Univerza v Ljubljani
Fakulteta za elektrotehniko

Computer Vision
04 – Image formation, part 3

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Quick recap of the previous week

- Perspective projection revisited
  - Nonlinearity, introduced because of “1/z”
- Homogeneous coordinates
- Projective geometry revisited
- Camera model revisited
  - Camera calibration matrix, projection matrix
- Lens distortions
- For the past 2 weeks, we dealt with *geometry* of image formation

Quick recap of the previous week

Rotation, translation, projection, discretization

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \mathbf{K}_{3\times3}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}_{3\times1}\end{bmatrix}\begin{bmatrix} {}^{C}X \\ {}^{C}Y \\ {}^{C}Z \\ 1 \end{bmatrix} = \mathbf{K}_{3\times3}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}_{3\times1}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{3\times3} & \mathbf{t} \\ \mathbf{0}^{\mathrm{T}}_{3\times1} & 1\end{bmatrix}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathbf{M}_{3\times4}\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Internal (intrinsic) parameters matrix

External (extrinsic) parameters matrix

Also note that:

$$\mathbf{K}_{3\times3}\begin{bmatrix}\mathbf{I}_{3\times3} & \mathbf{0}_{3\times1}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{3\times3} & \mathbf{t} \\ \mathbf{0}^{\mathrm{T}}_{3\times1} & 1\end{bmatrix} = \mathbf{K}_{3\times3}\begin{bmatrix}\mathbf{R}_{3\times3} & \mathbf{t}_{3\times1}\end{bmatrix} = \mathbf{M}_{3\times4}$$

Outline

- This week, we are dealing with photometric aspects of the image formation!
- Photometric lens equation
- A few facts about cameras and lenses
- Some lighting techniques

Light - scene – camera

How was this image obtained?

What does that tell us?

How well can we model image formation?

Image credit: Giles Tran

Photometric aspects of image formation

- Geometry of image formation
  - *Where* does the image *p* of the point *P* appear in the image?
  - 2 weeks of this behind you

- Photometry of image formation
  - Given the brightness of the point *P*, what will be the measured brightness of its image *p* ?
  - This is the subject of today's lecture!

Photometric lens equation

- Image on the sensor is a consequence of:
  - Light source, emitting light
  - Object surface, reflecting light
- Let's assume grayscale sensor and image
  - For now, for the sake of simplicity
- Image pixel values are proportional
  - to the illuminance (E) of the sensor
  - caused by the luminance (L) of the scene surface.

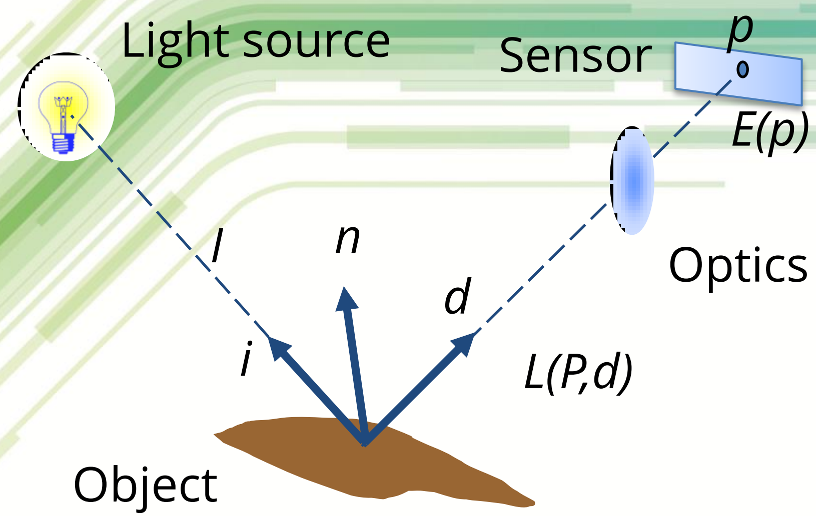
(scene luminance) $L \rightarrow E$ (image illuminance)

Photometric lens equation

- We are studying the effect of reflected light from the scene surfaces on the image sensor (CCD, CMOS).

- Only a fraction of that light gets captured by the sensor!

- Lambertian model – each surface point *P* is equally bright from all directions (*Lambertian* surface)



L – radiance, luminance
E – irradiance, illuminance
I – intensity
P – surface point (3D)
p – image plane point
n – surface normal
i – incoming light direction
d – outgoing light direction

Photometry refresher

- Φ = radiometric/photometric flux:
  - Radiometric unit [W] (watt), photometric unit [lm] (lumen)
  - 1 W of monocromatic 555 nm light == 683 lm
- flux density
  - [W / m^2], [lx = lm / m^2] (lux)
- E = irradiance / illuminance
  - [W / m^2], [lx =lm / m^2]
- I = intensity
  - [W / sr], [cd = lm / sr] (candela)
- L = radiance/luminance
  - [W / sr / m^2], [cd / m^2]

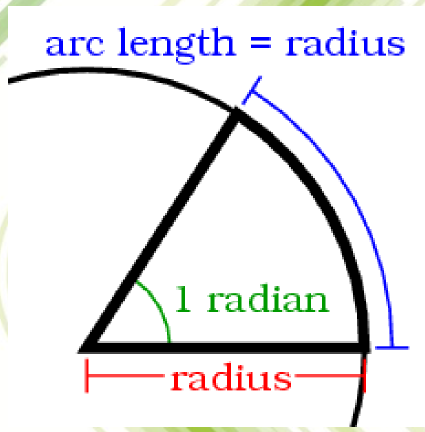
sr
(steradian)

Unit of *solid angle*!

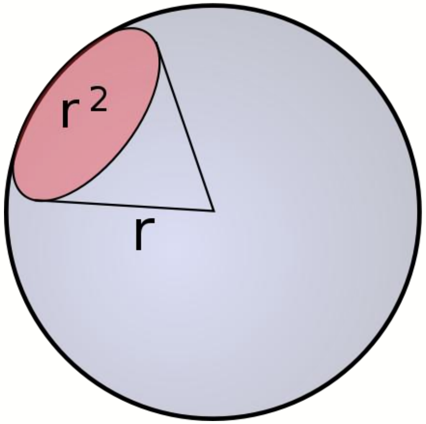
Solid angle [sr]

- 2D geometry – planar (1D) angles in radians
  - 1 radian = angle where length of the arc is equal to the radius
- 3D geometry – solid (2D) angles in steradians
  - 1 steradian = 2D angle where surface area equals r^2

