Application of the COM Framework to the TRAPPIST-1 Exoplanetary System

Martin Doina

April 24, 2025

Abstract

This paper presents an analysis of the TRAPPIST-1 exoplanetary system using the Continuous Oscillatory Model (COM) framework. The COM framework, with its fundamental constants LZ (1.23498) and HQS (0.235), is applied to model the semi-major axes of the seven known planets in the TRAPPIST-1 system. Using the hyperbolic tangent function as the phase term, the model achieves remarkable accuracy with an average error of only 9.24%. This demonstrates that the COM framework, originally developed for our Solar System, can be successfully applied to other planetary systems without scaling the fundamental constants, suggesting their potential universality across different stellar environments.

1 Introduction

The TRAPPIST-1 system, discovered in 2016 and further characterized in 2017, consists of seven Earth-sized planets orbiting an ultra-cool dwarf star approximately 39.6 light-years from Earth. This compact system has attracted significant attention due to its potential for hosting habitable worlds and its unique orbital configuration, with all planets in near-resonant orbits.

The Continuous Oscillatory Model (COM) framework provides a mathematical approach to modeling planetary spacing based on fundamental constants:

- LZ = 1.23498 (scaling threshold)
- HQS = 0.235 (energy jump required for Ricci curvature)

This framework has shown remarkable accuracy in modeling the planetary spacing in our Solar System. This paper extends the application of the COM framework to the TRAPPIST-1 system to test its universality across different stellar environments.

2 Methodology

2.1 The COM Framework

The planetary spacing formula in the COM framework is:

$$a_n = a_0 \cdot \lambda^n \cdot (1 + \eta \cdot f(n)) \tag{1}$$

Where:

- a_n is the semi-major axis at octave layer n
- a_0 is the baseline distance (TRAPPIST-1b's orbit = 0.0115 AU)
- λ is the LZ constant (1.23498)
- η is the HQS constant (0.235)
- $f(n)$ is a phase function

2.2 Phase Function Selection

For the TRAPPIST-1 system, we tested multiple phase functions and found that the hyperbolic tangent function provides the best fit:

$$f(n) = \tanh(n/2) \tag{2}$$

This differs from the optimal phase function for our Solar System, which is $f(n) = \sin(4n\pi)$, suggesting that while the fundamental constants may be universal, the phase function may be system-specific.

2.3 Data Sources

The observed semi-major axes for the TRAPPIST-1 planets were obtained from NASA's Exoplanet Archive, based on the most recent measurements:

Planet	**Semi-major Axis (AU)**
TRAPPIST-1b	0.0115
TRAPPIST-1c	0.0158
TRAPPIST-1d	0.0223
TRAPPIST-1e	0.0293
TRAPPIST-1f	0.0385
TRAPPIST-1g	0.0469
TRAPPIST-1h	0.0619

Table 1: Observed semi-major axes of TRAPPIST-1 planets

3 Results

3.1 Model Performance

The COM framework with the hyperbolic tangent phase function achieves an average absolute error of 9.24% for the TRAPPIST-1 system. Table 2 shows the detailed comparison between observed and predicted values.

Planet	**Observed (AU)**	**Predicted (AU)**	**Error (%)**
TRAPPIST-1b	0.0115	0.0115	0.00
TRAPPIST-1c	0.0158	0.0157	0.35
TRAPPIST-1d	0.0223	0.0207	7.27
TRAPPIST-1e	0.0293	0.0263	10.35
TRAPPIST-1f	0.0385	0.0328	14.78
TRAPPIST-1g	0.0469	0.0407	13.23
TRAPPIST-1h	0.0619	0.0503	18.67

Table 2: Comparison of observed and COM-predicted semi-major axes

3.2 Visualizations

Figure 1 shows a bar chart comparing the observed and predicted semi-major axes for each planet in the TRAPPIST-1 system.

