Mersennes Exponents Equations & visuals on Spiral Geometry

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Abstract

This study is a continuation of previous research in Geometric Decomposition of Mersenne Exponents: A Novel Additive Representation System (1).

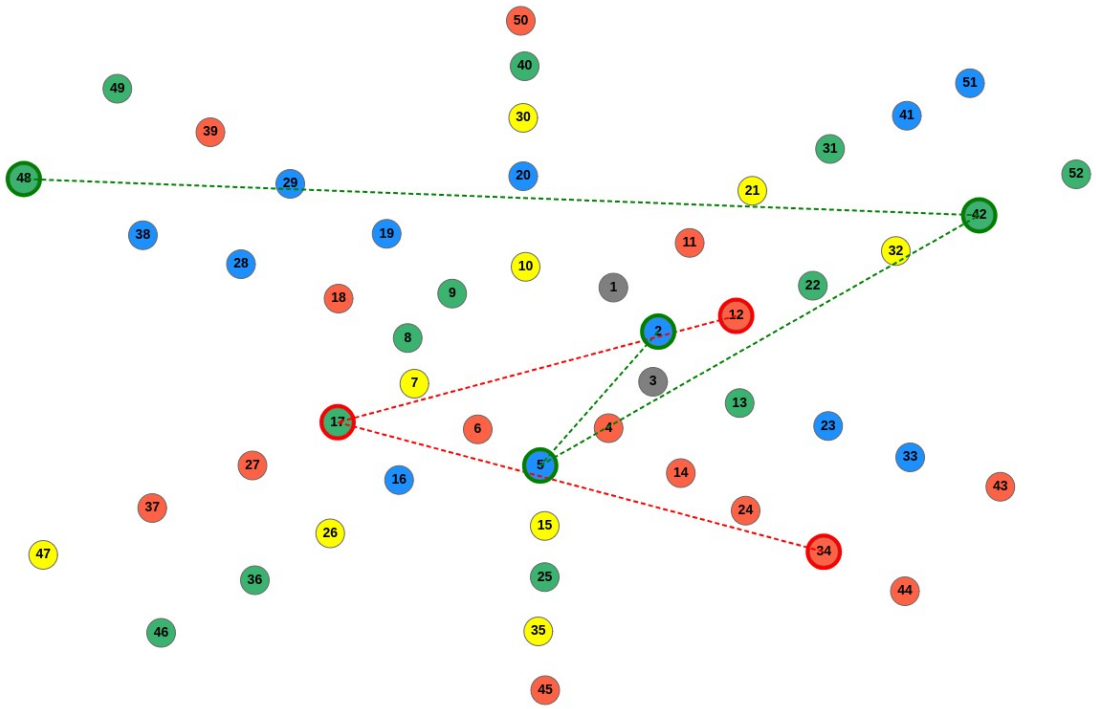
This paper presents an observed geometric pattern in the sequence of known Mersenne prime exponents (M1–M51). When plotted in a two-dimensional spiral arrangement based on index order and last digit, a remarkable structure emerges.

Each Mersenne exponent corresponds to a unique point in the spiral

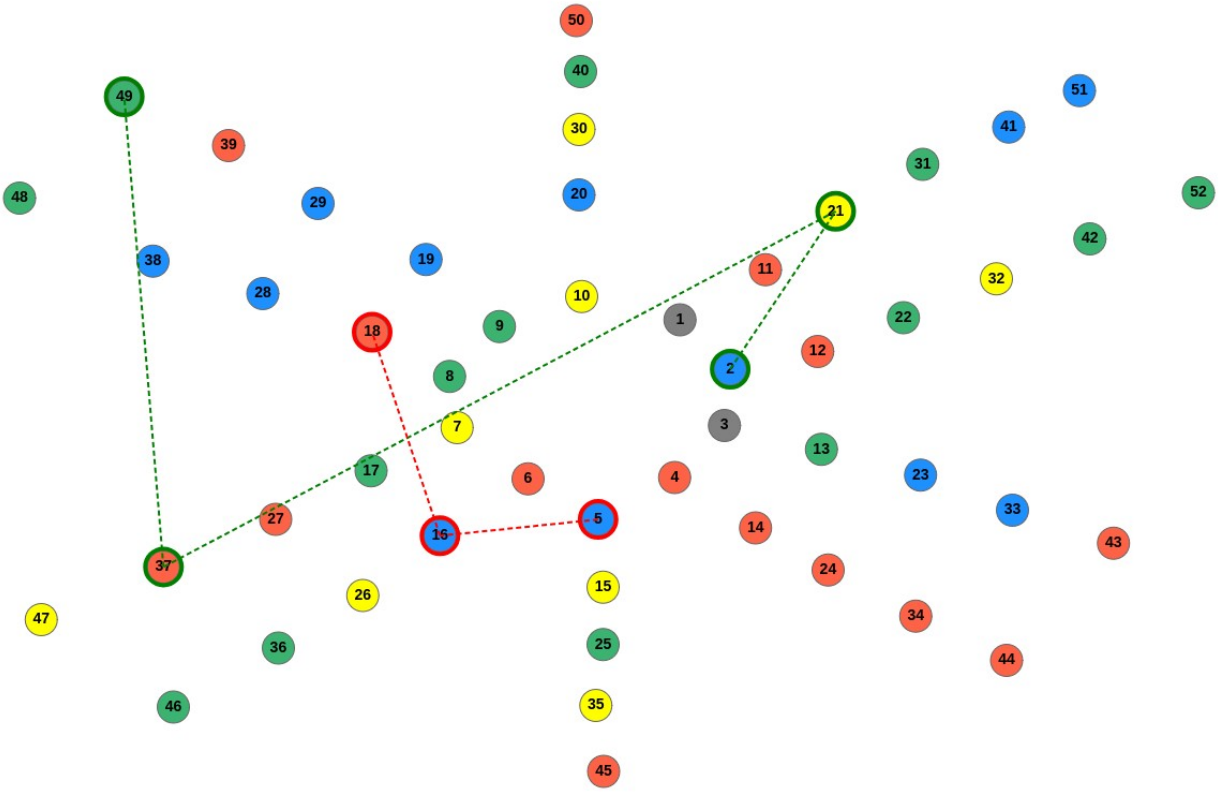
- For every known Mersenne prime up to M51, there exist sets of 4 positive and 3 negative indices whose vector sum (in spiral coordinates) lands precisely on the target point
- These geometric visuals helps to identify patterns while the sums are revealing relationship
- The pattern reveals an unexpected additive structure linking earlier and later Mersenne primes

"For any Mersenne exponent Mn (with a few exceptions), there exist sets of indices P and N, with |P|=4, |N|=3, such that in any ordinal embedding, the points corresponding to P form a triangle, the points corresponding to N form another triangle, and these triangles are balanced around Mn."

