Quantum Collatz Process in COM Framework

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Quantum Collatz Function

$$\text{quantum\_collatz}(n, \eta, \lambda) = \begin{cases} \dfrac{n}{2}\left(1-\eta\cdot\sin\left(\dfrac{2\pi\cdot\log(n)}{\log(\lambda)}\right)\right) & \text{if } n \text{ is even} \\ (3n+1)\left(1+\eta\cdot\sin\left(\dfrac{2\pi\cdot\log(n)}{\log(\lambda)}\right)\right) & \text{if } n \text{ is odd} \end{cases}$$

where:

- $\eta = 0.235$ (HQS)
- $\lambda = 1.23498$ (LZ)

Modified Casimir Force with Octave-Structured Vacuum

$$F_{\text{modified}}(d, r_0, \eta, \lambda) = F_{\text{classical}}(d)\left(1+\eta\cdot\sin\left(\frac{2\pi\cdot\log(d/r_0)}{\log(\lambda)}\right)\right)\cdot\frac{\lambda^4}{(1-\eta)^2}$$

where:

- $F_{\text{classical}}(d) = -\dfrac{\pi^2\hbar cA}{240d^4}$
- r_0 is a fundamental length scale

Attractor Analysis

The attractor value after N iterations is defined as:

$$A(n_0, \eta, \lambda, N) = \text{quantum\_collatz}^N(n_0, \eta, \lambda)$$

where quantum_collatz^N represents applying the quantum Collatz function N times starting with initial value n_0.

Sequence Length Function

The sequence length until convergence (or maximum iterations) is:

$$L(n_0, \eta, \lambda, \varepsilon) = \min\{k \in N : |A(n_0, \eta, \lambda, k) - A(n_0, \eta, \lambda, k-1)| < \varepsilon\}$$

where ε is a convergence threshold.

Parameter Space Exploration

The parameter space for the quantum Collatz process is defined by:

$$P = \{(\eta, \lambda) : \eta \in [0.1, 0.4], \lambda \in [1.1, 1.4]\}$$

Lyapunov Exponent

The Lyapunov exponent for the quantum Collatz process is:

$$\Lambda(n_0, \eta, \lambda) = \lim_{N\to\infty} \frac{1}{N} \sum_{k=0}^{N-1} \log \left| \frac{d}{dn} \text{quantum\_collatz}(n_k, \eta, \lambda) \right|$$

where $n_k = \text{quantum\_collatz}^k(n_0, \eta, \lambda)$.

Bifurcation Analysis

The bifurcation set is defined as:

$$B = \{(\eta, \lambda) \in P : \exists n_0 \text{ such that } L(n_0, \eta, \lambda, \varepsilon) \text{ changes discontinuously}\}$$