# ROULETTE SIMULATION: STRATEGIES ANALYSIS

**Gastón Amengual**
Simulation
National Technological University
gastonamengual@icloud.com

**ABSTRACT**

Many strategies were proposed in the game of roulette that guarantee victory and a considerable amount of money. In this work, the application of some of these strategies, such as Martingale and d'Alembert, will be analyzed, and the flow of capital throughout the game and the percentage of wins at the end of the game will be evaluated. Moreover, the probabilistic definition of the martingale will also be verified.

## 1 Introduction

Both in popular culture and in more sophisticated works, roulette strategies which claim to have the winning formula for a definitive win in the game are described. In this work, the Martingale, d'Alembert, and All-in strategies will be analyzed on a 37-number French roulette wheel, and the flow of capital the relative frequency of winning bets in different rounds of the game, the percentage of wins at the end, and the final capital obtained will be evaluated. The application of Martingale will also be analyzed without considering capital limits, that is, assuming infinite capital. Finally, the probabilistic definition of Martingale will be verified.

## 2 Definitions and assumptions

The roulette wheel to be modeled consists of 37 numbers, including zero.

The roulette wheel is divided by color, red and black, as follows:

red = [1, 3, 5, 7, 9, 12, 14, 16, 18, 19, 21, 23, 25, 27, 30, 32, 34, 36]

black = [2, 4, 6, 8, 10, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 31, 33, 35]

with a total of 18 red numbers, and 18 black numbers.

The simulation models neither the error nor the human bias of whom throws the numbers, nor inaccuracies in the roulette materials, and assumes the total randomness of the variables.

The independent random variable $Y$ is defined, which is distributed in a discrete uniform way between $0$ and $36$, including the extremes, and represents a number extracted from the wheel.

The random variable *C* is defined as

$C = red$ if $Y \in [red]$

$C = black$ if $Y \in [black]$

*C* takes a value every time the roulette is launched.

*A* is defined as a bet, which has an amount, the bet slot, and the result (win, loss).

A win occurs when the bet slot *A* corresponds to the value of *C*, otherwise a loss.

The variable under study $X$ is defined as continuous random, and represents capital. $X$ takes a different value in each iteration, depending on the outcome of the bet. The way $X$ takes the value, as discussed below, depends on the strategy used.

If in a play the capital is less than zero or the amount of the bet is greater than the capital, the game is stopped and bankruptcy is considered .

## 3 Strategies

**Martingale Strategy**

It is a roulette strategy that emerged in France in the 18th century. In this strategy, a fixed amount is set on the initial bet. In case of losing a bet, the bet amount is doubled, in case of winning, the bet amount is set to the initial bet value.

On the other hand, in probability theory, a martingale is a sequence of random variables $X_1, ..., X_n$ for which, for at a particular time, the conditional expectation of the next value in the sequence is equal to the present value, regardless of all prior values.

$$E(|X_n|) < \infty$$

$$E(X_{n+1}|X_1, ..., X_n) = X_n$$

**d'Alembert Strategy**

It is a method developed by the mathematician Jean le Rond d'Alembert in the 18th century, and is based on the Law of Balance. It consists of adding a bet unit after a failure. In the same way, the same amount is subtracted in case of success. It is one of the most used European roulette strategies or systems in the casino and is also known under the name of the Pyramid system.

**All-in Strategy**

It is an extremely risky strategy that consists of betting all the initial capital on each play. Of the strategies mentioned in this work, it is historically the one that has the highest profit, and also the one with the least probability of achieving it. The casino's stake limits must be checked in order to place the last bet in the series, should it go all the way.

## 4 Methodology

A total of 4 experiments were carried out, the first for the Martingale strategies, the second for Infinite Martingale, the third for d'Alembert, and the fourth for All-in. For each one, 100 runs of 100 iterations were executed. In each iteration, a number was obtained from the roulette wheel, and for each run the result of each bet made and the capital flow were taken. In each experiment, the initial capital was fixed at 1000 and the initial bet amount at 100.