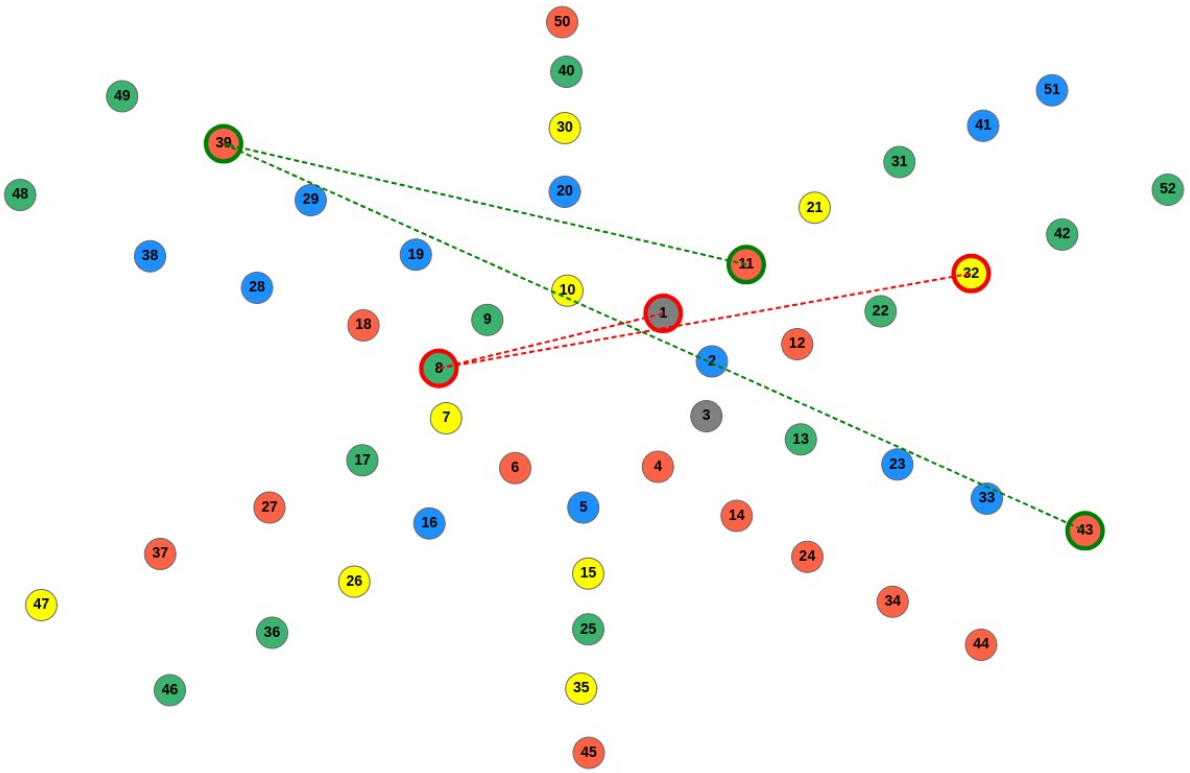
M51 = (M02 + M05 + M42 + M48) - (M12 + M17 + M34)



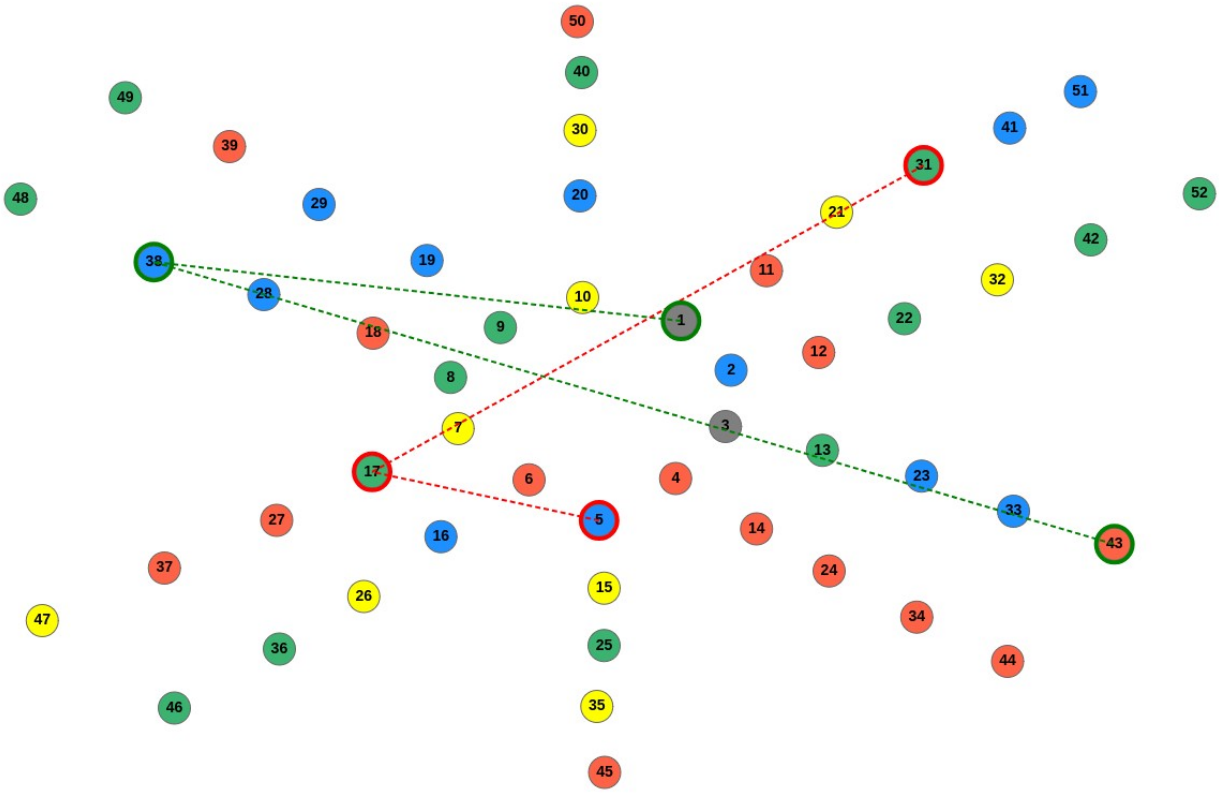
M50= (M02 + M21 + M37 + M49) - (M05 + M16 + M18)



