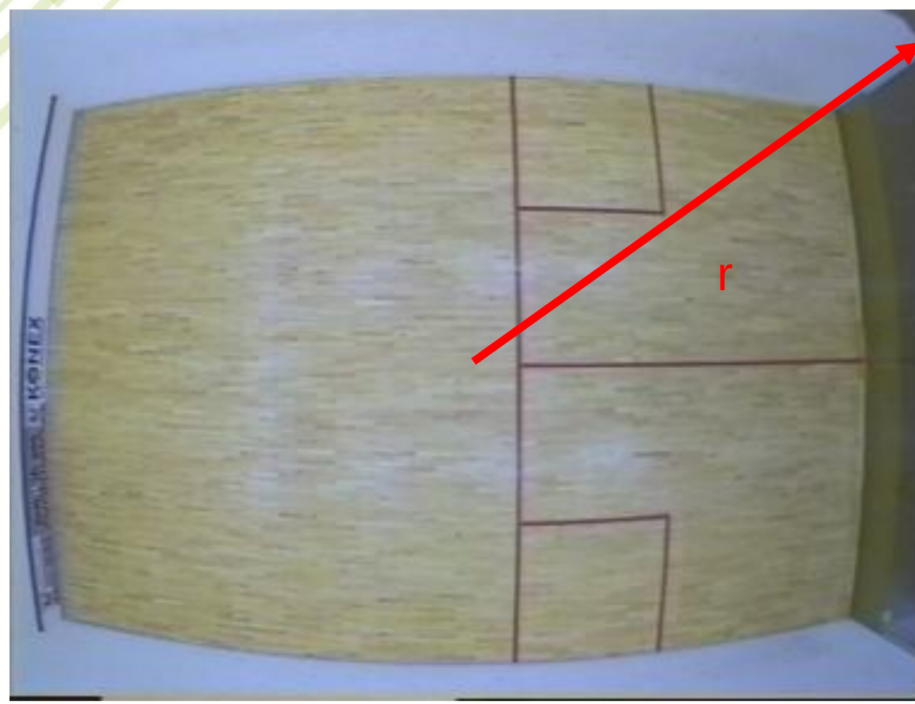
Somewhat confusing treatment of radial distortion (2/2)

- Background
  - In optical engineering, complex formulas (projection functions) used to design even “simple” lens
  - The actual formula not revealed to the end user
  - Most lenses on the market of compound type
  - Lens design a compromise between brightness, uniformity, viewing angle, distortion, etc.
- Low distortion lens (photogrammetry)
  - Radial distortion in “photogrammetric sense” likely only the residual after the (imperfect) compensation
- In computer vision
  - “Low quality” (“Off-the-shelf”) lenses, that were not designed for measurement are used for measurement!
  - No incentive for any distortion correction!

Radial distortion

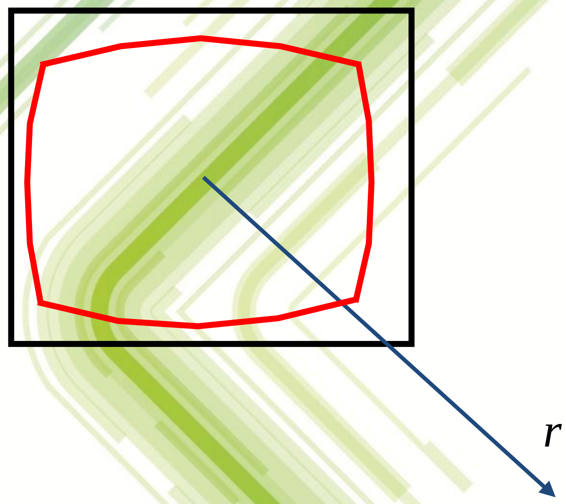
- In computer vision mostly barrel type ...
  - when using wide-angle lenses
  - most of the researchers tried to address this particular case

