Cutnorm Documentation

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CONTENTS

Welcome to the Cutnorm package documentation. Please read the introduction and checkout the documentation.

CHAPTER

ONE

INTRODUCTION

1.1 Cutnorm

1.1.1 Approximation via Gaussian Rounding and Optimization with Orthogonality Constraints

This package computes the approximations to the cutnorm using some of the techniques detailed by Alon and Noar *[ALON2004]* and a fast optimization algorithm by Wen and Yin *[WEN2013]*.

Read the documentation.

1.1.2 Installation

Use pip to install the package. Install from terminal as follows:

```
$ pip install cutnorm
```

1.1.3 Example Usage

Below is an example of using the cutnorm package and tools. Given two graphs A and B, we wish to compute a norm for the difference matrix (A - B) between the two graphs. An obvious example to represent the advantage of using a cutnorm over l1 norm is to consider A and B as Erdos-Renyi random graphs. Under a fixed vertex set, an Erdos-Renyi random graph is one where a fixed probability determines the presence of an edge.

Given two Erdos-Renyi random graphs with fix n and p=0.5, the edit distance (l1 norm) of the difference (after normalization) is 1/2 with large probability. However, these two graphs have the same global structure. The edit distance fails as a notion of 'distance' between the two graphs in the perspective of global structural similarity as discussed by Lovasz *[LOVASZ2009]*. The cutnorm is a measure of distance that reflects global structural similarity. In fact, the cutnorm of the difference for this example approaches 0 as n grows.

```
import numpy as np
from cutnorm import cutnorm, tools

# Generate Erdos Renyi Random Graph
n = 100
p = 0.5
erdos_renyi_a = tools.sbm.erdos_renyi(n, p)
erdos_renyi_b = tools.sbm.erdos_renyi(n, p)

# Compute l1 norm
normalized_diff = (erdos_renyi_a - erdos_renyi_b) / n**2
```

```
l1 = np.linalg.norm(normalized_diff.flatten(), ord=1)

# Compute cutnorm
cutn_round, cutn_sdp, info = cutnorm(erdos_renyi_a, erdos_renyi_b)

print("l1 norm: ", l1)                   # prints l1 norm value near ~0.5
print("cutnorm rounded: ", cutn_round) # prints cutnorm rounded solution near ~0
print("cutnorm sdp: ", cutn_sdp)        # prints cutnorm sdp solution near ~0
```

CHAPTER TWO

CUTNORM

2.1 cutnorm package

2.1.1 Subpackages

cutnorm.tools package

Submodules

cutnorm.tools.sbm module

`cutnorm.tools.sbm.`**`erdos_renyi`**`(`*n*, *p*`)`

Generates Erdos Renyi random graph size n with probability p

Parameters

- **`n`** – int, size of the output matrix
- **`p`** – float, edge probability

Returns Erdos Renyi random graph matrix 2d array, shape (n,n)

`cutnorm.tools.sbm.`**`make_symmetric_triu`**`(`*mat*`)`

Makes the matrix symmetric upper triangular

Parameters **`mat`** – 2d array, shape (n,n)

Returns upper triangular symmetric matrix of the input 2d array, shape (n,n)

`cutnorm.tools.sbm.`**`sbm`**`(`*community_sizes*, *prob_mat*`)`

Generates a stochastic block matrix

Community_sizes indicate the size of each community and the probability matrix indicate the probability that a 1 will be generated for each element within the community.

Parameters

- **`community_sizes`** – 1d array, shape (n) sizes of community
- **`prob_mat`** – 2d array, shape (n,n) probability of edges for each community

Returns stochastic block matrix, 2d array, shape depending on community sizes

`cutnorm.tools.sbm.`**`sbm_autoregressive`**`(`*community_sizes*, *prob_list*`)`

Generates an autoregressive SBM

An autoregressive SBM has edge probability according to the prob_list on the diagonal but (prob_list[i] * prob_list[j])**(abs(i - j)) for the off-diagonal blocks entries.

This idea is similar to the autoregressive models

Parameters

- **`community_sizes`** – 1d array, shape (n) sizes of community
- **`prob_list`** – 1d array, shape (n), where n is the number of diagonal blocks

Returns An autoregressive SBM, 2d array, shape depending on community sizes

`cutnorm.tools.sbm.`**`sbm_autoregressive_prob`**(*community_sizes*, *prob_list*)

Generates the underlying probability matrix thatgives rise to the autoregressive SBM

Parameters

- **`community_sizes`** – 1d array, shape (n) sizes of community
- **`prob_list`** – 1d array, shape (n), where n is the number of diagonal blocks

Returns A probability matrix for an autoregressive SBM, 2d array, shape depending on community sizes

`cutnorm.tools.sbm.`**`sbm_prob`**(*community_sizes*, *prob_mat*)

Generates a matrix indicating the underlying probability that gives rise to a stochastic block matrix

Parameters

- **`community_sizes`** – 1d array, shape (n) sizes of community
- **`prob_mat`** – 2d array, shape (n,n) probability of edges for each community

Returns probabilities of a stochastic block matrix, 2d array, shape depending on community sizes

Module contents

2.1.2 Submodules

2.1.3 cutnorm.OptManiMulitBallGBB module

`cutnorm.OptManiMulitBallGBB.`**`cutnorm_quad`**(*V: numpy.ndarray*, *C: numpy.ndarray) -> (<class 'numpy.float64'>, <class 'numpy.ndarray'>*)

Cutnorm function to compute objective function value and gradient

Parameters

- **`V`** – ndarray, Low rank model X = V' * V;
- **`C`** – ndarray, Objective matrix to compute maxcut

