Text Clustering: Motivation

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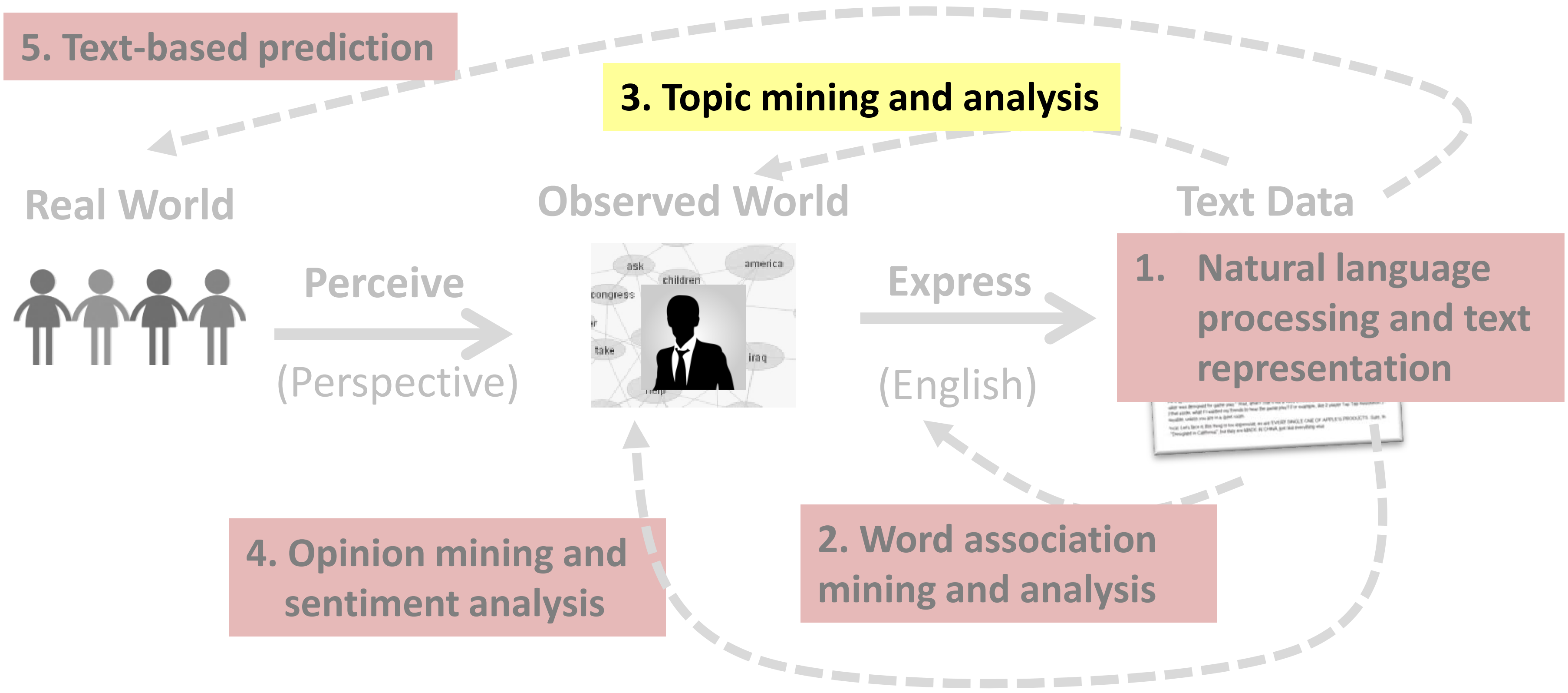
Text Clustering: Motivation



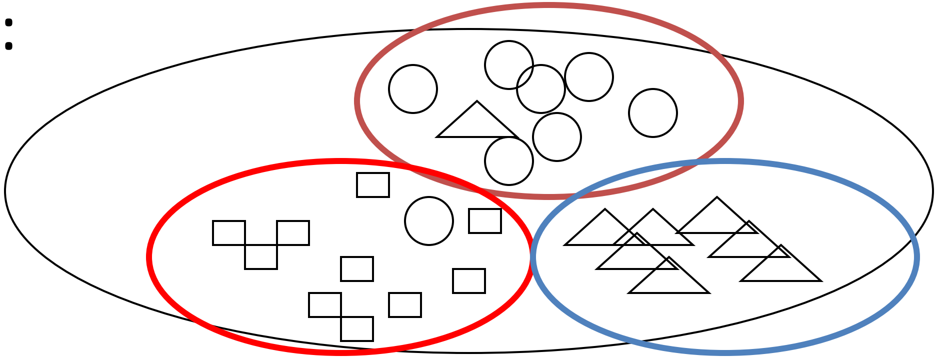
Overview

- What is text clustering?
- Why text clustering?
- How to do text clustering?
  - Generative probabilistic models
  - Other approaches
- How to evaluate clustering results?

This lecture

What Is Text Clustering?

- Discover "natural structure"
- Group similar objects together
- Objects can be documents, terms, passages, websites,…
- Example:

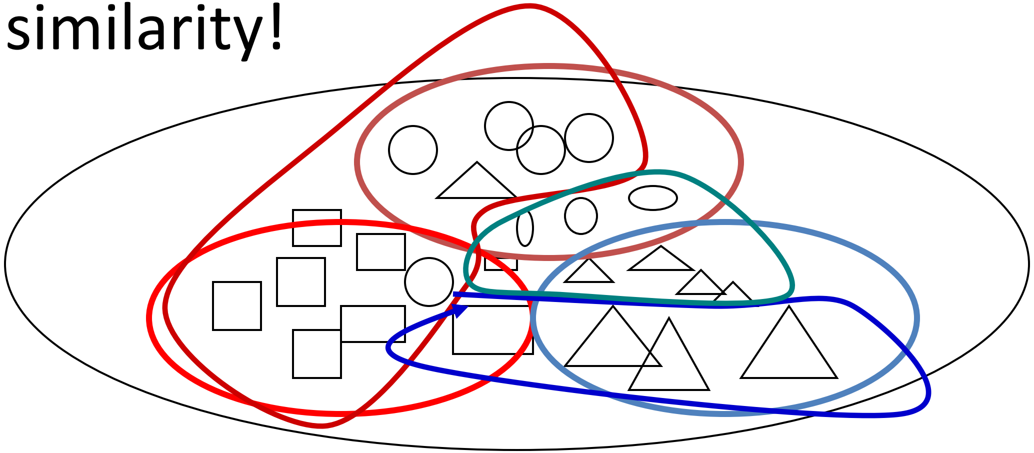


Not well defined! **What does "similar" mean?**

The "Clustering Bias"

- Any two objects can be similar, depending on how you look at them!
- Are "car" and "horse" similar?
- A user must define the **perspective** (i.e., a "**bias**") for assessing similarity!

Basis for evaluation



Examples of Text Clustering

- Clustering of documents in the whole collection
- Term clustering to define “concept”/“theme”/“topic”
- Clustering of passages/sentences or any selected text segments from larger text objects (e.g., all text segments about a topic discovered using a topic model)
- Clustering of websites (text object has multiple documents)
- Text clusters can be further clustered to generate a hierarchy

Why Text Clustering?

- In general, very useful for text mining and <u>exploratory</u> text analysis:
  - ➔Get a sense about the overall content of a collection (e.g., what are some of the “typical”/representative documents in a collection?)
  - ➔Link (similar) text objects (e.g., removing duplicated content)
  - ➔Create a structure on the text data (e.g., for browsing)
  - ➔As a way to induce additional features (i.e., clusters) for classification of text objects
- Examples of applications
  - Clustering of search results
  - Understanding major complaints in emails from customers

Text Clustering: Generative Probabilistic Models

Part 1

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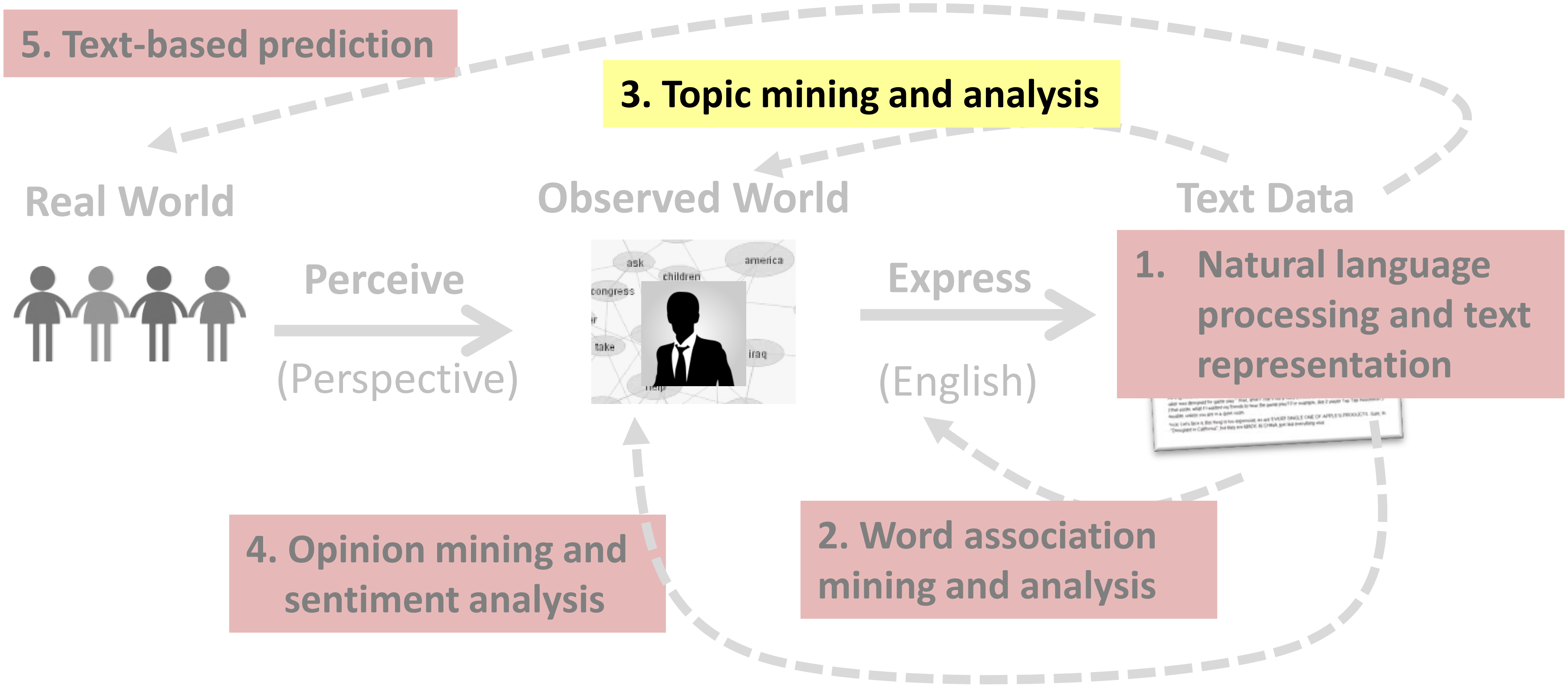
Text Clustering: Generative Probabilistic Models (Part 1)



