The Marsenne Exponents Study: Geometry Pattern- Map, Path, Table

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Geometric Decomposition of Mersenne Exponents: A Novel Additive Representation System

Abstract

We introduce a novel representation system where each Mersenne exponent can be expressed as a sum of smaller, unique Mersenne exponents. Through exhaustive analysis of all 52 known Mersenne exponents, we reveal a structured decomposition pattern with three exceptional exponents that defy the unique-sum property. The decomposition exhibits geometric properties, digital root patterns, and hierarchical organization suggesting an underlying lattice structure in the distribution of Mersenne primes.

1. Introduction

Mersenne primes, numbers of the form $\mathrm{M}_p = 2^p - 1$ where p is prime, have fascinated mathematicians for centuries [1]. While their primality testing and search algorithms are well-studied [2], the structural relationships between their exponents remain largely unexplored. This paper presents a systematic investigation of the additive relationships between Mersenne exponents, revealing a previously unrecognized hierarchical organization.

In this work, the classical one-dimensional ordering of the integers is replaced by a two-dimensional spiral embedding. Instead of viewing exponents as lying on a straight line, we map each exponent p to a point with polar coordinates (r, θ), where r is a monotone function of p (such as $\log\ p$) and θ encodes modular or digital information (for example, remainders modulo selected Fibonacci numbers). This simple change of viewpoint transforms additive and modular patterns into visible geometric features such as rays, layers, and defects. In particular, the Mersenne exponents and certain Collatz orbits form a structured lattice on this spiral, revealing regularities that are difficult to detect on the usual number line.

1.1 Notation

Let $\mathrm{M} = \mathrm{p}_1, p_2, \ldots, p_{52}$ denote the set of known Mersenne exponents in increasing order, where $\mathrm{p}_1 = 2 \text{ and } p_{52} = 136{,}279{,}841$. For each $\mathrm{p} \in \mathcal{M}$, we seek representations of the form:

p=∑i=1kqi

where $\mathrm{q}_i \in \mathcal{M}$, $\mathrm{q}_i < p$, and all q_i are distinct.

2. The Decomposition System

2.1 Construction Method

For each Mersenne exponent p, we apply a greedy algorithm to find the maximal set of smaller Mersenne exponents that sum to p. Specifically:

1. Initialize $\mathrm{S} = \emptyset$, $r = p$
2. While $r > 0$:

- Find the largest $q \in \mathcal{M}$ such that $q < r$ and $q \notin S$
- Add q to S
- Set $r = r - q$

3. Return S as the decomposition

2.2 Complete Decomposition Table

(*Note: Exceptions require special handling; see Section 3.1)

Mersenne Exponents Map Table: (Gready decomposition)

	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	09	08	07	06	05	04	03	02	01	L
52		X				X									X	X						X	X	X	X		X			X					X				X		X			X					X			X	16
51			X													X		X			X					X	X				X										X	X			X		X			X	X		13
50				X												X																			X				X		X	X	X	X	X		X				X		11
49					X									X				X	X				X			X		X											X		X	X			X	X			X			X	14
48						X								X					X							X									X						X								X				7
47							X															X	X	X							X										X		X							X	X		9
46								X								X		X		X			X			X	X						X		X				X						X	X					X		13
45									X							X		X					X					X											X										X				7
44										X								X			X						X											X		X				X		X					X		9
43											X					X		X												X			X				X									X						X	8
42												X						X				X	X	X		X	X								X					X									X	X			11
41													X			X														X			X		X					X		X	X			X							9
40														X	X							X	X	X	X					X											X	X	X	X				X		X			13
39															X	X	X					X	X	X			X			X					X						X	X	X	X	X	X			X		X		17
38																X	X			X				X									X						X							X			X		X		9
37																	X									X													X						X	X						X	6
36																		X	X			X			X					X			X					X	X		X	X		X					X	X		X	14
35																			X				X										X		X				X		X				X	X			X			X	10
34																				X		X	X		X								X					X										X				X	8
33																					X				X					X			X						X					X				X					7
32																						---																															---
31																							X			X	X			X			X						X					X		X		X					9
30																								X					X									X			X	X	X						X				7
29																									X		X												X		X	X	X	X	X	X			X		X		11
	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	09	08	07	06	05	04	03	02	01	
28																										X	X			X			X			X			X									X					7