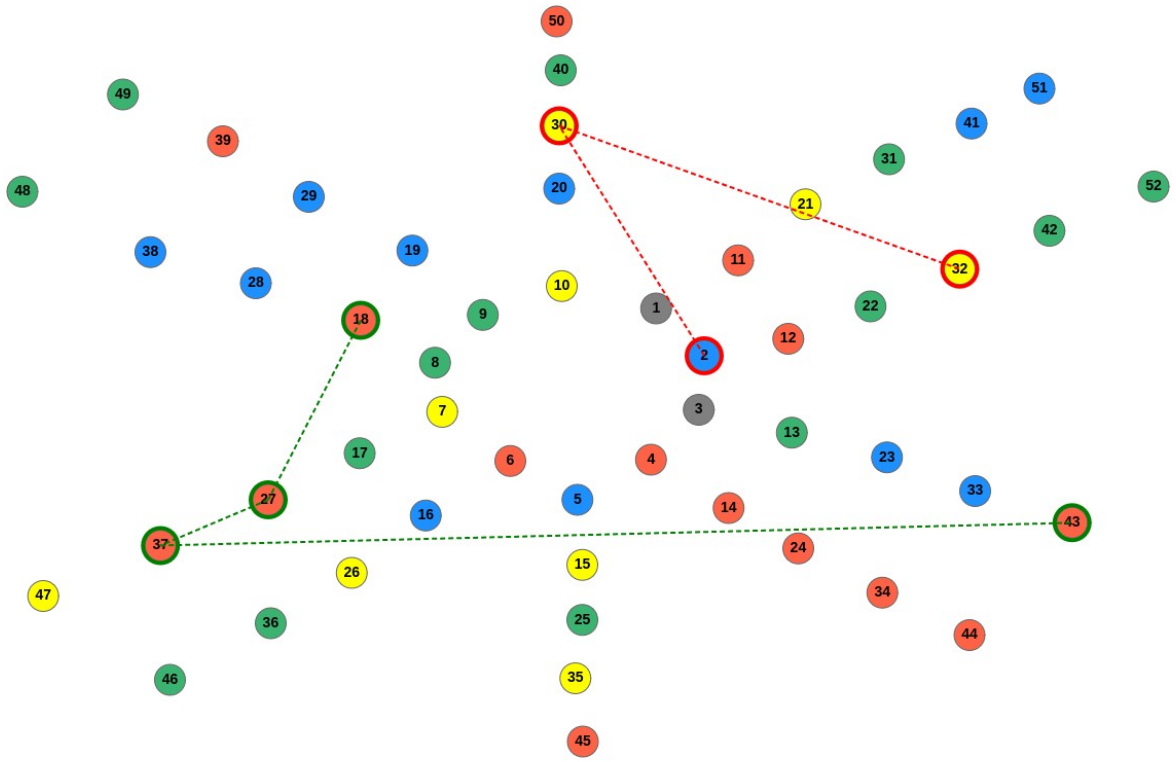
M47= (M11 + M39 + M43) - (M01 + M08 + M32)



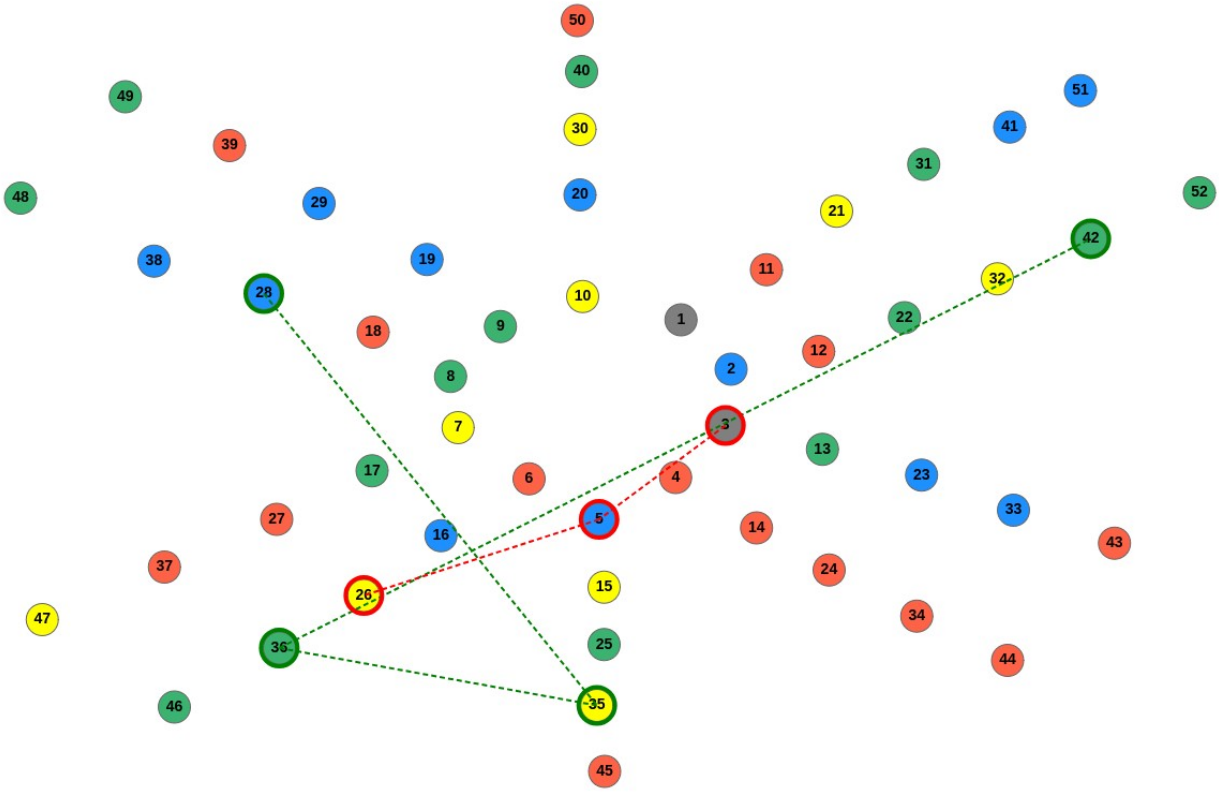
M45= (M01 + M38 + M43) - (M05 + M17 + M31)



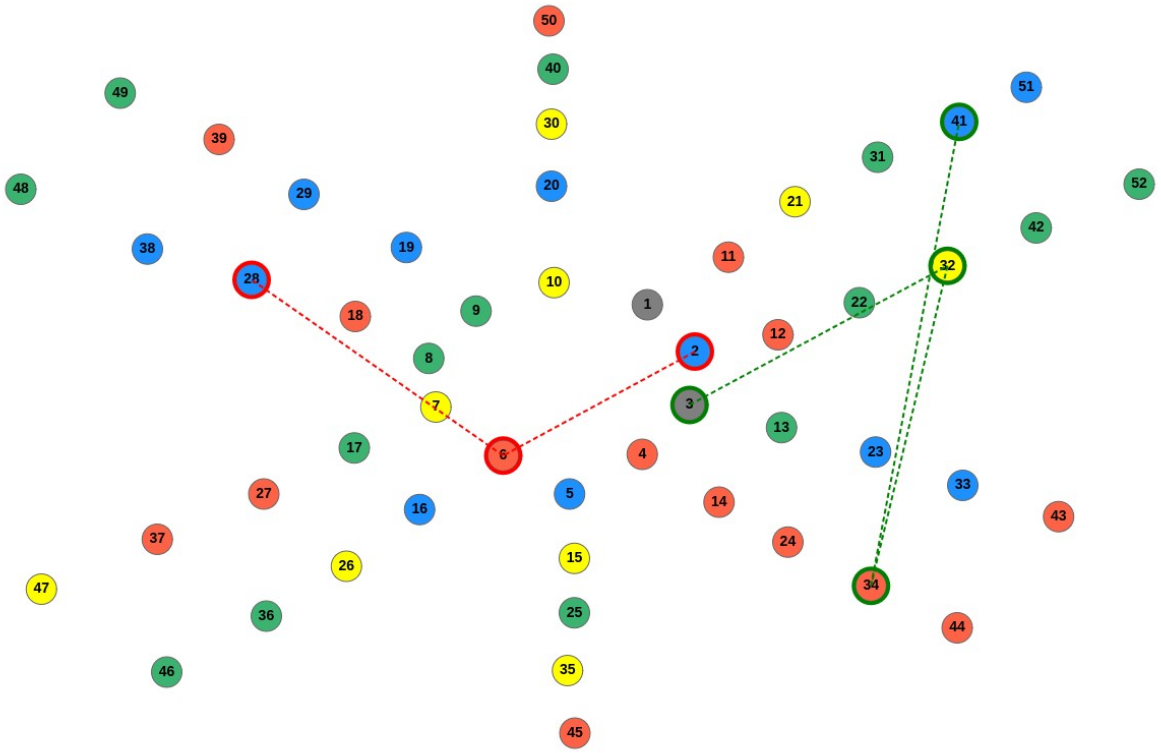
M44= (M18 + M27 + M37 + M43) - (M02 + M30 + M32)