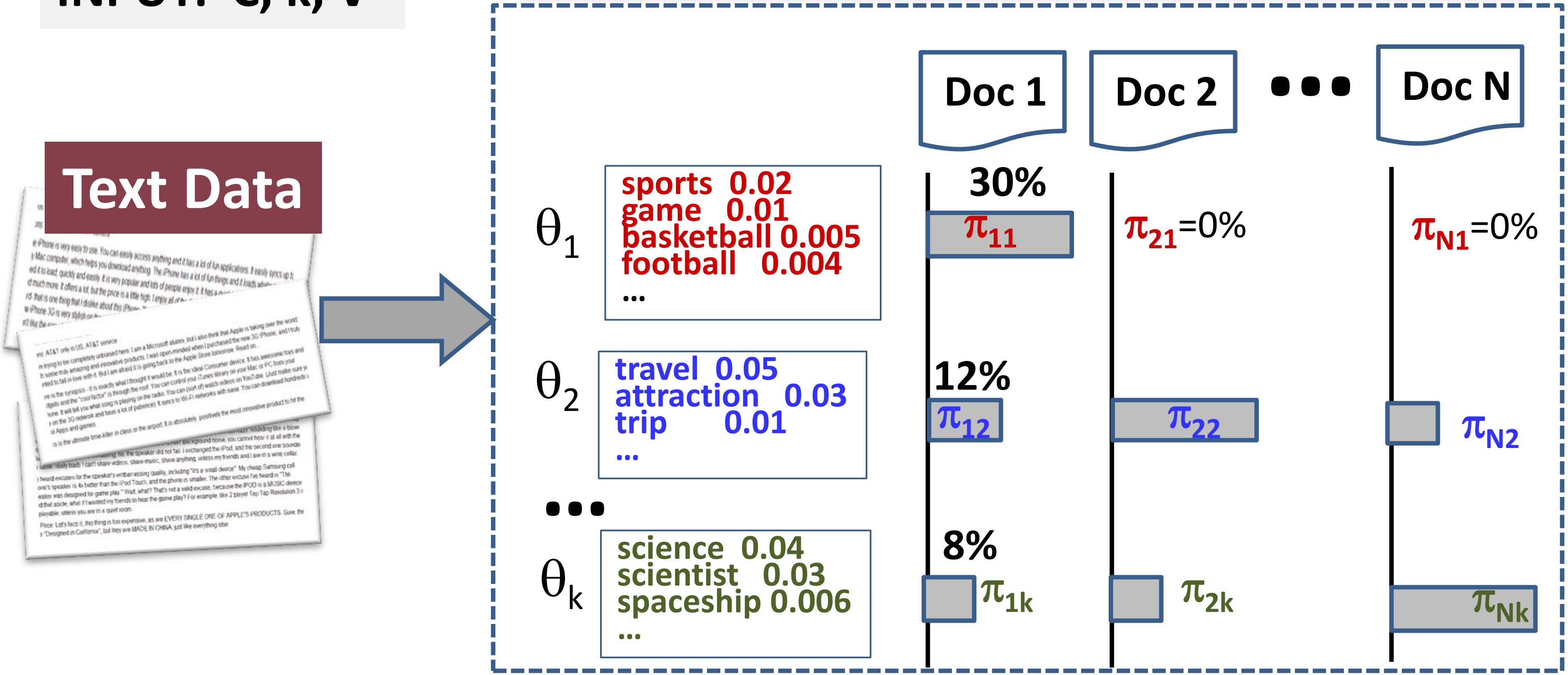
Overview

- What is text clustering?
- Why text clustering?
- How to do text clustering?
  - **Generative probabilistic models**
  - Similarity-based approaches
- How to evaluate clustering results?

Topic Mining Revisited

INPUT: C, k, V

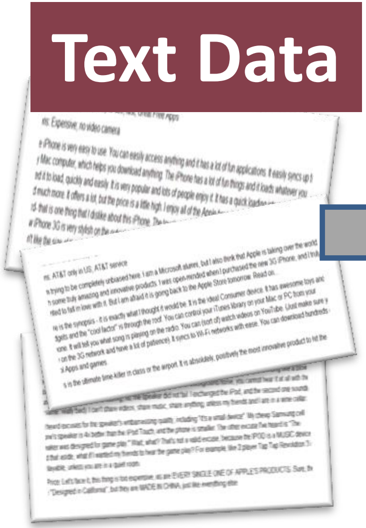
OUTPUT: $\{\theta_1, \ldots, \theta_k\}, \{\pi_{i1}, \ldots, \pi_{ik}\}$

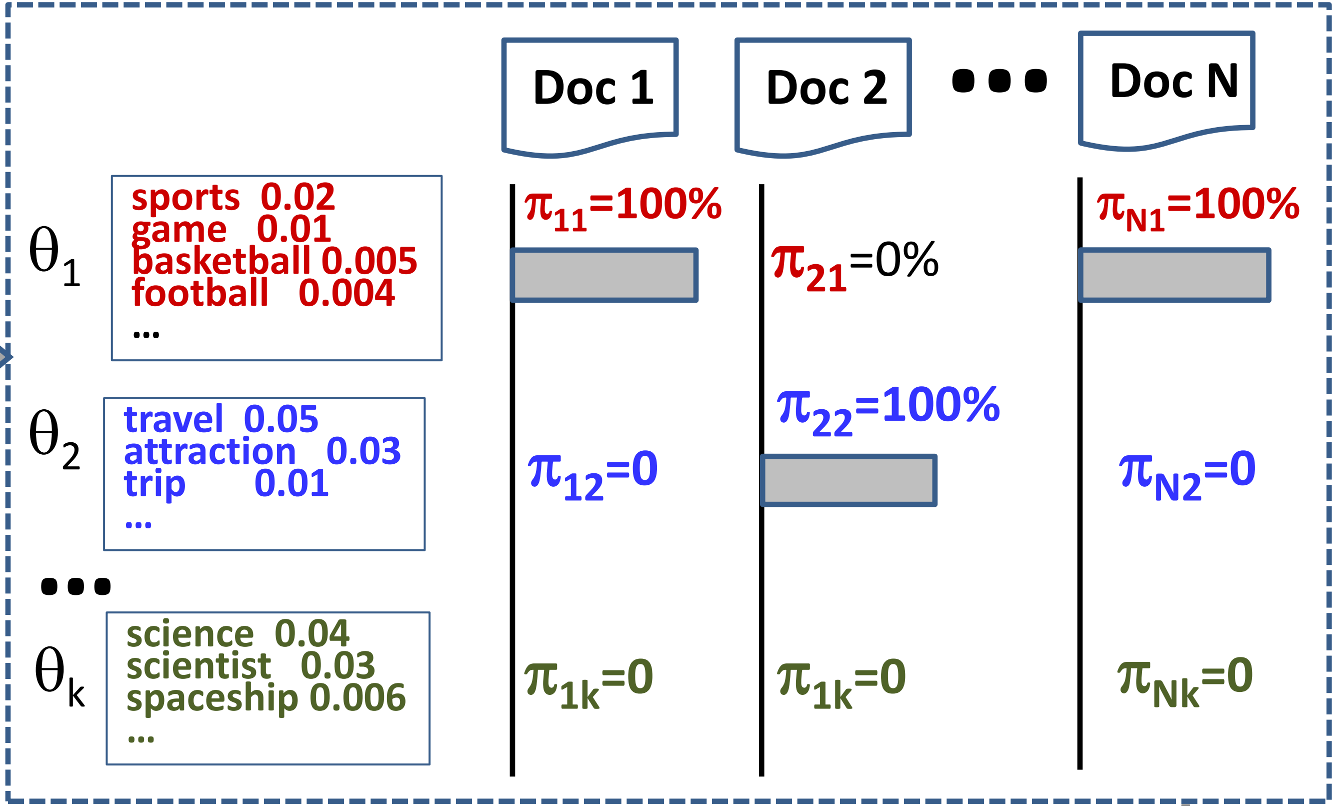


One Topic(=cluster) Per Document

INPUT: C, k, V

OUTPUT: $\{ \theta_1, \ldots, \theta_k \}$, $\{ c_1, \ldots, c_N \}$ $c_i \in [1,k]$



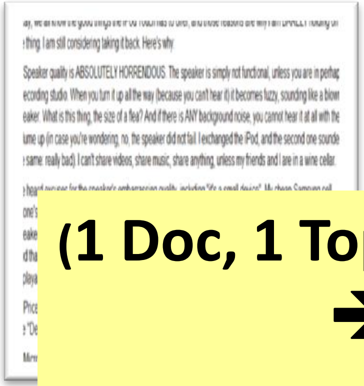


Mining One Topic Revisited

INPUT: C={d}, V

OUTPUT: { θ}

Text Data



$P(w|\theta)$

θ

text ?
mining ?
association ?
database ?

Doc d

100%

(1 Doc, 1 Topic)