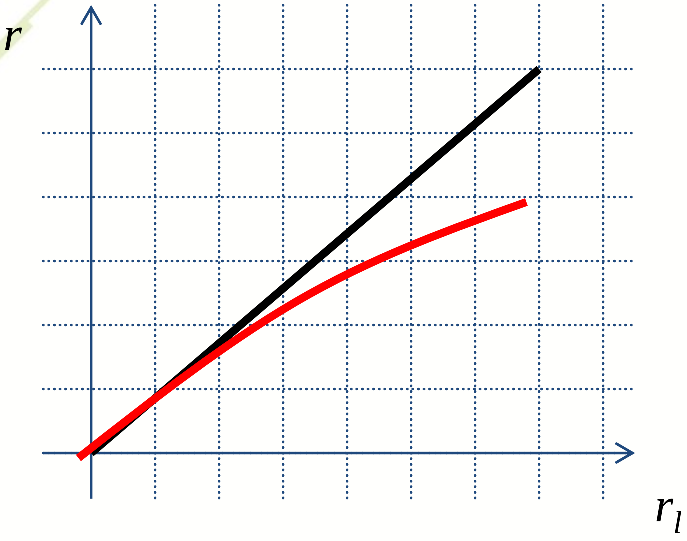




Radial distortion model

- In general, we seek non-linear function of the form
  $r = f(r_l)$ or $r - r_l = f(r_l)$
  which should describe transformation of radius r_l into r.





Radial distortion model

- Polynomial approximation of the form:
  $\Delta r = f(r_l) = k_0 + k_1 r_l + k_2 r_l^2 + k_3 r_l^3 + \ldots + k_n r_l^n$
  - Inherited from the field of photogrammetry
  - … where the distortion usually is of unpredictable nature
  - Alternative/equivalent formulations (e.g. for Δx, Δy) exist.
- Advantage:
  - General model, does not assume type of the distortion
- Disadvantages:
  - With significant distortion, many coefficients are needed!
  - Closed-form inverse cannot be obtained

Radial distortion model

- Redundant for pure barrel type distortion
  - coefficients are used to better fit the monotonically increasing slope of $f(r_l)$, instead of modelling the unpredictable form of $f(r_l)$, as envisioned in photogrammetry)
  - Obtaining parameters is an optimization process, which can be unstable and/or slow.

- That's why people invented many competing models for radial distortion

Alternative models

- Alternative models have appeared
  - To model particular kind of distortion (monotonic, barrel) with less coefficients than in polynomial model
  - To provide closed-form inverse
  - To provide model with parameters, related to physically measurable lens properties
    - viewing angle
    - focal length
- Polynomial model is still state-of-the art
  - Sometimes it is inadequate
  - Optimization may not converge, or is unstable
  - Small number of parameters or inverse needed
  - Sometimes, simpler model is preferred, even with lower accuracy

Alternative models

- Fish-Eye Transform (FET)

$$r = s \log(1 + \lambda r_l)$$

  - Anup Basu, Sergio Licardie, Alternative models for fish-eye lenses, *Pattern Recognition Letters*, Volume 16, Issue 4, April 1995, pages 433-441,