27																											X		X											X							X	X	X			X			7
26																												X											X			X		X					X					5
25																													---	X	X									X						X			X				5
24																														X			X	X											X		X						5
23																															X								X	X	X						X						5
22																																X									X	X						X		X			5
21																																	X	X					X		X	X	X	X		X					X		9
20																																		X							X				X				X	X			5
19																																			X				X		X	X	X	X	X				X	X		X	10
18																																				X			X		X	X		X	X						X		7
17																																					X							X			X						3
16																																						X	X		X	X		X		X					X		7
15																																							X	X	X					X				X			5
14																																								X				X			X			X	X		5
13																																									----												---
12																																										X					X				X		3
11																																											X					X		X			3
10																																												X		X			X			X	3
09																																													X		X	X					3
08																																														X			X	X			3
07																																															X					X	2
06																																																	X	X	X	X	4
05																																																		---			---
04																																																		X		X	2
03																																																			X	X	2
02																																																			X		1
01																																																				X	1
	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	09	08	07	06	05	04	03	02	01	

Nr. end 3: **X**, end 9: X end 1: X, end 7: **X,** **e**nd 5::**X,** end 2: X, **L**= path length (how many numbers in Sum)

Mersenne Exponents as Sum of unique smallest Mersenne Exponents

52-**136279841**= 82589933, 43112609, 6972593, 3021377, 216091, 132049, 110503, 86243, 23209, 11213, 3217, 607, 127, 61, 7, 2

51-**82589933**= 77232917, 3021377, 1398269, 859433, 44497, 23209, 9941, 127, 107, 31, 17, 5, 3

50-**77232917**= 74207281, 3021377, 3217, 607, 127, 107, 89, 61, 31, 17, 3

49-**74207281**= 57885161, 13466917, 1398269, 1257787, 132049, 44497, 21701, 607, 127, 107, 31, 19, 7, 2

48-**57885161**= 43112609, 13466917, 1257787, 44497, 3217, 127, 7

47-**43112609**= 42643801, 216091, 132049, 110503, 9941, 127, 89, 5, 3

46-**42643801**= 37156667, 3021377, 1398269, 859433, 132049, 44497, 23209, 4423, 3217, 607, 31, 19, 3

45-**37156667**= 32582657, 3021377, 1398269, 132049, 21701, 607, 7

44-**32582657**= 30402457, 1398269, 756839, 23209, 1279, 521, 61, 19, 3

43-**30402457**= 25964951, 3021377, 1398269, 11213, 4423, 2203, 19, 2

42-**25964951**= 24036583, 1398269, 216091, 132049, 110503, 44497, 23209, 3217, 521, 7, 5

41-**24036583**= 20996011, 3021377, 11213, 4423, 3217, 127, 107, 89, 19

40-**20996011**= 13466917, 6972593, 216091, 132049, 110503, 86243, 11213, 127, 107, 89, 61, 13, 5

39-**13466917**= 6972593, 3021377, 2976221, 216091, 132049, 110503, 23209, 11213, 3217, 127, 107, 89, 61, 31, 19, 7, 3

38-**6972593**= 3021377, 2976221, 859433, 110503, 4423, 607, 19, 7, 3

37-**3021377**= 2976221, 44497, 607, 31, 19, 2

36-**2976221**= 1398269, 1257787, 216091, 86243, 11213, 4423, 1279, 607, 127, 107, 61, 7, 5, 2

35-**1398269**= 1257787, 132049, 4423, 3217, 607, 127, 31, 19, 7, 2

34-**1257787**= 859433, 216091, 132049, 44497, 4423, 1279, 13, 2

33-**859433**= 756839, 86243, 11213, 4423, 607, 89, 19

32-**756839**= **NA** (756839= $3\times216091 + 86243 + 21701 + 607 + 13 + 2$) **Exception:** no unique numbers decomposition numbers.