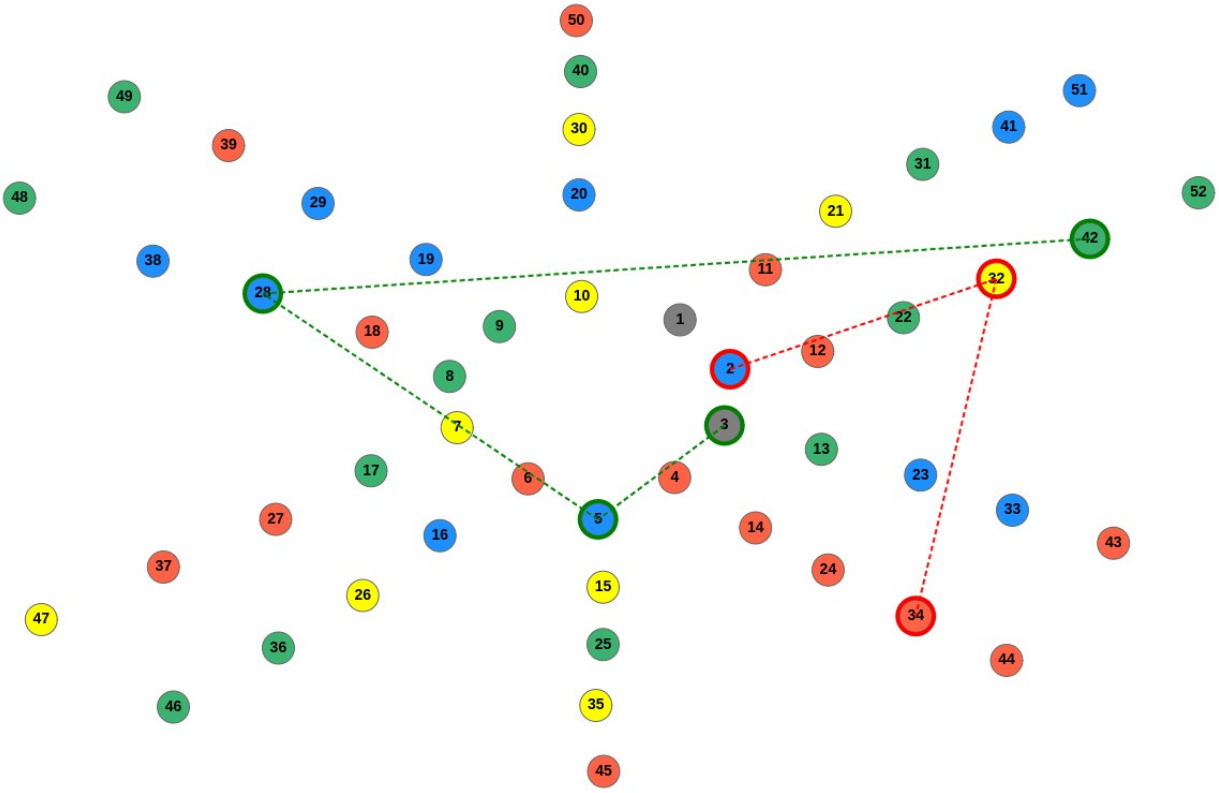
M43= (M28 + M35 + M36 + M42) - (M03 + M05 + M26)



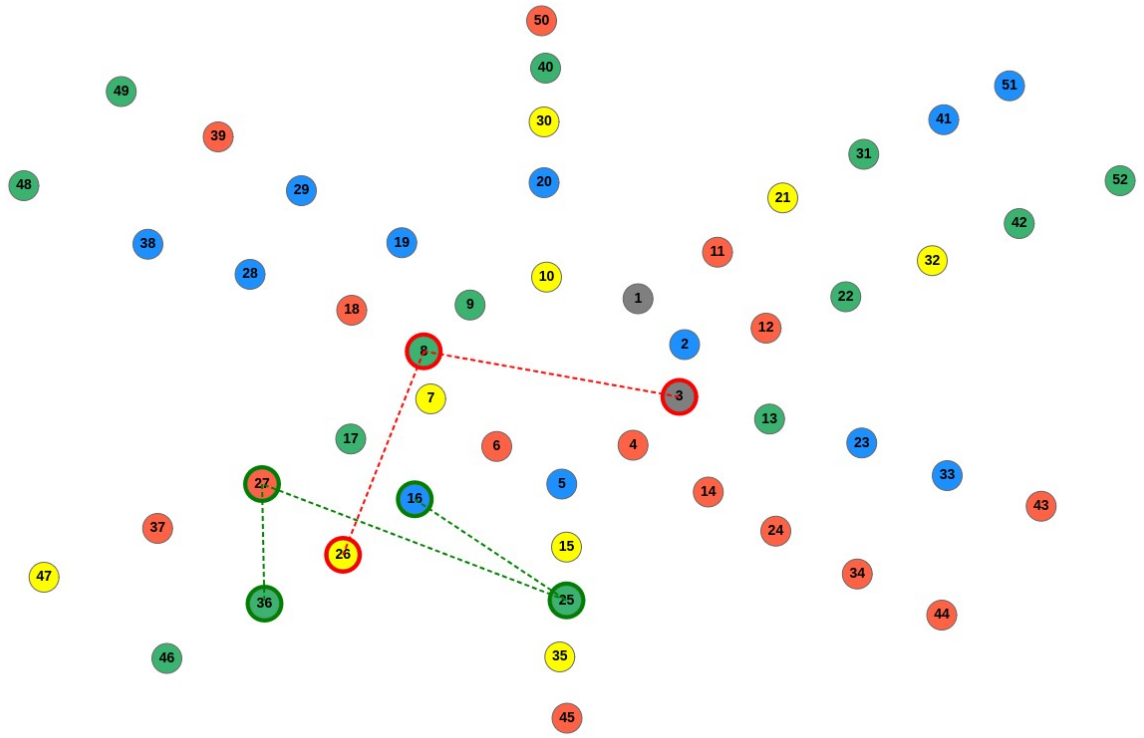
M42= (M03 + M32 + M34 + M41) - (M02 + M06 + M28)