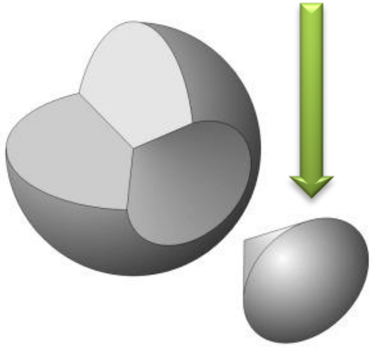
1 radian



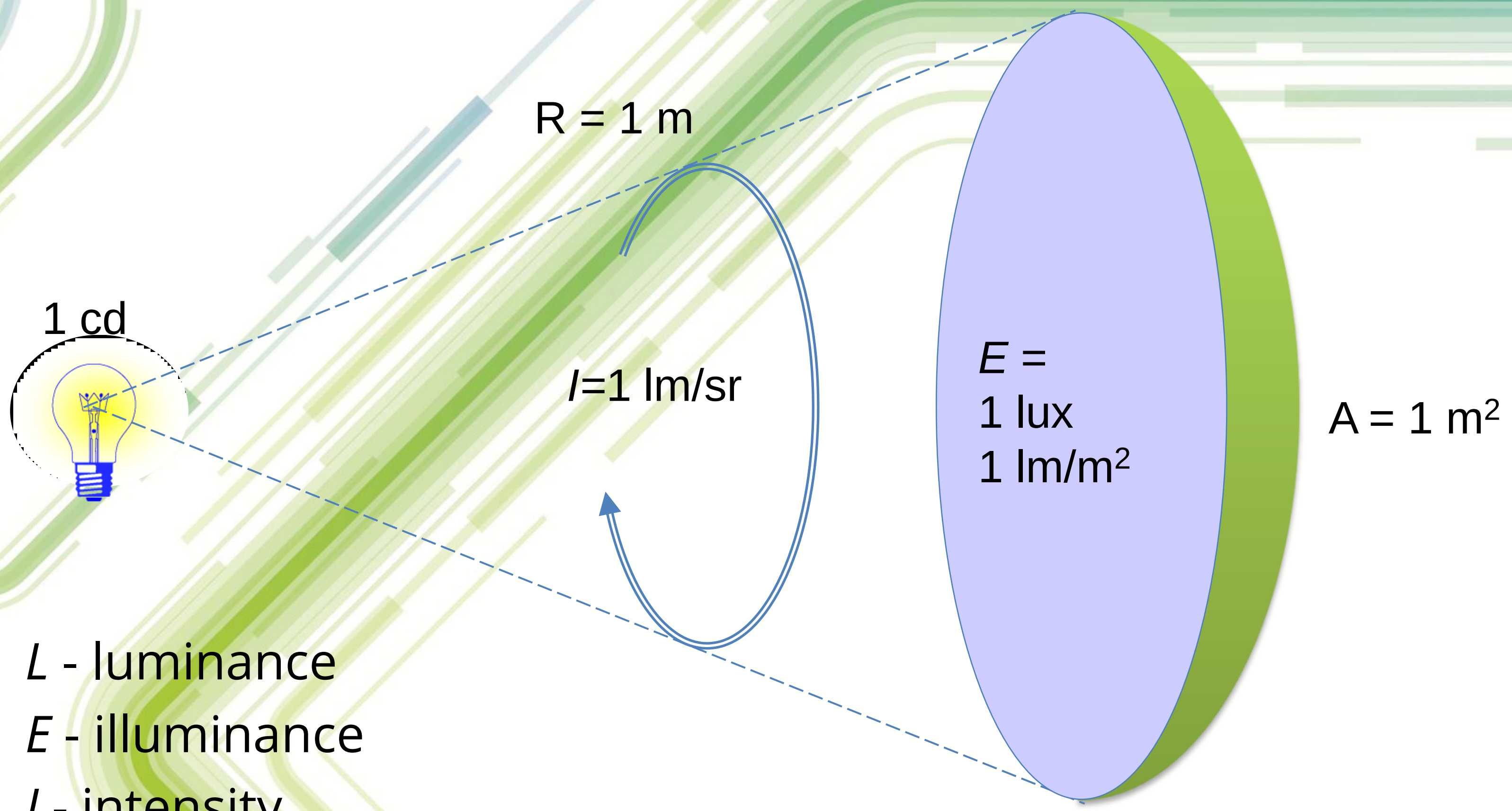
1 steradian



Solid angle of 1 steradian

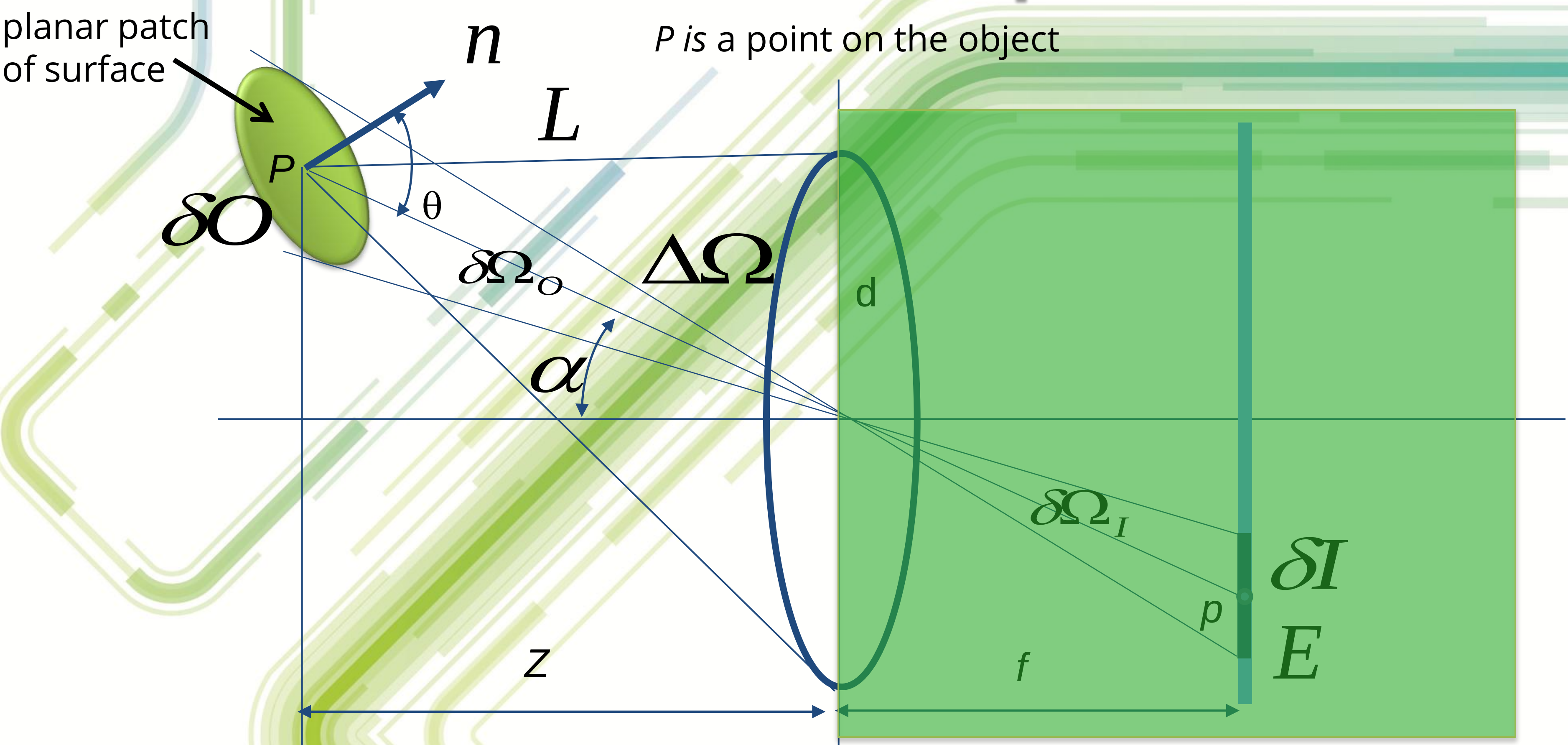


Measuring light



- *L* - luminance
- *E* - illuminance
- *I* - intensity

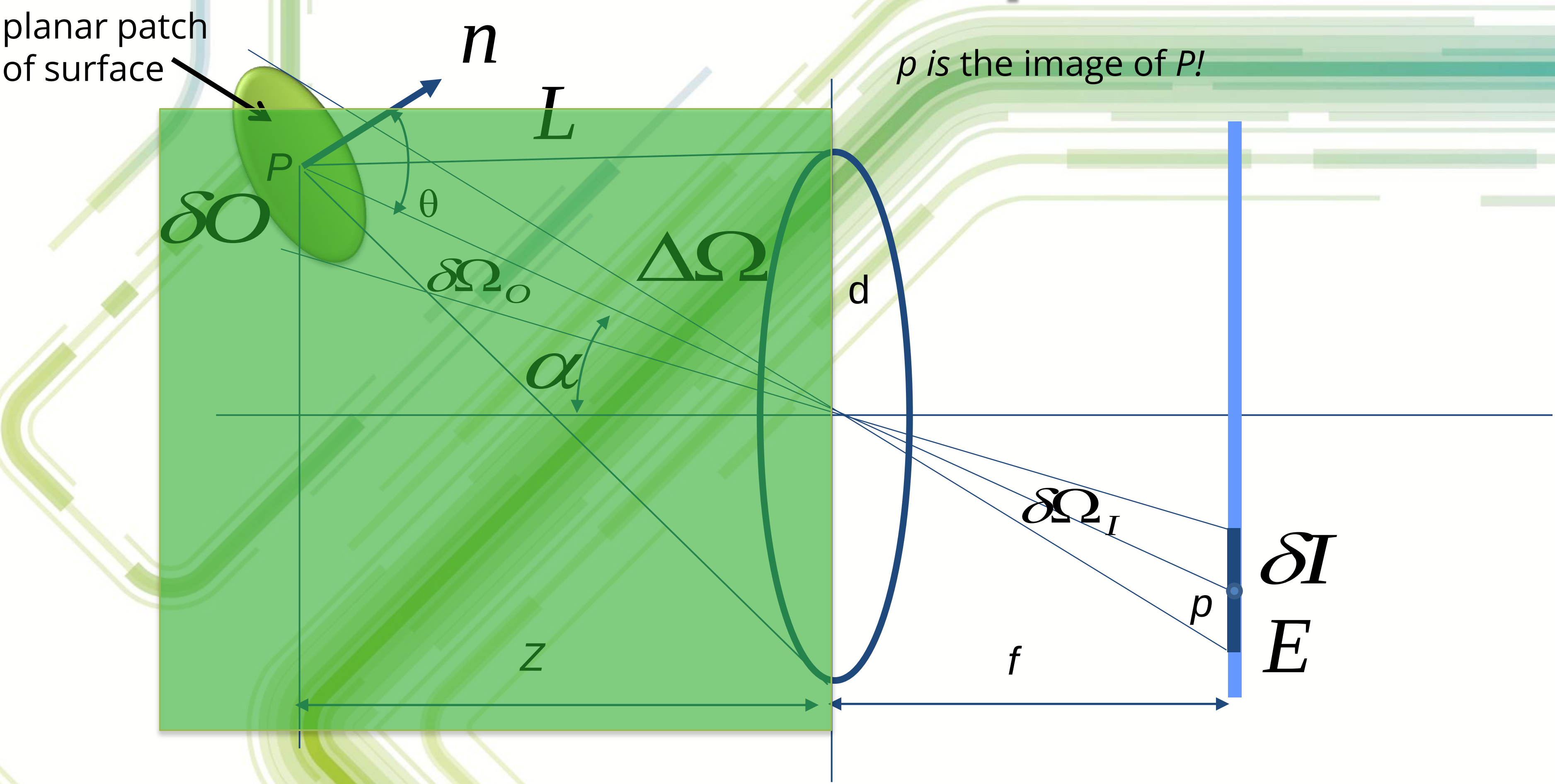
Photometric lens equation



$\Delta\Omega$ Solid angle subtended by lens
δO Area of the surface patch around *P*
L Scene radiance at *P*

$\delta\Omega_O$ Solid angle, subtended by δO
θ angle between *n* and principal ray
Z – object distance to the lens

Photometric lens equation

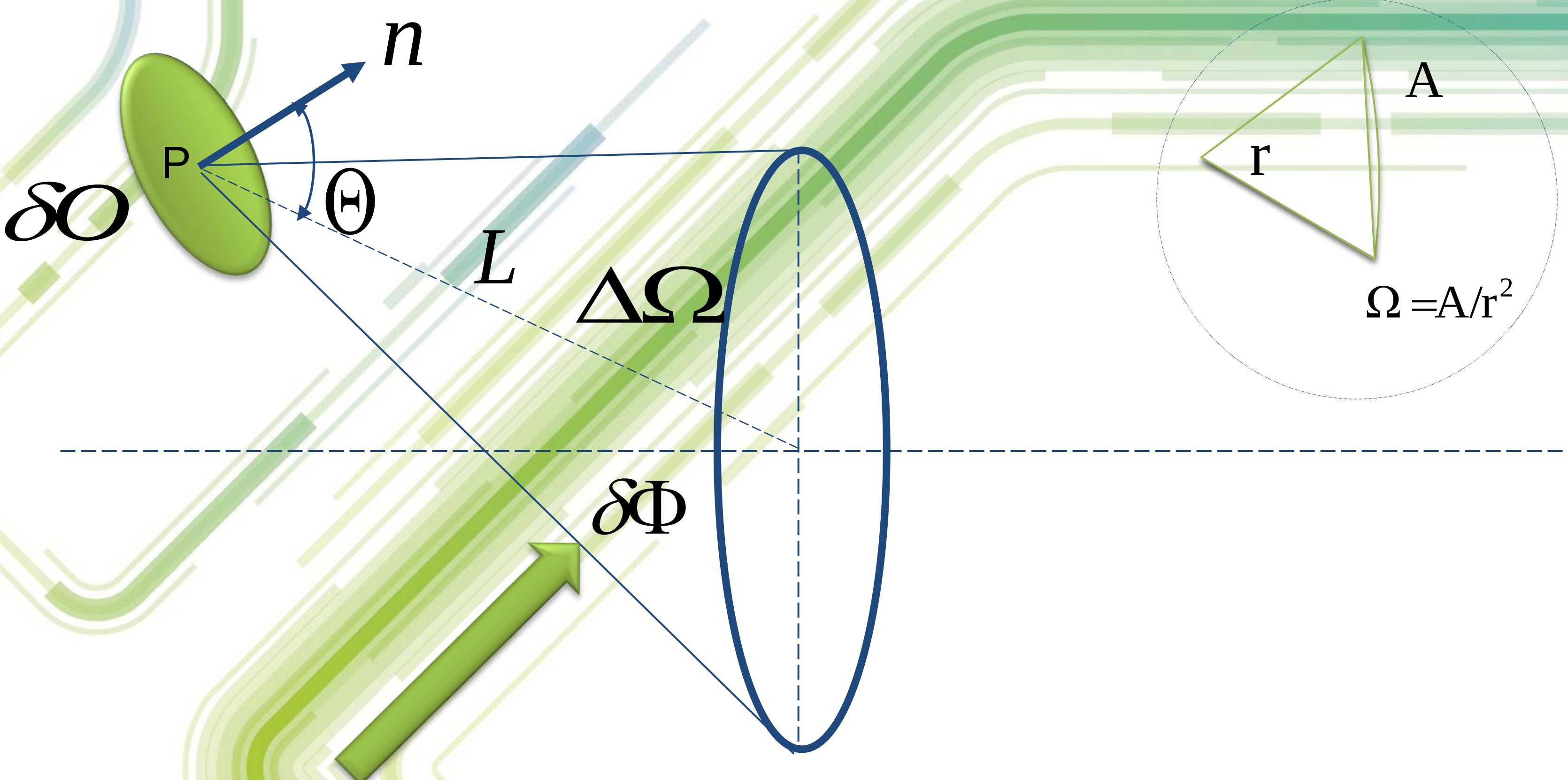


$\delta\Omega_I$ Solid angle, subtended by image patch δI

f focal length (distance between lens and the image)

