

Standard Mathematics Equations and Concepts

This document compiles fundamental mathematical equations and concepts that will be redefined under the COM (Continuous Oscillatory Model) framework.

Number Theory and Arithmetic Operations

Basic Operations

- Addition: $a + b = c$
- Subtraction: $a - b = c$
- Multiplication: $a \times b = c$
- Division: $a \div b = c$, $b \neq 0$
- Exponentiation: $a^b = c$
- Logarithm: $\log_a(b) = c$, where $a^c = b$

Number Sets

- Natural Numbers (\mathbb{N}): $\{1, 2, 3, \dots\}$
- Integers (\mathbb{Z}): $\{\dots, -2, -1, 0, 1, 2, \dots\}$
- Rational Numbers (\mathbb{Q}): $\{p/q \mid p, q \in \mathbb{Z}, q \neq 0\}$
- Real Numbers (\mathbb{R}): Complete ordered field
- Complex Numbers (\mathbb{C}): $\{a + bi \mid a, b \in \mathbb{R}, i^2 = -1\}$

Number Theory Concepts

- Prime numbers: Numbers divisible only by 1 and themselves
- Modular arithmetic: $a \equiv b \pmod{n}$ if n divides $(a - b)$
- Fibonacci sequence: $F_0 = 0, F_1 = 1, F_n = F_{n-1} + F_{n-2}$ for $n > 1$
- Golden ratio: $\phi = (1 + \sqrt{5})/2 \approx 1.618033988749895$

Algebra and Functions

Equations and Inequalities

- Linear equation: $ax + b = 0$
- Quadratic equation: $ax^2 + bx + c = 0$, with solution $x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$
- Polynomial equation: $a_0 + a_1x + a_2x^2 + \dots + a_nx^n = 0$
- Linear inequality: $ax + b < 0$ (or $>$, \leq , \geq)

Functions

- Function definition: $f: X \rightarrow Y$, where for each $x \in X$, there exists exactly one $y \in Y$
- Composition: $(f \circ g)(x) = f(g(x))$
- Inverse function: $f^{-1}(y) = x$ if $f(x) = y$
- Domain and range: Domain is input set, range is output set

Algebraic Structures

- Groups: Set with binary operation satisfying closure, associativity, identity, and inverse properties
- Rings: Set with two binary operations (addition and multiplication) with specific properties
- Fields: Commutative ring where every non-zero element has a multiplicative inverse

Calculus

Limits

- Definition: $\lim_{x \rightarrow a} f(x) = L$ if for every $\epsilon > 0$, there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$
- Properties: $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$

Derivatives

- Definition: $f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h$
- Power rule: $d/dx [x^n] = nx^{n-1}$
- Product rule: $d/dx [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- Chain rule: $d/dx [f(g(x))] = f'(g(x))g'(x)$
- Exponential: $d/dx [e^x] = e^x$
- Logarithmic: $d/dx [\ln(x)] = 1/x$

Integrals

- Indefinite integral: $\int f(x)dx = F(x) + C$, where $F'(x) = f(x)$
- Definite integral: $\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$
- Fundamental theorem of calculus: $\int_a^b f(x)dx = F(b) - F(a)$, where $F'(x) = f(x)$

Differential Equations

- First-order linear: $dy/dx + P(x)y = Q(x)$
- Second-order linear: $a(d^2y/dx^2) + b(dy/dx) + cy = f(x)$
- Partial differential equations: $u_t = \kappa u_{xx}$ (Heat equation)

Geometry and Trigonometry

Euclidean Geometry

- Pythagorean theorem: $a^2 + b^2 = c^2$
- Area of triangle: $A = (1/2)bh$
- Area of circle: $A = \pi r^2$
- Volume of sphere: $V = (4/3)\pi r^3$

Trigonometric Functions

- Sine: $\sin(\theta) = \text{opposite/hypotenuse}$
- Cosine: $\cos(\theta) = \text{adjacent/hypotenuse}$
- Tangent: $\tan(\theta) = \text{opposite/adjacent} = \sin(\theta)/\cos(\theta)$
- Fundamental identity: $\sin^2(\theta) + \cos^2(\theta) = 1$

Trigonometric Equations

- Law of sines: $a/\sin(A) = b/\sin(B) = c/\sin(C)$
- Law of cosines: $c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$

Complex Numbers and Functions

Complex Number Operations

- Addition: $(a + bi) + (c + di) = (a + c) + (b + d)i$
- Multiplication: $(a + bi)(c + di) = (ac - bd) + (ad + bc)i$
- Complex conjugate: $\bar{z} = a - bi$ for $z = a + bi$
- Modulus: $|z| = \sqrt{a^2 + b^2}$ for $z = a + bi$

Complex Functions

- Euler's formula: $e^{i\theta} = \cos(\theta) + i \cdot \sin(\theta)$
- Complex exponential: $e^{az} = e^{a(\cos(b) + i \cdot \sin(b))}$ for $z = a + bi$

Linear Algebra

Vectors and Matrices

- Vector addition: $u + v = (u_1 + v_1, u_2 + v_2, \dots, u_n + v_n)$
- Dot product: $u \cdot v = u_1 v_1 + u_2 v_2 + \dots + u_n v_n$
- Matrix multiplication: $(AB)_{ij} = \sum_k A_{ik} B_{kj}$
- Determinant: $\det(A) = \sum (\pm) a_{1i} a_{2j} \dots a_{nk}$ (where i,j,\dots,k is a permutation of $1,2,\dots,n$)

Linear Transformations

- Eigenvalue equation: $Av = \lambda v$
- Characteristic polynomial: $\det(A - \lambda I) = 0$

Discrete Mathematics

Set Theory

- Union: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- Intersection: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$
- Complement: $A' = \{x \in U \mid x \notin A\}$
- Cartesian product: $A \times B = \{(a, b) \mid a \in A, b \in B\}$

Combinatorics

- Permutations: $P(n,r) = n!/(n-r)!$
- Combinations: $C(n,r) = n!/[r!(n-r)!]$
- Binomial theorem: $(a + b)^n = \sum C(n,k)a^{(n-k)}b^k$

Graph Theory

- Euler's formula for planar graphs: $V - E + F = 2$
- Handshaking lemma: $\sum \deg(v) = 2E$

Statistics and Probability

Probability

- Probability of event: $P(A) = |A|/|S|$
- Conditional probability: $P(A|B) = P(A \cap B)/P(B)$
- Bayes' theorem: $P(A|B) = P(B|A)P(A)/P(B)$

Statistics

- Mean: $\bar{x} = (1/n)\sum x$
- Variance: $s^2 = (1/n)\sum (x - \bar{x})^2$
- Standard deviation: $s = \sqrt{s^2}$
- Normal distribution: $f(x) = (1/(\sqrt{2\pi}))e^{-(x-\bar{x})^2/(2s^2)}$