## 5 Results

| | **Wins** | **Final Capital** | **Maximum Capital** |
|---|---|---|---|
| **Martingale** | 5% | \$6180 | \$6800 |
| **Infinite Martingale** | 97% | \$5695 | \$7100 |
| **d'Alembert** | 53% | \$1708 | \$3331 |
| **All-in** | 0% | \$51200 | \$70400 |

Table 1: Win percentage and final and maximum capital for each strategy

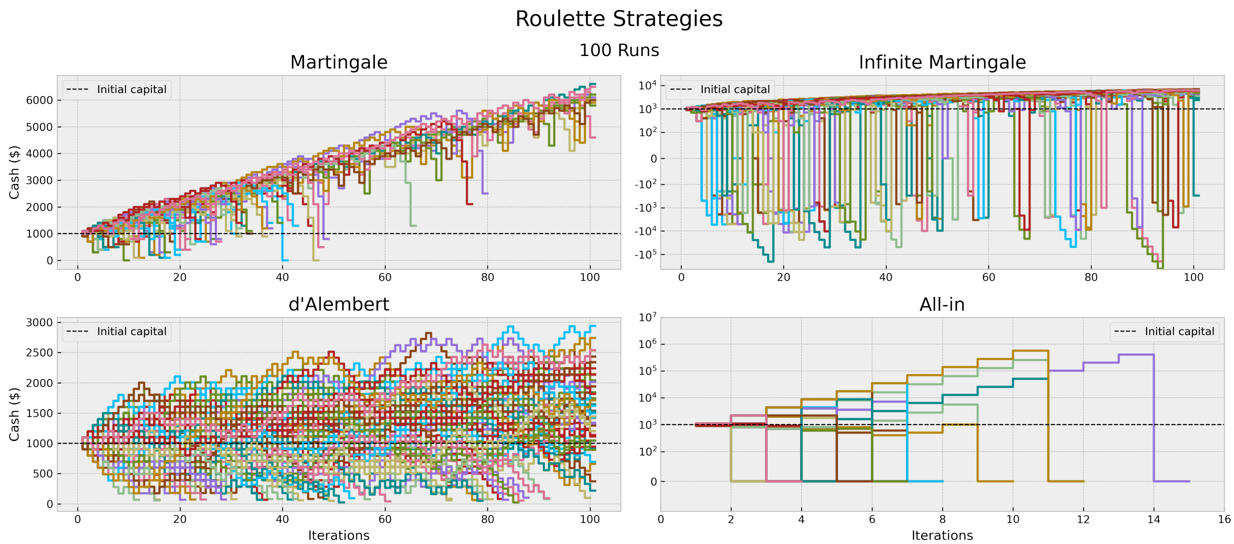


Figure 1: Capital flow in each strategy

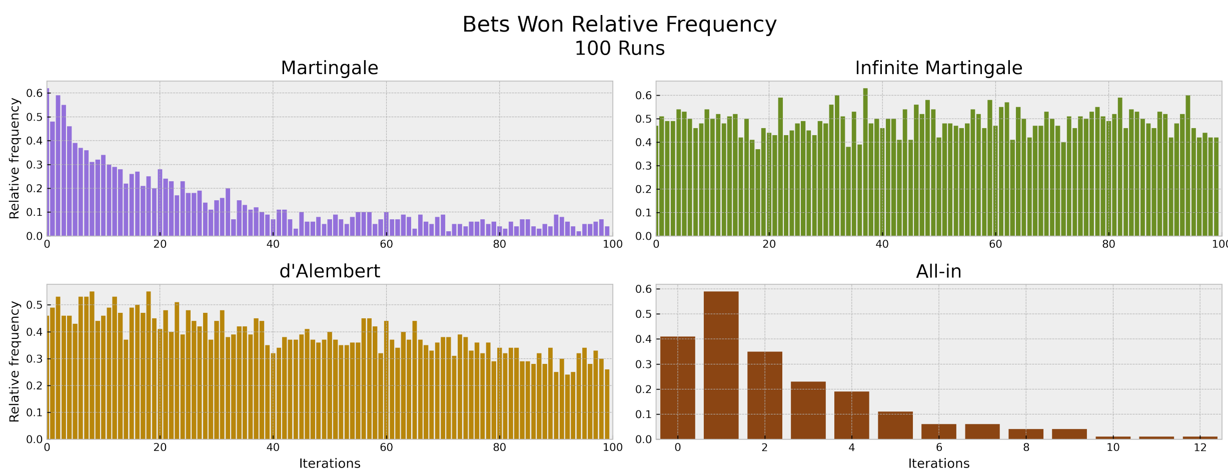


Figure 2: Relative frequency of bets won

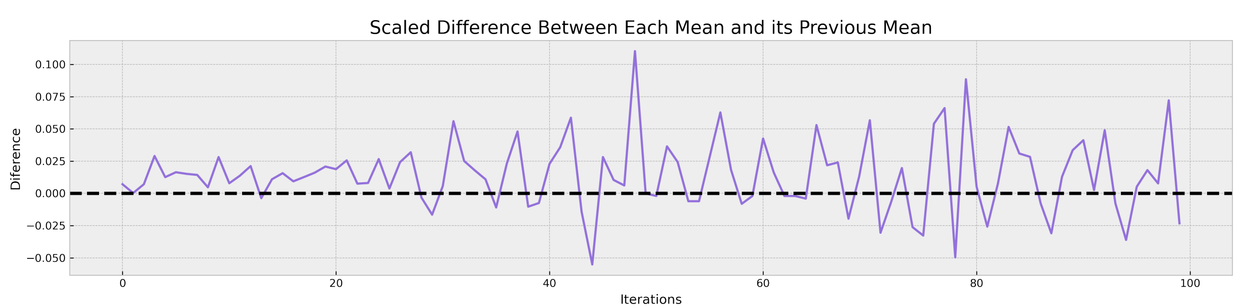


Figure 3: Difference between the mean of the capital in $i$ and the mean in $i-1$

Figure 1 illustrates the capital flow of each run for each strategy described. Figure 2 shows the relative frequency of bets won, taking the percentage of bets won in each iteration for all runs.

Figure 3 shows the difference between the mean capital flow per iteration of all runs and the mean in the previous iteration, divided by the mean to obtain the relative value. As can be seen, this difference tends to remain at $0$. To affirm this observation, credibility intervals were constructed according to different errors:

$P(|E(X_{n+1} - X_n)| < 0.0001 = 0.01$

$P(|E(X_{n+1} - X_n)| < 0.001 = 0.02$

$P(|E(X_{n+1} - X_n)| < 0.01 = 0.31$

$P(|E(X_{n+1} - X_n)| < 0.05 = 0.88$

$P(|E(X_{n+1} - X_n)| < 0.1 = 0.99$

$P(|E(X_{n+1} - X_n)| < 0.15 = 1$

In Table 1 three results are illustrated, the first column shows the percentage of wins in the last play, the second column shows the average of the capital level in the last play (on play 12 for All-in, maximum number of iterations that was obtained without going bankrupt) for all the runs, and the third column shows the maximum capital level that was obtained from all the runs regardless of the number of plays.