E Illumination of the image at *p*

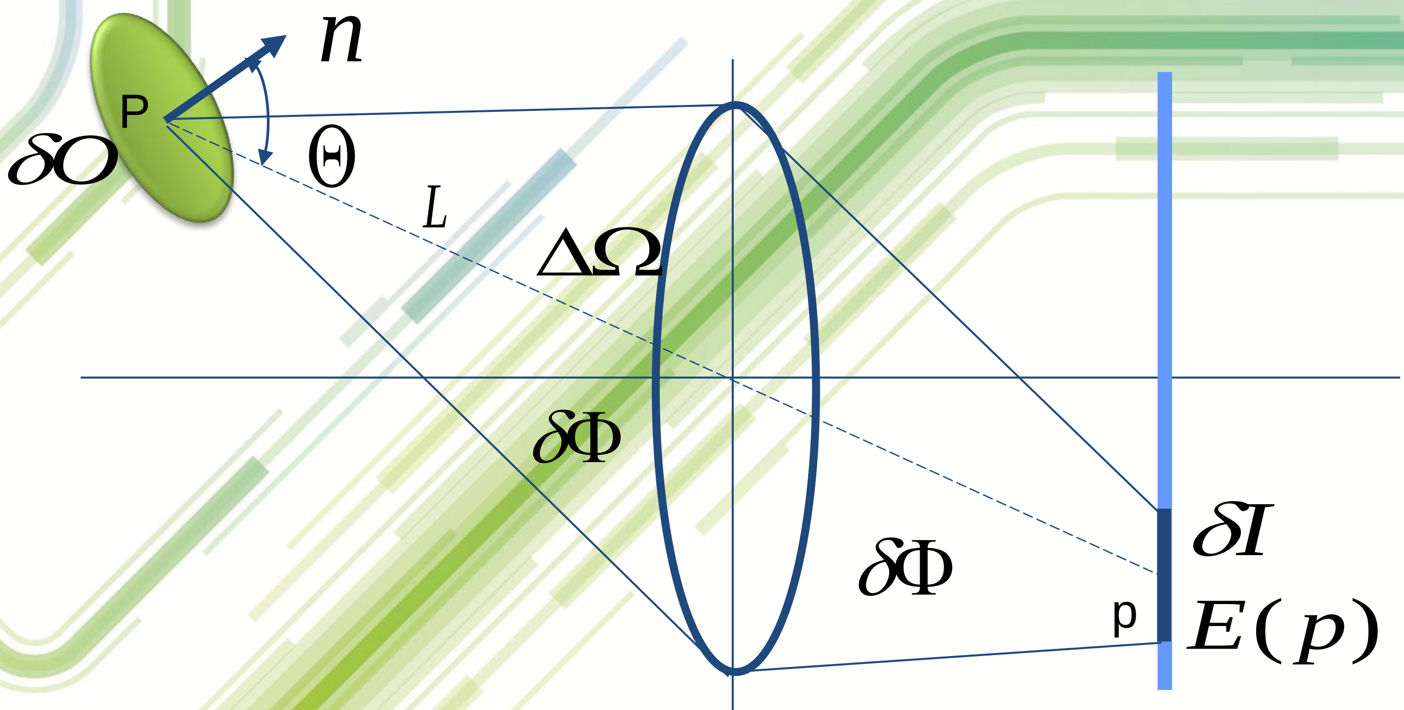
Photometric lens equation



Light energy (power) hitting the lens:

$$\delta\Phi = L(P) \times \Delta\Omega \times (\delta O \times \cos\Theta)$$

Photometric lens equation

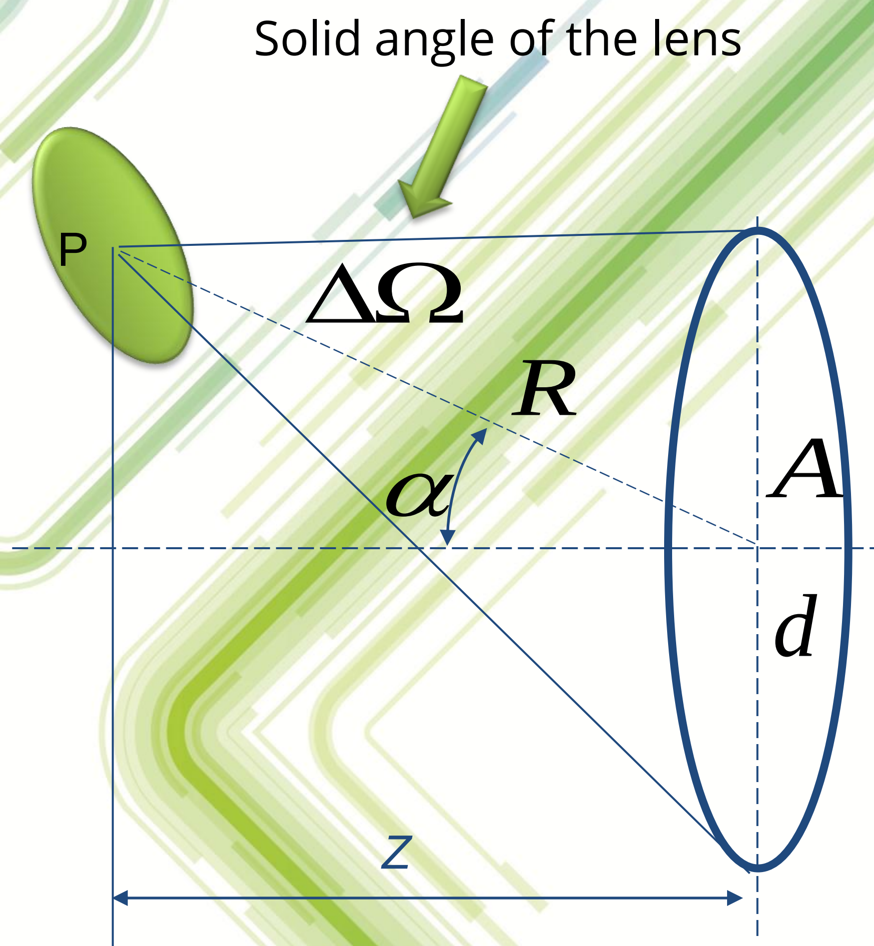


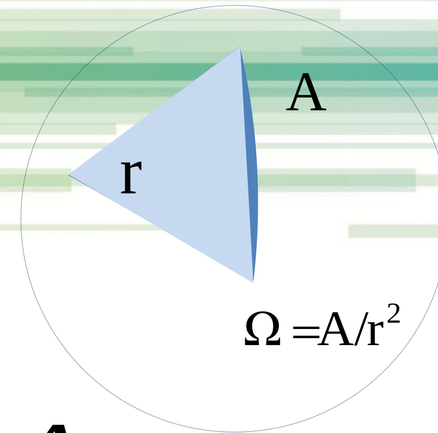
$$E(p) = \frac{\delta\Phi}{\delta I}$$

Illuminance of a pixel (power density)

$$E(p) = L(P) \times \Delta\Omega \times \cos\Theta \times \frac{\delta O}{\delta I}$$

Photometric lens equation





$$\Delta\Omega = \frac{A}{R^2}\cos\alpha$$

$$A = \tfrac{\pi}{4} d^2$$

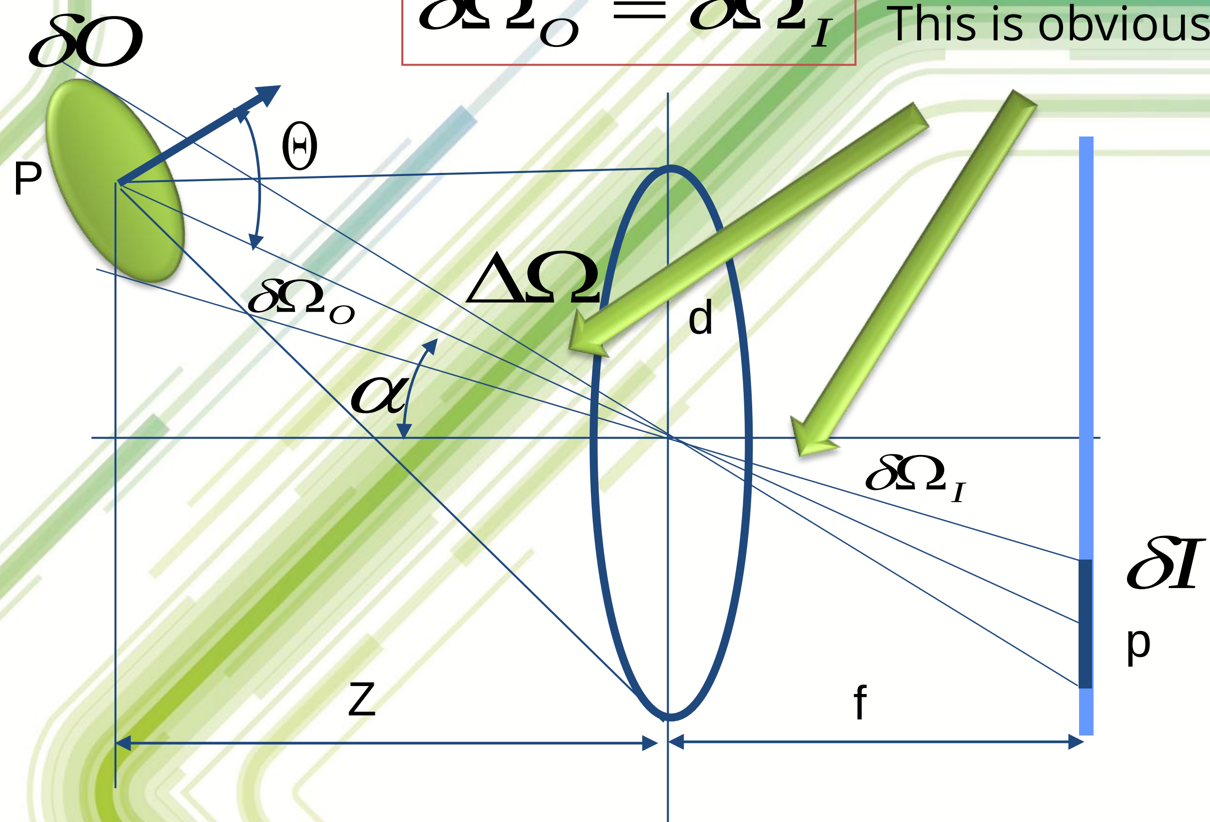
$$R = Z / \cos\alpha$$

$$\Delta\Omega = \tfrac{\pi}{4} d^2 \frac{\cos^3\alpha}{Z^2}$$

Photometric lens equation

$$\boxed{\delta\Omega_O = \delta\Omega_I}$$

This is obvious from the figure!



$$\delta\Omega_O = \frac{\delta O \cos\Theta}{(Z/\cos\alpha)^2} \qquad \delta\Omega_I = \frac{\delta I \cos\alpha}{(f/\cos\alpha)^2}$$

Photometric lens equation

$$\delta\Omega_O = \delta\Omega_I$$

$$\frac{\delta O \cos\Theta}{(Z/\cos\alpha)^2} = \frac{\delta I \cos\alpha}{(f/\cos\alpha)^2}$$

Area of the surface patch around *P*

Angle of the principal ray (relative to the optical axis)

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\Theta}\left(\frac{Z}{f}\right)^2$$

Area of the image patch around *p*

Angle between *n* and the principal ray

Photometric lens equation

$$E(p) = L(P) \times \Delta\Omega \times \cos\Theta \times \frac{\delta O}{\delta I}$$

$$\Delta\Omega = \frac{\pi}{4} d^2 \frac{\cos^3\alpha}{z^2}$$

$$\frac{\delta O}{\delta I} = \frac{\cos\alpha}{\cos\Theta}\left(\frac{Z}{f}\right)^2$$

$$\boxed{E(p) = L(P)\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha}$$

This is the photometric lens equation!