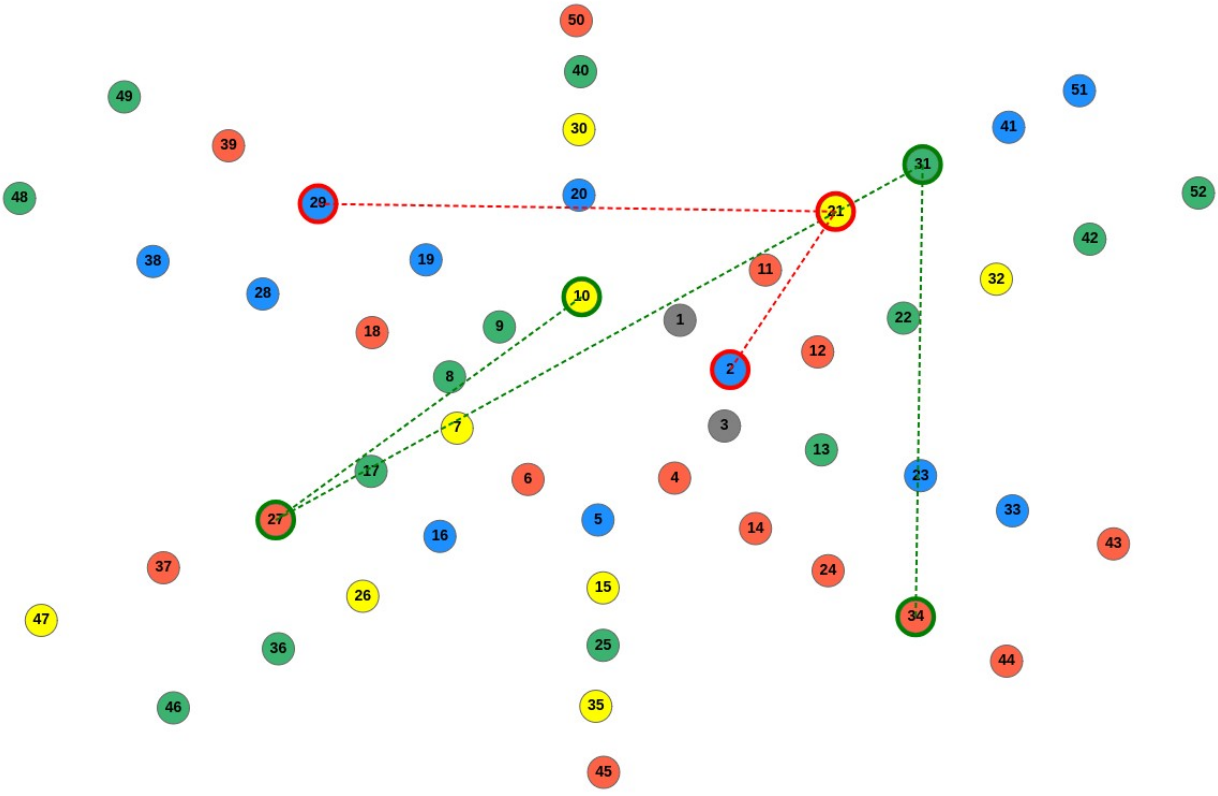
M41= (M03 + M05 + M28 + M42) - (M02 + M32 + M34)



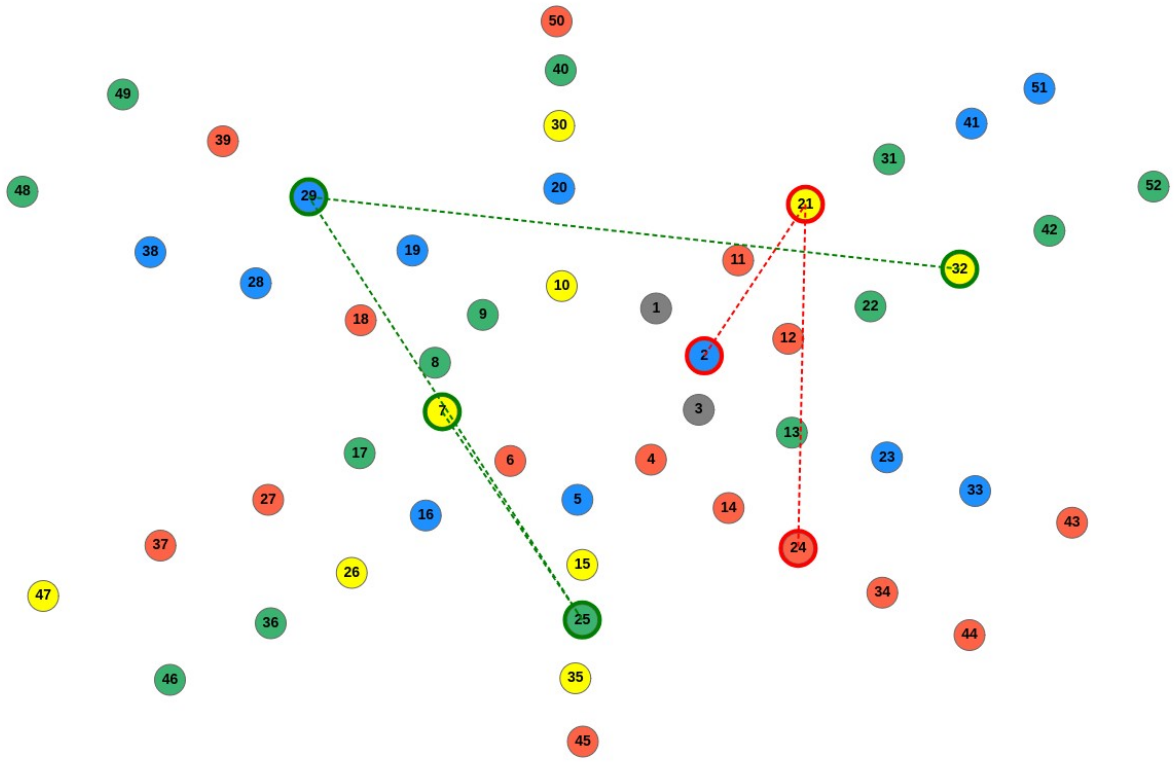
M37= (M16 + M25 + M27 + M36) - (M03 + M08 + M26)