Regarding bets won, the strategy with the highest percentage of wins was the Infinite Martingale, although, since this strategy is not feasible in the real game, it is d'Alembert who has the best chances of winning.

In relation to the level of capital, after the 100 iterations Martingale presents the highest capital with $\$6,180$, although it is All-in the one that historically presents the highest profits.

## 6 Conclusions

In conclusion, all strategies went bankrupt at some point, so there is no guarantee of ultimate success. In general terms, no strategy turned out to be better or worse than another, since all have their advantages in terms of percentage and frequency of wins, amounts obtained, and associated risk. The choice of the strategy will then depend on the characteristics of the player, and his or hers particular goal between obtaining lower profit with lower risk, or higher profit with higher risk. The All-in, for example, allows few plays in a row, is the riskiest of all, and is the one with the highest level of capital to obtain. d'Alembert, on the other hand, has a winning percentage of almost $53\%$, but it is the one with the lowest earnings.

In relation to the Martingale probabilistic definition, it is concluded that the difference between the mean in $i$ and in $i-1$ is less than $0.15$, approximating with sufficient precision what is proposed by the theoretical definition.

## 7 Future Work

In the present work, only three roulette strategies were modeled. It would be of interest to perform the analysis for others, such as the Reverse Martingale, Labouchère, and Fibonacci, to compare them with those previously analyzed.