Photometric lens equation

$$E(p) = L(P)\frac{\pi}{4}\left(\frac{d}{f}\right)^2 \cos^4\alpha$$

- Image illuminance at *p* is proportional to the object luminance (brightness) at *P*
- Illuminance goes down with the 4th power of cosine of the *angle of incidence* α !
  - ($\cos^4 11^o = 0.93$, $\cos^4 22^o = 0.74$, $\cos^4 45^o = 0.25$)
- Image illuminance depends on the optical power (‚speed‘) of the lens, "F number"

F-number (F#)

$$\boxed{F\# = \frac{f}{d}}$$

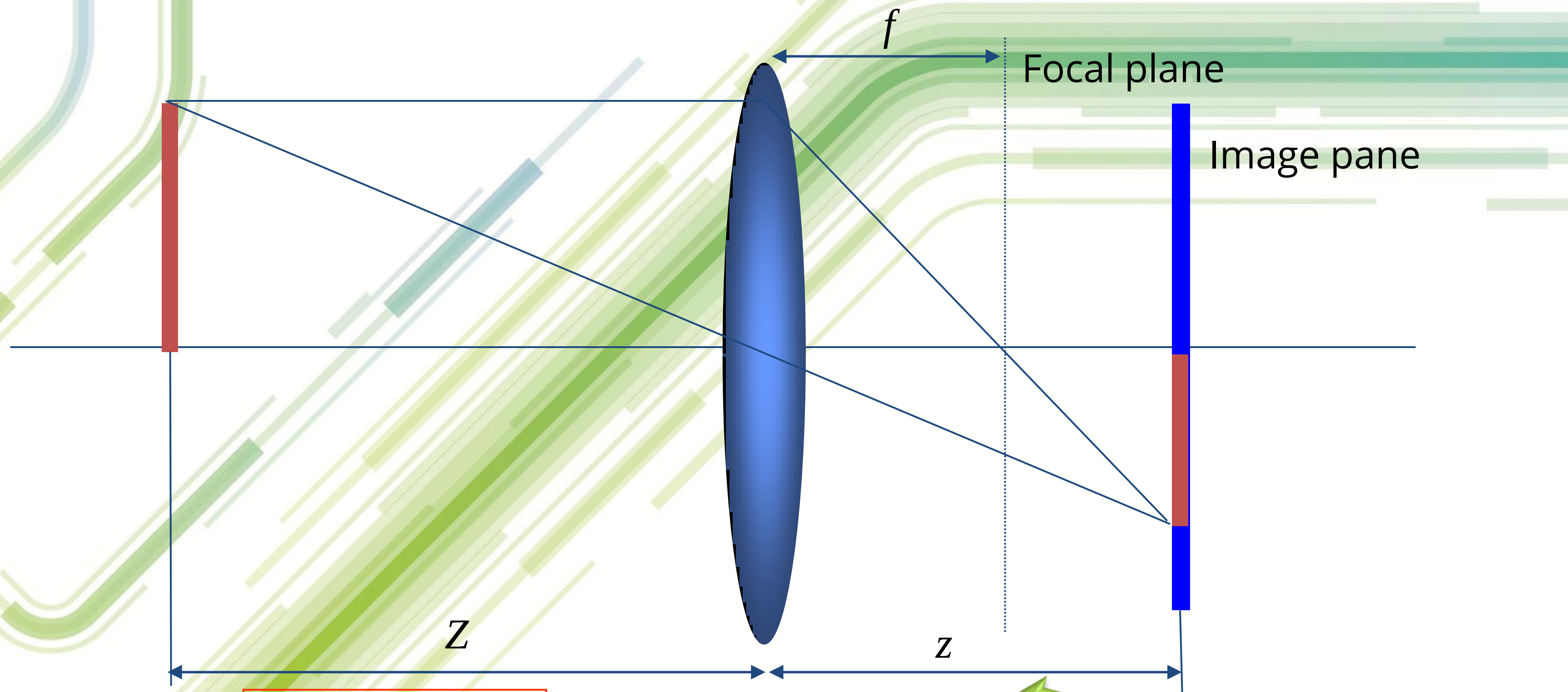
- – *f* – effective focal length, *d* – lens aperture diameter

- various notations for *F#* are in use:
  - F/1.4, F-1.4, 1:1.4,...
- Scale on the lens aperture ring:
  - 1.4, 2, 2.8, 4, 5.6, 8, 11, 16, 22,...

$$\boxed{E(p) = L(P)\frac{\pi}{4}\left(\frac{1}{F\#}\right)^2 \cos^4 \alpha}$$

  - Light energy/power gets halved from mark to mark
  - Required exposition time doubles from mark to mark

Thin lens equation



$$\boxed{\frac{1}{Z}+\frac{1}{z}=\frac{1}{f}} \qquad Z >> f \Rightarrow z \approx f$$

Image / image sensor is very close to the focal plane!

Phtometric lens equation again

$$\alpha = 0, \quad m << 1$$

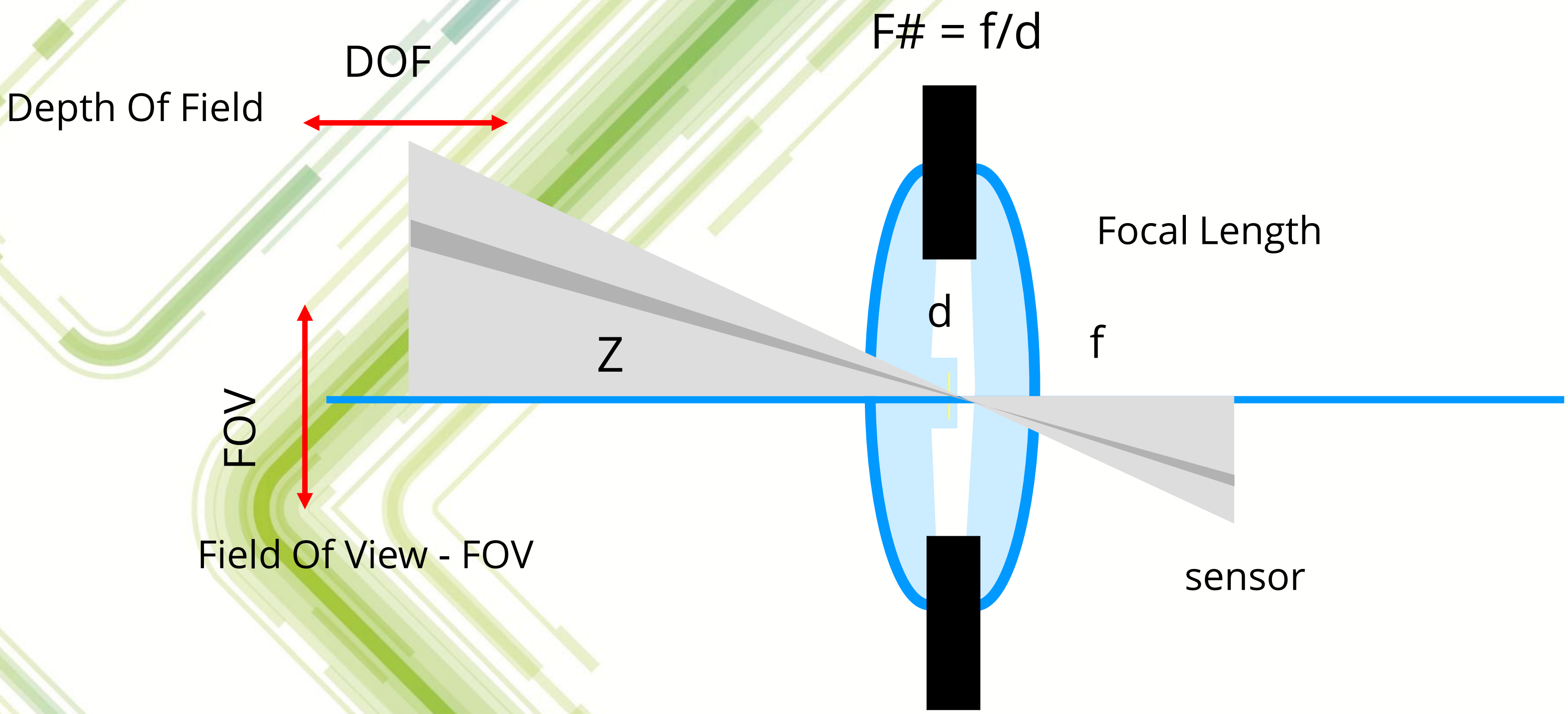
$$\alpha = 0, \quad m = \frac{z-f}{f}, \rightarrow \frac{z}{f} = m+1$$

$$'F\# = \frac{z}{d} = \frac{z\,f}{d\,f} = F\#\ \frac{z}{f} = F\#(m+1)$$

$$\boxed{E(p) = \frac{\pi}{4}\frac{1}{[F\#(m+1)]^2}L(P)}$$

Lens parameters

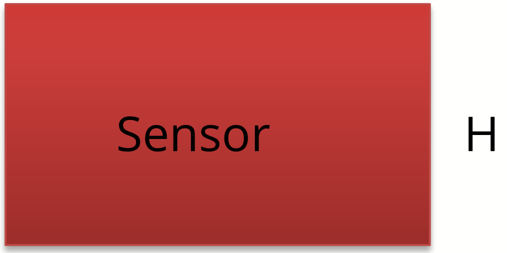
- Conventional lens:
  - image size depends on the object-to-lens distance



Field of view - FOV

- FOV is parameter specified by the manufacturer.
  – Nevertheless, we should always consider the size of the sensor.

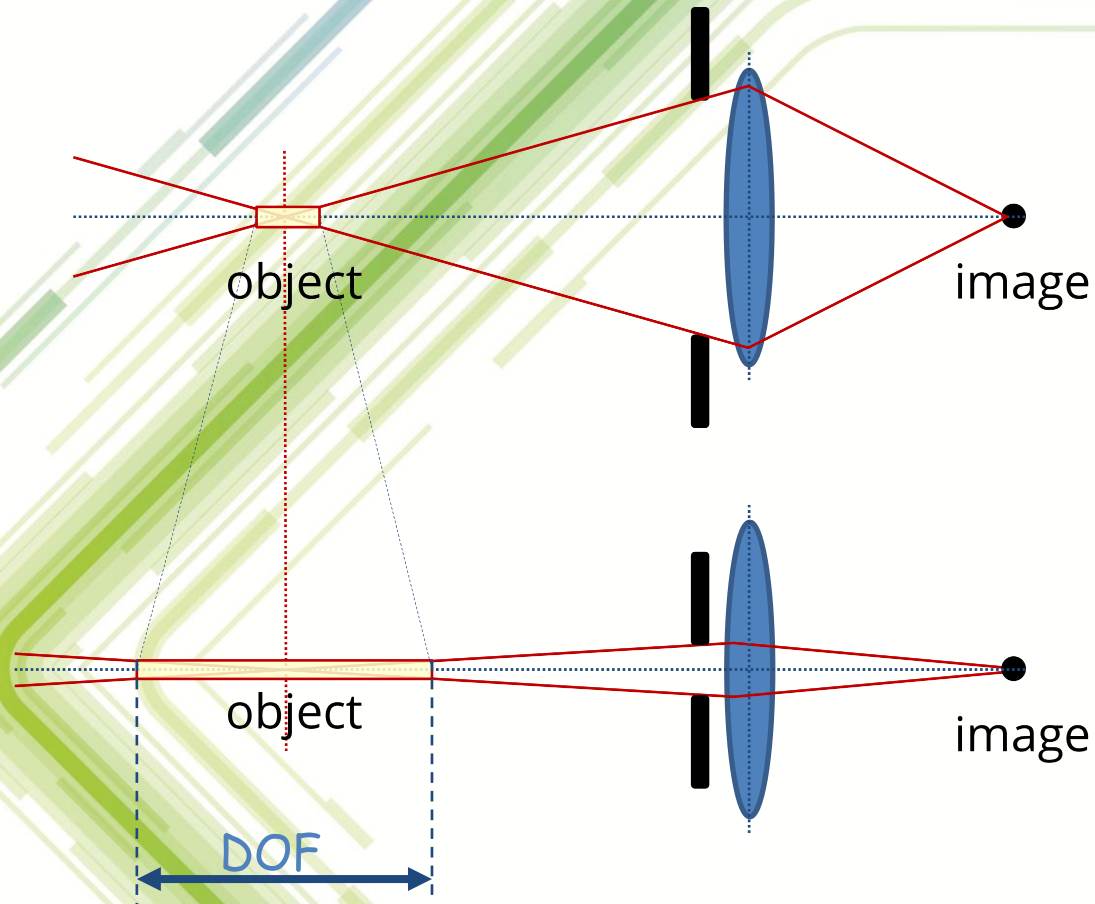
$$FOV = 2 \times \arctan \frac{H}{2f}$$



  – Wide angle lens – small f
  – H is the smaller sensor dimension

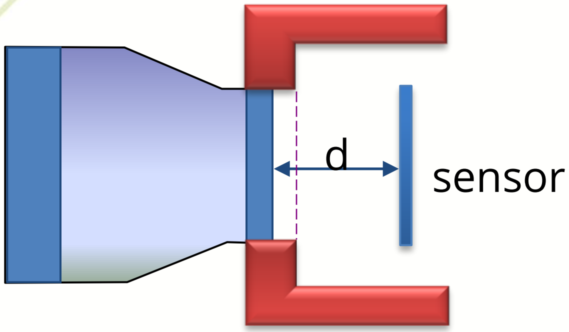
Depth of field - DOF

- DOF largely depends on the lens aperture.