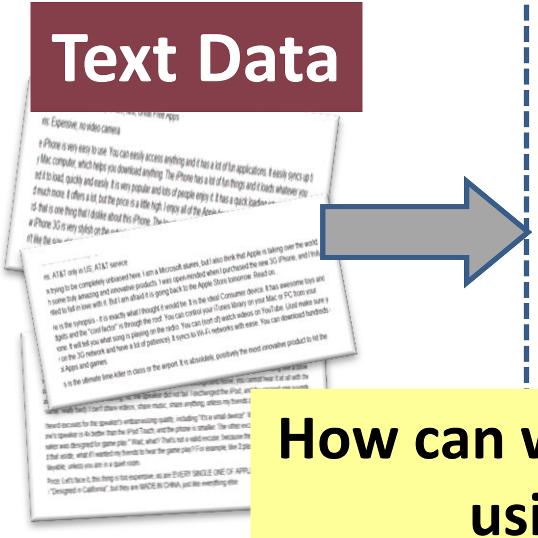
➔ (N Docs, N Topics) **k<N**

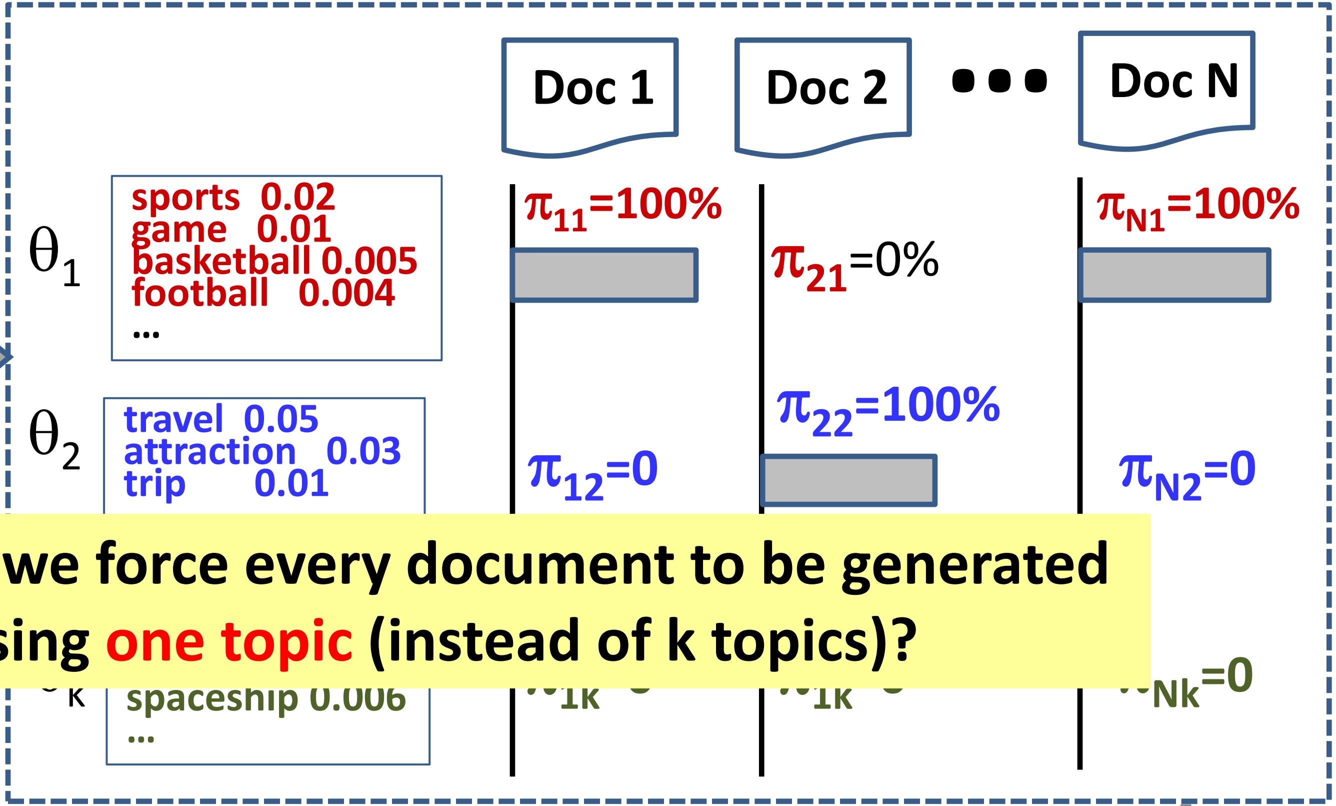
➔ (N Docs, k <u>Shared</u> Topics)=Clustering!

What Generative Model Can Do Clustering?

INPUT: C, k, V

OUTPUT: $\{ \theta_1, \ldots, \theta_k \}$, $\{ c_1, \ldots, c_N \}$ $c_i \in [1,k]$

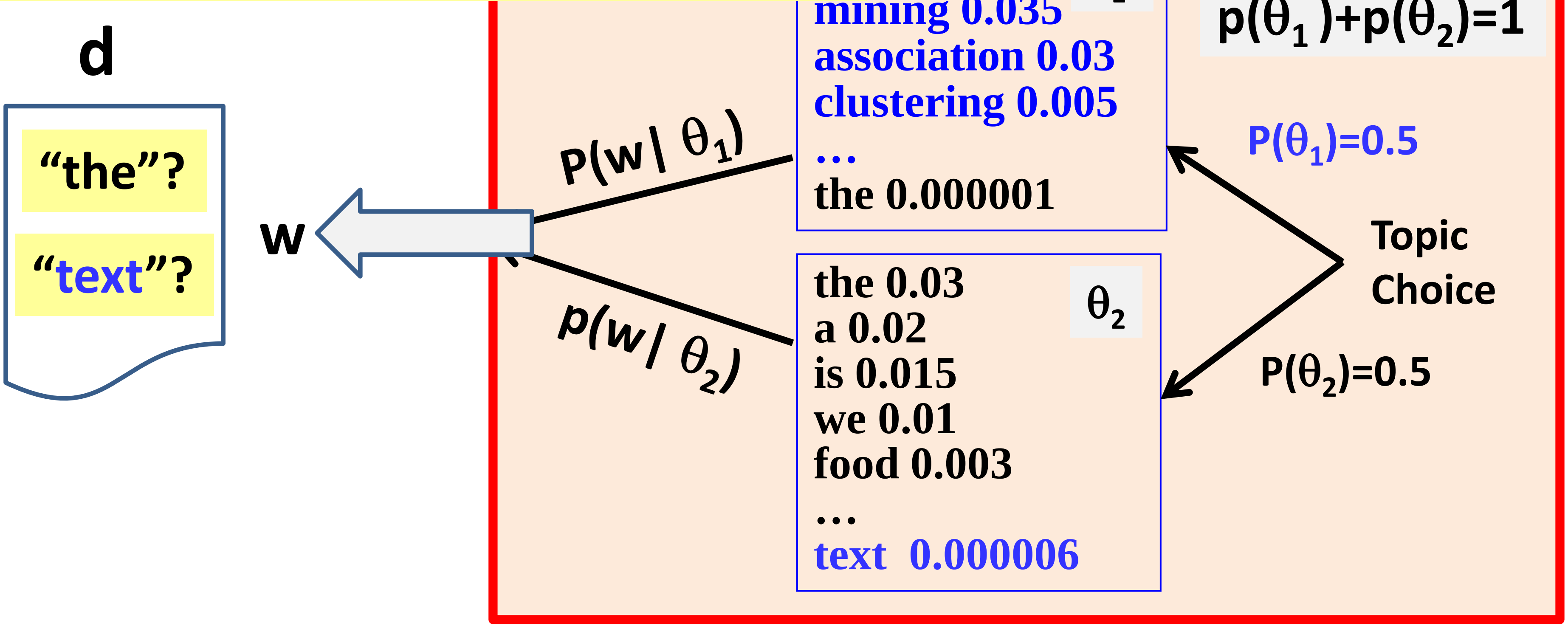




How can we force every document to be generated using one topic (instead of k topics)?

Generative Topic Model Revisited

Why can't this model be used for clustering?

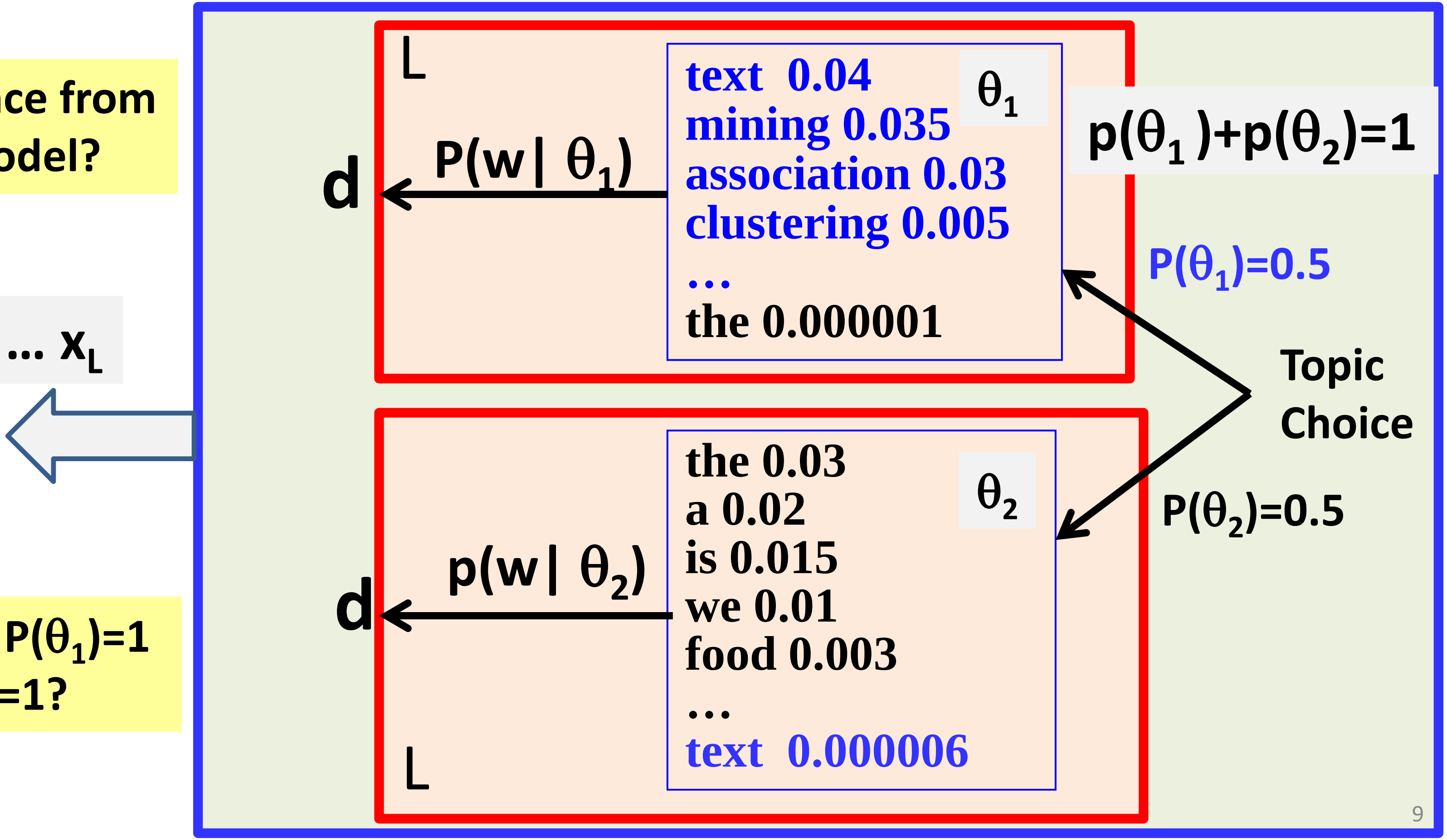


Mixture Model for Document Clustering

Difference from topic model?

$d = x_1\ x_2 \dots x_L$

What if $P(\theta_1)=1$ or $P(\theta_2)=1$?

