## Automatic Calibration of Large Traffic Models

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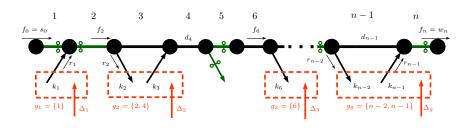
#### Introduction

and motivation of this work

- Calibrate large traffic models input:
  - Constants of the scenario
  - ▶ Source and sink flows depending on day of week & moment of year
- for the output to match:
  - Congestion
  - VMT and VHT
  - Credible ramp flows
- Goal: model accurately a usual traffic situation to make predictions.

## Setup

#### Freeway and traffic model, data, notation



$$\begin{split} M &= \llbracket 1, n \rrbracket \\ R &\subset M \\ T &\subset M \\ K &\subset R \\ G &= (g_i)_{i \in \llbracket 1, \gamma \rrbracket} \\ \vec{k} &= (k_{i_1}, k_{i_2} ..., k_{i_\kappa}) \\ \sigma &= (\sigma_{i_1}, \sigma_{i_2} ..., \sigma_{i_\kappa}) \end{split}$$

Mainline link indexes Ramp link indexes Monitored mainline links Non-Monitored ramps Knob group indexes Knobs value vector on/off-ramp indicator 
$$\begin{split} &(f_i)_{i \in M} \\ &(d_i)_{i \in M} \\ &(r_i)_{i \in R} \\ &(f_i)_{i \in R} \\ &(\widetilde{f_i})_{i \in T} \\ &(\widetilde{d_i})_{i \in T} \\ &(\widetilde{r_i})_{i \in (R \setminus K)} \end{split}$$

Mainline links exit flows Mainline links densities Ramps exit flows Flows inputted to the model Measured mainline exit flows Measured mainline densities Measured ramps exit flows

### Constraints

#### Physical Boundaries

Let  $(t_i(t))_{i \in K}$  the templates:

$$m_i = \frac{[\textit{Capacity of ramp associated to } k_i]}{\max_t t_i(t)}$$

$$\forall i \in K$$
,  $0 \le k_i \le m_i \Leftrightarrow \vec{k} \in \mathscr{B}$ 

#### Constraints

#### Knob groups and refined constraints

Each ramp is closely monitored by mainline sensors.

$$egin{aligned} orall i \in \llbracket 1, \gamma 
rbracket, & \Delta_i = \sum_{t \in au} \left[ \widetilde{f}_{eta_i}(t) - \widetilde{f}_{\eta_i}(t) + \sum_{j \in (R \setminus K) \cap S_i} \sigma_j. \widetilde{r_j}(t) 
ight] \ & \ldots \ & \ldots \ & \ldots \ & \ldots \end{aligned}$$

$$\Leftrightarrow$$
  $\Delta_i = -\sum_{j \in g_i} \sigma_j.k_j.\Theta$ 

### Constraints

#### Loosening the constraints: uncertainty

- Umul, Uadd and Uglobal
- $\forall i \in [1, \gamma]$ , let the most permissive flow demands:

$$\begin{array}{l} \Delta_i^- = \max\left\{|\Delta_i| - U^{add}; |\Delta_i|.(1 - U^{mul})\right\} \\ \Delta_i^+ = \min\left\{|\Delta_i| + U^{add}; |\Delta_i|.(1 + U^{mul})\right\} \end{array}$$

Loosened constraints:

$$\forall i \in [1, \gamma], \ \Delta_i^- \le |\sum_{j \in g_i} \sigma_j.k_j.\Theta| \le \Delta_i^+$$

### Performance and error calculation

Vehicles Hour Traveled (VHT)

Value computation from model output and data :

$$TVH = \sum_{l \in M} \sum_{t \in [0, \frac{24h}{dt}]} d_l^{(p)}(t) * \frac{dt}{1 \text{ hour}}$$

• Relative difference :  $\tau_p(TVH) = 100 * \frac{TVH^{BeATS}(p) - TVH^{PeMS}(p)}{TVH^{PeMS}(p)}$ 

