Automatic Calibration of Large Traffic Models

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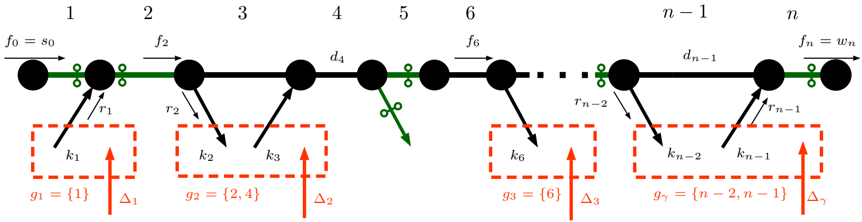
Introduction

and motivation of this work

- Calibrate large traffic models input:
  - Constants of the scenario
  - Source and sink flows depending on day of week & moment of year
- for the output to match:
  - Congestion
  - VMT and VHT
  - Credible ramp flows
- Goal: model accurately a usual traffic situation to make predictions.

Setup

Freeway and traffic model, data, notation



$M = [\![1, n]\!]$	Mainline link indexes	$(f_i(t))_{i \in M}$	Mainline links exit flows
$R \subset M$	Ramp link indexes	$(d_i(t))_{i \in M}$	Mainline links densities
$T \subset M$	Monitored mainline links	$(r_i(t))_{i \in R}$	Ramps exit flows
$K \subset R$	Non-Monitored ramps	$(\widetilde{f_i}(t))_{i \in T}$	Measured mainline exit flows
$G = (g_i)_{i \in [\![1, \gamma]\!]}$	Knob group indexes	$(\widetilde{d_i}(t))_{i \in T}$	Measured mainline densities
$\vec{k} = (k_{i_1}, k_{i_2} \ldots, k_{i_\kappa})$	Knobs value vector	$(\widetilde{r_i}(t))_{i \in (R \setminus K)}$	Measured ramps exit flows
$\sigma = (\sigma_{i_1}, \sigma_{i_2} \ldots, \sigma_{i_\kappa})$	on/off-ramp indicator	τ, D	Set of time-steps, Duration

Constraints

Physical Boundaries

Let $(t_i(t))_{i \in K}$ the templates:

$$m_i = \frac{[Capacity\ \ of\ \ ramp\ \ associated\ \ to\ \ k_i]}{\max_t t_i(t)}$$

$$\forall i \in K, \qquad 0 \leq k_i \leq m_i \qquad \Leftrightarrow \qquad \vec{k} \in \mathscr{B}$$

Constraints

Knob groups and refined constraints

Each ramp is closely monitored by mainline sensors.

$$\forall i \in [\![1, \gamma]\!], \qquad \Delta_i = \sum_{t \in \tau} \left[\widetilde{f}_{\beta_i}(t) - \widetilde{f}_{\eta_i}(t) + \sum_{j \in (R \backslash K) \cap S_i} \sigma_j . \widetilde{r}_j(t) \right]$$

$$\ldots$$

$$\ldots$$

$$\ldots$$

$$\Leftrightarrow \qquad \Delta_i = - \sum_{j \in g_i} \sigma_j . k_j . \Theta$$

Constraints

Loosening the constraints: uncertainty

- U^{mul}, U^{add} and U^{global}
- $\forall i \in [\![1, \gamma]\!]$, let the most permissive flow demands:

$$\Delta_i^- = \max\{|\Delta_i| - U^{add}; |\Delta_i|.(1 - U^{mul})\}$$
$$\Delta_i^+ = \min\{|\Delta_i| + U^{add}; |\Delta_i|.(1 + U^{mul})\}$$

- Loosened constraints:

$$\forall i \in [\![1, \gamma]\!],\ \Delta_i^- \leq |\sum_{j \in g_i} \sigma_j.k_j.\Theta| \leq \Delta_i^+$$

Performance and error calculation

Vehicles Hour Traveled (VHT)

- Value computation from model output and data :

$$VHT(\vec{k}) = \frac{dt}{[1\ hour]} \sum_{i \in T} L_i \sum_{t \in \tau} d_i(t)$$

- Error :

$$E_{VHT}(\vec{k}) = \frac{|VHT(\vec{k}) - \widetilde{VHT}|}{\widetilde{VHT}}$$

Performance and error calculation

Vehicles Mile Traveled (VMT) (1)

- Value computation from BeATS output and PeMS:

$$VMT(\vec{k}) = \sum_{i \in T} L_i \sum_{t \in \tau} f_i(t)$$

- Error :

$$E_{VMT}(\vec{k}) = \frac{|VMT(\vec{k}) - \widetilde{VMT}|}{\widetilde{VMT}}$$

Performance and error calculation

Vehicles Mile Traveled (VMT) (2)

Initial conditions hypothesis $\Rightarrow$ VMT can be computed a-priori.

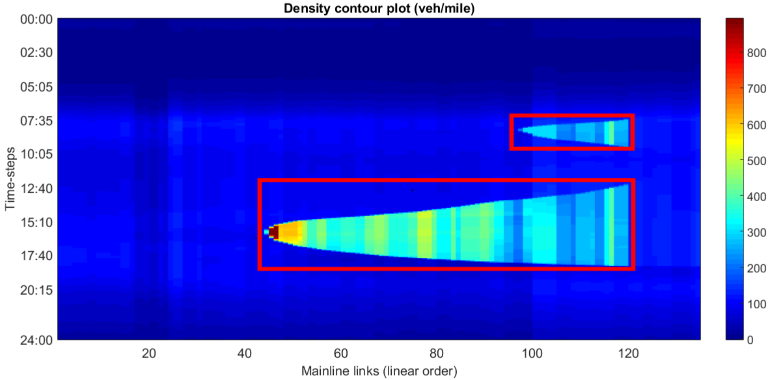
$\Rightarrow$ new constraint:

$$\widetilde{VMT} = VMT^{a\ priori}(\vec{k}) = VMT^{ref} + \sum_{i \in K} \left[\sigma_i . k_i . \Theta . \sum_{\substack{j \in T \\ j > i}} L_j \right] \approx VMT(\vec{k})$$

with $VMT^{ref} = VMT((1, 1, ..., 1))$ and $(L_j)_{j \in M}$ the mainline links lengths.