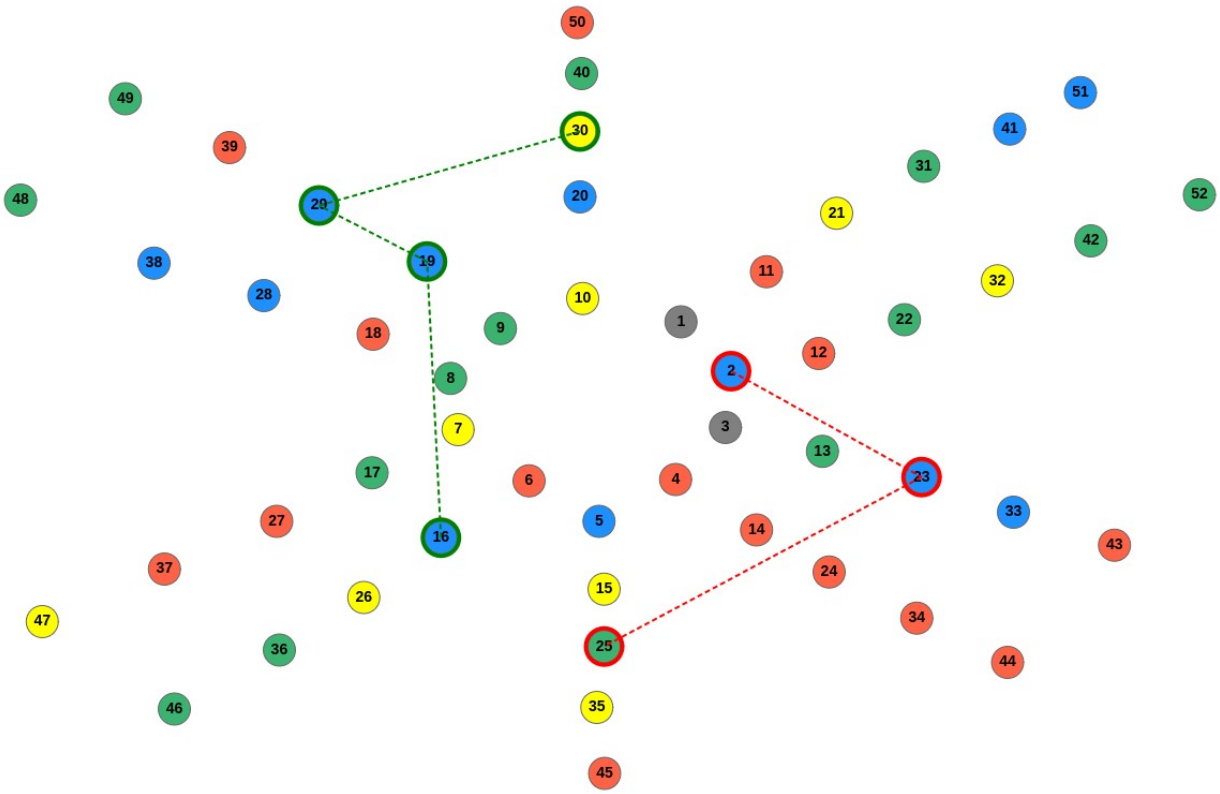
M35= (M10 + M27 + M31 + M34) - (M02 + M21 + M29)



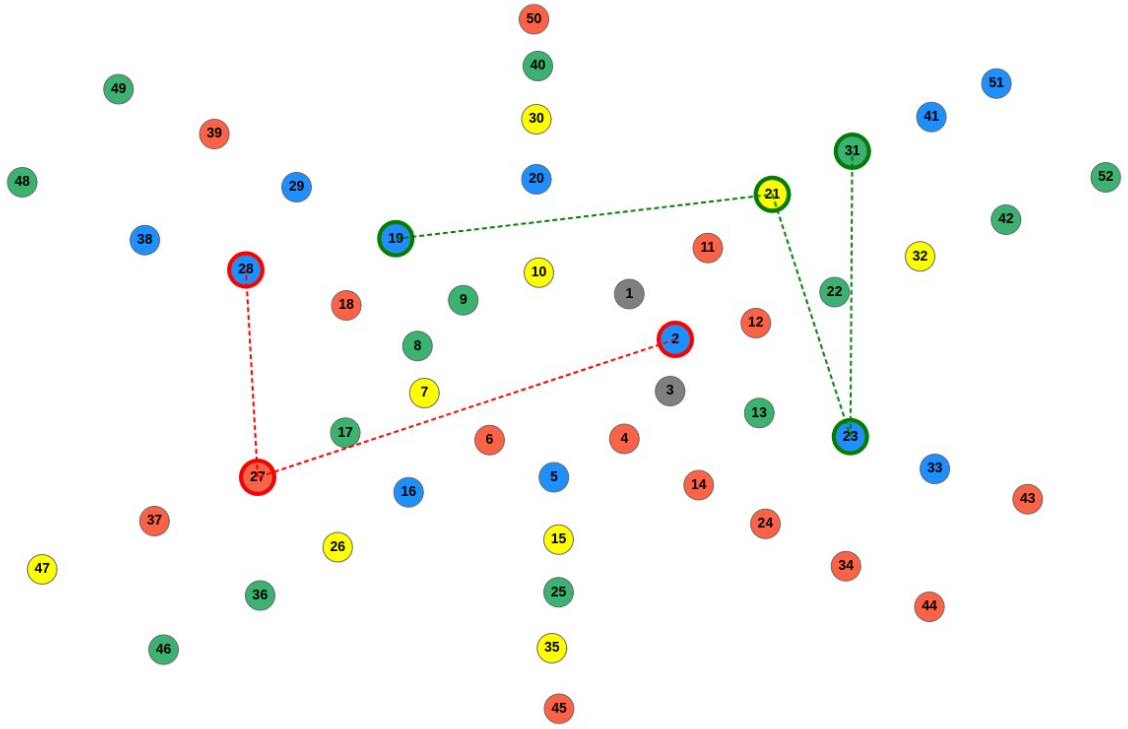
M33= (M07 + M25 + M29 + M32) - (M02 + M21 + M24)