Alternative models

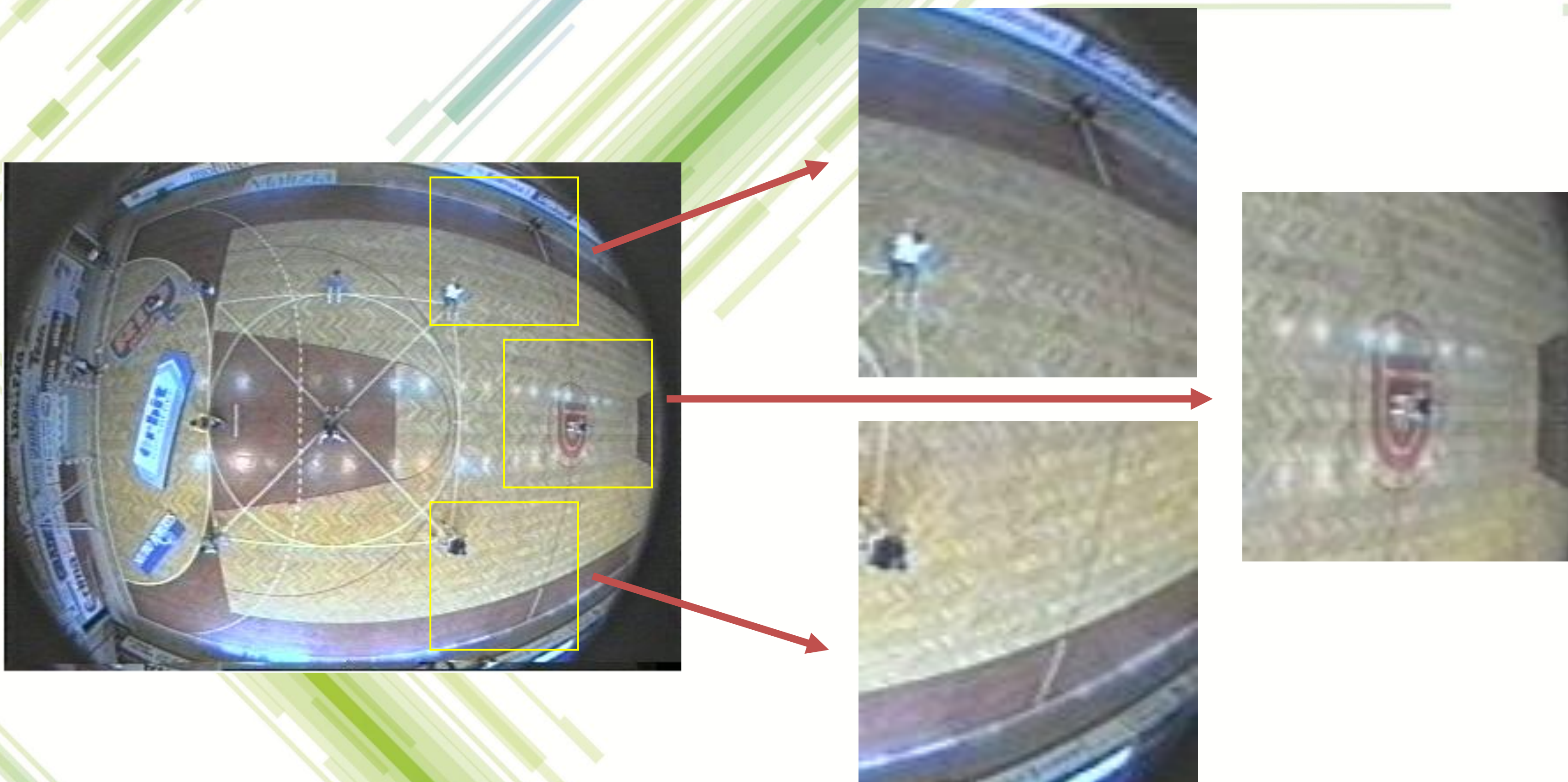
- Field Of View model (FOV)

