MATRIKE IN DIMENZIJE

$A_1, \dots A_u$ in $[d_1, \dots d_{u+1})$

dim $A_i = a_i \times a_{i+1}$

Izhod: min št. množenj realnih števil za izračun produkta danih matrik $A_1 \cdot A_2 \dots A_u$

$N(i,j) \dots *$ od A_i do A_j

$$N(i,j) = \min_{i \le u < j} \{ N(i,k) + N(k+1, j) + d_i * d_k * d_j$$

	3×5	5×4	4×2	2×3	3×5	5×4	4×6	6×3
i\j	1	2	3	4	5	6	7	8
1	0	60	70_1					
2		0	40	70_3				
3			0	24	70_4			
4				0	30	76_5		
5					0	60	132_6	
6						0	120	132_6
7							0	72
8								0

$$N(2,4) = \begin{cases} k=2; & 0+24 + 5\cdot4\cdot3 = 84 \\ k=3; & 40+ 5\cdot2\cdot3 = 70 \end{cases}$$

$$N(3,5) = \begin{cases} k=3; & 24+0 + 4\cdot3\cdot5 = 84 \\ k=4; & 30 +0+ 4\cdot2\cdot5 = 70 \end{cases}$$

$$N(4,6) = \begin{cases} k=4; & 0 +60 + 2\cdot3\cdot4 = 84 \\ k=5; & 30 +0 + 2\cdot5\cdot4 = 70 \end{cases}$$

$$N(5,7) = \begin{cases} k=5: & 0 + 120 + 3 \cdot 5 \cdot 6 = 210 \\ k=6: & 0 + 72 + 60 + 132 \end{cases}$$

$$U(6,9) = \begin{cases} k=6: & 0 + 72 + 60 = 132 \\ k=7: & 120 + 0 + 5 \cdot 6 \cdot 3 = 210 \end{cases}$$

NALOGA 2.

$O(i,j) = \#$ opt. prodnktor matrik $A_i \ldots A_j$

$$\overset{k=\ell}{(A_1 \ldots A_\ell)(A_{\ell+1} \ldots A_u)}$$

$$O(1,\ell) \cdot O(\ell+1, u)$$

$O(i,i) = 1$

$O(i, i+1) = 1$

$O(i,j) = \sum_{k \in N(i,j)[1]} O(i,k) \, O(k+1, j)$

$\underset{idx}{\downarrow}$

Časovna zahtevnost $\underset{\text{st stanj}}{\overset{\uparrow}{O(u^2 \cdot u)}} \overset{\nwarrow}{\text{izračun stanja}}$