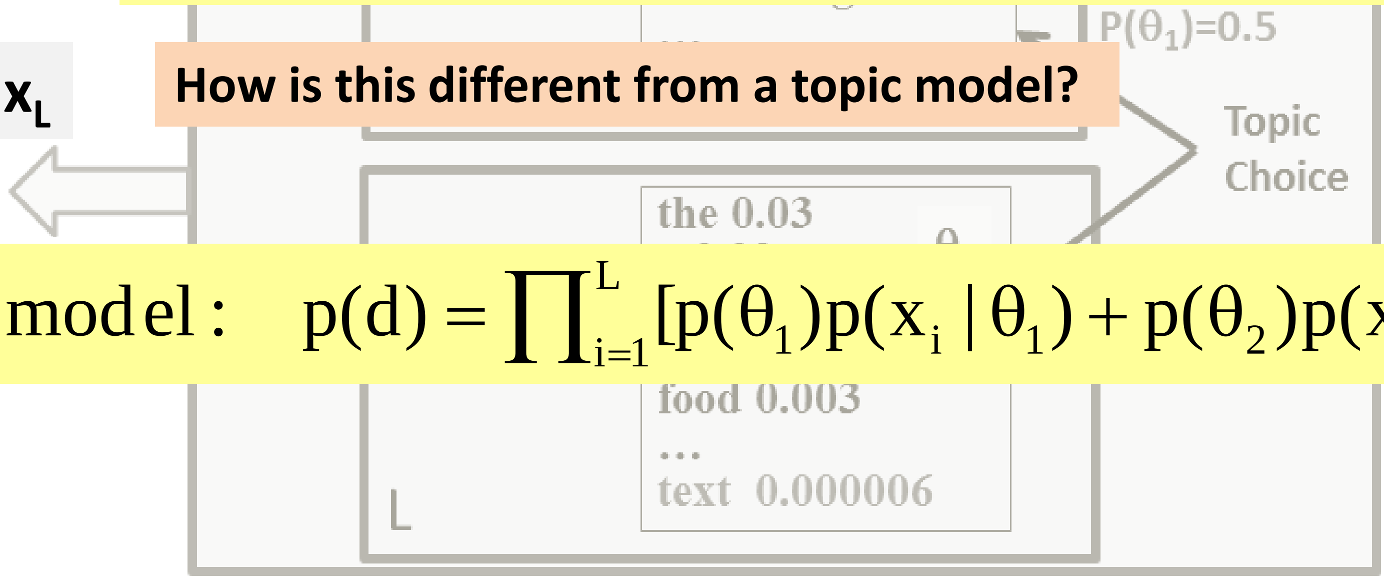


Likelihood Function: p(d)=?

$$p(d) = p(\theta_1)p(d \mid \theta_1) + p(\theta_2)p(d \mid \theta_2)$$
$$= p(\theta_1)\prod_{i=1}^{L} p(x_i \mid \theta_1) + p(\theta_2)\prod_{i=1}^{L} p(x_i \mid \theta_2)$$

$d = x_1\ x_2 \ldots x_L$

How is this different from a topic model?



$$\text{topic model:} \quad p(d) = \prod_{i=1}^{L} [p(\theta_1)p(x_i \mid \theta_1) + p(\theta_2)p(x_i \mid \theta_2)]$$

Text Clustering: Generative Probabilistic Models

Part 2

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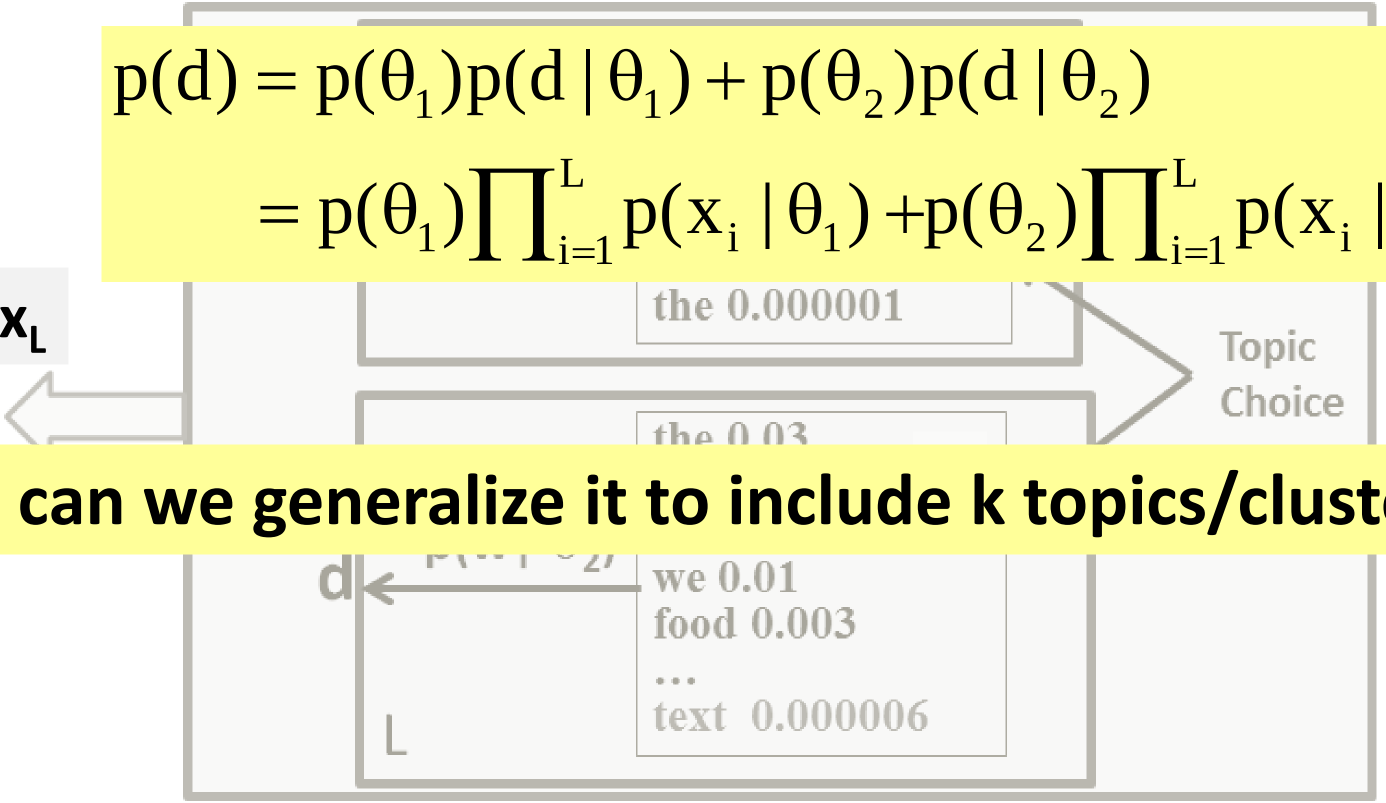
Likelihood Function: p(d)=?

$$p(d) = p(\theta_1)p(d \mid \theta_1) + p(\theta_2)p(d \mid \theta_2)$$
$$= p(\theta_1)\prod_{i=1}^{L} p(x_i \mid \theta_1) + p(\theta_2)\prod_{i=1}^{L} p(x_i \mid \theta_2)$$

$d=x_1\ x_2 \ldots x_L$



How can we generalize it to include k topics/clusters?

Mixture Model for Document Clustering

- Data: a collection of documents $C=\{d_1, \ldots, d_N\}$
- Model: mixture of k unigram LMs: $\Lambda=(\{\theta_i\}; \{p(\theta_i)\})$, $i\in[1,k]$
  - To generate a document, first **choose a θ_i** according to $p(\theta_i)$, and then generate **all** words in the document using $p(w|\theta_i)$
- Likelihood:

$$p(d|\Lambda) = \sum_{i=1}^{k}[p(\theta_i)\prod_{j=1}^{|d|} p(x_j|\theta_i)]$$

$$= \sum_{i=1}^{k}[p(\theta_i)\prod_{w\in V} p(w|\theta_i)^{c(w,d)}]$$

- Maximum Likelihood estimate

$$\Lambda^* = \arg\max_{\Lambda} p(d|\Lambda)$$

Cluster Allocation After Parameter Estimation

- **Parameters** of the mixture model: $\boldsymbol{\Lambda=(\{\theta_i\}; \{p(\theta_i)\}), i\in[1,k]}$
  - Each θ_i represents the **content of cluster i** : $\mathbf{p(w|\ \theta_i)}$
  - $\mathbf{p(\theta_i)}$ indicates the **size of cluster i**
  - Note that unlike in PLSA, $p(\theta_i)$ doesn't depend on d!
- Which cluster should document d belong to? c_d=?
  - **Likelihood only**: Assign d to the cluster corresponding to the topic θ_i that most likely has been used to generate d

$$c_d = \arg\max_i p(d\,|\,\theta_i)$$

  - **Likelihood + prior $\mathbf{p(\theta_i)}$ (Bayesian):** favor large clusters

$$c_d = \arg\max_i p(d\,|\,\theta_i)p(\theta_i)$$

Text Clustering: Generative Probabilistic Models

Part 3

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How Can We Compute the ML Estimate?

- Data: a collection of documents $C=\{d_1, \ldots, d_N\}$
- Model: mixture of k unigram LMs: $\Lambda=(\{\theta_i\}; \{p(\theta_i)\}), i \in [1,k]$
  - To generate a document, first **choose a θ_i** according to $p(\theta_i)$ and then generate **all** words in the document using $p(w|\theta_i)$
- Likelihood:

$$p(d|\Lambda) = \sum_{i=1}^{k} [p(\theta_i) \prod_{w \in V} p(w|\theta_i)^{c(w,d)}]$$

$$p(C|\Lambda) = \prod_{j=1}^{N} p(d_j|\Lambda)$$

- Maximum Likelihood estimate

$$\Lambda^* = \arg\max_{\Lambda} p(C|\Lambda)$$

EM Algorithm for Document Clustering

- Initialization: Randomly set Λ**=({θ_i}; {p(θ_i)}), i$\in$[1,k]**
- **Repeat until likelihood p(C|Λ) converges**
  - **E-Step: Infer which distribution has been used to generate document d: hidden variable $Z_d \in [1, k]$**

$$p^{(n)}(Z_d = i \mid d) \propto p^{(n)}(\theta_i) \prod_{w \in V} p^{(n)}(w \mid \theta_i)^{c(w,d)}$$

$$\sum_{i=1}^{k} p^{(n)}(Z_d = i \mid d) = 1$$

  - **M-Step: Re-estimation of all parameters**

$$p^{(n+1)}(\theta_i) \propto \sum_{j=1}^{N} p^{(n)}(Z_{d_j} = i \mid d_j)$$

$$\sum_{i=1}^{k} p^{(n+1)}(\theta_i) = 1$$

$$p^{(n+1)}(w \mid \theta_i) \propto \sum_{j=1}^{N} c(w, d_j) p^{(n)}(Z_{d_j} = 1 \mid d_j)$$

$$\sum_{w \in V} p^{(n+1)}(w \mid \theta_i) = 1, \quad \forall i \in [1, k]$$

An Example of 2 Clusters

Random Initialization

$$p(\theta_1)=p(\theta_2)= 0.5$$

	$p(w\|\theta_1)$	$p(w\|\theta_2)$
text	0.5	0.1
mining	0.2	0.1
medical	0.2	0.75
health	0.1	0.05

E-step **Document d**

	c(w,d)
text	2
mining	2
medical	0
health	0

Hidden variables:

$$Z_d \in \{1, 2\}$$

$$p(Z_d = 1|d) = \frac{p(\theta_1)p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2}{p(\theta_1)p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2 + p(\theta_2)p(\text{"text"}|\theta_2)^2 p(\text{"mining"}|\theta_2)^2}$$

$$= \frac{0.5 * 0.5^2 * 0.2^2}{0.5 * 0.5^2 * 0.2^2 + 0.5 * 0.1^2 * 0.1^2} = \frac{100}{101}$$

$$p(Z_d = 2|d) = ?$$

Normalization to Avoid Underflow

	$p(w\|\theta_1)$	$p(w\|\theta_2)$	$p(w\|\bar{\theta})$
text	0.5	0.1	(0.5+0.1)/2
mining	0.2	0.1	(0.2+0.1)/2
medical	0.2	0.75	(0.2+0.75)/2
health	0.1	0.05	(0.1+0.05)/2

Average of $p(w|\theta_i)$ as a possible normalizer

$$p(Z_d=1|d)=\frac{\dfrac{p(\theta_1)p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2 p(\text{"mining"}|\bar{\theta})^2}}}{\dfrac{p(\theta_1)p(\text{"text"}|\theta_1)^2 p(\text{"mining"}|\theta_1)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2 p(\text{"mining"}|\bar{\theta})^2}}+\dfrac{p(\theta_2)p(\text{"text"}|\theta_2)^2 p(\text{"mining"}|\theta_2)^2}{\boxed{p(\text{"text"}|\bar{\theta})^2 p(\text{"mining"}|\bar{\theta})^2}}}$$

An Example of 2 Clusters (cont.)

