ROULETTE SIMULATION: STATISTICAL ANALYSIS

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ABSTRACT

In the present work, the statistical properties of a 37-slot casino roulette will be analyzed, to verify whether the sample statistics approximate the population parameters and to analyze how this approximation depends on the sample size. After running the simulation, it is observed that, after an initial period of dispersion and randomness, the variables enter a steady-state and their values tend to the parameters proposed by the analytical methods.

1. Introduction

One of the main uses of simulation is to approximate population values through random samples, a study favored by recent computational advances, which greatly increased the size of iterations, runs, and values that can be generated. In the present work, the outputs of a program that simulates the operation of a 37-number French roulette dish (including zero) will be studied, more precisely, the relative frequency, and the sample mean, variance, and standard deviation.

2. Definitions and assumptions

- The roulette to be modeled consists of 37 numbers, from 0 to 36, and there is no distinction between colors.
- The sample space S is defined as S = 0,1,2,3, ..., 36.
- The event of obtaining a number is defined as any element of S.
- The events are equally likely.
- The variables under study are discrete random variables that follow a discrete uniform distribution.
- Independence between variables is assumed, both in the consecutive generation of numbers and between different runs of the program.
- The simulation does not model the human error of who throws the numbers, nor in the roulette materials, and assumes the total randomness of the variables.

In subsequent subsections, two theories will be described that constitute the analytical model by which the population parameters will be obtained.

2.1. Classical probability

Classical probability states that the probability of an event is equal to the quotient between the number of favorable outcomes s and the total number of possible outcomes n, as long as all outcomes have an equal chance of occurring. The procedure for obtaining numbers in roulette has $n = 37$ different events, and each event can occur in only one way s. Then, the probability of occurrence is defined as $\frac{s}{n}$, that is, $\frac{1}{37}$.

2.2. Frequentist probability

The probability of an event x is the estimate of the relative frequency of x when the number of trials approaches infinity. As more times the experiment is repeated, eventually, the chances of each event occurring will be regular, even though the behavior of the events is random.

If *n* is the number of times an event occurs, and *N* the number of events, then

$$Fr(x) = lim_{n \to \infty} \frac{n}{N} = P(x)$$

3. Law of Large Numbers

The law of large numbers encompasses several theorems that describe the behavior of the average of a sequence of random variables as the number of trials increases.

The **Weak Law of Large Numbers** states that if $X_1, X_2, ...$ is is an infinite sequence of independent random variables that have the same expected value μ and variance σ^2, then the average

$$\overline{X}_n = X_1 + \cdots + X_n n$$

converges in probability to μ.

The **Strong Law of Large Numbers** states that if $X_1, X_2, ...$ is an infinite sequence of independent and identically distributed random variables that satisfy $E(|X_i|) < \infty$ and have the expected value μ, then

$$P\left(\lim_{n \to \infty} \overline{X}_n = \mu\right) = 1$$

that is, the average of the random variables converges to μ almost certainly.

4. Population parameters

The roulette model has, among others, the following population parameters:

Population relative frequency $\rightarrow Fr(x) = P(x) = \frac{1}{37} \approx 0{,}027$

Population sample mean $\rightarrow \bar{x} = \sum x \cdot p(x) = \frac{1}{37} \cdot (0 + 1 + 2 + 3 + ... + 36) = 18$

Population sample variance $\rightarrow S^2 = \frac{\sum (x_i - \mu)^2}{n} = 114$

Population standard deviation $\rightarrow S = \sqrt{\sigma^2} = 10{,}677$

5. Methodology

Two experiments of 30 runs were carried out, the first one with 100 iterations, and the second one with 15000 iterations. In each iteration, a number was obtained from the roulette wheel, and the set of the evolution of the statistics that occurred with each number generated was taken, as well as the relative frequency of a randomly generated number.

6. Results

Figure 1 shows the evolution of the statistics for 30 runs of 100 iterations, while Figure 2 illustrates the evolution of the statistics for 30 runs of 15000 iterations.