Optimalno število operacij je $N(1,8) = 1932$

$(A_1 A_2)(A_3 \cdots A_8)$

$\downarrow$

$(A_1 A_2)$

$\searrow$

$k=6: (A_3 \cdots A_6)(A_7 A_8)$

$\downarrow \qquad \qquad \searrow A_7 A_8$

$(A_3 A_4 A_5) A_6 \qquad \downarrow \quad \downarrow$

$\qquad \qquad A_7 \quad A_8$

$\searrow A_6$

$\downarrow$

$(A_3 A_4 A_5$

$\downarrow \qquad \searrow A_5$
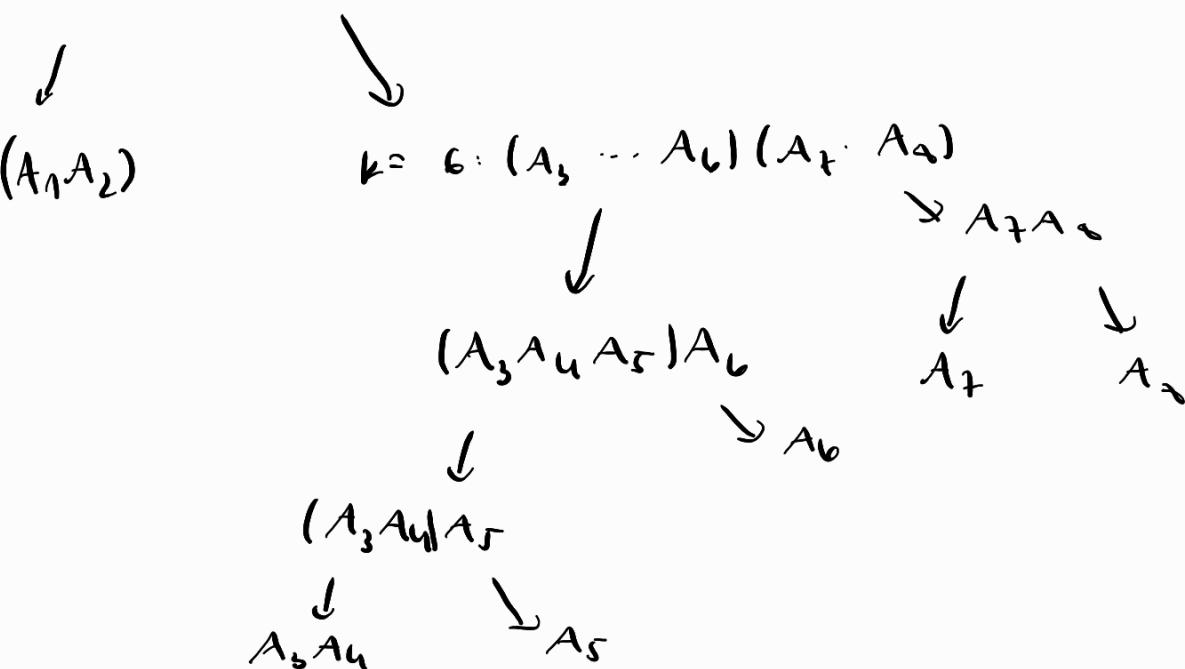
$A_3 A_4$

$$A_1 \cdot A_2 \cdot ((((A_3 A_4) A_5) A_6)(A_7 A_8))$$

$k = 7$: $(A_3 \cdots A_7) A_8$

$$\downarrow \qquad\qquad \searrow A_8$$

$(A_3 \cdots A_6) A_7$

$$\downarrow \qquad \searrow A_7$$

$(A_3 A_4 A_5) A_6$

$$\downarrow \qquad\qquad \searrow A_6$$

$(A_3 A_4) A_5$

$$\downarrow \qquad \searrow A_5$$

$A_3 A_4$

$$(A_1 A_2)((((A_3 \cdot A_4) A_5) A_6) A_7) A_8)$$

korensko   binarno   drevo $\mathcal{z}$ n listi

$$\Big\{$$

členi $\mathcal{z}$ n členi   in   pravilno   postavljenimi

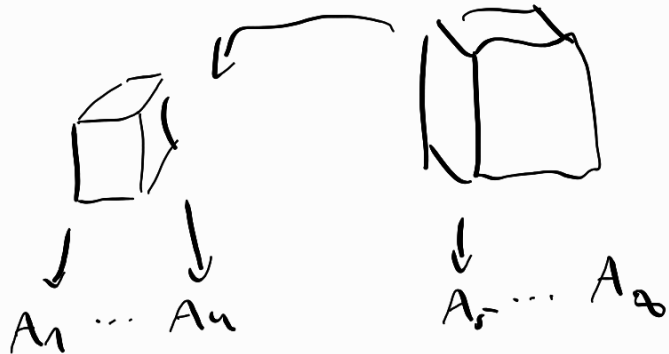oklepaji $\rightarrow$ CATALANOVA   ŠTEVILA

a) $2n2$ operacij

b) $(A_1(A_2 A_3)(A_4 A_5) A_6) A_7) A_8)$

c) $A_3(((A_4 A_5) A_6) A_7)$

d) 130 operacij $N(115) =$

$$\tilde{N}(1,8) = \min_{1 \leq u \leq 8} \{ \max (N(1,k), N(k+1,8)) + d_1 d_k + d_8 \}$$

↓
"neomejen dostop"

$A_1 \cdots A_4 \qquad A_5 \cdots A_8$

2 računalnikih   2 "omejenim" dostopom N^3

$$\tilde{N}(1,8) = \max \{ N(1,4), N(5,8) \} + \underbrace{d_1 \cdot d_{15} \cdot d_8 +}_{\text{izračun } O(\min \{ d_1 \cdot d_4, d_5 \cdot d_8 \})}$$
N^2

↓   ↓
"end"   samo en
                 računalnik

L·8

$N = \max d_i$