From E-Step

	$P(Z_d=1 \mid d)$
d1	0.9
d2	0.1
d3	0.8

	c("text")	c("mining")
d1	2	3
d2	1	2
d3	4	3

M-Step

$p(\theta_1)$=? $p(\theta_2)$= ?

$$p(\theta_1) = \frac{p(Z_{d_1}=1|d_1)+p(Z_{d_2}=1|d_2)+p(Z_{d_3}=1|d_3)}{3}$$
$$= \frac{0.9+0.1+0.8}{3} = 0.6$$

	$p(w \mid \theta_1)$	$p(w \mid \theta_2)$
text	?	?
mining	?	?
medical	?	?
health	?	?

$$p(\text{"text"}|\theta_1) \propto c(\text{"text"},d_1)*p(Z_{d_1}=1|d_1)+\ldots+c(\text{"text"},d_3)*p(Z_{d_3}=1|d_3)$$
$$= 2*0.9+1*0.1+4*0.8$$
$$p(\text{"mining"}|\theta_1) \propto 3*0.9+2*0.1+3*0.8$$

$$p(\text{"text"}|\theta_1)+p(\text{"mining"}|\theta_1)+p(\text{"medical"}|\theta_1)+p(\text{"health"}|\theta_1)=1$$

Summary of Generative Model for Clustering

- A slight variation of topic model can be used for clustering documents
  - Each **cluster** is represented by a **unigram LM** **$p(w|\theta_i)$** ➔ **Term cluster**
  - A document is generated by first choosing a unigram LM and then generating **ALL words** in the document using this **single LM**
  - Estimated model parameters give both a topic characterization of each cluster and a probabilistic assignment of a document into each cluster
  - "Hard" clusters can be obtained by forcing a document into the cluster corresponding to the unigram LM most likely used to generate the document
- EM algorithm can be used to compute the ML estimate
  - Normalization is often needed to avoid underflow

Text Clustering: Similarity-based Approaches

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Overview

- What is text clustering?
- Why text clustering?
- How to do text clustering?
  - Generative probabilistic models
  - **Similarity-based approaches**
- How to evaluate clustering results?

Similarity-based Clustering: General Idea

- Explicitly define a similarity function to measure similarity between two text objects (i.e., providing "clustering bias")
- Find an optimal partitioning of data to
  - maximize intra-group similarity and
  - minimize inter-group similarity
- Two strategies for obtaining optimal clustering
  - Progressively construct a hierarchy of clusters (hierarchical clustering)
    - Bottom-up (agglomerative): gradually group similar objects into larger clusters
    - Top-down (divisive): gradually partition the data into smaller clusters
  - Start with an initial tentative clustering and iteratively improve it ("flat" clustering, e.g., k-Means)

Similarity-based Clustering Methods

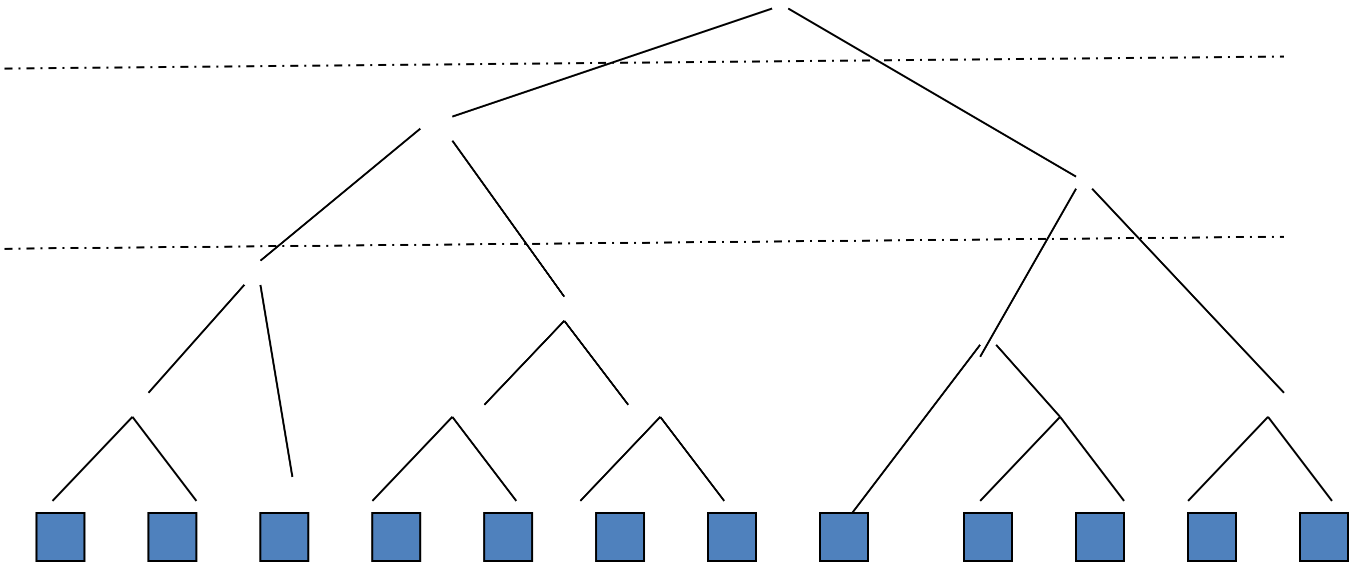
- Many general clustering methods are available!

- Two representative methods
  - Hierarchical Agglomerative Clustering (HAC)
  - k-means

Agglomerative Hierarchical Clustering

- Given a similarity function to measure similarity between two objects
- Gradually group similar objects together in a bottom-up fashion to form a hierarchy
- Stop when some stopping criterion is met
- Variations: different ways to compute group similarity based on individual object similarity

Similarity-induced Structure



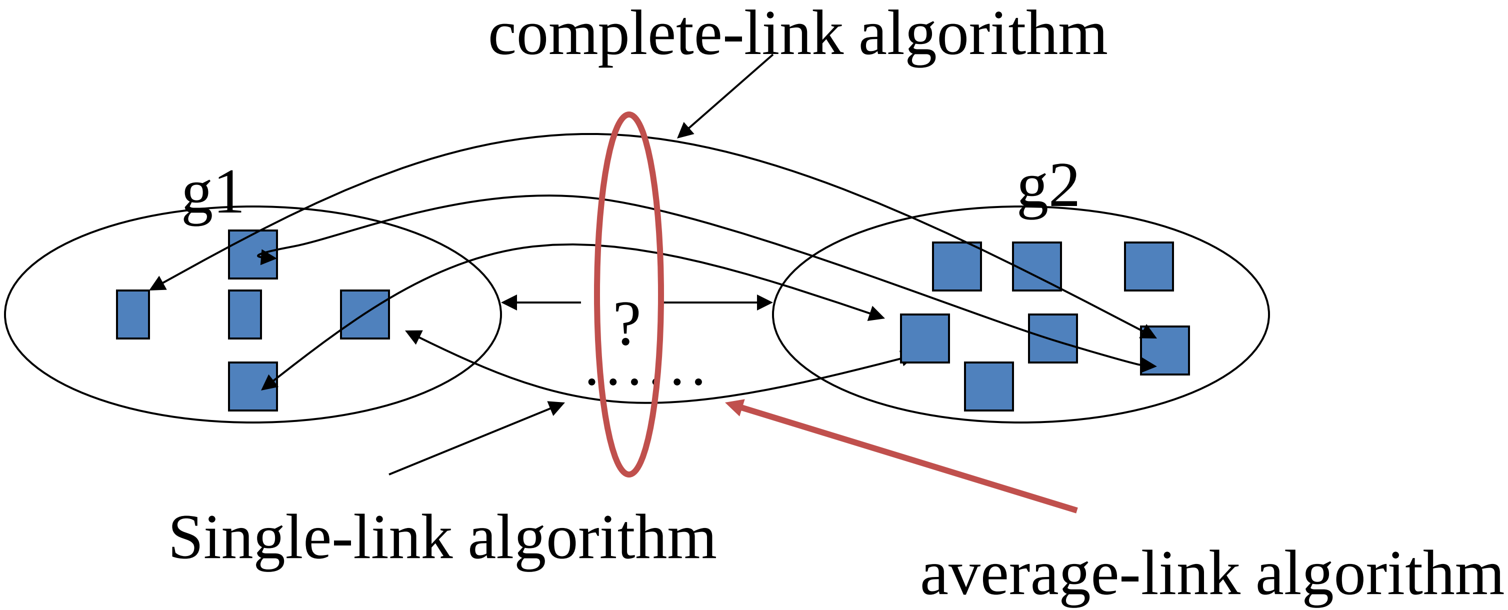
How to Compute Group Similarity

Three popular methods:

Given two groups g1 and g2,

- Single-link algorithm: s(g1,g2)= similarity of the closest pair
- Complete-link algorithm: s(g1,g2)= similarity of the farthest pair
- Average-link algorithm: s(g1,g2)= average of similarity of all pairs

Group Similarity Illustrated



Comparison of Single-Link, Complete-Link, and Average-Link

- Single-link
  - “Loose” clusters
  - Individual decision, sensitive to outliers
- Complete-link
  - “Tight” clusters
  - Individual decision, sensitive to outliers
- Average-link
  - “In between”
  - Group decision, insensitive to outliers
- Which one is the best? It depends on what you need!