Returns

(f, g)

f: float, objective funciton value

g: ndarray, gradient

`cutnorm.OptManiMulitBallGBB.`**`maxcut_quad`**(*V: numpy.ndarray*, *C: numpy.ndarray) -> (<class 'numpy.float64'>, <class 'numpy.ndarray'>*)

Maxcut function to compute objective function value and gradient

maxcut SDP: X is n by n matrix max Tr(C*X), s.t., X_ii = 1, X psd

Parameters

- **V** – ndarray, Low rank model X = V' * V;
- **C** – ndarray, Objective matrix to compute maxcut

Returns

(f, g)

f: float, objective funciton value

g: ndarray, gradient

`cutnorm.OptManiMulitBallGBB.`**`opt_mani_mulit_ball_gbb`**(*x: numpy.ndarray*, *fun*, **args*, *xtol=1e-06*, *ftol=1e-12*, *gtol=1e-06*, *rho=0.0001*, *eta=0.1*, *gamma=0.85*, *tau=0.001*, *nt=5*, *mxitr=1000*, *record=0*)

Line search algorithm for optimization on manifold Reinterpreted directly from Zaiwen Wen and Wotao Yin's Matlab implementation of their paper on 'A feasible method for optimizationwith orthogonality constraints'

Parameters

- **x** – Numpy array where each column lies on the unit sphere ||x_i||_2 = 1
- **fun** – Function that returns the objective function value and its gradient. Params: [x, args] Returns: [f, g]
- **args** – args to be used in fun
- **kwargs** – Options record = 0, no print out mxitr max number of iterations xtol stop control for ||X_k - X_{k-1}|| gtol stop control for the projected gradient ftol stop control for abs(F_k - F_{k-1})/(1+|F_{k-1}|) usually, max{xtol, gtol} > ftol

Returns

(x, g, out)

x: solution

g: gradient of x

Out: output information

2.1.4 cutnorm.compute module

`cutnorm.compute.`**`compute_cutnorm`**(*A: numpy.ndarray*, *B: numpy.ndarray*, *w1=None*, *w2=None*, *max_round_iter=100*, *logn_lowrank=False*, *extra_info=False) -> (<class 'numpy.float64'>, <class 'numpy.float64'>, <class 'dict'>*)

Computes the cutnorm of the differences between the two matrices

Parameters

- **A** – ndarray, (n, n) matrix
- **B** – ndarray, (m, m) matrix
- **w1** – ndarray, (n, 1) array of weights for A
- **w2** – ndarray, (m, 1) array of weights for B
- **max_round_iter** – int, number of iterations for gaussian rounding
- **logn_lowrank** – boolean to toggle log2(n) low rank approximation

- **`extra_info`** – boolean, generate extra computational information

Returns

(cutnorm_round, cutnorm_sdp, info)

cutnorm_round: objective function value from gaussian rounding

cutnorm_sdp: objective function value from sdp solution

S: Cutnorm set axis = 0

T: Cutnorm set axis = 1

info: dictionary containing computational information

Computational information from OptManiMulitBallGBB: sdp_augm_n: dimension of augmented matrix sdp_relax_rank_p: rank sdp_tsolve: computation time sdp_itr, sdp_nfe, sdp_feasi, sdp_nrmG: information from OptManiMulitBallGBB

Computational information from gaussian rounding: round_tsolve: computation time for rounding round_approx_list: list of rounded objf values round_uis_list: list of uis round_vjs_list: list of vjs round_uis_opt: optimum uis round_vjs_opt: optimum vjs

Computational information from processing the difference: weight_of_C: weight vector of C, the difference matrix

Raises `ValueError` – if A and B are of wrong dimension, or if weight vectors does not match the corresponding A and B matrices

`cutnorm.compute.`**`cutnorm_sets`**(*uis: numpy.ndarray*, *vjs: numpy.ndarray) -> (<class 'numpy.ndarray'>*, *<class 'numpy.ndarray'>*)

Generates the cutnorm sets from the rounded SDP solutions

Parameters

- **`uis`** – ndarray, (n+1,) shaped array of rounded +- 1 solution
- **`vis`** – ndarray, (n+1,) shaped array of rounded +- 1 solution

Returns

(S, T) Reconstructed S and T sets that are {1, 0}^n

S: Cutnorm set axis = 0

T: Cutnorm set axis = 1

`cutnorm.compute.`**`gaussian_round`**(*U: numpy.ndarray*, *V: numpy.ndarray*, *C: numpy.ndarray*, *max_round_iter: int*, *logn_lowrank=False*, *extra_info=False) -> (<class 'numpy.float64'>*, *<class 'numpy.ndarray'>*, *<class 'numpy.ndarray'>*, *<class 'dict'>*)

Gaussian Rounding for Cutnorm

The algorithm picks a random standard multivariate gaussian vector w in R^p and computes the rounded solution based on sgn(w dot ui).

Adopted from David Koslicki's cutnorm rounding code https://github.com/dkoslicki/CutNorm and Peter Diao's modifications

Parameters

- **`U`** – ndarray, (p, n) shaped matrices of relaxed solutions
- **`V`** – ndarray, (p, n) shaped matrices of relaxed solutions
- **`C`** – ndarray, original (n, n) shaped matrix to compute cutnorm

- **`max_round_iter`** – maximum number of rounding operations
- **`logn_lowrank`** – boolean to toggle log2(n) low rank approximation
- **`extra_info`** – boolean, generate extra computational information

Returns

(approx_opt, uis_opt, vjs_opt, round_info)

approx_opt: approximated objective function value

uis_opt: rounded u vector

vis_opt: rounded v vector

round_info: information for rounding operation

2.1.5 Module contents

CHAPTER

THREE

INDICES AND TABLES

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BIBLIOGRAPHY

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