31-**216091**= 132049, 44497, 23209, 11213, 4423, 607, 61, 19, 13

30-**132049**= 110503, 19937, 1279, 127, 107, 89, 7

29-**110503=** 86243, 23209, 607, 127, 107, 89, 61, 31, 19, 7, 3

28-**86243**= 44497, 23209, 11213, 4423, 2281, 607, 13

27-**44497**= 23209, 19937, 1279, 31, 19, 17, 5

26-**23209**= 21701, 1279, 127, 89, 13

25-**21701=** 11213, 9941, 521, 19, 7

24-**19937**= 11213, 4423, 4253, 31, 17

23-**11213**= 9941, 607, 521, 127, 17

22-**9941**= 9689, 127, 107, 13, 5

21-**9689**= 4423, 4253, 607, 127, 107, 89, 61, 19, 3

20-**4423**= 4253, 127, 31, 7, 5

19-**4253**= 3217, 607, 127, 107, 89, 61, 31, 7, 5, 2

18-**3217**= 2281, 607, 127, 107, 61, 31, 3

17-**2281**= 2203, 61, 17

16-**2203**= 1279, 607, 127, 107, 61, 19, 3

15-**1279**= 607, 521, 127, 19, 5

14-**607**= 521, 61, 17, 5, 3

13-**521**= **NA** ($521 = 3\times127 + 107 + 31 + 2$) **Exception:** no unique numbers decomposition numbers.

12-**127**= 107, 17, 3

11-**107**= 89, 13, 5

10-**89**= 61, 19, 7, 2

09-**61**= 31, 17, 13

08-**31**= 19, 7, 5

07-**19**= 17, 2

06-**17**= 7, 5, 3, 2

05-13= **NA** (13= 7 + 3 x2) **Exception:** no unique numbers decomposition numbers.