$$r = \frac{1}{\omega}\arctan\left(2r_l \tan\frac{\omega}{2}\right)$$

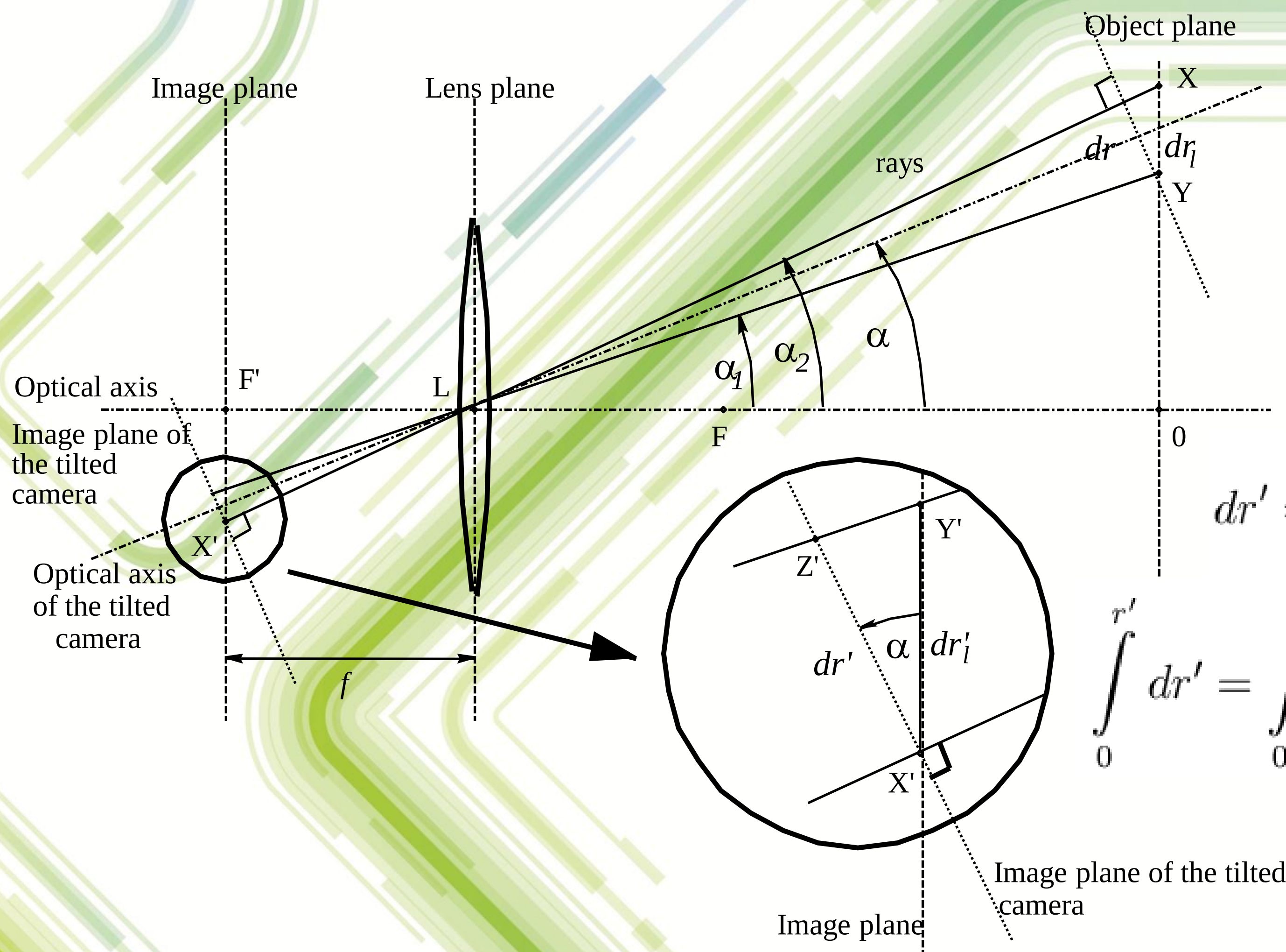
  – Frédéric Devernay, Olivier Faugeras, Straight lines have to be straight, *Machine Vision and Applications*, Volume 13, Issue 1, pages 14-24, 2001.