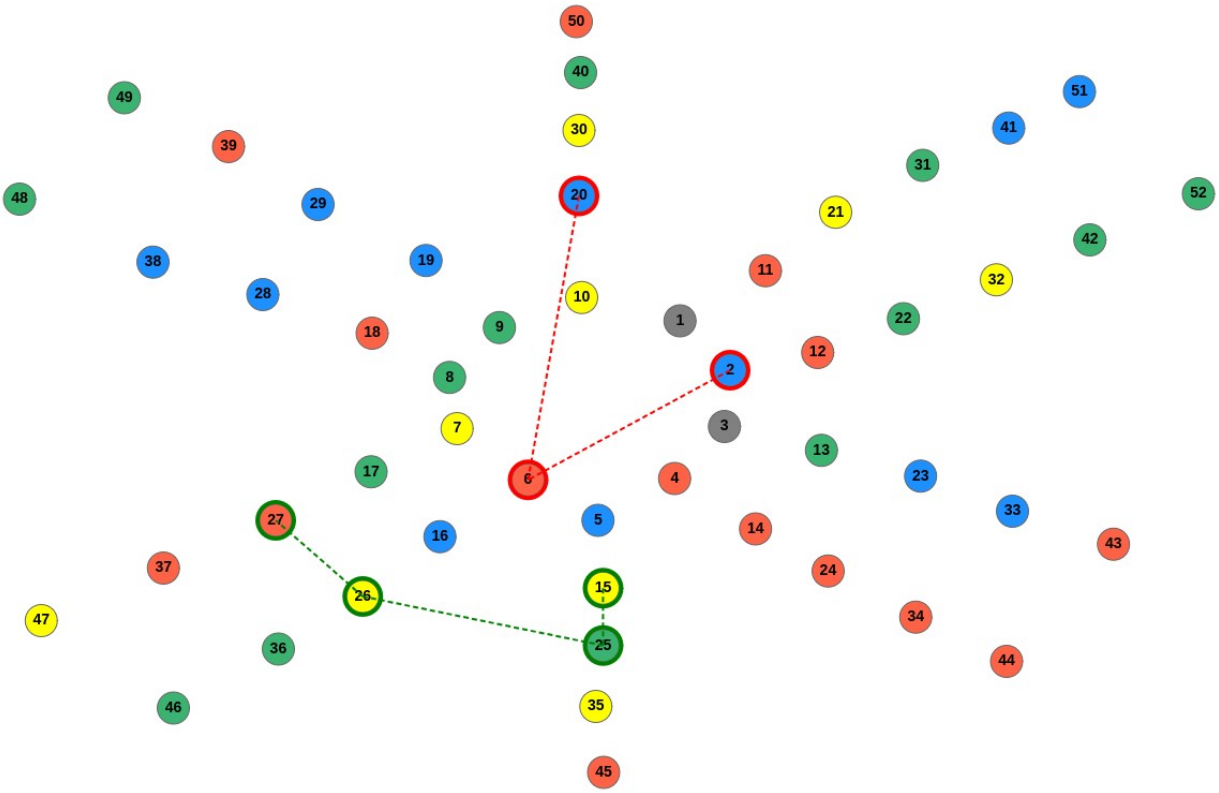
M31= (M16 + M19 + M29 + M30) - (M02 + M23 + M25)



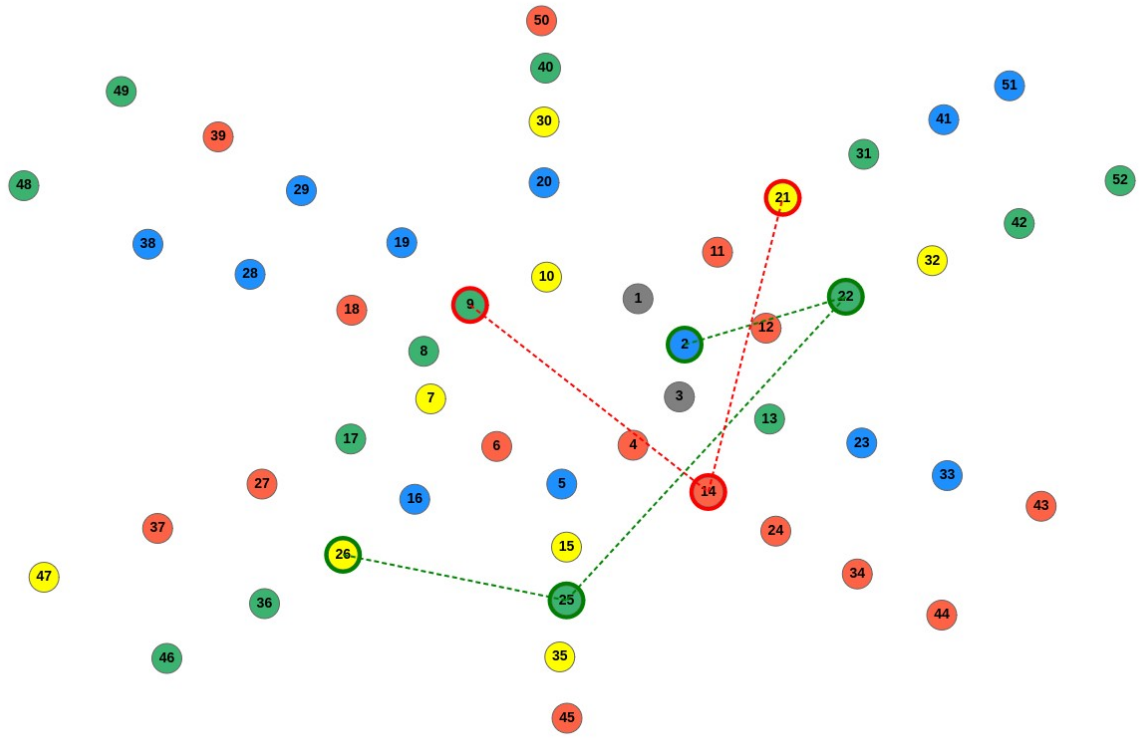
M29= (M19 + M21 + M23 + M31) - (M02 + M27 + M28)



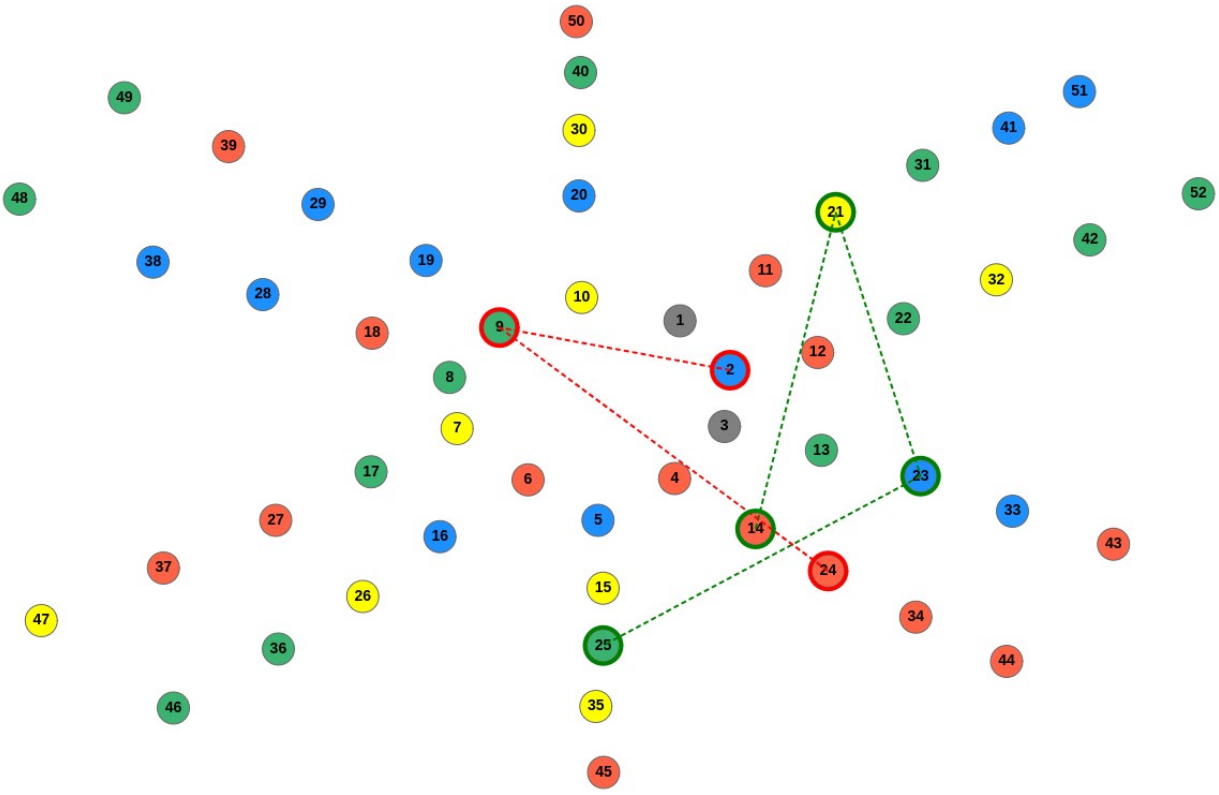
M28= (M15 + M25 + M26 + M27) - (M02 + M06 + M20)