C mount and CS mount

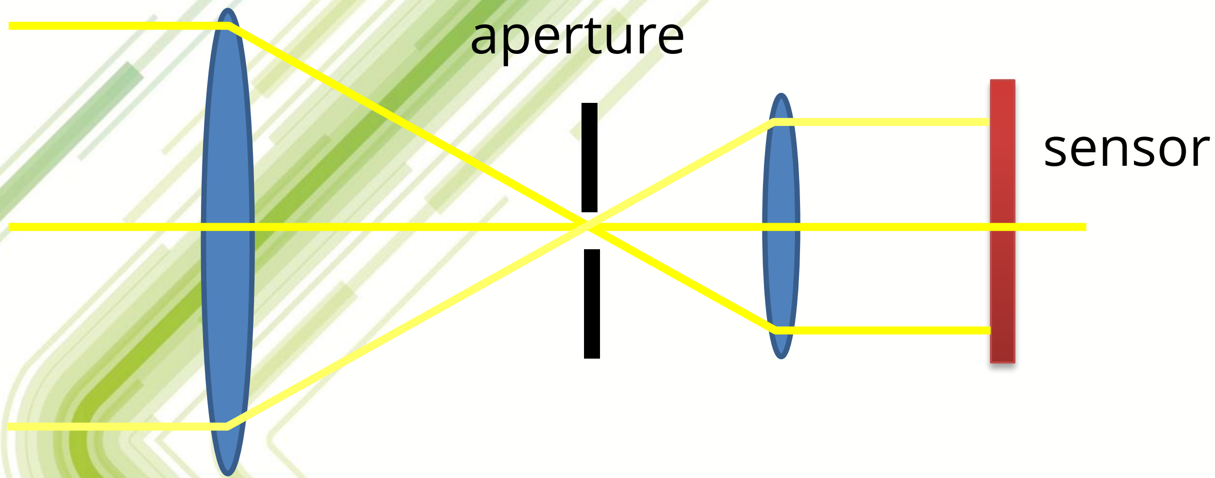
- The difference between the two is in *the distance d* between the lens and the sensor.
  - d = 12.5 mm for CS mount
  - d = 17.5 mm for C mount



  - It should be taken into account when selecting lens for a particular camera (lens and camera pairing)

Telecentric lens

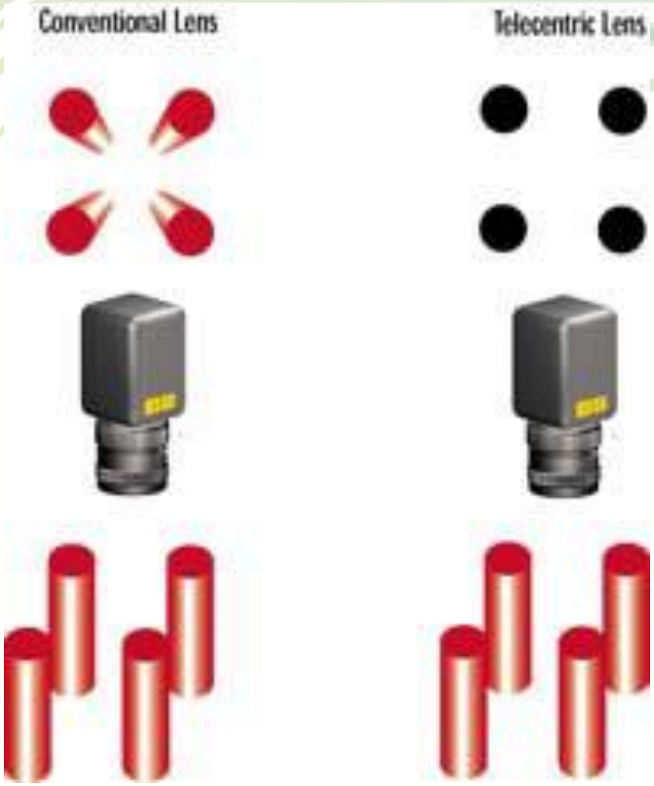
- Image size does not depend on object distance
  - Image sharpness does not change with object distance (to some extent)



  - Lens can be telecentric outside (object side) or inside (sensor side)

Telecentric lens

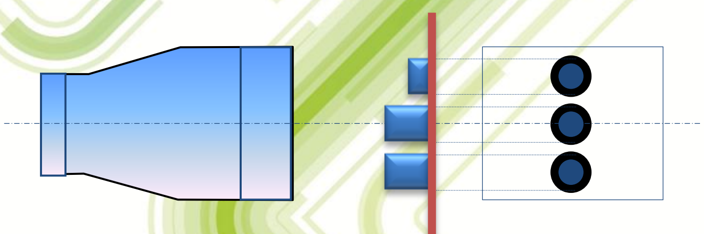
- Ideally:
  - No parallax
  - No size difference
- In practice
  - Telecentric only to certain limit
  - And within certain error
- Important parameters
  - Working distance (e.g. 100 mm)
  - Lens diameter (e.g. 70 mm)
  - Telecentric range
  - The distance difference that causes 1 micrometer size difference



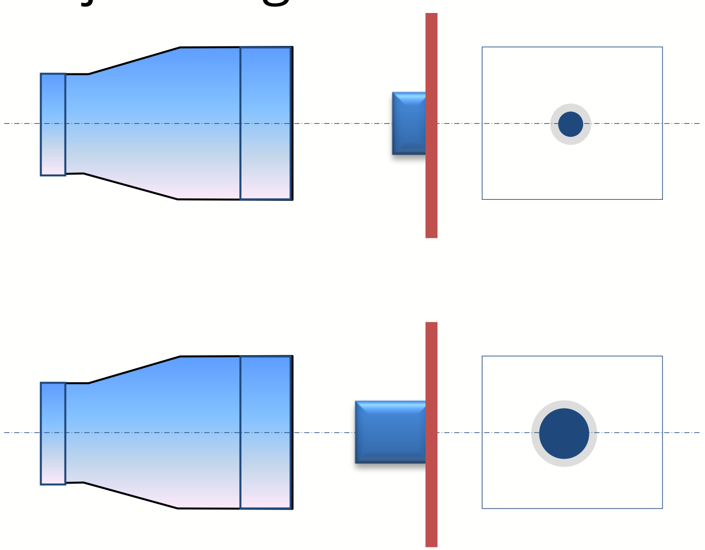
Telecentric lens

Entocentric lens

- The effect of perspective projection is extremely emphasized
  - There is strong dependence between image size and object distance
  - Used for quality control - e.g. of object height



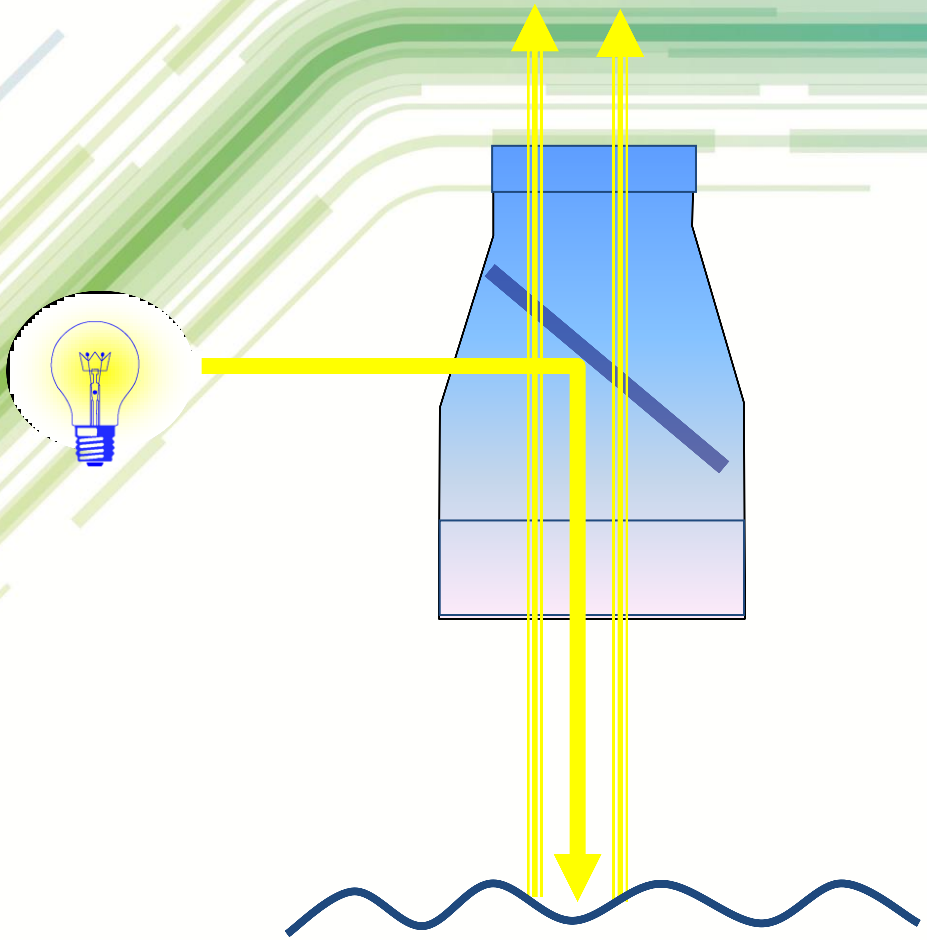
Telecentric lens



Entocentric lens

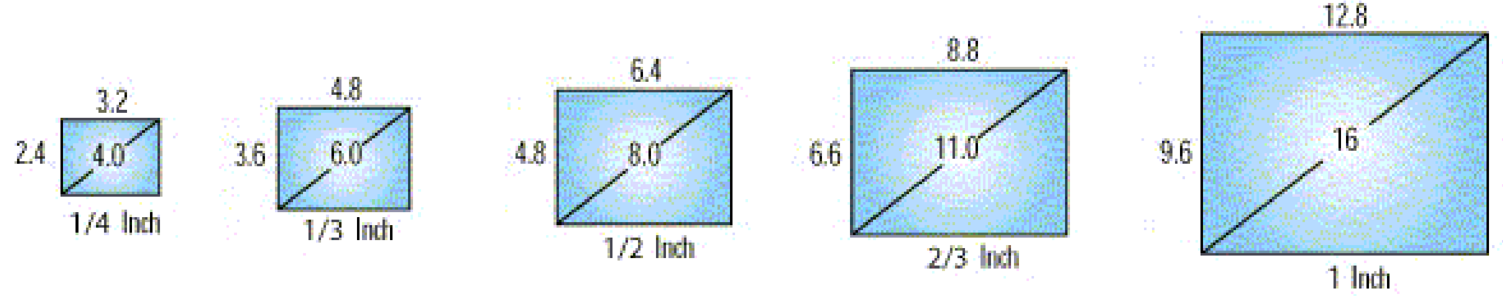
Special lenses

- Co-axial lighting
  - No shadows!



Array sensors ("array cameras")

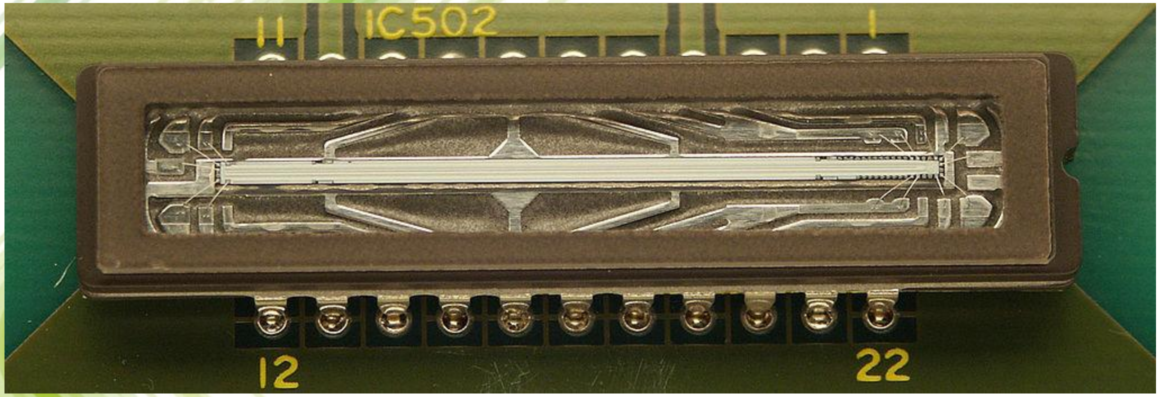
- This is the most common type
  - Sensor sizes are not intuitive!
  - 1 inch = 25.4 mm



  - "inches" in sensor sizes refer to obsolete Vidicon tubes
  - This made sense – for a while & long time ago!
- Lens for larger sensor size can be used with smaller sensor (affects magnification!)
  - But not vice versa!