Another model – our derivation

- Based on the properties of the distorted image
  - “Tilted camera” analogy



Derivation



$$dr' = \cos(\alpha) \cdot dr'_l,$$

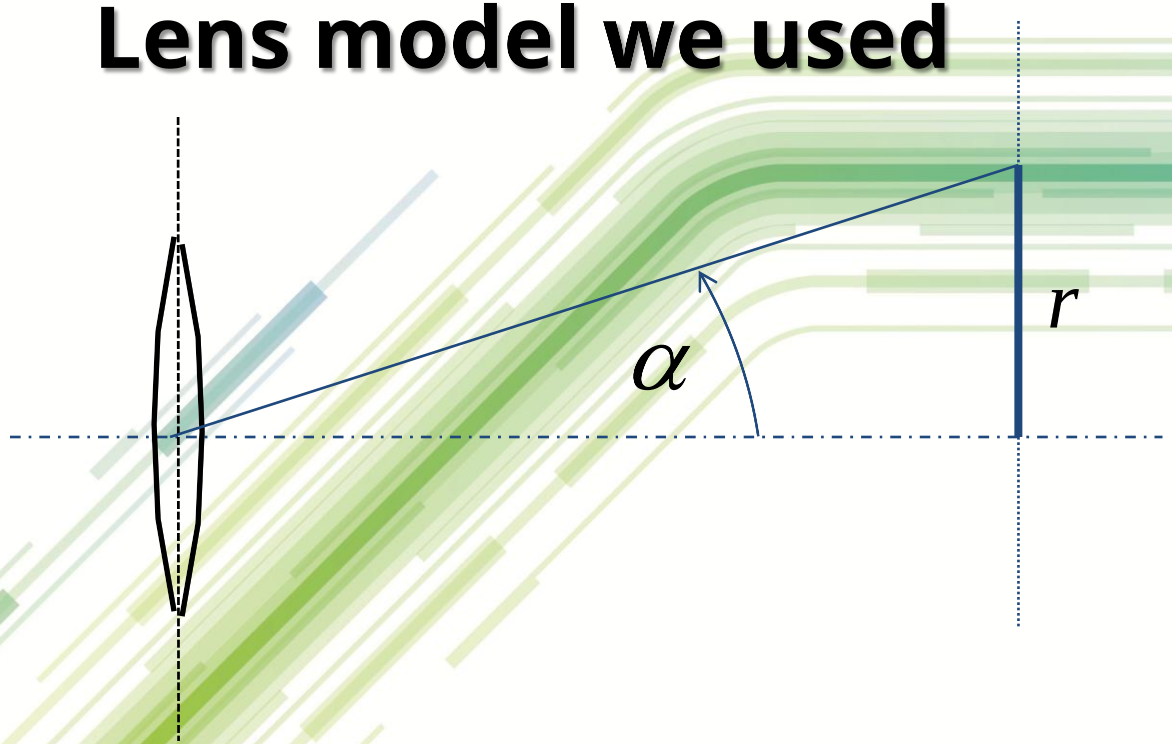
$$r'_l = f \tan \alpha,$$

$$\alpha = \arctan \frac{r'_l}{f}.$$

$$dr' = \cos \left[\arctan \frac{r'_l}{f} \right] dr'_l.$$

$$\int_0^{r'} dr' = \int_0^{r'_l} \cos \left[\arctan \frac{r'_l}{f} \right] dr'_l,$$

Lens model we used



- Radial projection functions (lens specific!):
  - **perspective**, $r(\alpha) = f\tan(\alpha)$
  - stereographic, $r(\alpha) = f\tan(\alpha/2)$
  - equidistant, $r(\alpha) = f\alpha$
  - equi-solid angle, $r(\alpha) = f\sin(\alpha/2)$
  - sine law $r(\alpha) = f\sin(\alpha)$

Our radial distortion model

- Radial distortion function:
  - Based on the perspective radial projection function

$$r' = f \cdot ln\left(\frac{r'_l}{f} + \sqrt{1 + \frac{r'^2_l}{f^2}}\right) = f \cdot \operatorname{arcsinh}\left(\frac{r'_l}{f}\right)$$

- The inverse:

$$r'_l = f \cdot \sinh\left(r'/f\right).$$

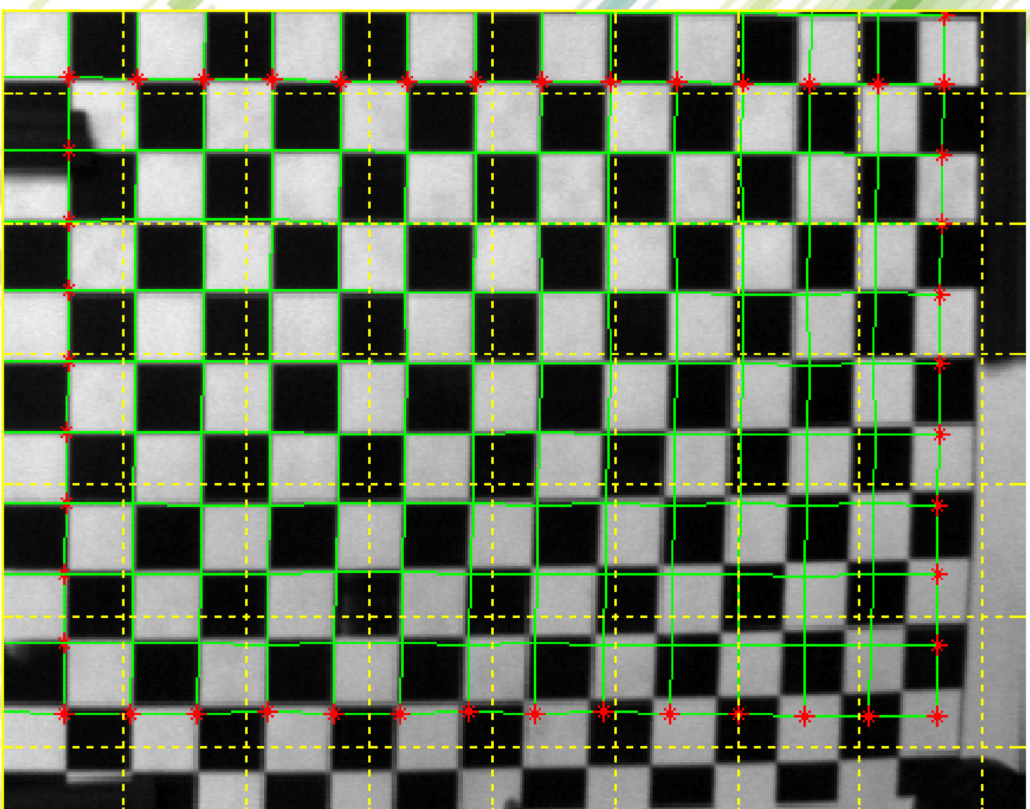
- Only parameter is the focal length *f*!
  - Already a parameter of linear calibration