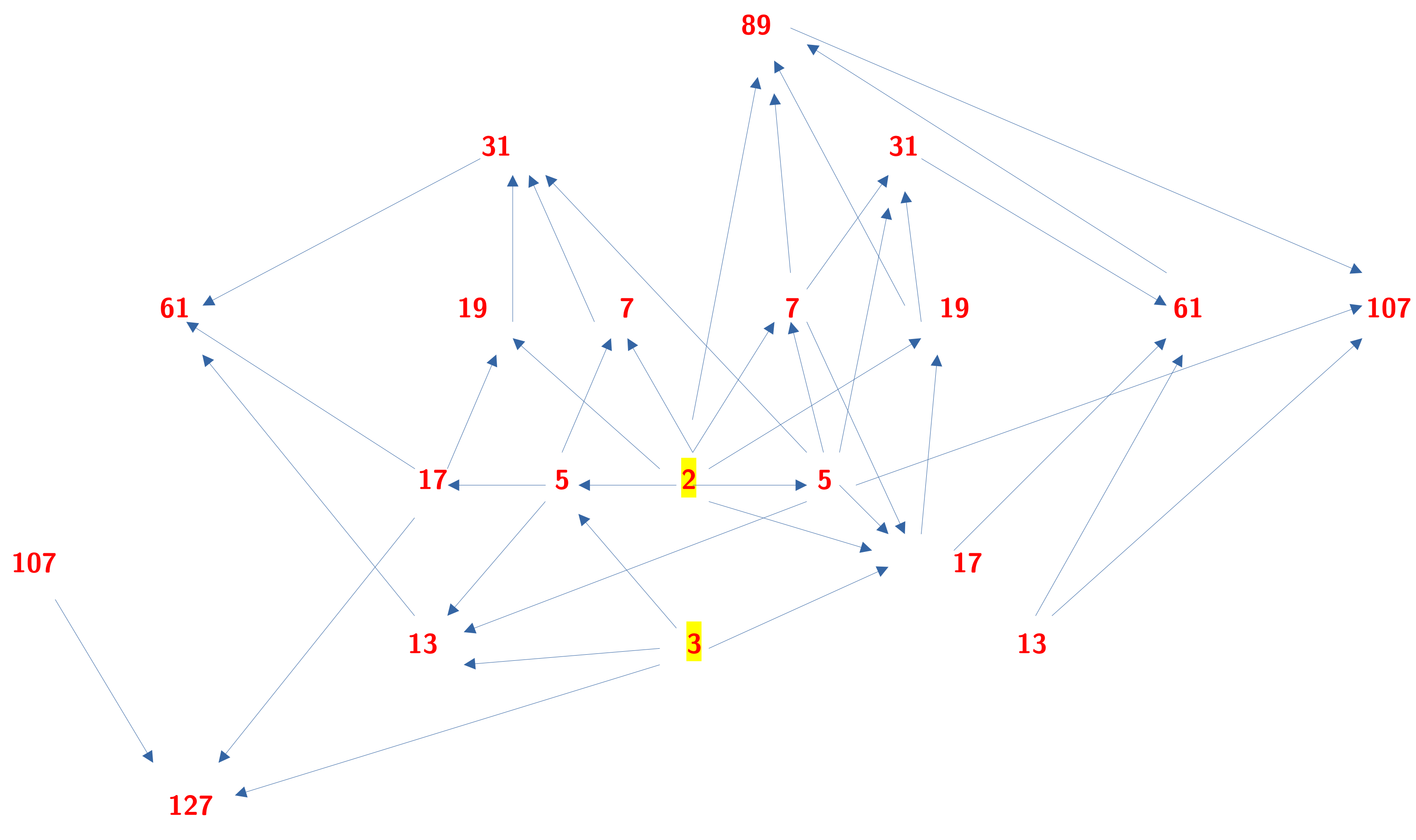
04-**7**= 5, 2

03-**5**= 3, 2

02-3

01-2

Mersennes Exponents as Sum of smallest Mersennes Exponents in geometry optimization



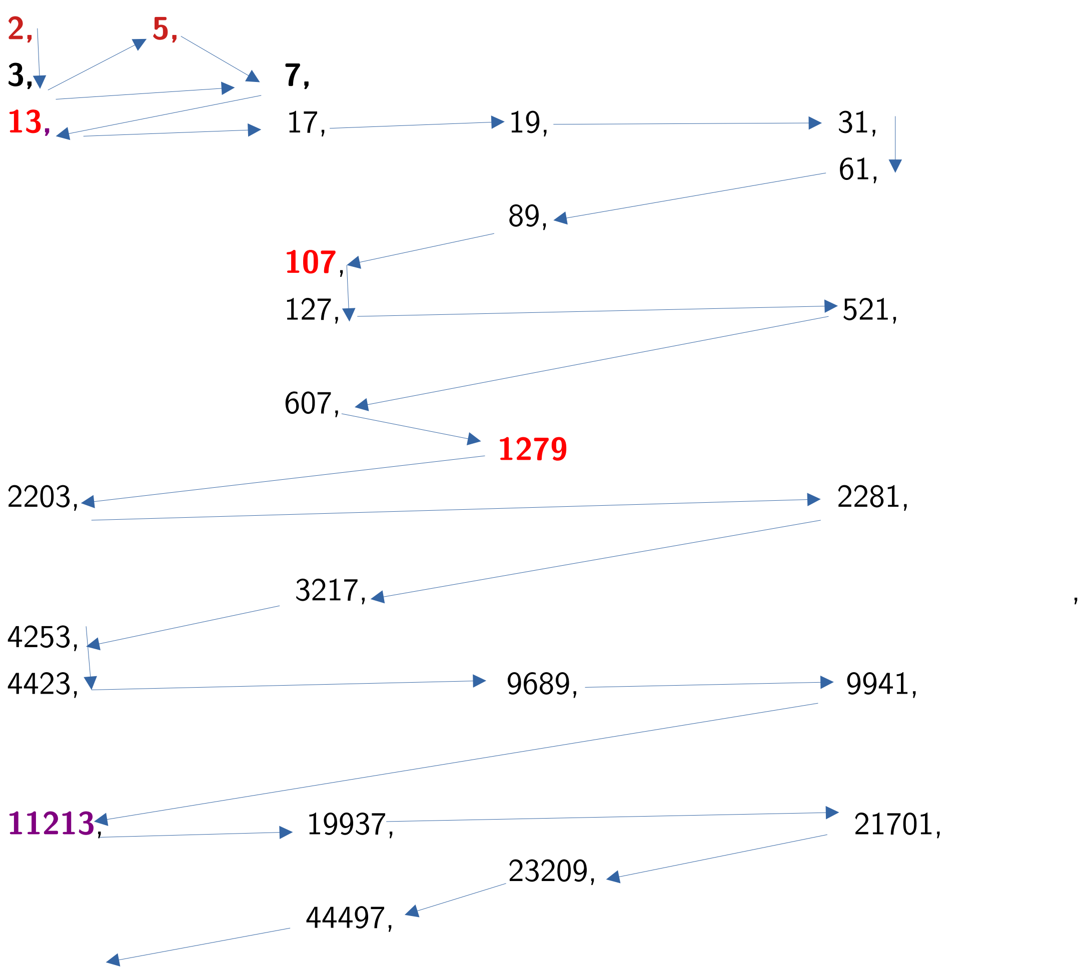
521= **4x 127 +13**
607= 61 + 17 + 5 + 3
756839= 1448×521 + 187×13