Performance and error calculation

Congestion Pattern matching (CP) (1)



Performance and error calculation

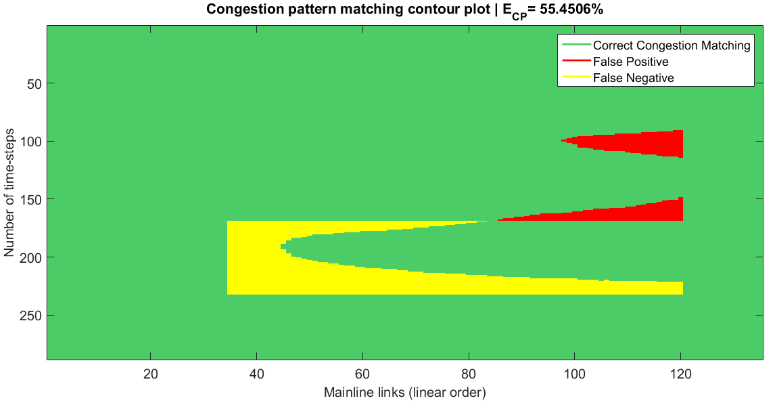
Congestion Pattern matching (CP) (2)

- A congestion density threshold is defined for each mainline link.
- Error calculation:

$$E_{CP}(\vec{k}) = \frac{\sum_{t \in \tau} \sum_{i \in M} \mathbb{1}_{Wrong\ \ congestion\ \ state\ \ pixels}}{\sum_{t \in \tau} \sum_{i \in M} \mathbb{1}_{Pixels\ \ supposed\ \ to\ \ be\ \ congested}}$$

Performance and error calculation

Congestion Pattern matching (CP) (3)



Problem Formulation

Fitness function

- Components:
  - $\phi_{VHT}(\vec{k}) = w_1.E_{VHT}(\vec{k}).\mathbb{1}_{(E_{VHT} > U^{global})}$
  - $\phi_{VMT}(\vec{k}) = w_2.E_{VMT}(\vec{k}).\mathbb{1}_{(E_{VMT} > U^{global})}$
  - $\phi_{CP}(\vec{k}) = w_3.E_{CP}(\vec{k}).\mathbb{1}_{(E_{CP} > U^{global})}$
- Objective function:

$$\Phi : \left| \begin{array}{ccc} \mathscr{B} & \longrightarrow & [0, 100] \\ \vec{k} & \longmapsto & \phi_{VHT}(\vec{k}) + \phi_{VMT}(\vec{k}) + \phi_{CP}(\vec{k}) \end{array} \right.$$

Problem Formulation

Optimization problem statement

$$
\begin{aligned}
&\textit{minimize} \quad \Phi(\vec{k}) \\
&s.t. \quad \forall i \in [\![1,\gamma]\!],\ \Delta_i^- \leq |\sum_{j \in g_i} \sigma_j.k_j.\Theta| \leq \Delta_i^+ \\
&\textit{and } \widetilde{VMT}^- \leq \sum_{i \in K} \left[\sigma_i.k_i.\Theta. \sum_{\substack{j \in T \\ j>i}} L_j \right] + VMT^{ref} \leq \widetilde{VMT}^+ \\
&\textit{and } \vec{k} \in \mathscr{B}
\end{aligned}
$$

Numerical method

Requirements

- Non-linear, non-convex black-box imputation problem in continuous domain
- Need for adaptive method
- Execution time does not matter

Numerical method

Covariance Matrix Adaptation - Evolution Strategy (CMA-ES)

- One of the most powerful evolutionary algorithms for single-objective real-valued optimization (very used)
- "Designed for difficult non-linear non-convex black-box optimisation problems in continuous domain"
- "Typically applied to unconstrained or bounded constraint optimization problems, and search space dimensions between three and a hundred"
- Does not presume existence of approximate gradients : feasible on our non-smooth problem
- *Adaptive* algorithm : almost no parameter tuning $\rightarrow$ suitable to be used on several different freeways and days.
- Time does not matter for this initial study : not the fastest ES but good solution quality

Numerical method

Linear constraints implementation: Single knob groups equations

$$\forall \ i \in G \ s.t. \ Card(g_i) = 1, \ i.e. \ g_i = \{j\} \ :$$

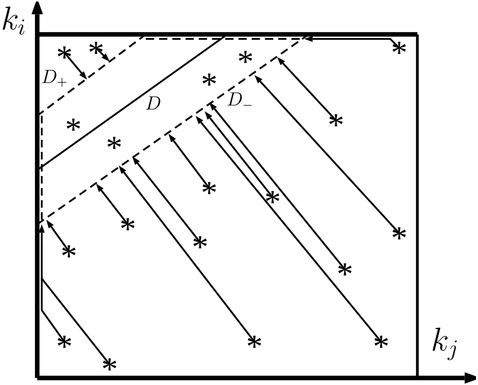
$$\frac{\max\left\{0; |\Delta_i^-|\right\}}{\Theta} \leq k_j \leq \frac{\min\left\{m_j; |\Delta_i^+|\right\}}{\Theta} \qquad (1)$$

- Linear constraints not implemented in CMA-ES source code $\Rightarrow$ project & penalize.
- Projection:

$$\begin{aligned} &\textit{minimize} \quad \left\| \vec{k}^{(p)} - \underline{\vec{k}}^{(p)} \right\|_2 \\ &s.t. \quad \forall i \in G, \quad \Delta_i^- < \left| \textstyle\sum_{j \in g_i} \sigma_j . k_j . \Theta \right| < \Delta_i^+ \\ &\textit{and} \ \widetilde{VMT}^- \leq \sum_{i \in K} \left[\sigma_i . k_i . \Theta . \sum_{\substack{j \in T \\ j > i}} L_j \right] + VMT^{ref} \leq \widetilde{VMT}^+ \\ &\textit{and} \quad \vec{k}^{(p)} \in \mathscr{B} \end{aligned}$$

Numerical method

Constraints implementation : multiple knob groups and $VMT^{a-priori}$ equations (2)



$D : \Theta.k_i - \Theta.k_j = \Delta_l$
$D_+ : \Theta.k_i - \Theta.k_j = \Delta_l^+$
$D_- : \Theta.k_i - \Theta.k_j = \Delta_l^-$

- Penalization:

$$E_{proj}(\underline{\vec{k}}^{(p)}, \vec{k}^{(p)}) = \frac{\left\| \vec{k}^{(p)} - \underline{\vec{k}}^{(p)} \right\|_2}{\left\| \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_\kappa \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \right\|_2}$$

- The fitness function becomes:

$$J : \left| \begin{array}{ccc} \mathscr{B} & \longrightarrow & [0, 100] \\ \underline{\vec{k}}^{(p)} & \longmapsto & \Phi(\vec{k}^{(p)}) + w_4.E_{proj}(\underline{\vec{k}}^{(p)}, \vec{k}^{(p)}) \end{array} \right.$$

Experiment settings

Data

- Freeway 210 East in Los Angeles, average of 5 Tuesdays in fall 2104.
- 5-min measurements distributed as follows:
  - 33/135 monitored mainline links
  - 26/28 monitored on-ramps
  - 15/25 monitored off-ramps

  $\Rightarrow$ 12 knobs.
- Duration: 24 hours i.e. 289 time-steps (5min).