K-Means Clustering

- Represent each text object as a term vector and assume a similarity function defined on two objects
- Start with k randomly selected vectors and assume they are the centroids of k clusters (initial tentative clustering) → **Initialization**
- Assign every vector to a cluster whose centroid is the closest to the vector **≈ E-step difference?**
- Re-compute the centroid for each cluster based on the newly assigned vectors in the cluster **≈ M-step difference?**
- Repeat this process until the similarity-based objective function (i.e., within cluster sum of squares) converges (to a local minimum)

Very similar to clustering with EM for mixture model!

Summary of Clustering Methods

- Model based approaches (mixture model)
  - Uses an implicit similarity function (model ➔ clustering bias)
  - Cluster structure is “built” into a generative model
  - Complex generative models can discover complex structures
  - Prior can be leveraged to further customize the clustering algorithm
  - However, no easy way to directly control the similarity measure
- Similarity-based approaches
  - Allows for direct and flexible specification of similarity
  - Objective function to be optimized is not always clear
- Both approaches can generate both term clusters and doc clusters

Text Clustering: Evaluation

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- Why text clustering?
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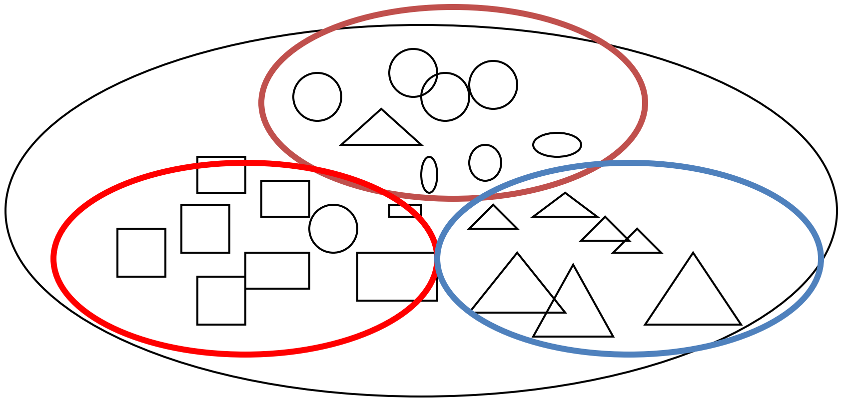
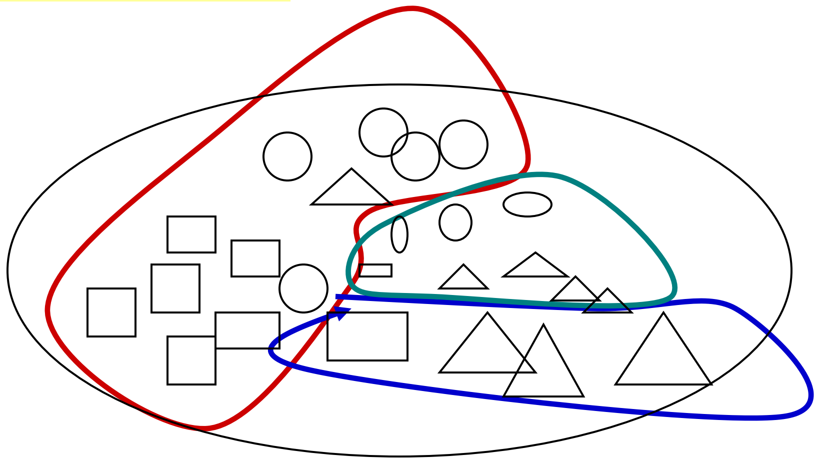
  - Generative probabilistic models
  - Similarity-based approaches
- **How to evaluate clustering results?**

The "Clustering Bias"

- Any two objects can be similar, depending on how you look at them!
- A user must define the **perspective** (i.e., a "**bias**") for assessing similarity!

Basis for evaluation





Direct Evaluation of Text Clusters

- Question to answer: How close are the system-generated clusters to the ideal clusters (generated by humans)?
  - “Closeness” can be assessed from multiple perspectives
  - “Closeness” can be quantified
  - “Clustering bias” is imposed by the human assessors
- Evaluation procedure:
  - Given a test set, have humans to create an ideal clustering result (i.e., an ideal partitioning of text objects or “gold standard”)
  - Use a system to produce clusters from the same test set
  - Quantify the similarity between the system-generated clusters and the gold standard clusters
  - Similarity can be measured from multiple perspectives (e.g., purity, normalized mutual information, F measure)

Indirect Evaluation of Text Clusters

- Question to answer: how useful are the clustering results for the intended applications?
  - "Usefulness" is inevitably application specific
  - "Clustering bias" is imposed by the intended application
- Evaluation procedure:
  - Create a test set for the intended application to quantify the performance of any system for this application
  - Choose a baseline system to compare with
  - Add a clustering algorithm to the baseline system ➔ "clustering system"
  - Compare the performance of the clustering system and the baseline in terms of any performance measure for the application

Summary of Text Clustering

- Text clustering is an unsupervised general text mining technique to
  - obtain an overall picture of the text content (exploring text data)
  - discover interesting clustering structures in text data
- Many approaches are possible
  - Strong clusters tend to show up no matter what method used
  - Effectiveness of a method highly depends on whether the desired clustering bias is captured appropriately (either through using the right generative model or the right similarity function)
  - Deciding the optimal number of clusters is generally a difficult problem for any method due to the unsupervised nature
- Evaluation of clustering results can be done both directly and indirectly

Suggested Reading

- Manning, Chris D., Prabhakar Raghavan, and Hinrich Schütze. *Introduction to Information Retrieval*. Cambridge: Cambridge University Press, 2007. (Chapter 16)

Text Categorization: Motivation

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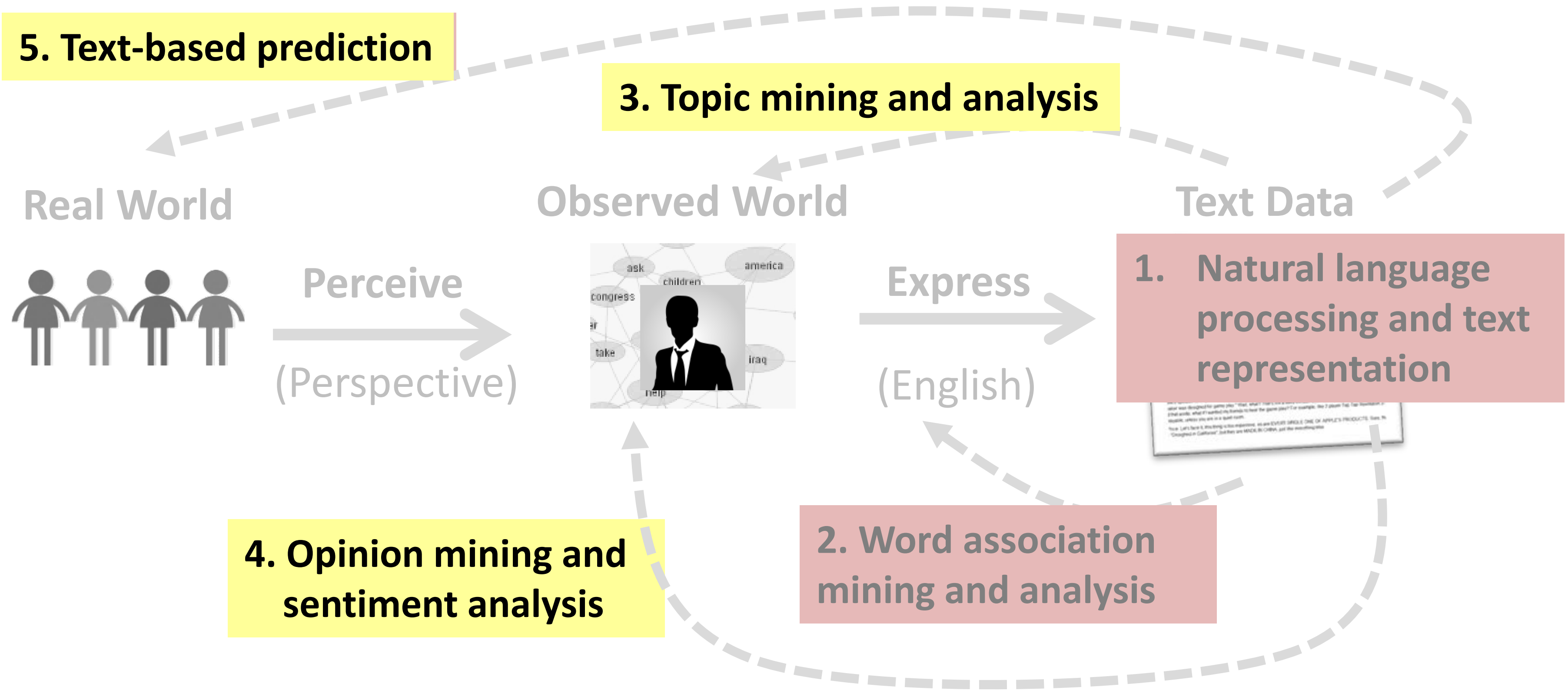
Text Categorization



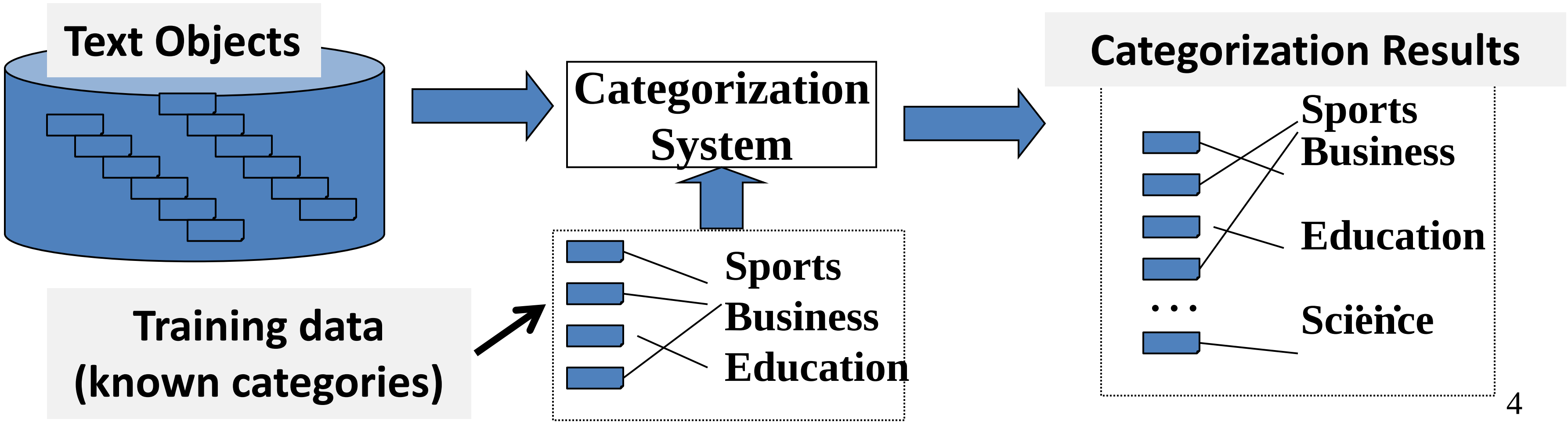
Overview

- What is text categorization?
- Why text categorization?
- How to do text categorization?
  - Generative probabilistic models
  - Discriminative approaches
- How to evaluate categorization results?