Experiment

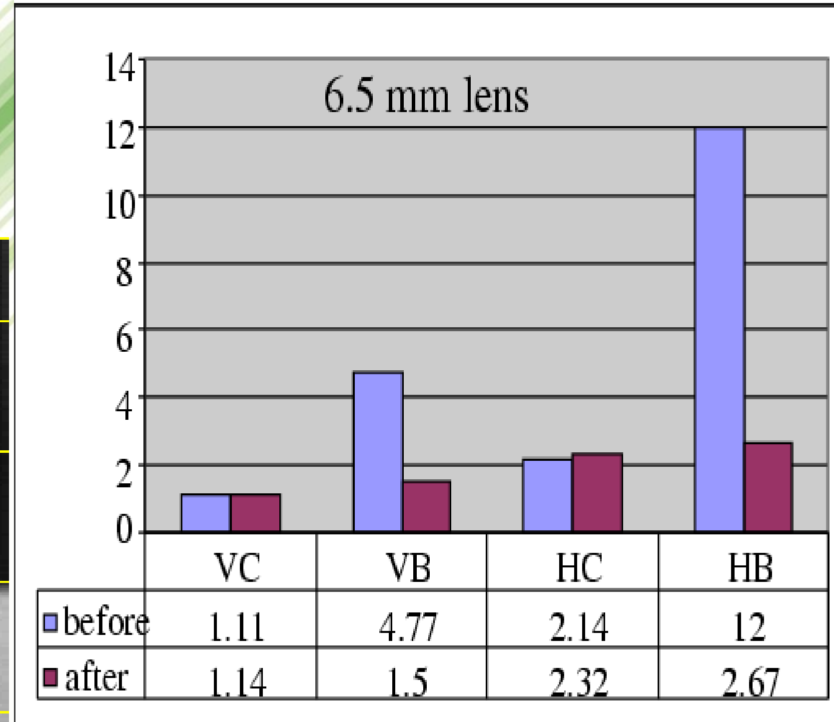
- Protocol:
  - Used 6.5 mm and 8 mm lens
    - We show results only for 6.5 mm TAMRON lens
  - Used DLT for calibration, regardless of the distortion
  - DLT-based calibration provided focal length *f* (in pixels)
  - Acquired chessboard image, and extracted grid of points
  - Used the calculated focal length *f* to correct the distortion and "flatten" the grid.
  - The distortion before and after the correction was measured by:
    - Fitting of lines across rows and columns of points
    - Measuring the residual error = residual nonlinearity

Results

- f = 818 pixels



1/4 of the image!



	VC	VB	HC	HB
before	1.11	4.77	2.14	12
after	1.14	1.5	2.32	2.67

VC - vertical center,

VB - vertical border,

HC - horizontal center

HB - horizontal border

Further reading

- By doing the radial distortion *correction* “the established way”, we are enforcing the *perspective projection.*
  - This is usually expressed as *“Straight lines have to be straight”* paradigm.
  - And it is rarely questioned.
  - But then we have to accept that “*circles are not circular”.*
  - Why should be straightness of lines more important than the shape of circles?
- If you are interested, the TR on this topic:
  - **Margaret M. Fleck (1994) "Perspective Projection: the Wrong Imaging Model," technical report 95-01, Computer Science, University of Iowa**

Questions?