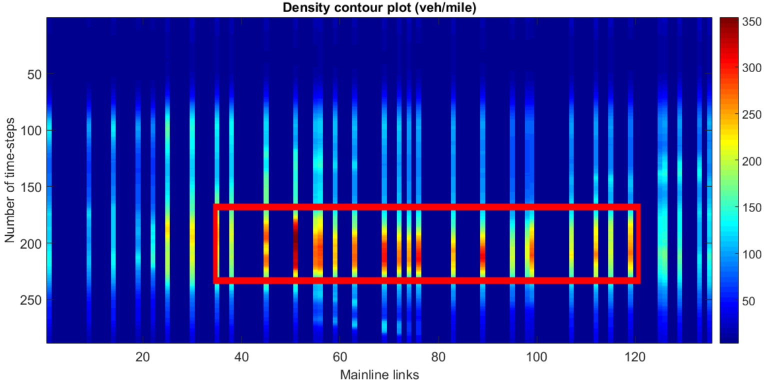
Experiment settings

Model and simulator

- Cell-transmission model
- Simulator: BeATS

Experiment settings

Implementation of congestion pattern



$$d_i^* = \frac{Link\ \ capacity}{Link\ \ free\ \ flow\ \ speed} + \delta$$

Experiment results

General observations

- Always converges
- Limited result quality
- No uniqueness

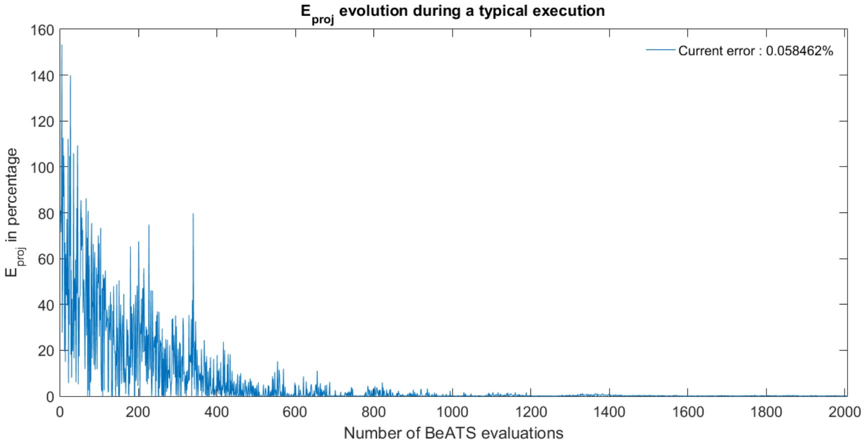
Experiment results

Overview

U^{global}	Population Size : λ	Initial standard deviation : σ	$\frac{U^{add}}{\widetilde{F}}$	U^{mul}	Number of BeATS Evaluations	Number of CMA-ES generations	J minimum : J*
5%	11	2	2.5%	25%	2004	182	**43.5%**
				50%			**28.2%**
				75%			**25.4%**
				100%			**23.7%**
			5.0%	25%			**35.1%**
				50%			**25.3%**
				75%			**25.3%**
				100%			**22.8%**
			10.0%	25%			**22.2%**
				50%			**21.6%**
				75%			**21.9%**
				100%			**21.8%**
		5	2.5%	25%			**44.7%**
				50%			**27.3%**
				75%			**27.0%**
				100%			**23.5%**
			5.0%	25%			**36.1%**
				50%			**24.5%**
				75%			**23.7%**
				100%			**25.5%**
			10.0%	25%			**22.8%**
				50%			**22.2%**
				75%			**23.8%**
				100%			**21.4%**
	12	2	2.5%	50%	3002	250	**27.1%**
							27.8%
							27.7%
	24				3002	125	**27.0%**
							27.1%
							27.3%
	36				3026	84	**27.1%**
							27.1%
							27.2%