This lecture

Text Categorization

- Given the following:
  – A set of **predefined categories**, possibly forming a hierarchy
  and often
  – A **training set** of labeled text objects
- Task: **Classify** a text object into **one or more** of the **categories**



Examples of Text Categorization

- **Text objects can vary** (e.g., documents, passages, or collections of text)
- **Categories can also vary**
  - **“Internal”** categories that characterize a text object (e.g., topical categories, sentiment categories)
  - **“External”** categories that characterize an entity associated with the text object (e.g., author attribution or any other meaningful categories associated with text data)
- Some **examples of applications**
  - News categorization, literature article categorization (e.g., MeSH annotations)
  - Spam email detection/filtering
  - Sentiment categorization of product reviews or tweets
  - Automatic email sorting/routing
  - Author attribution

Variants of Problem Formulation

- **Binary** categorization: Only two categories
  - Retrieval: {relevant-doc, non-relevant-doc}
  - Spam filtering: {spam, non-spam}
  - Opinion: {positive, negative}
- **K-category** categorization: More than two categories
  - Topic categorization: {sports, science, travel, business,…}
  - Email routing: {folder1, folder2, folder3,…}
- **Hierarchical** categorization: Categories form a hierarchy
- **Joint** categorization: Multiple **related** categorization tasks done in a joint manner

Binary categorization can potentially support all other categorizations

Why Text Categorization?

- To **enrich text representation** (more understanding of text)
  - Text can now be represented in multiple levels (keywords + categories)
  - Semantic categories assigned can be directly or indirectly useful for an application
  - Semantic categories facilitate aggregation of text content (e.g., aggregating all positive/negative opinions about a product)
- To **infer properties of entities** associated with text data (discovery of **knowledge about the world**)
  - As long as an entity can be associated with text data, we can always use the text data to help categorize the associated entities
  - E.g., discovery of non-native speakers of a language; prediction of party affiliation based on a political speech

Text Categorization: Methods

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Overview

- What is text categorization?
- Why text categorization?
- **How to do text categorization?**
  - Generative probabilistic models
  - Discriminative approaches
- How to evaluate categorization results?

Categorization Methods: Manual

- Determine the category based on rules that are carefully designed to reflect the domain knowledge about the categorization problem
- Works well when
  - The categories are very well defined
  - Categories are easily distinguished based on surface features in text (e.g., special vocabulary is known to only occur in a particular category)
  - Sufficient domain knowledge is available to suggest many effective rules
- Problems
  - Labor intensive ➔ doesn't scale up well
  - Can't handle uncertainty in rules; rules may be inconsistent ➔ not robust
- Both problems can be solved/alleviated by using machine learning

Categorization Methods: "Automatic"

- Use **human experts** to
  - Annotate data sets with **category labels** ➔ Training data
  - Provide a set of **features** to represent each text object that can potentially provide a "clue" about the category
- Use **machine learning** to learn "soft rules" for categorization from the training data
  - Figure out **which features are most useful** for separating different categories
  - **Optimally combine the features** to **minimize the errors** of categorization on the training data
  - The trained classifier can then be applied to a new text object to predict the most likely category (that a human expert would assign to it)

Machine Learning for Text Categorization

- **General setup**: Learn a classifier f: X➔Y
  - Input: X = all text objects; Output: Y = all categories
  - Learn a classifier function, f: X➔Y, such that f(x)=y ∈Y gives the correct category for x∈X ("correct" is based on the training data)
- **All methods**
  - Rely on discriminative features of text objects to distinguish categories
  - Combine multiple features in a weighted manner
  - Adjust weights on features to minimize errors on the training data
- **Different methods** tend to vary in
  - Their way of measuring the errors on the training data (may optimize a different objective/loss/cost function)
  - Their way of combining features (e.g., linear vs. non-linear)

Generative vs. Discriminative Classifiers

- **Generative** classifiers (learn **what the data "looks" like in each category**)
  - Attempt to model $p(X,Y) = p(Y)p(X|Y)$ and compute $p(Y|X)$ based on $p(X|Y)$ and $p(Y)$ by using Bayes Rule
  - Objective function is likelihood, thus indirectly measuring training errors
  - E.g., Naïve Bayes
- **Discriminative** classifiers (learn **what features separate categories**)
  - Attempt to model $p(Y|X)$ directly
  - Objective function directly measures errors of categorization on training data
  - E.g., Logistic Regression, Support Vector Machine (SVM), k-Nearest Neighbors (kNN)

Text Categorization: Generative Probabilistic Models

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Overview

- What is text categorization?
- Why text categorization?
- How to do text categorization?
  - **Generative probabilistic models**
  - Discriminative approaches
- How to evaluate categorization results?

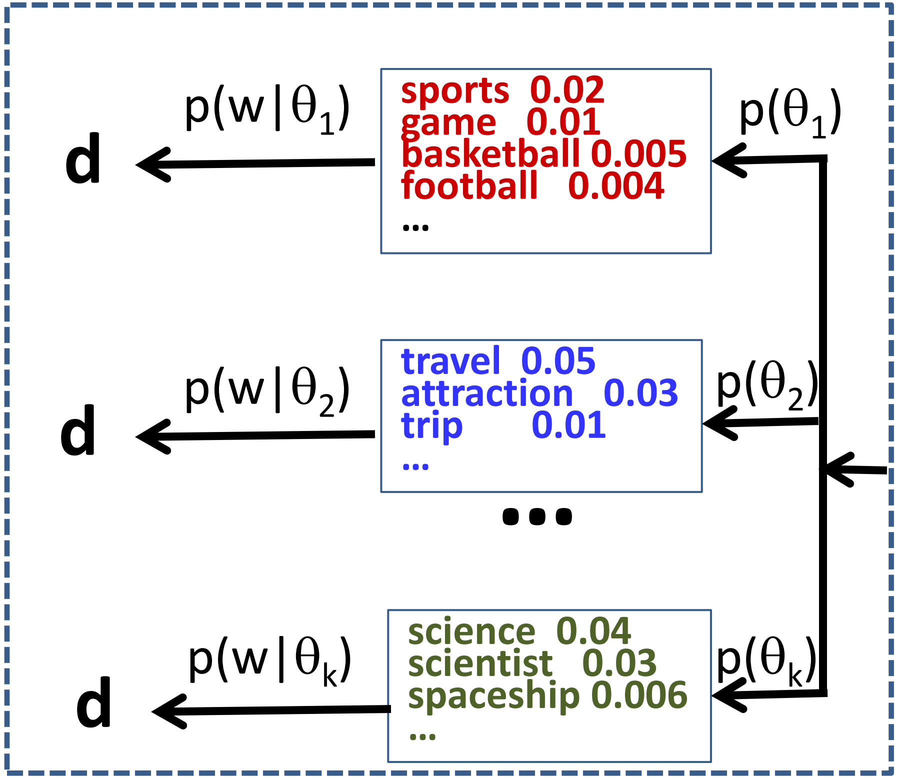
Document Clustering Revisited

Which cluster does d belong to? ➔ Which θ_i was used to generate d?

$\mathbf{d=x_1x_2\ldots x_L}$ where $x_i \in V$

$$\begin{aligned}\text{cluster}(d) &= \arg\max_i p(\theta_i \mid d) \\ &= \arg\max_i p(d \mid \theta_i) p(\theta_i) \\ &= \arg\max_i \prod_{j=1}^{L} p(x_j \mid \theta_i) p(\theta_i) \\ &= \arg\max_i \prod_{w \in V} p(w \mid \theta_i)^{c(w,d)} p(\theta_i)\end{aligned}$$

$$\begin{aligned}p(\theta_i \mid d) &= \frac{p(d \mid \theta_i) p(\theta_i)}{p(d)} \\ &= \frac{p(d \mid \theta_i) p(\theta_i)}{\sum_{j=1}^{k} p(d \mid \theta_j) p(\theta_j)}\end{aligned}$$



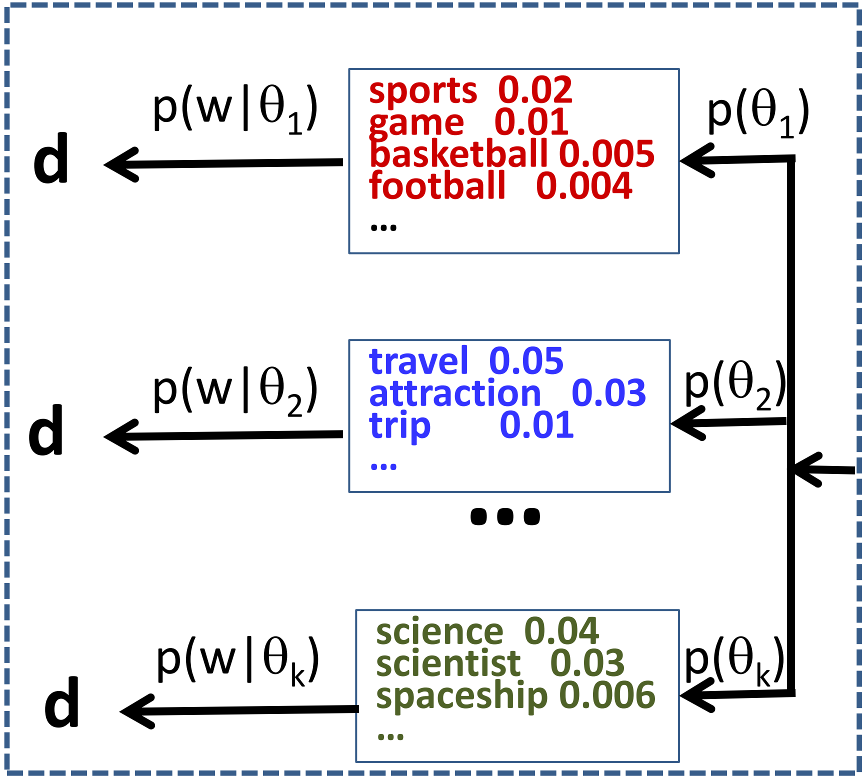
Text Categorization with Naïve Bayes Classifier