Line sensors (“line cameras”)

- Somewhat exotic
  - Only 1 row of image sensing elements
  - Makes very high resolutions possible
  - But we need to move/scan either the object or the camera to acquire 2D image



Lighting techniques

You must have been asking yourself…

- Why do we need to know this?
- Why camera geometry?
  - All you wanted was to make robots that see!
- Why photometric lens equation?
- And now, LIGHTING TECHNIQUES?!

The reason

- In general, computer vision is tricky
  - Methods often don't work as you would expect
    (At least the basic methods you will hear about in the remaining of the semester)
- We make it work by controlling *all* aspects of the vision system
  - Honestly, algorithms come *after* you choose and set up cameras, lighting, etc.
  - Also known as "machine vision". Approach widely used in industrial deployment. With success measured in $$$
- To do this, you need to know a lot about image formation

The lighting

- Lighting is one of elements of a computer vision system
  - Manipulating lighting can have dramatic effect on the performance of computer vision methods
  - Often problem is solved when you *finally* find proper way to light the object you are trying to observe
- Manipulating lighting can be often easier than changing other components
  - And lighting elements may be less expensive than other elements of the optical system (lens, camera, actuators).
  - Lighting optics can be of lesser quality than imaging optics

Interaction between light & surface

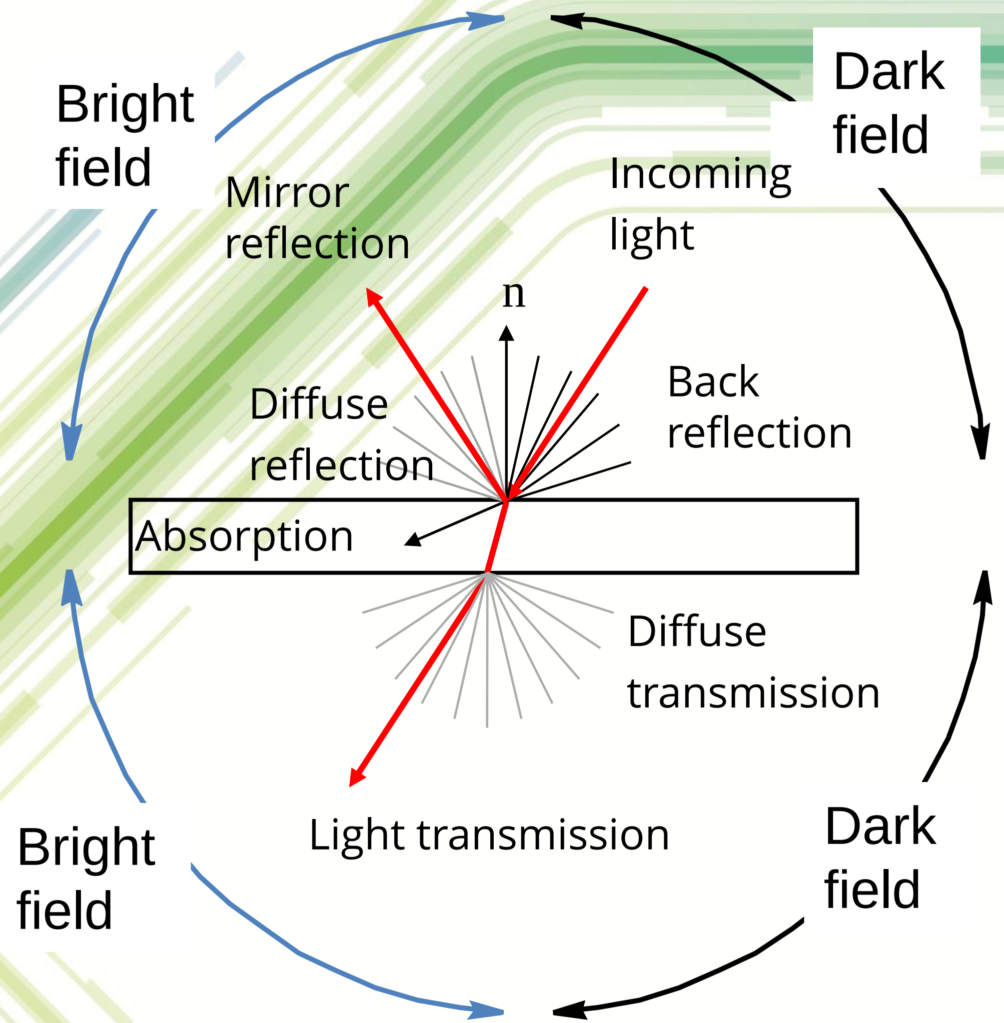
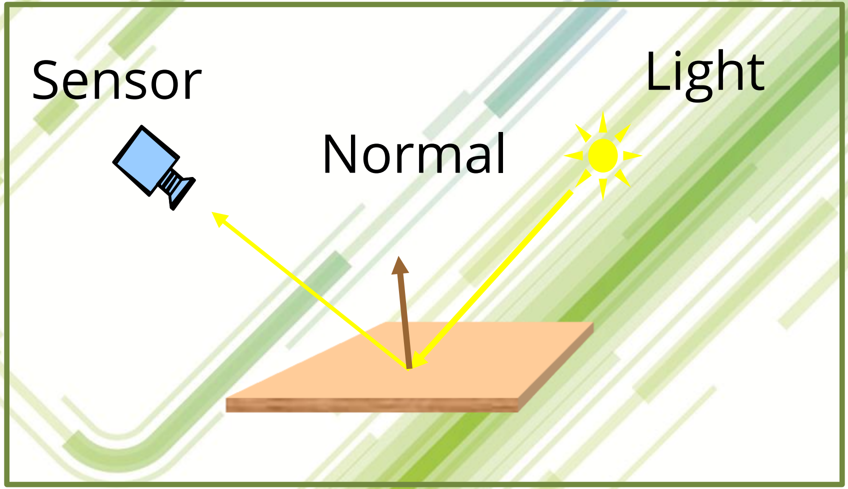


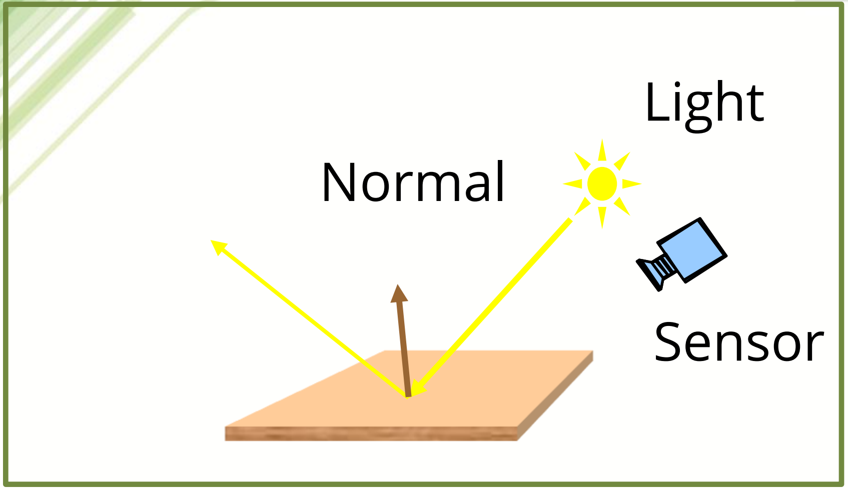
Image credit:
Illumination Technologies
Inc., 2000

Dark field – bright field

Bright field



Dark field

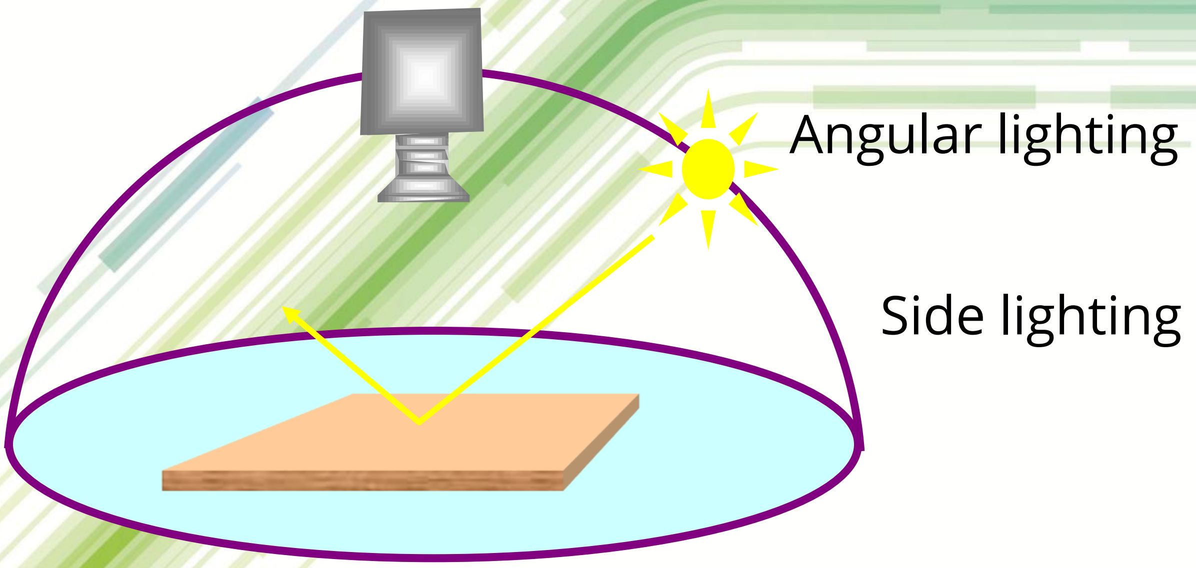


- Why would anyone use dark field?
  - The point of manipulating light is to make things well visible
  - Some features are visible in dark field, e.g. relief

Dark field illumination

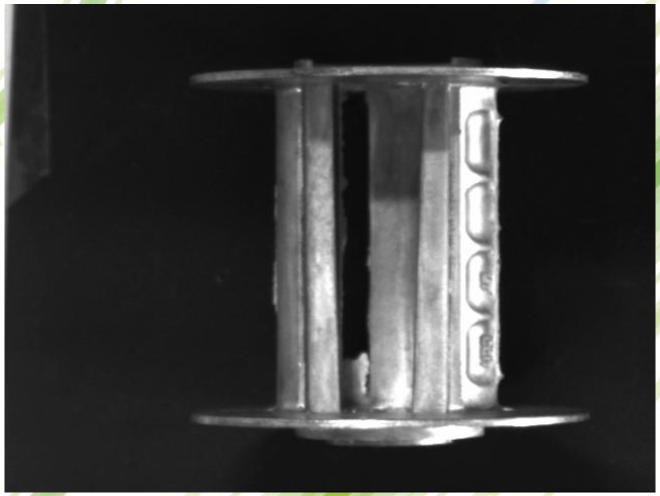
Images credit: Stemmer Imaging

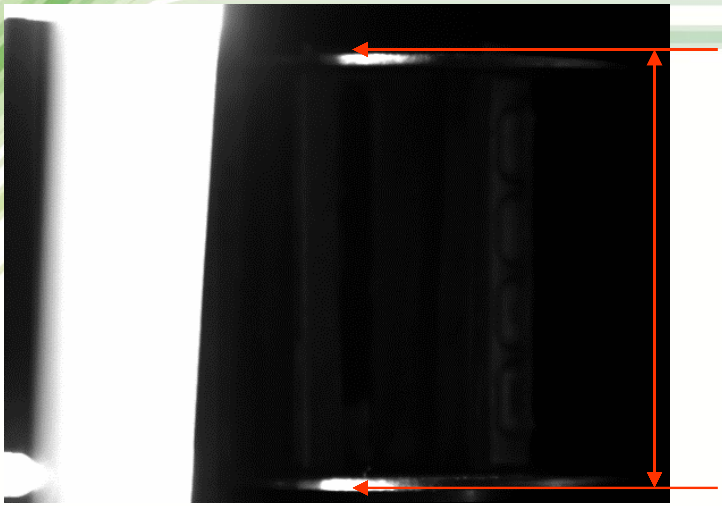
Front/angula/side lighting



- Depending on an angle:
  - Produces shadows, and reflections due to surface defects
  - Useful for controlling flattness