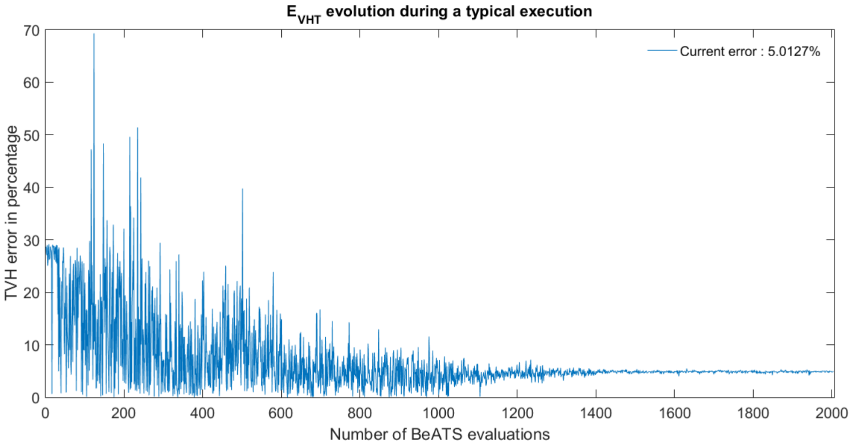
Experiment results

Typical execution



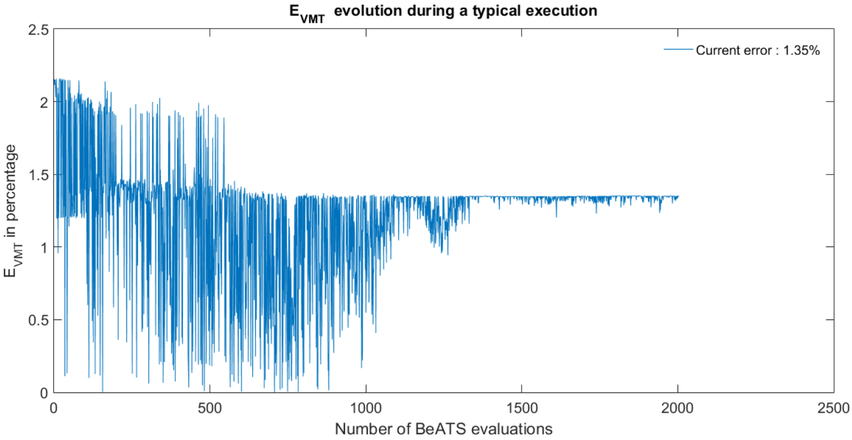
Experiment results

Typical execution



Experiment results

Typical execution

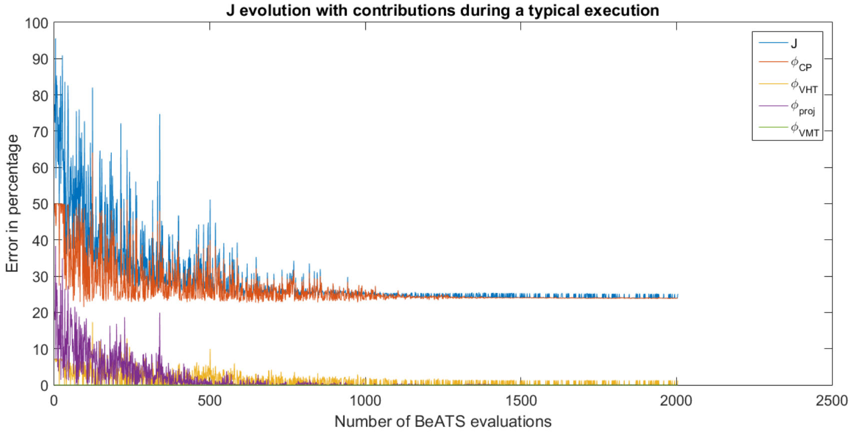


Experiment results

Typical execution

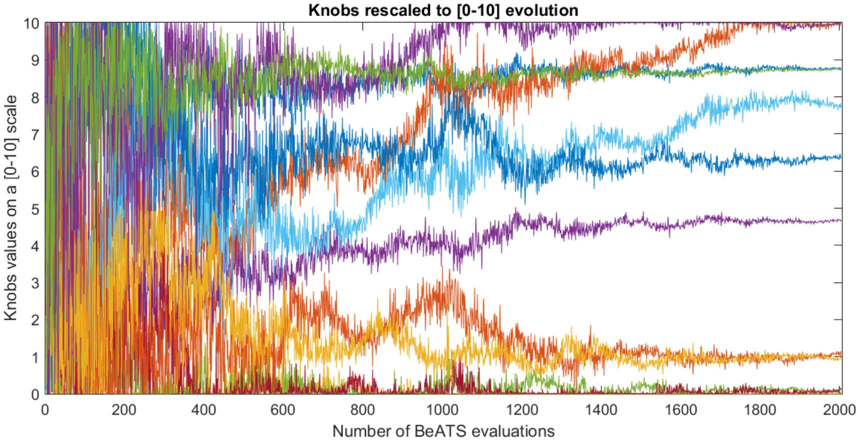
Experiment results

Typical execution



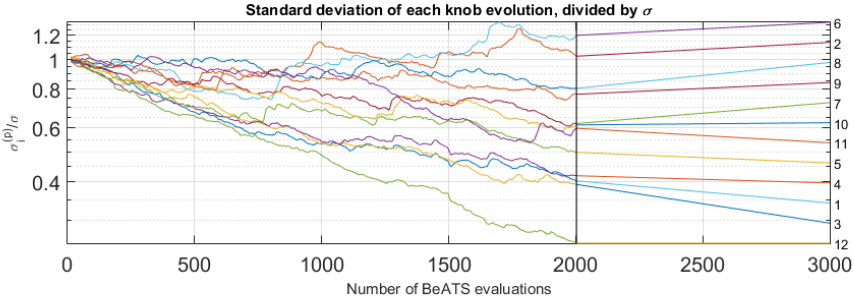
Experiment results

Typical execution



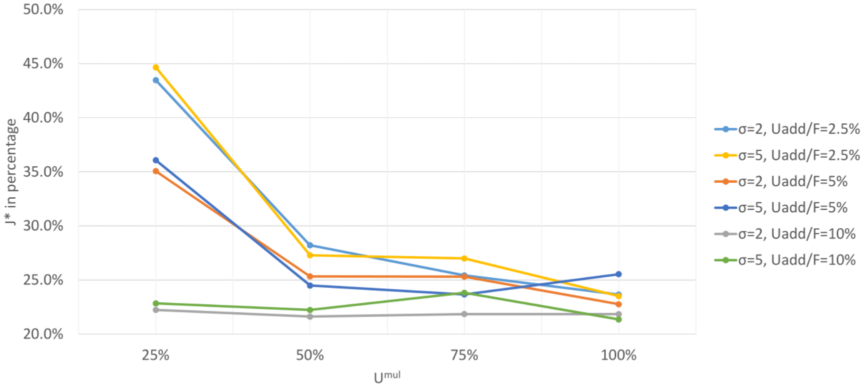