$\mathbf{d=x_1x_2...x_L}$ where $x_i \in V$

IF θ_i represents category i accurately, then…

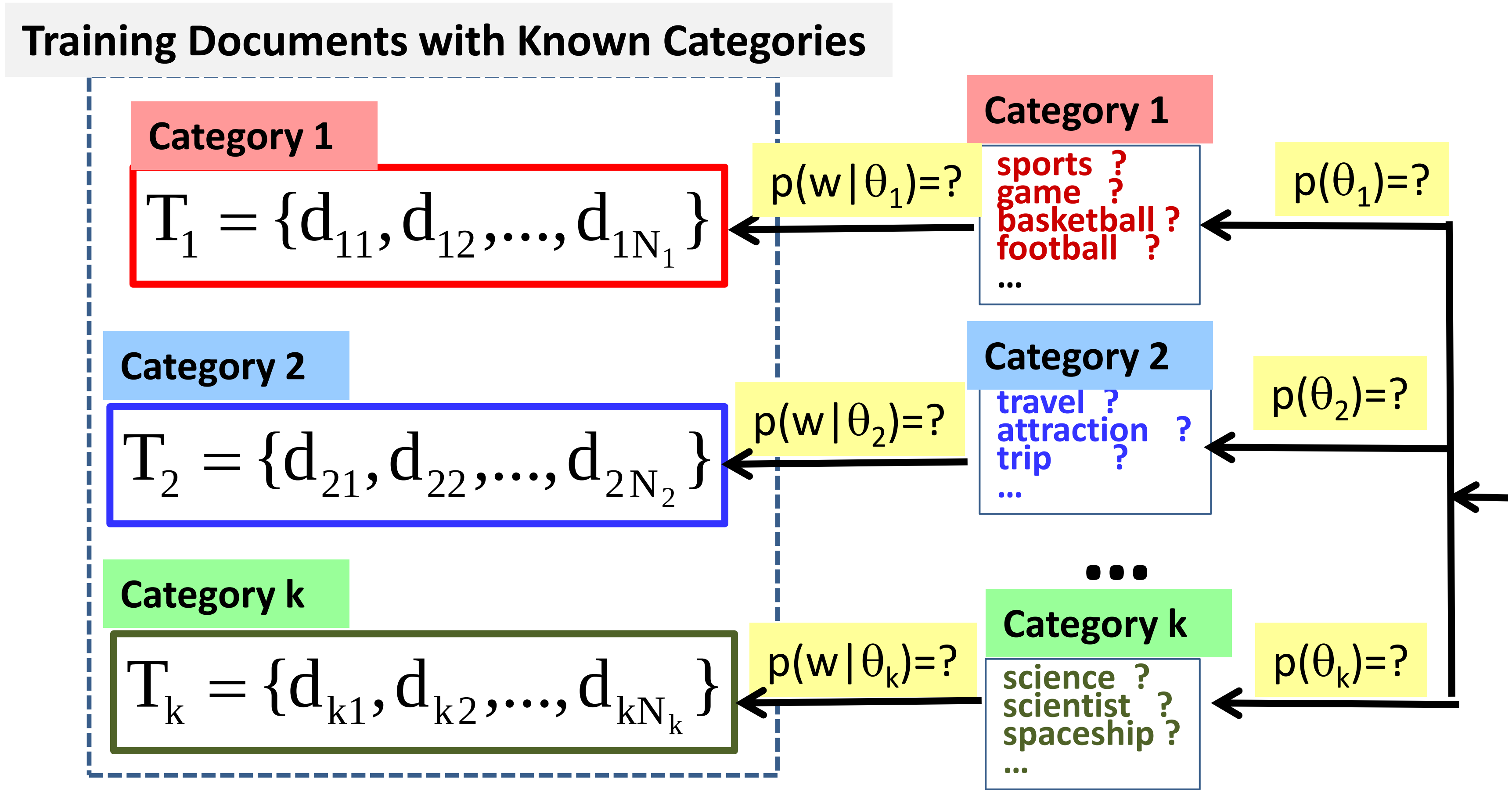
How can we make this happen?

$$\text{category}(d) = \arg\max_i p(\theta_i \mid d)$$
$$= \arg\max_i p(d \mid \theta_i)p(\theta_i)$$
$$= \arg\max_i \prod_{w \in V} p(w \mid \theta_i)^{c(w,d)} p(\theta_i)$$

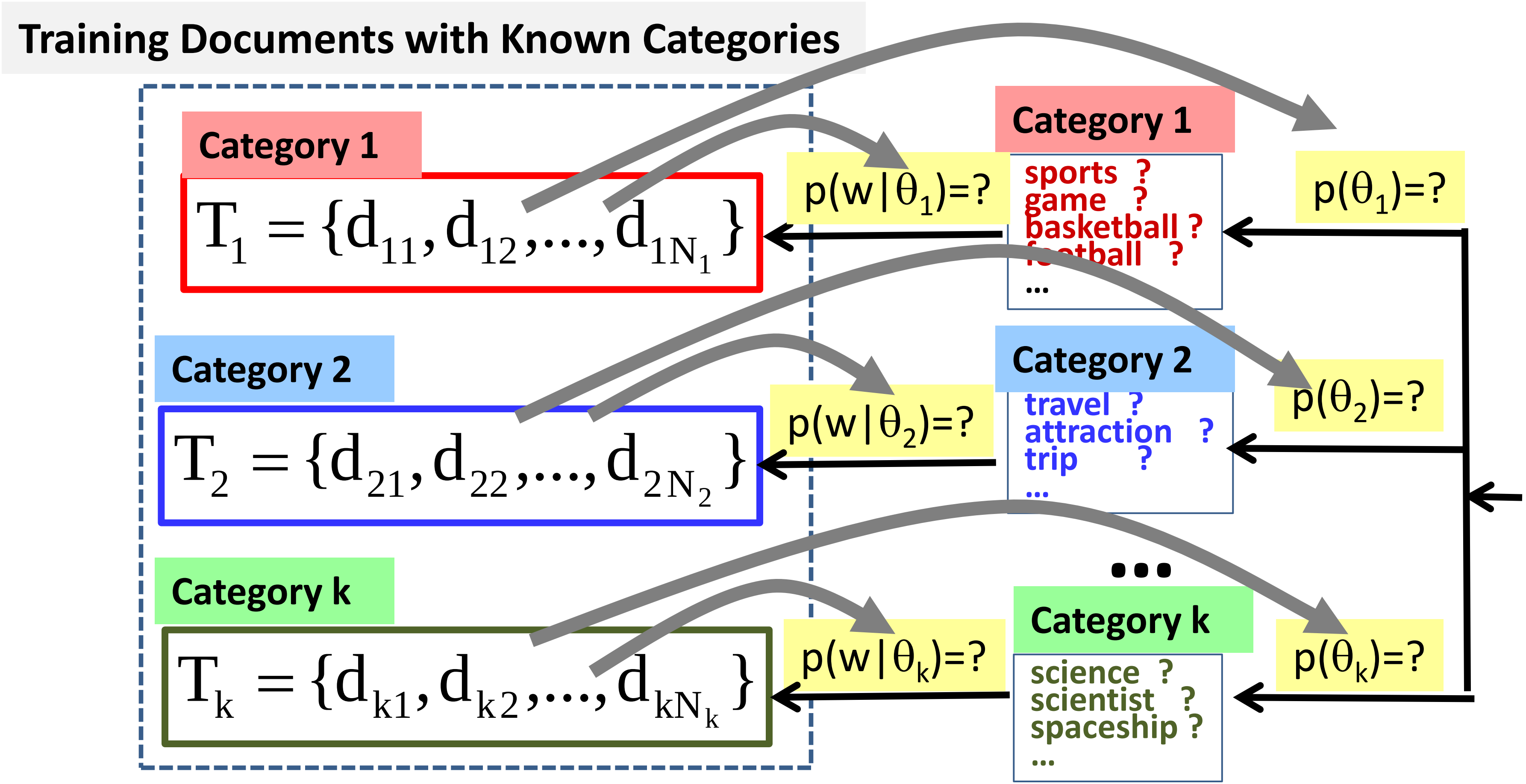


$$\boxed{\text{category}(d) = \arg\max_i \log p(\theta_i) + \sum_{w \in V} c(w,d) \log p(w \mid \theta_i)}$$

Learn from the Training Data



How to Estimate $p(w|\theta_i)$ and $p(\theta_i)$



Naïve Bayes Classifier: $p(\theta_i)$=? and $p(w|\theta_i)$=?

Category 1

$$T_1 = \{d_{11}, d_{12}, \ldots, d_{1N_1}\}$$

Category 2

$$T_2 = \{d_{21}, d_{22}, \ldots, d_{2N_2}\}$$

Category k

$$T_k = \{d_{k1}, d_{k2}, \ldots, d_{kN_k}\}$$

Which category is most popular?

$$p(\theta_i) = \frac{N_i}{\sum_{j=1}^{k} N_j} \propto |T_i|$$

$$p(w|\theta_i) = \frac{\sum_{j=1}^{N_i} c(w, d_{ij})}{\sum_{w' \in V} \sum_{j=1}^{N_i} c(w', d_{ij})} \propto c(w, T_i)$$

Which word is most frequent in category i?

What are the constraints on $p(\theta_i)$ and $p(w|\theta_i)$?

Smoothing in Naïve Bayes

- Why smoothing?
  - Address data sparseness (training data is small ➔ zero prob.)
  - Incorporate prior knowledge
  - Achieve discriminative weighting (i.e., IDF weighting)
- How?

$$p(\theta_i) = \frac{N_i + \delta}{\sum_{j=1}^{k} N_j + k\delta} \qquad \delta \geq 0$$

What if $\delta \rightarrow \infty$?

$p(w|\theta_B)$: background LM

$$p(w \mid \theta_i) = \frac{\sum_{j=1}^{N_i} c(w, d_{ij}) + \mu p(w \mid \theta_B)}{\sum_{w' \in V} \sum_{j=1}^{N_i} c(w', d_{ij}) + \mu} \qquad \mu \geq 0$$

$p(w|\theta_B)=1/|V|$?

What if $\mu \rightarrow \infty$?

Anatomy of Naïve Bayes Classifier

Two categories: θ_1 and θ_2

$$\text{score}(d) = \log \frac{p(\theta_1 | d)}{p(\theta_2 | d)} = \log \frac{p(\theta_1) \prod_{w \in V} p(w | \theta_1)^{c(w,d)}}{p(\theta_2) \prod_{w \in V} p(w | \theta_2)^{c(w,d)}}$$

$$= \boxed{\log \frac{p(\theta_1)}{p(\theta_2)}} + \sum_{w \in V} \underline{c(w,d)} \boxed{\log \frac{p(w | \theta_1)}{p(w | \theta_2)}}$$

Category bias (β_0) doesn't depend on d!

Sum over all words (features $\{f_i\}$)

Feature value: f_i=c(w,d)

Weight on each word (feature) β_i

Generalize

$$d = (f_1, f_2, \ldots, f_M), \;\; f_i \in \Re$$

$$\text{score}(d) = \beta_0 + \sum_{i=1}^{M} f_i \beta_i \quad \beta_i \in \Re$$

= Logistic Regression!