Side lighting - example



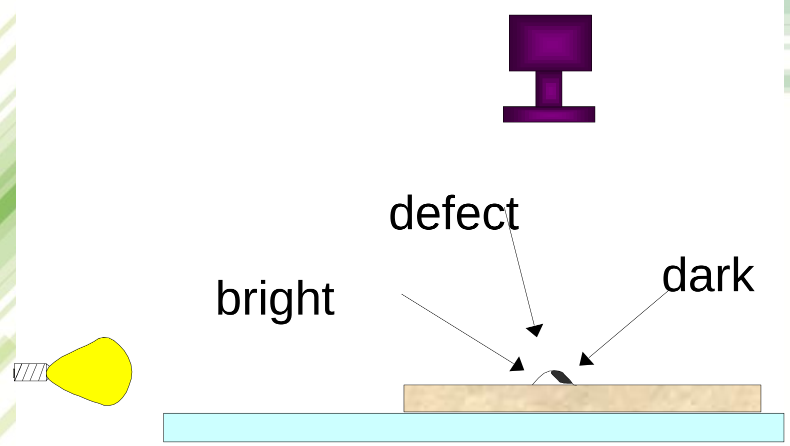


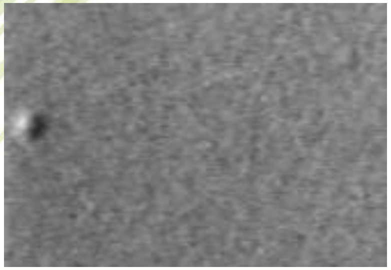
side lighting, directed

- Very important
  - The image does not have to *look good* to human!
  - It needs to show what we need to observe/measure!

Angular lighting - example

- Task:
  - Find defects in ceramic tiles (bumps)

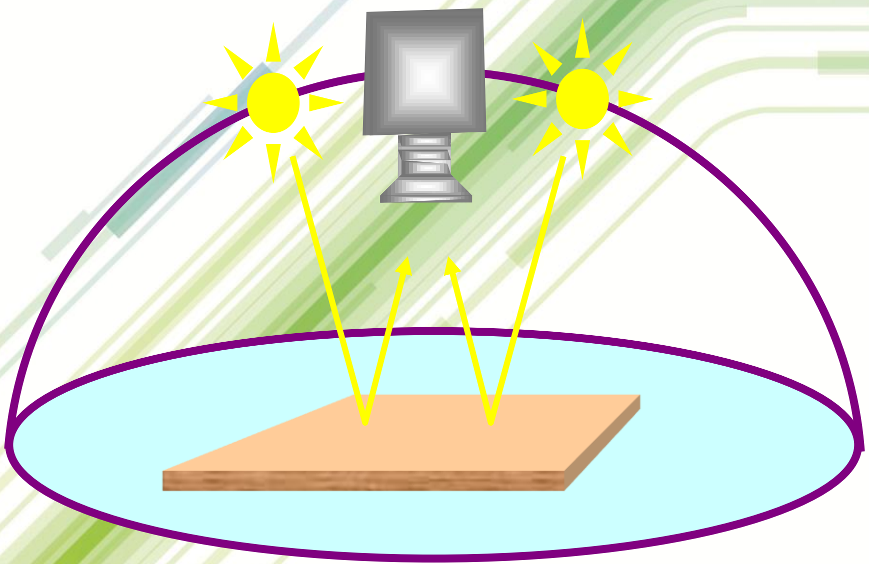




tile

defect

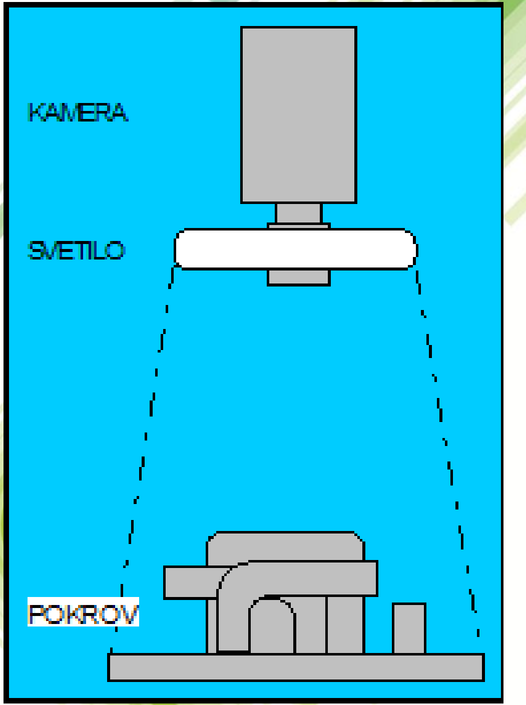
Top lighting



- Top lighting, directed or diffuse, bright field, does not cause shadows
  - Co-axial illumination for flat surfaces (e.g. PCB)

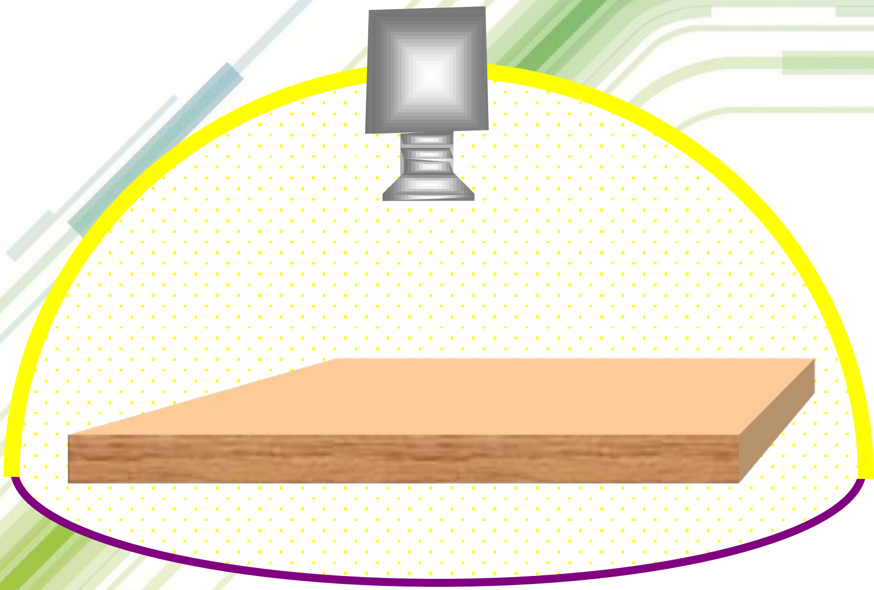
Top lighting – an example

- Light ring (lighting from the top)
  - To prevent shadows



Fuel filter - top view

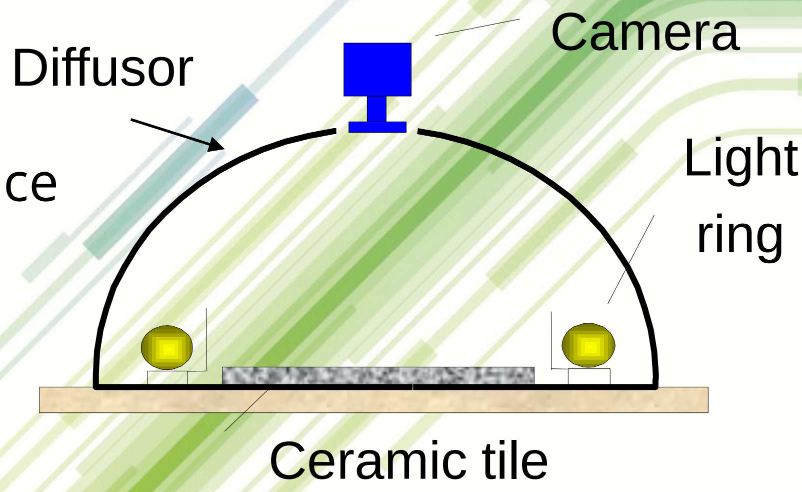
Diffuse lighting

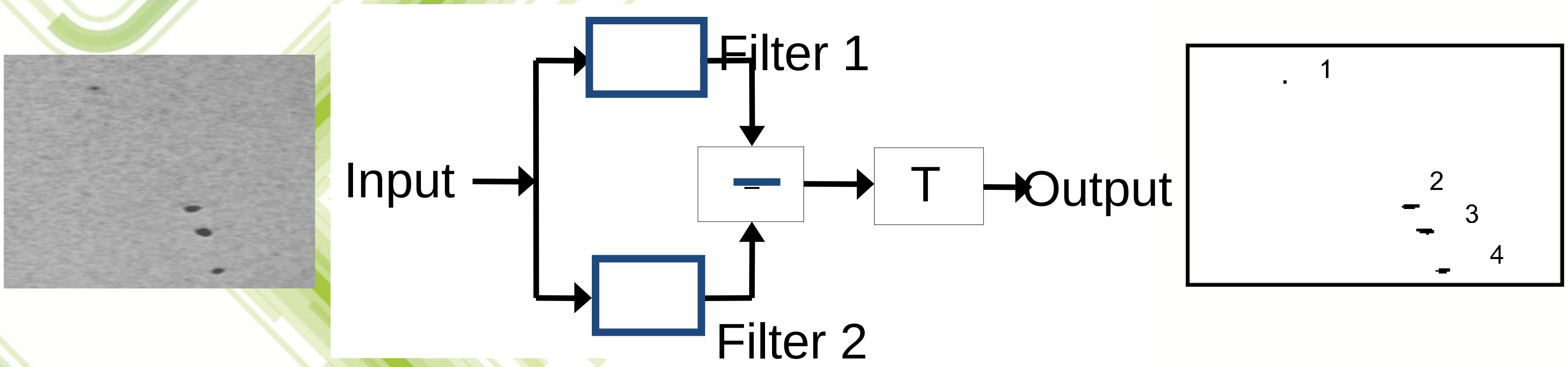


- Does not cause shadows, less reflections
  - Appropriate for metallic/reflective surfaces