Experiment results

Typical execution



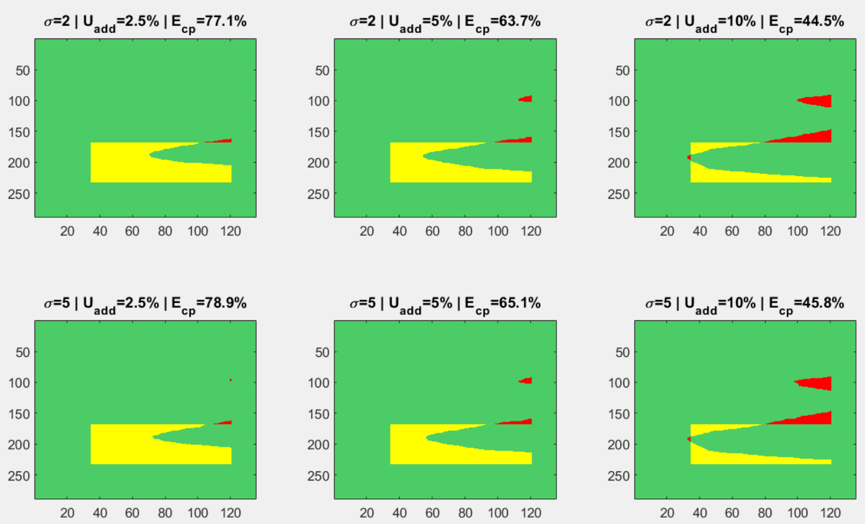
Experiment results

Effect of U^{mul}



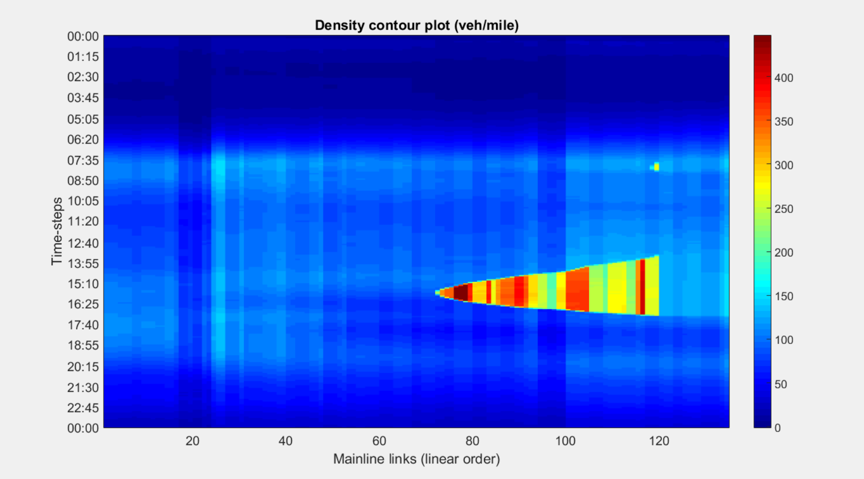
Experiment results

$U^{mul} = 25\%$



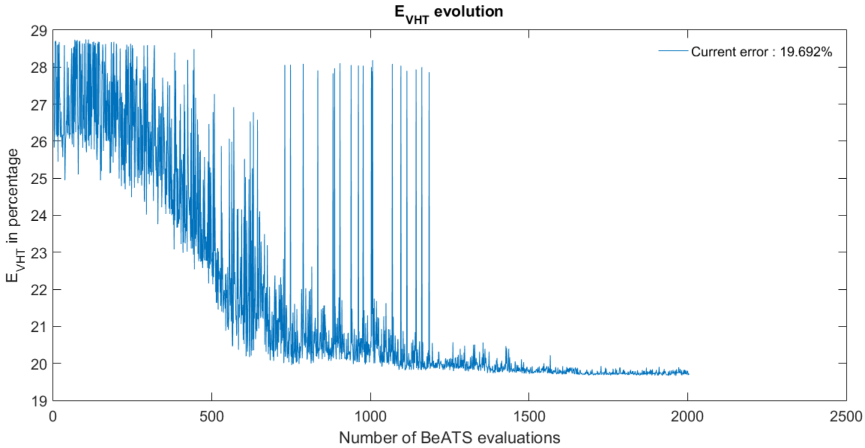
Experiment results

$U^{mul} = 25\%$



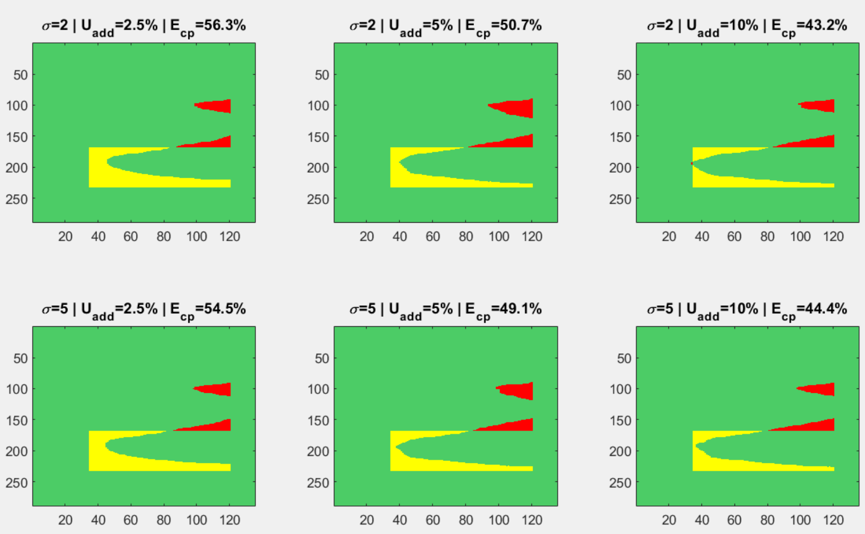
Experiment results

$U^{mul} = 25\%$



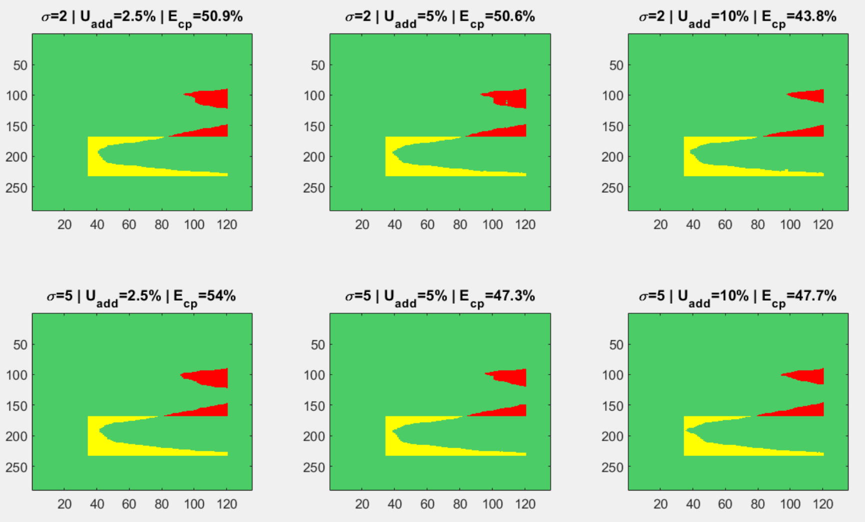
Experiment results

$U^{mul} = 50\%$



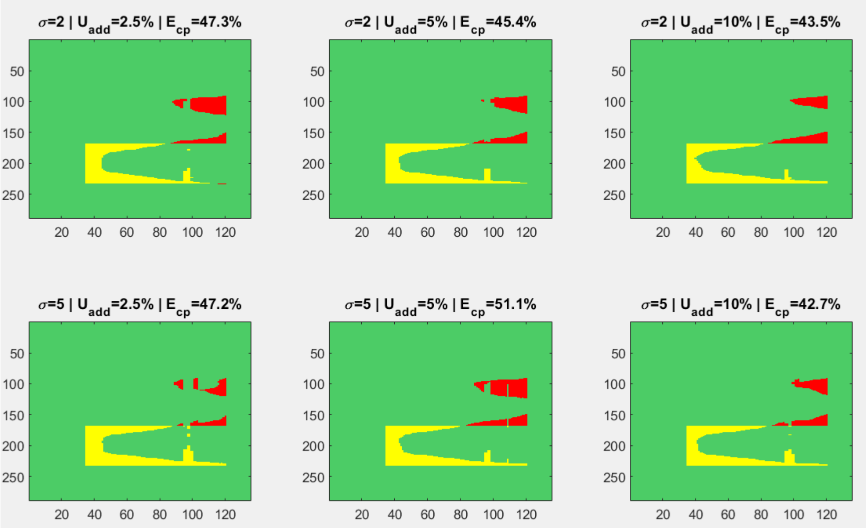