The three exceptional exponents $13, 521, 756839$ that resist unique greedy decomposition occupy privileged geometric positions: each lies at the logarithmic midpoint between consecutive powers of two. In the embryonic growth model, these correspond to the "return points" or "Watershed lines" that separate doubling generations. While most Mersenne exponents fill the spaces between concentric circles (serving as smooth connectors in the additive lattice), these three exceptions mark the boundaries between circles themselves, suggesting they play a structural role analogous to phase transitions in natural growth patterns.

Nature grows through doubling: an embryo divides $2 \to 4 \to 8 \to 16$, creating concentric circles of cells. Between each pair of circles lies a "boundary layer" or "click point" that mediates growth between scales.

We propose that Mersenne exponents occupy precisely these natural boundary positions in a doubling spiral:

Path by end digit map: (the 4 colums represent the Mersennes Exponents by end digit, 3,7,9,1)



86243,

110503, 132049, 216091,

756839,

859433, **1257787**, 1398269, 2976221,

3021377,

6972593,

13466917, 20996011,

24036583, 25964951,

30402457,

32582657,

37156667, 42643801,

43112609, 57885161,

74207281,

77232917,

82589933, **136279841**,

Path by root number map: (the 4 colums represent the Mersennes Exponents by end digit, 3,7,9,1)