Diffuse lighting – an example

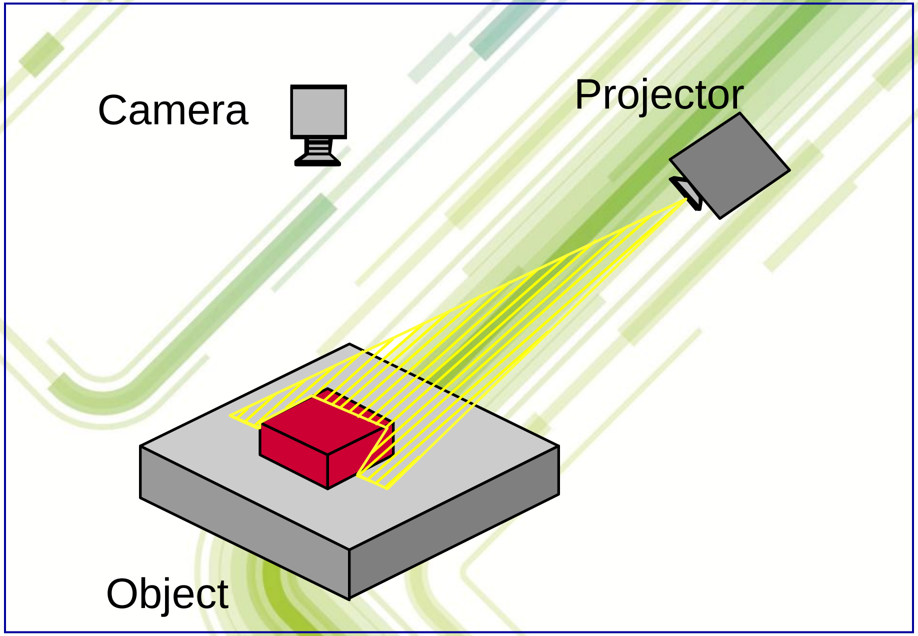
– Task:
find surface spots



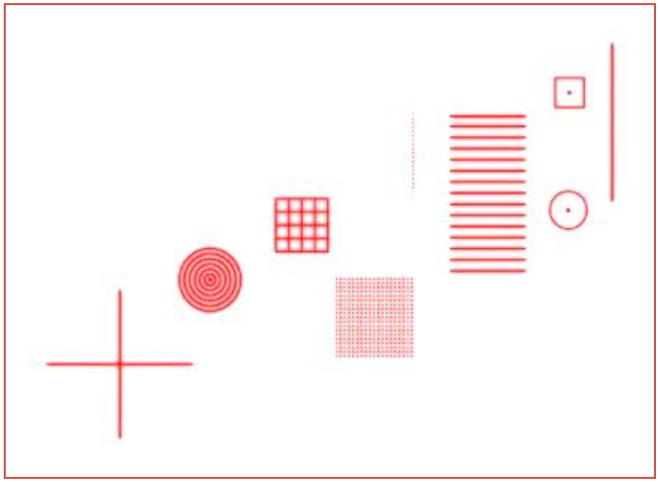


Structured lighting

- Used for 3D shape/surface analysis

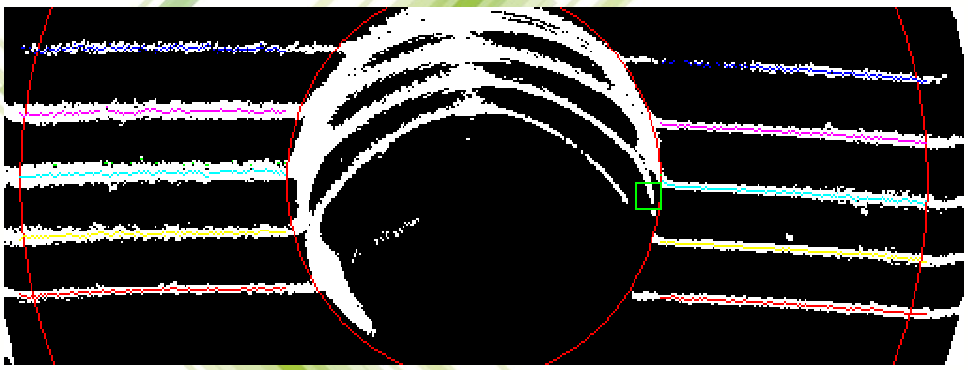


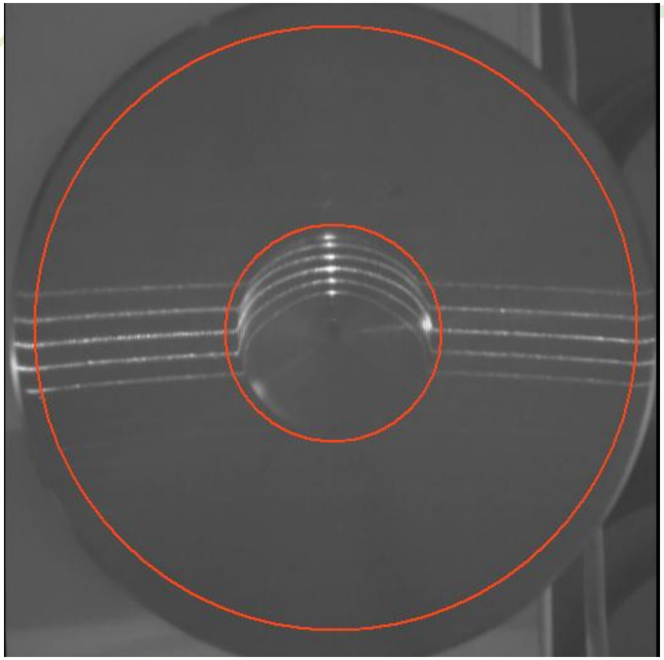
Light patterns



Structured lighting – an example

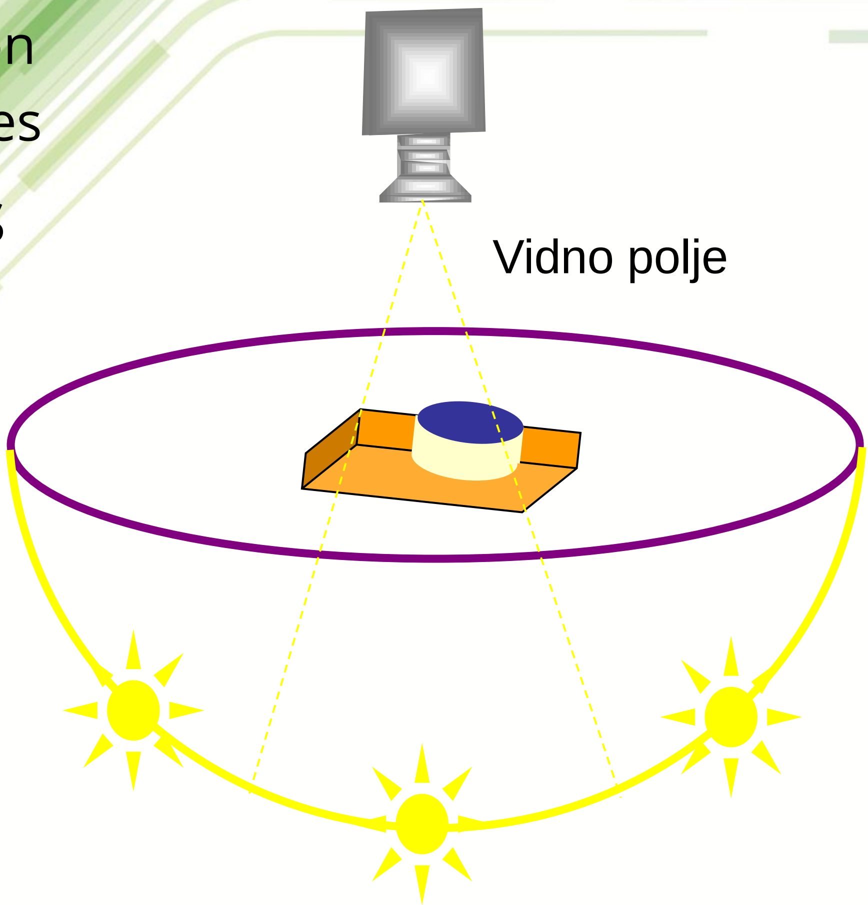
- Measurement of concavity (cooking plates)
  - Select area of interest
  - Binarize the image
  - Calculate the parameters of straight lines





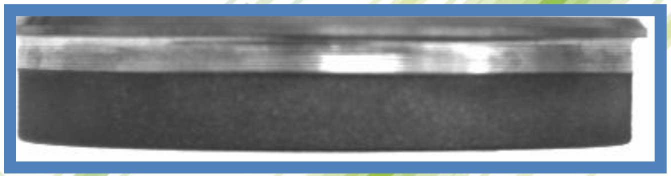
Back lighting

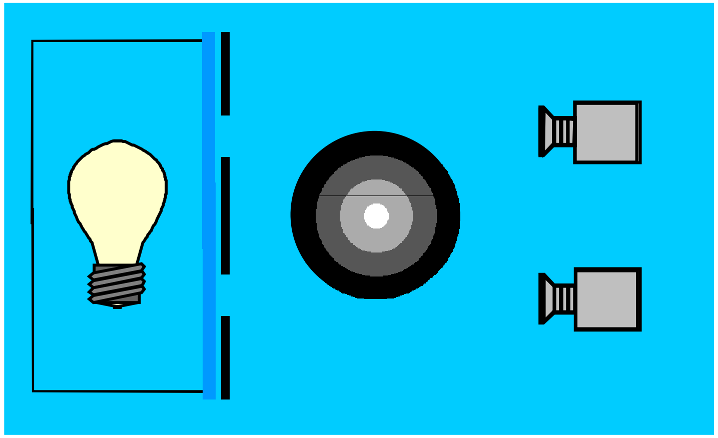
- For opaque objects
  - Shape & silhouette acqusition
  - Provides sharp & stable edges
- For translucent objects
  - analysis of composition



Back lighting – an example

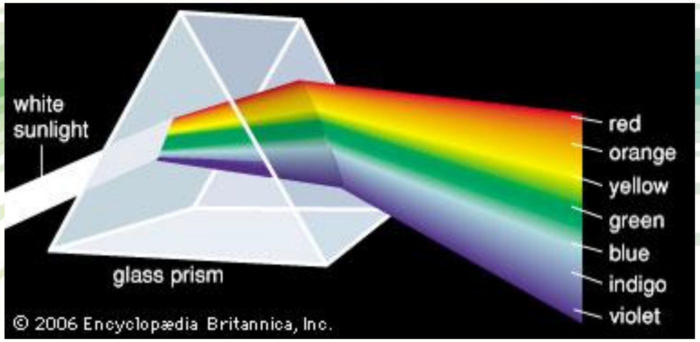
- Dimensional measurement of cooking plates
  - Collimated, directional lighting
  - Telecentric lighting
  - Telecentric lens!





Next lecture

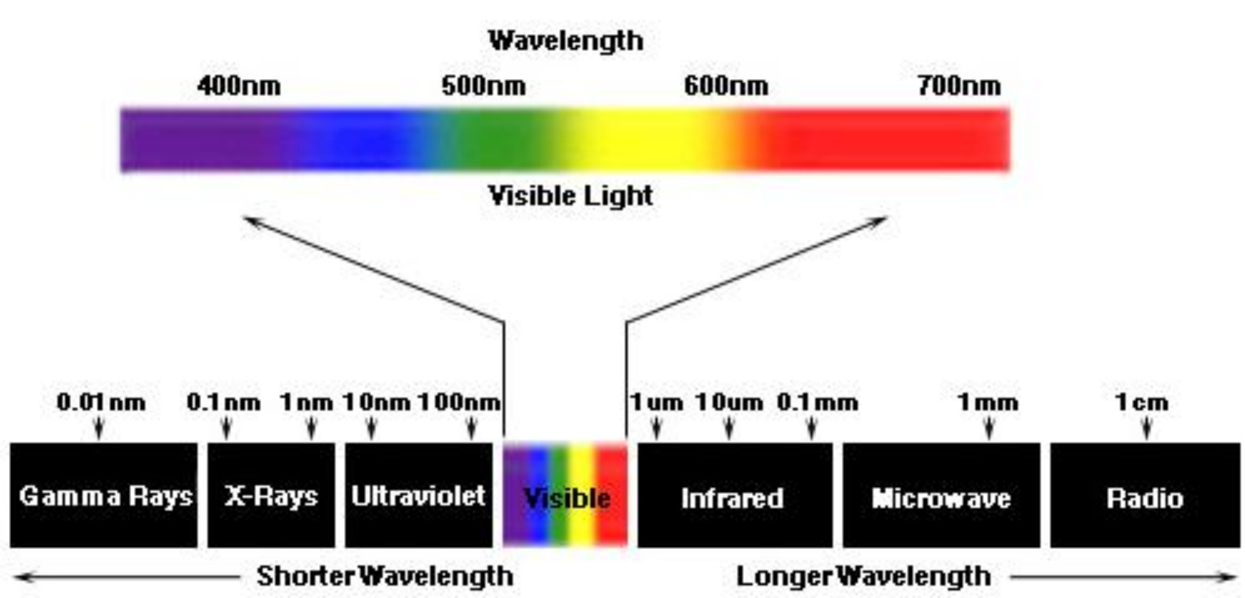
Light is electromagnetic radiation



But colors only exist due to the human visual system

Visible light

Vijolična	380-420	Violet
Modra	440-490	Blue
Zelena	490-560	Green
Rumena	560-590	Yellow
Oranžna	590-630	Orange
Rdeča	630-760	Red



Questions?