$U^{mul} = 75\%$

Effect of U^{mul}



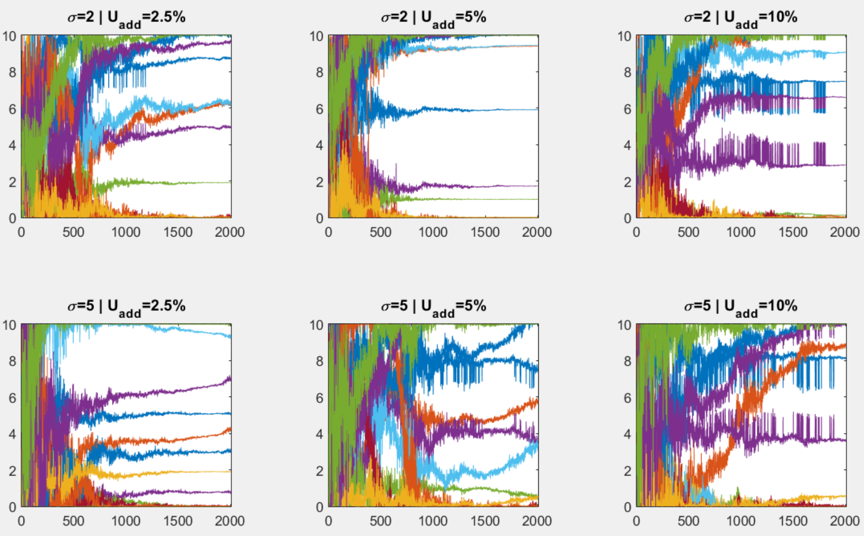
$U^{mul} = 100\%$

Effect of U^{mul}



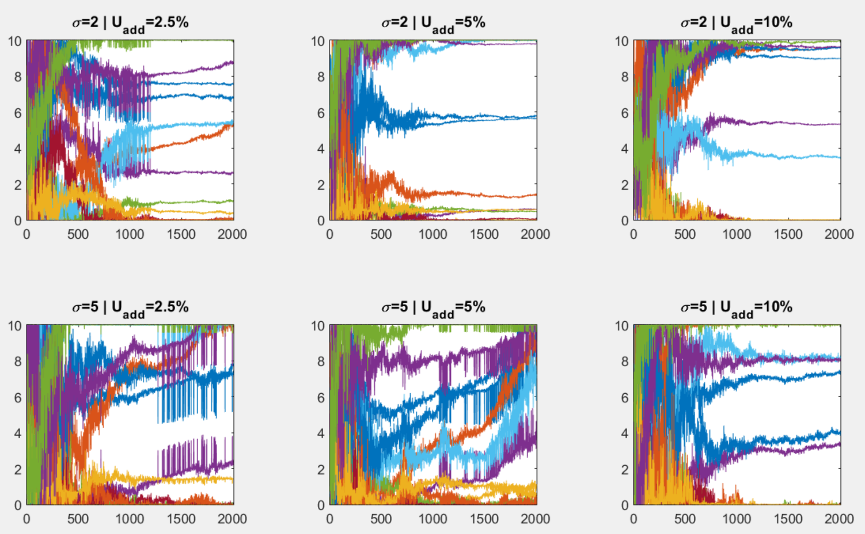
$U^{mul} = 25\%$

Effect of U^{mul}



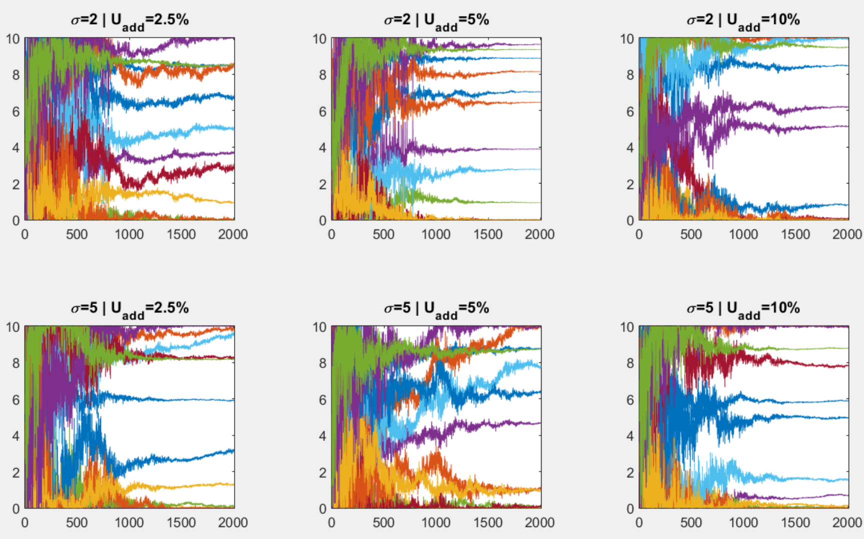
$U^{mul} = 25\%$

Effect of U^{mul}



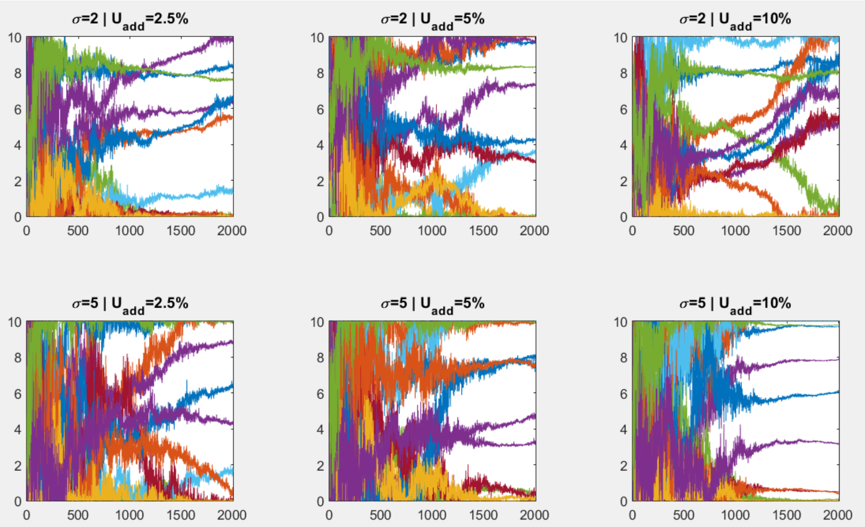
$U^{mul} = 25\%$

Effect of U^{mul}



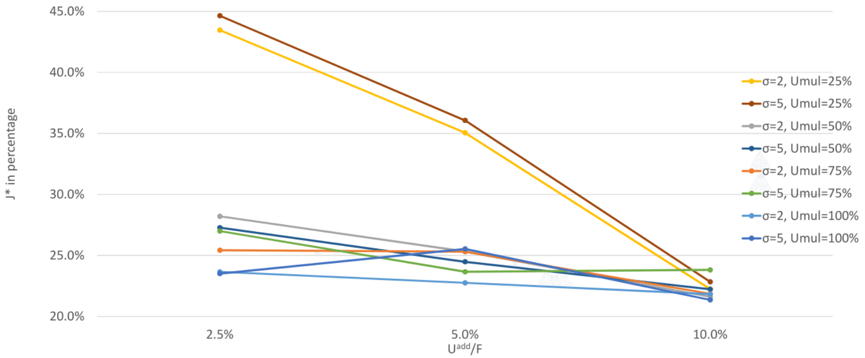
Experiment results

Effect of U^{mul}



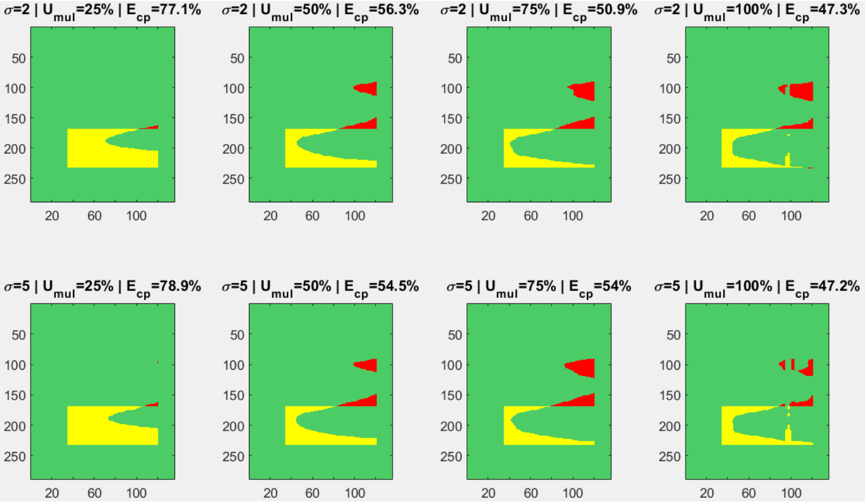
Experiment results