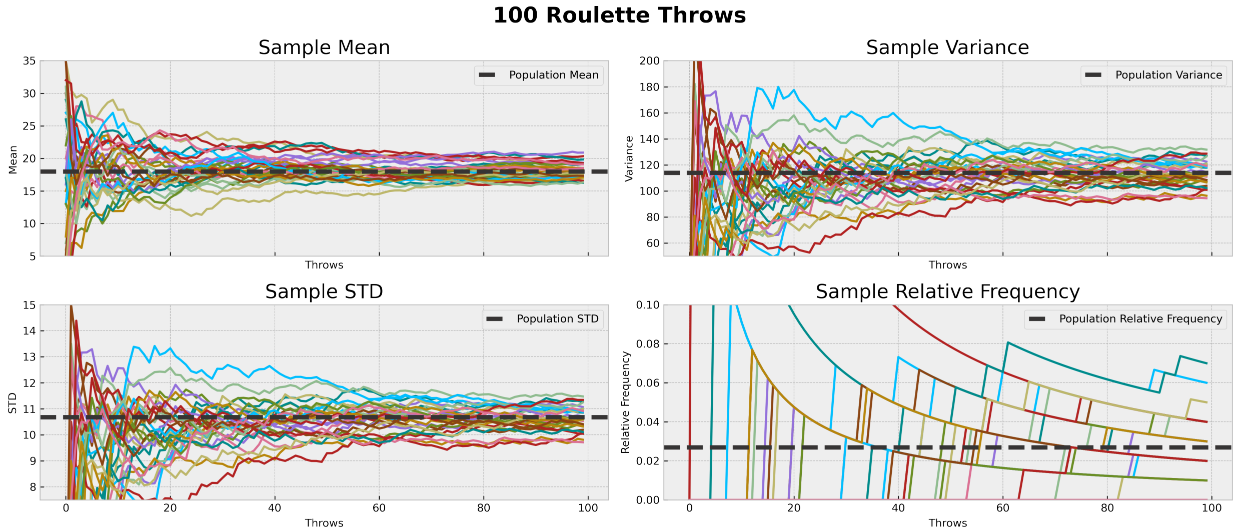

Figura 1: 100 throws

It was observed that, in all runs, the variables present a similar pattern. In the first iterations, there is some instability, and the variables present a random and dispersed behavior and begin to stabilize and enter a steady-state as the number of iterations increases, that is, as more numbers are thrown. At a very large value, 15000 in this case, the sample statistics begin to converge towards the population parameters.

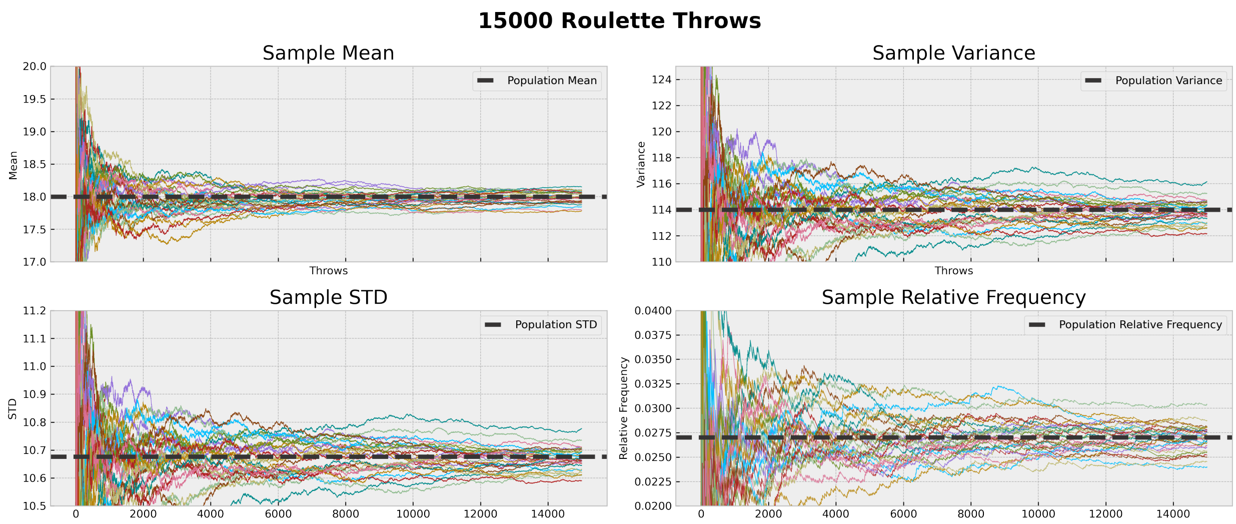

Figura 2: 15000 throws

7. Conclusions

The law of large numbers was empirically demonstrated since initially the variables presented a random, chaotic and unpredictable behavior, but, as the number of iterations increased, the sample statistics entered a steady-state and accurately approximated the parameters of the population, even at a value far from the *infinity* proposed by the law.