### Performance and error calculation

#### Vehicles Mile Traveled (VMT)

Value computation from BeATS output and PeMS:

$$TVM(p) = \sum_{I \in \mathcal{M}} \sum_{t} f_{I}^{(p)}(t) * length(I)$$

•  $\forall i \in L, f_i(midnight) \approx 0 \Rightarrow \text{TVM}$  can be computed before BeATS: Let  $TVM^{ref} = TVM((1,...,1))$  outputed by BeATS

$$TVM^{a \; priori}(k_p) = TVM^{ref} + \sum_{i \in K} \left[ \sigma_i.T_i.k_i.\sum_{j \in M, j > i} length(j) \right]$$

ightarrow Same project & penalize as KD, within  $I^{global}$ : minimize  $\left\|k_p - \underline{k_p}\right\|_2$  s.t.

$$TVM^{PeMS}.(1 - I^{global}) < TVM^{a \ priori}(k_p) < TVM^{PeMS}.(1 + I^{global})$$
  
 $k_p \in [k^{naMin}, k^{naMax}]$ 

• Relative difference :  $\tau_p(TVM) = 100 * \frac{TVM^{PeATS}(p) - TVM^{PeMS}(p)}{TVM^{PeMS}(p)} = 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 200 * 20$ 

### Performance and error calculation

#### Congestion Pattern matching (CP)

- Principle: match the congested links and times into a box estimated from PeMS contour plot
- Error computation from BeATS output:
  - A congestion treshold is defined for each mainline link :

$$\frac{\textit{Link capacity}}{\textit{Link freeflow speed}} + \delta$$

► The number of pixels of the contour plot that are in the wrong state is the error:

$$\mathit{CP}(p) = \sum_{t \in rac{24h}{l}} \sum_{l \in L} \mathbb{1}_{\mathit{wrong}}$$
 congestion state links

• Relative difference :  $\tau_p(\mathit{CP}) = 100 * \frac{\mathit{CP}(p)}{\mathit{Total area of the boxes}}$ 

### **Problem Formulation**

Optimization problem statement



#### Requirements

- ullet Continuous search space: here,  $\mathcal{S}$  =12-D hypercube
- Function to minimize: mix of correlated and uncorrelated functions with non-differentiable congestion effects
- → Continuous black-box imputation problem

#### Covariance Matrix Adaptation - Evolution Strategy (CMA-ES)

- Most powerful evolutionary algorithms for single-objective real-valued optimization (very used)
- "Designed for difficult non-linear non-convex black-box optimisation problems in continuous domain"
- "Typically applied to unconstrained or bounded constraint optimization problems, and search space dimensions between three and a hundred"
- Does not presume existence of approximate gradients : feasible on our non-smooth problem
- Adaptive algorithm : almost no parameter tuning  $\rightarrow$  suitable to be used on several different freeways and days.
- Time does not matter for this initial study : not the fastest ES but excellent solution quality
- Principle: 12\*12 covariance matrix adapts the stochastic sampling direction and step size on-the-go

#### Constraints implementation (1)

For the single-knob groups, we can refine the knob boundaries using these two uncertainties as a limit:

$$\begin{aligned} k_i^{min} &= \max(\{\min(\{\alpha.k_i^*; k_i^* - \frac{I^{local}}{\sum_t t_i(t)}\}); 0\}) \\ k_i^{max} &= \min(\{\max(\{\beta.k_i^*; k_i^* + \frac{I^{local}}{\sum_t t_i(t))}\}); \mu_i\}) \end{aligned}$$

#### Refined boundaries on multiple-knob groups

Repairing: Here, we project the knobs of the group on a space delimited by the two "tolerance hyperplans" for  $\Delta_k$ :

Let  $k_p$  the knobs vector to be evaluated at iteration p before projection Let  $J = \{j, card(g_i > 1)\}\$ and  $\forall i \in K, T_i = \sum_t t_i(t)$ 

minimize 
$$\left\|k_p - \overline{k_p}\right\|_2$$

s.t.

$$\forall j \in J \ \min(\{\alpha.\Delta_j; \Delta_j - I^{local}\}) < \sum_{i \in g_j} \sigma_i.k_i.T_i < \max(\{\beta.\Delta_j; \Delta_j + I^{local}\})$$
  
 $k \in [k^{naMin}, k^{naMax}]$ 

# Experiment settings

Data

## **Experiment settings**

Model and simulator

## **Experiment settings**

Implementation of congestion pattern

General observations

Typical execution

Effect of  $U^{mul}$ 

Effect of  $U^{add}$ 

Effects of  $\sigma$ 

Effects of  $\lambda$ 

### Further work

Next steps and ideas