Effect of U^{add}



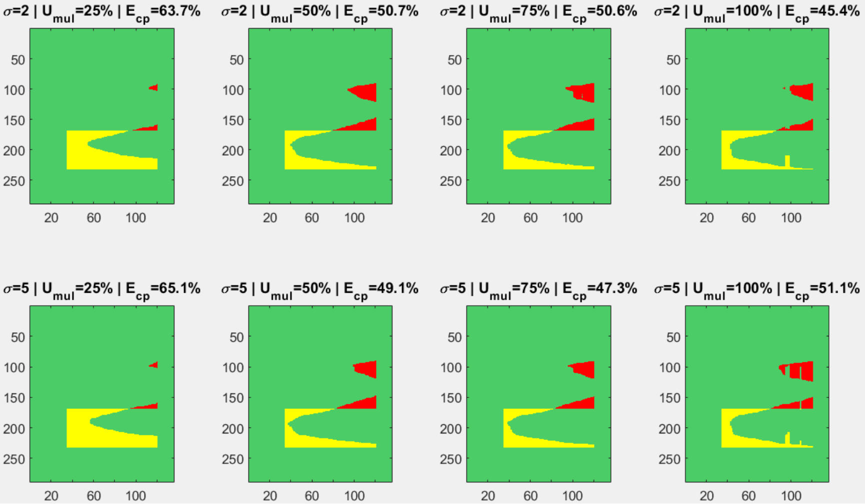
Experiment results

Effect of U^{add}



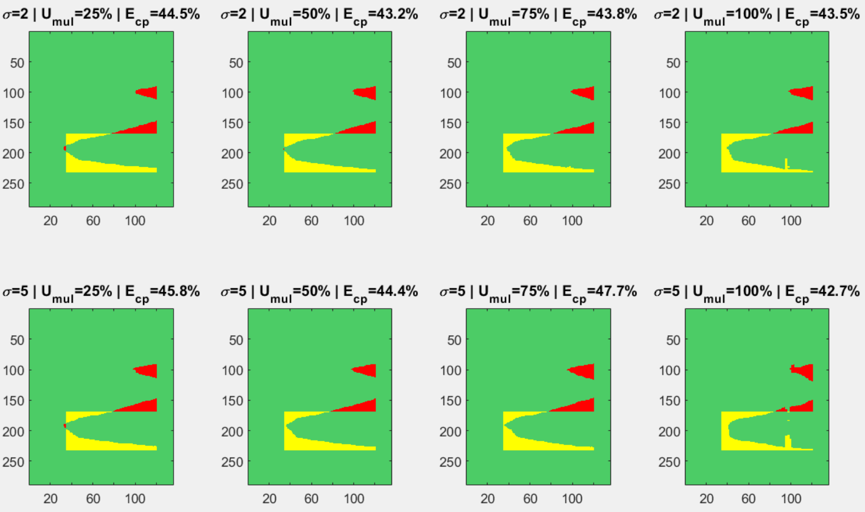
Experiment results

Effect of U^{add}



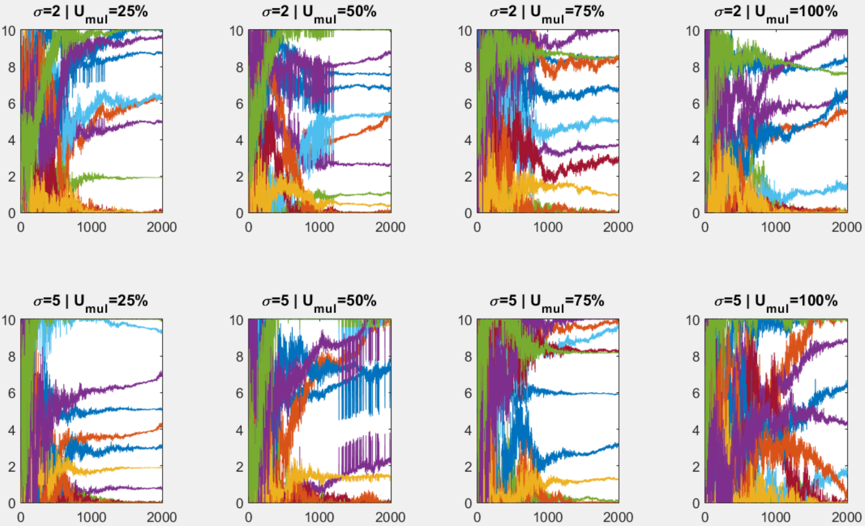
Experiment results

Effect of U^{add}



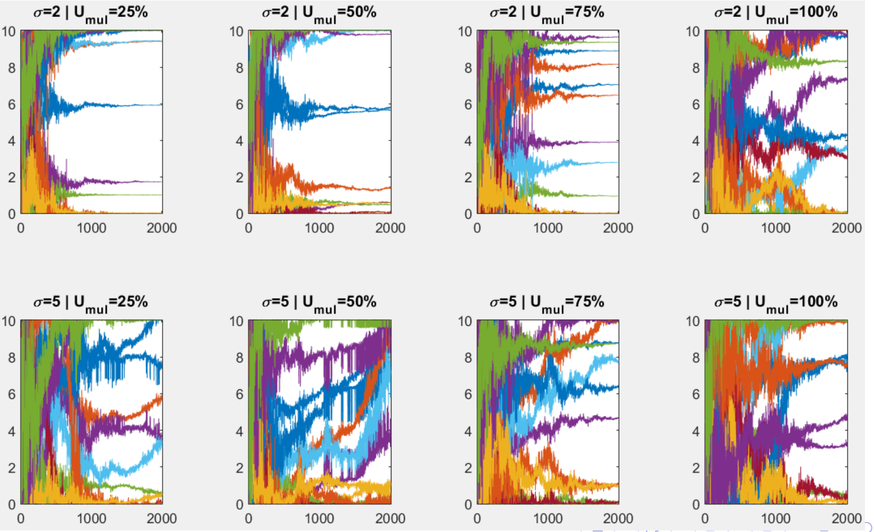
Experiment results

Effect of U^{add}



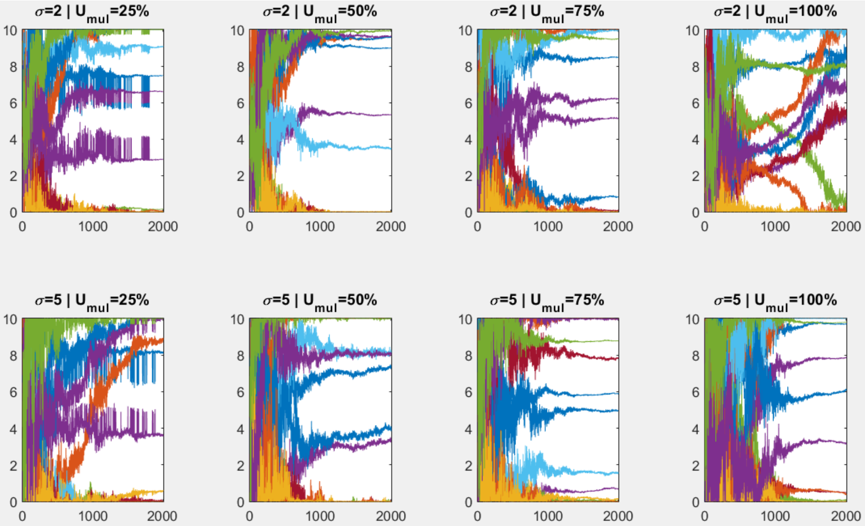
Experiment results

Effect of U^{add}



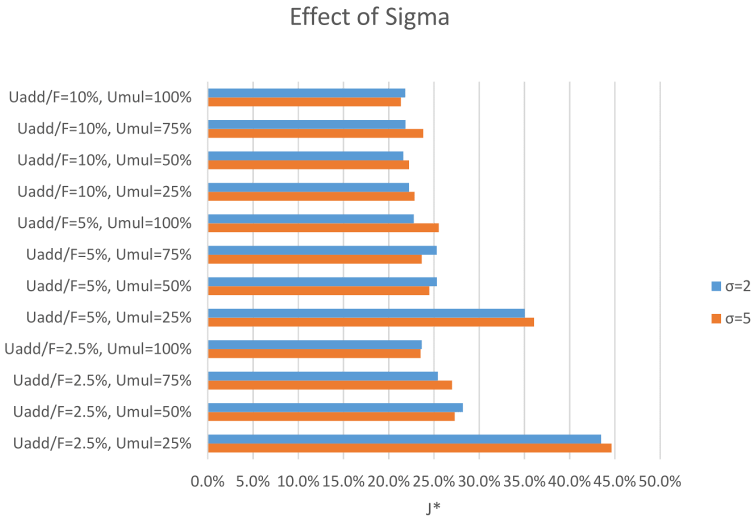
Experiment results