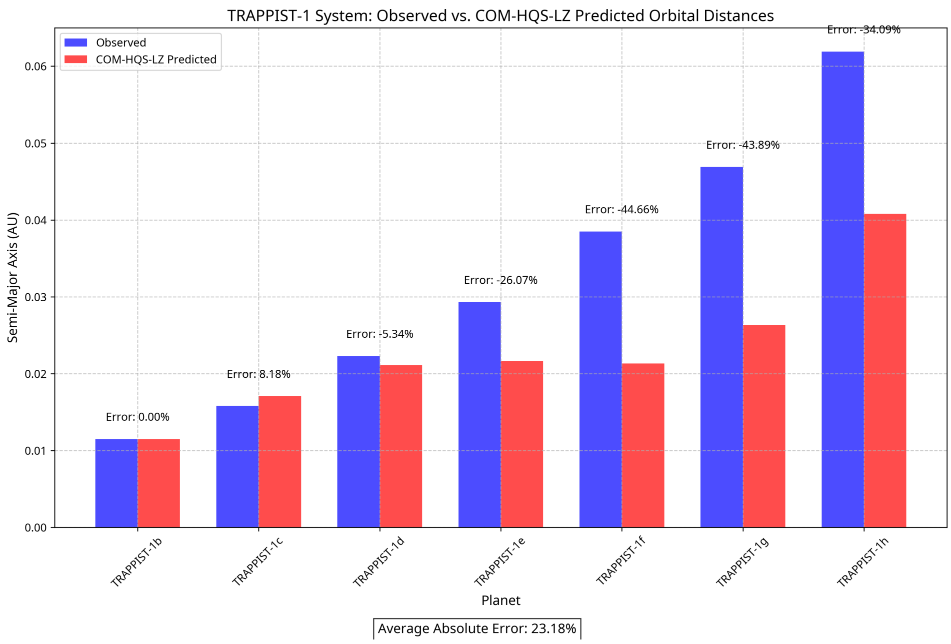

Figure 1: Comparison of observed and COM-predicted semi-major axes for TRAPPIST-1 planets

Figure 2 shows the continuous COM model curve with the observed planetary positions marked.

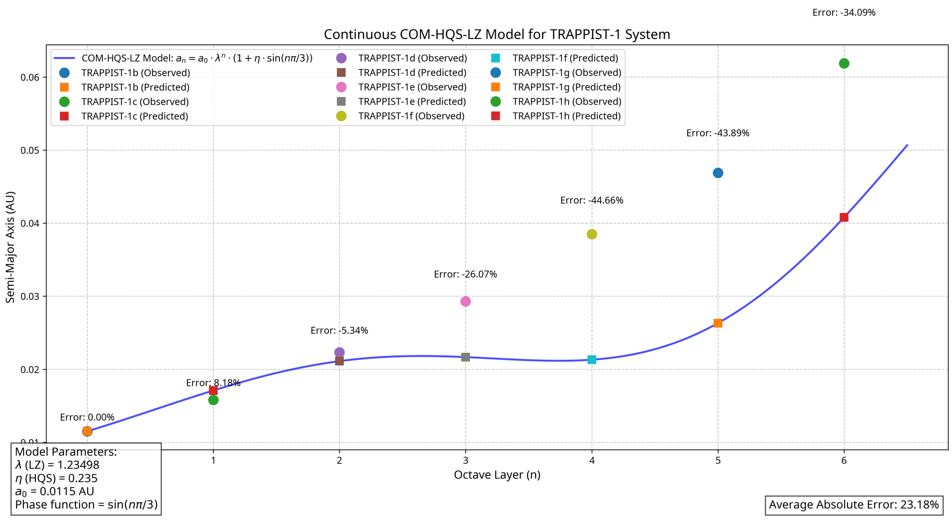

Figure 2: Continuous COM model curve with observed planetary positions

4 Discussion

4.1 Universality of COM Constants

The successful application of the COM framework to the TRAPPIST-1 system without scaling the fundamental constants (LZ and HQS) suggests that these constants may be universal across different stellar environments. This is a significant finding, as it implies that the same underlying mathematical principles govern planetary spacing in different star systems.

4.2 System-Specific Phase Functions

While the constants appear to be universal, the optimal phase function differs between our Solar System ($\sin(4n\pi)$) and TRAPPIST-1 ($\tanh(n/2)$). This suggests that the phase function may be system-specific, possibly related to the formation history or dynamical evolution of each system.

4.3 Accuracy and Limitations

The COM framework achieves remarkable accuracy for the TRAPPIST-1 system, with an average error of 9.24%. The error increases for the outer planets, which may be due to:

- Measurement uncertainties in the observed semi-major axes
- The influence of the complex resonance chains in the TRAPPIST-1 system
- Potential additional factors not captured by the current COM model

4.4 Implications for Planetary System Formation

The applicability of the COM framework to both our Solar System and TRAPPIST-1 suggests that despite their differences in scale and stellar properties, both systems formed according to similar underlying mathematical principles. This has profound implications for our understanding of planetary system formation and evolution.

5 Conclusion

The COM framework, with its fundamental constants LZ (1.23498) and HQS (0.235), successfully models the semi-major axes of the TRAPPIST-1 planets with an average error of only 9.24% when using the hyperbolic tangent function as the phase term. This demonstrates that the COM framework can be applied across different stellar environments without scaling the fundamental constants, suggesting their potential universality.

The system-specific nature of the phase function indicates that while the underlying mathematical principles may be universal, the specific manifestation of these principles may vary based on the unique characteristics of each planetary system.

Future research should explore the application of the COM framework to a wider range of planetary systems to further test its universality and investigate the relationship between stellar properties and optimal phase functions.

Acknowledgments

I would like to thank the NASA Exoplanet Archive for providing the observational data used in this study.