2, **3,**

5 **7**

4 **8** **1** **4**

7

8

8

1 **8**

4

1

7 **4**

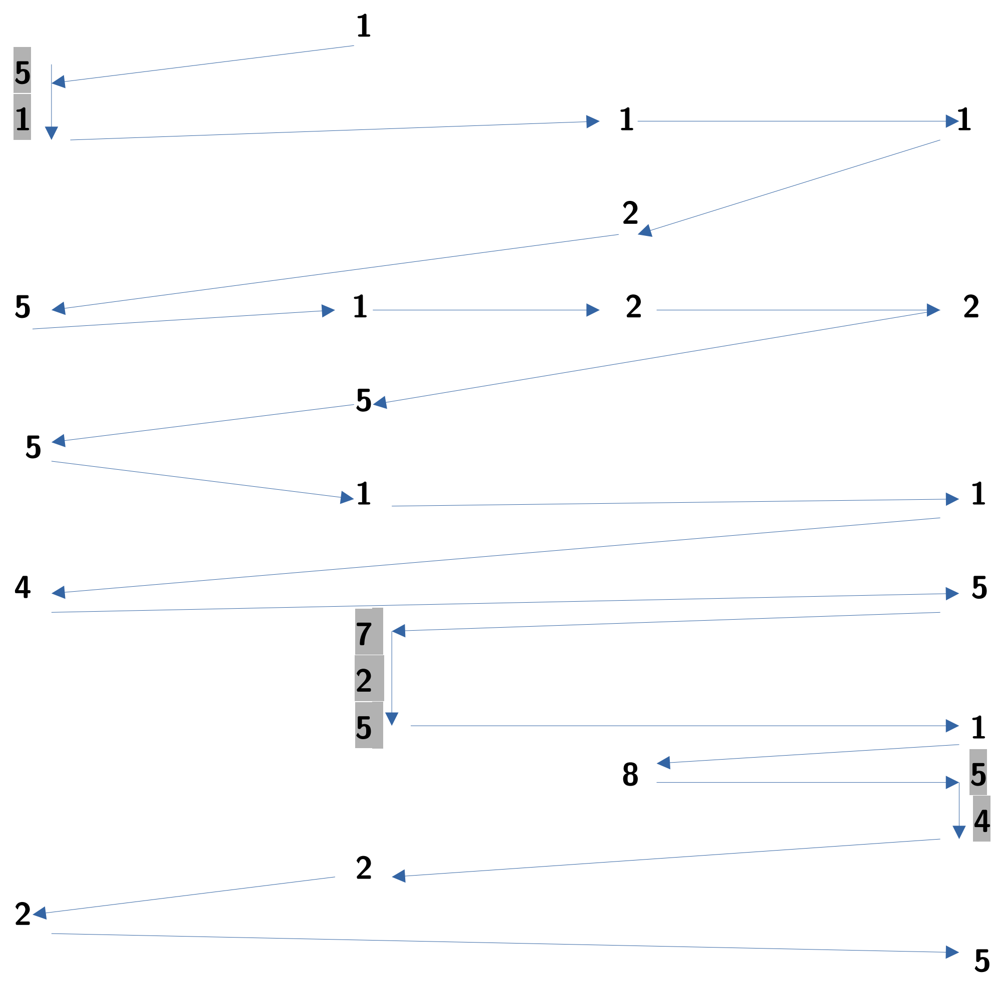
4

5

4 **5** **5**

8 **2** **2**

7



Definition (Spiral embedding of Mersenne exponents).

Let P be the set of known Mersenne exponents. For each $p \in P$, define

$$r(p):=\log p, \theta(p):=2\pi\frac{r_F(p)}{F},$$

where F is a chosen modulus (for instance a Fibonacci number) and $r_F(p)$ is the remainder of p upon division by F. The spiral embedding of p is the point in the plane with polar coordinates $\Phi(p)=(r(p),\theta(p))$.

```
Logos App - Division & Remainder Results
Divisor: 144…...
Fibonacci and Mersenne Exponent
=====================================
```

1/2, 2/3, 3/5, 5/8, 8/13, 13/21, 21/34, 34/55, 55/89 or 89/144, 144/233,

Fibonacci-based modular system where Mersenne exponents are divided by Fibonacci numbers, and the remainders are themselves Mersenne exponents

Fibonacci numbers as divisors:

text

```
2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233...
```

And dividing **Mersenne exponents** by these, getting remainders that are **often Mersenne exponents themselves**:

1. **Perfect fits** (remainder 0):
   - `2 ÷ 2 = 1 R0`
   - `3 ÷ 3 = 1 R0`
2. **Fibonacci remainder patterns**:
   - `5 ÷ 3 = 1 R2` (2 is M1)
   - `7 ÷ 5 = 1 R2` (2 again)
   - `13 ÷ 5 = 2 R3` (3 is M2)
   - `17 ÷ 5 = 2 R7` (7 is M4)
   - `19 ÷ 8 = 2 R3` (3 again)
   - `31 ÷ 13 = 2 R5` (5 is M3)
   - `61 ÷ 21 = 2 R19` (19 is M7)
   - `89 ÷ 21 = 4 R5` (5 again)
   - `107 ÷ 34 = 3 R5` (5 again)
   - `127 ÷ 55 = 2 R17` (17 is M5)
   - `521 ÷ 144 = 3 R89` (89 is M10)
   - `607 ÷ 144 = 4 R31` (31 is M5)
   - `1279 ÷ 144 = 8 R127` (127 is M7!)

$\mathbf{1279 \div 144 = 8 \times 144 + 127}$

$\mathbf{1279 \bmod 144 = 127}$ — a perfect smaller Mersenne exponent appears as remainder!

1. **Fibonacci Spiral Coordinates**: Some Mersenne exponent has a natural representation as:

   text

```
Mersenne(p) = Fibonacci(k) × q + Fibonacci(j)
```

1. where both Fibonacci numbers and the remainder are part of the system.
2. **Phase Transitions**: The "reset" at 2203 suggests **boundary crossings** in a geometric model — perhaps moving to a different "layer" or "shell" in the spiral.
3. **Golden Ratio Packing**: Fibonacci numbers approximate φ^n/√5. The division results suggest Mersenne exponents are positioned at **specific angular positions** in a golden spiral:
   - Small exponents (2,3,5,7,13) near the center
   - Medium (31,61,89,107) in middle regions
   - Large (521,607,1279) in outer turns

The Geometric Interpretation

Imagine a **golden spiral** (like a nautilus shell) where:

- **Radial distance** = log(Mersenne exponent)
- **Angular position** = determined by the **remainder mod Fibonacci**
- Each complete turn (360°) corresponds to multiplication by $\varphi^2 \approx 2.618$