Effect of U^{add}



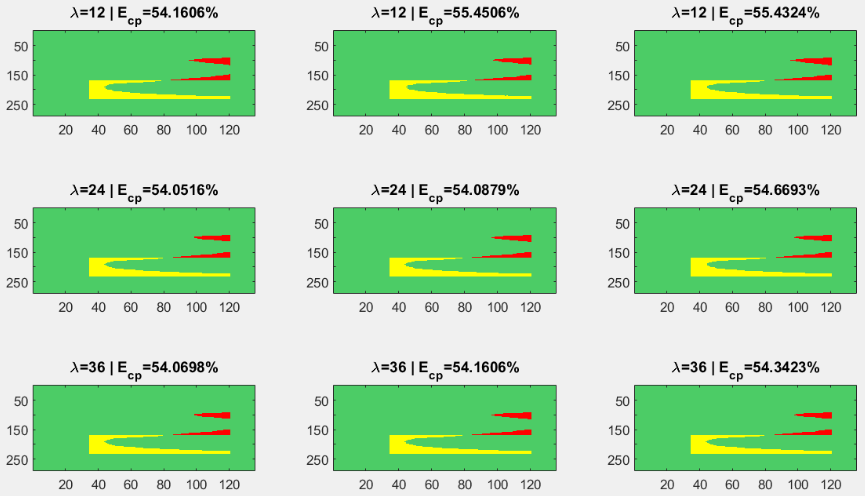
Experiment results

Effects of σ



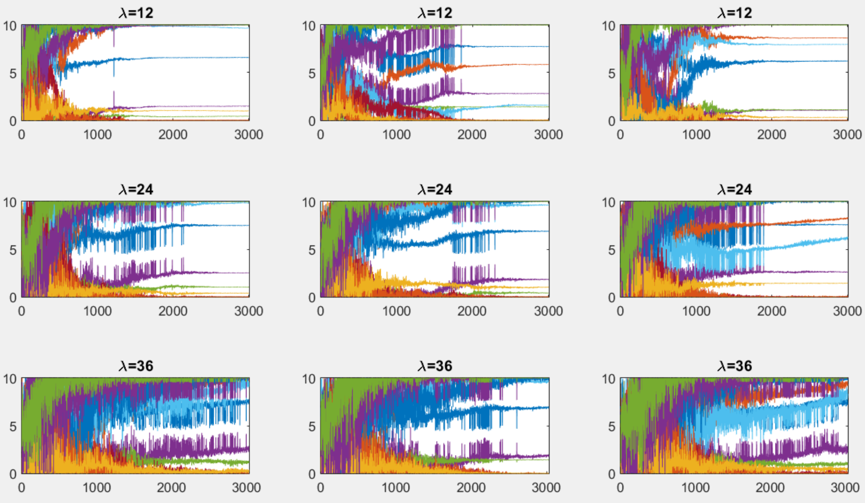
Experiment results

Effects of λ



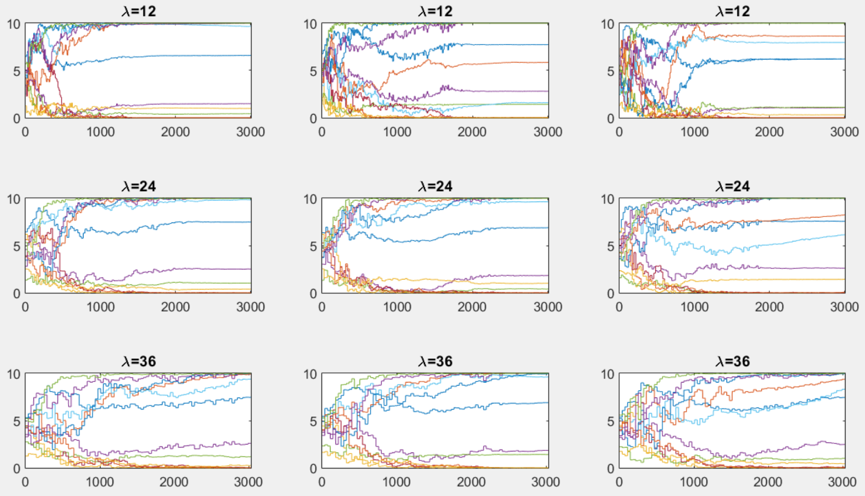
Experiment results

Effects of λ



Experiment results

Effects of λ



Further work

Next steps and ideas

- Modulate the templates shapes
- More accurate knob boundaries
- New constraints/objectives
- MO-CMAES
- Parallelization
- Wider loop with fundamental diagrams