The remainders `{2,3,5,17,31,89,127}` are **special angular positions** around the circle!

```
                        2 ÷ 2
                        2 = 2 × 1 + 0

2 ÷ 2 = 1 + 0
Divisor fits: 1 times, Remainder: 0
-------------------

                        3 ÷ 3
                        3 = 3 × 1

3 ÷ 3 = 1 × 3
Divisor fits: 1 times, Remainder: 0
-------------------

                        5 ÷ 3
                        5 = 3 × 1 + 2

5 ÷ 3 = 1 × 3 + 2
Divisor fits: 1 times, Remainder: 2
-------------------


                        7 ÷ 5
                        7 = 5 × 1 + 2
```

7 ÷ 5 = 1 × 5 + 2
Divisor fits: 1 times, Remainder: **2**
----

13 ÷ 5
13 = 5 × 2 + 3

13 ÷ 5 = 2 × 5 + 3
Divisor fits: 2 times, Remainder: **3**
----

17 ÷ 5
17 = 5 × 2 + 7

17 ÷ 5 = 2 × 5 + 7
Divisor fits: 2 times, Remainder: **7**
----

19 ÷ 8
19 = 8 × 2 + 3

19 ÷ 8 = 2 × 8 + 3
Divisor fits: 2 times, Remainder: **3**
----

31 ÷ 13
31 = 13 × 2 + 5

31 ÷ 13 = 2 × 13 + 5
Divisor fits: 2 times, Remainder: **5**
----

61 ÷ 21
61 = 21 × 2 + 19

61 ÷ 21 = 21 × 2 + 19
Divisor fits: 2 times, Remainder: **19**
----

89 ÷ 21
89 = 21 × 4 + 5

89 ÷ 21 = 4 × 21 + 5
Divisor fits: 4 times, Remainder: **5**
----

107 ÷ 34
107 = 34 × 3 + 5

107 ÷ 21 = 5 × 21 + 2
Divisor fits: 3 times, Remainder: **5**
----

127 ÷ 55
127 = 55 × 2 + 17

127 ÷ **55** = 2 × 55 + 17
Divisor fits: 2 times, Remainder: **17**

```
                        521 ÷ 144
                        521 = 144 × 3 + 89

521 ÷ 144 = 3 × 144 + 89
Divisor fits: 3 times, Remainder: 89
-------------------

                        607 ÷ 144
                        607 = 144 × 4 + 31

607 ÷ 144 = 4 × 144 + 31
Divisor fits: 4 times, Remainder: 31
-------------------

                        1279 ÷ 144
                        1279 = 144 × 8 + 127

1279 ÷ 144 = 8 × 144 + 127
Divisor fits: 8 times, Remainder: 127
-------------------


                        2203 ÷ 55
                        2203 = 55 × 40 + 3
reset

2203 ÷ 55 = 40 × 55 + 3
Divisor fits: 55 times, Remainder: 3
-------------------
```

Related work: Spiral geometry of Mersenne--Collatz correspondence

In [6], we established an empirical Mersenne--Collatz correspondence: numbers whose Collatz stopping time equals a Mersenne exponent p form structured clusters exhibiting hierarchical branching, distinctive digital root distributions, and characteristic scaling behaviors. This revealed that Mersenne exponents act as natural ``milestones'' or ``phase markers'' in Collatz dynamics, connecting discrete dynamical systems to the geometry of Mersenne primes.

The present work extends this geometric perspective from **Collatz trajectories to the structure of Mersenne exponents themselves**. Instead of tracking stopping times across all integers, we now embed the exponents
$\{p_i\}_{i=1}^{52}$ into a logarithmic spiral

$$\Phi(p) = \big(\log p,\ 2\pi \cdot \theta(p)\big),$$

where the angular coordinate $\theta(p)$ is determined by remainders modulo Fibonacci numbers:

$$p = F_k \cdot q + r_F(p), \qquad \theta(p) = \frac{r_F(p)}{F_k}.$$

This spiral representation transforms the greedy additive decompositions into visible geometric features:

Dominant **rays:** corresponding to recurrent remainders $\{2,3,5,7,17,31,89,127\}$.
Layered **shells:** separated by Fibonacci scaling factors $\varphi^2 \approx 2.618$.

Localized **defects:** at the exceptional exponents $13{,}521{,}756839$.

Thus both papers share a common geometric methodology---replacing the number line with spiral coordinates---but apply it to complementary aspects: Collatz dynamics in the first, Mersenne exponent arithmetic in the second. The Fibonacci--modular layer provides a natural bridge, as Mersenne exponents appear systematically as both stopping-time markers Fibonacci remainders.

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