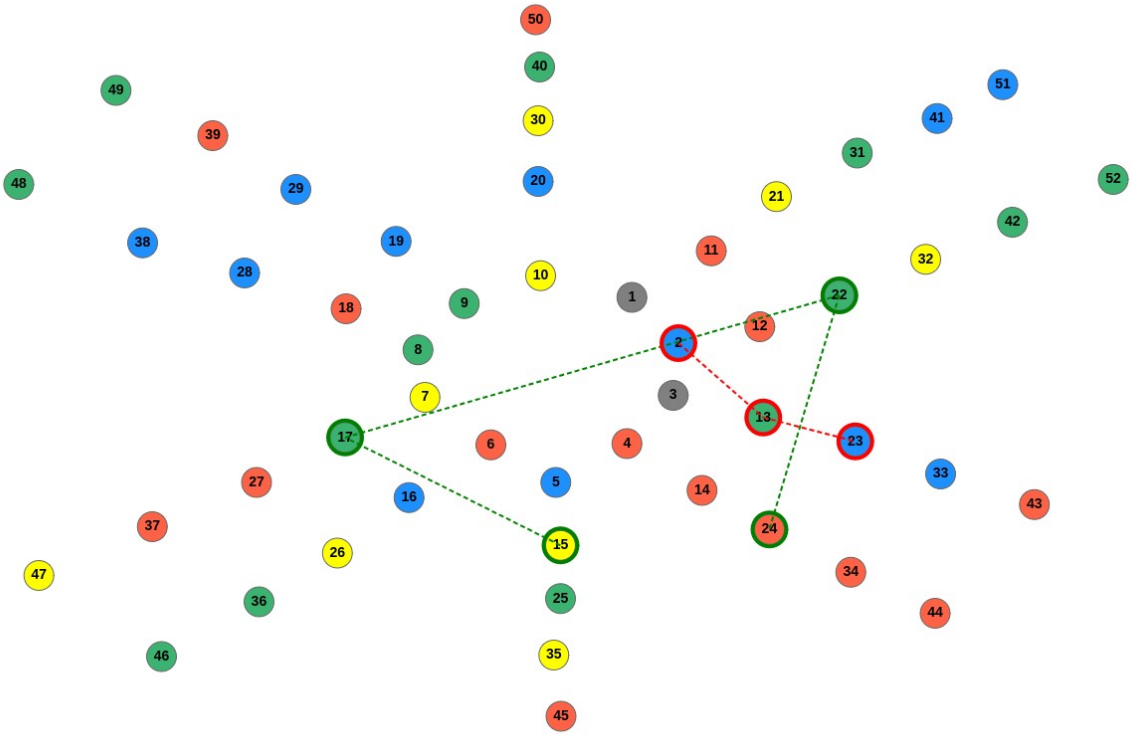
M27= (M02 + M22 + M25 + M26) - (M09 + M14 + M21)



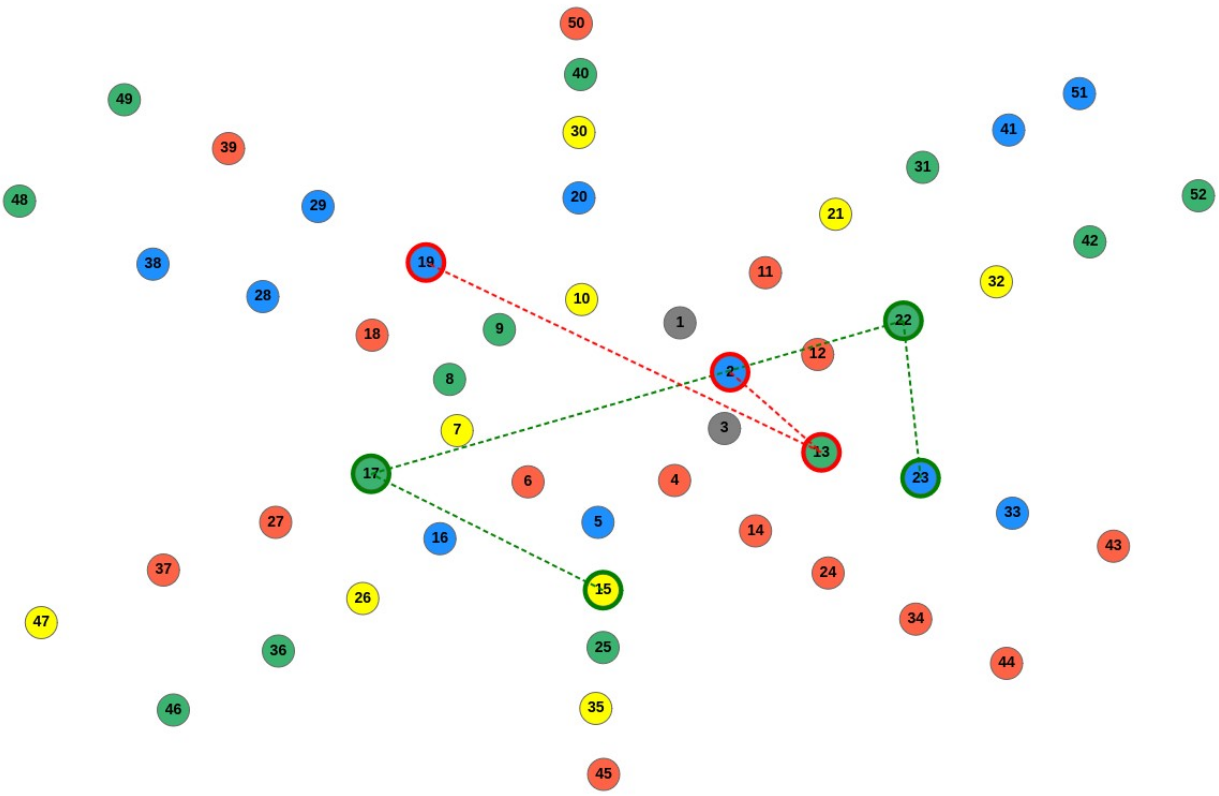
M26= (M14 + M21 + M23 + M25) - (M02 + M09 + M24)



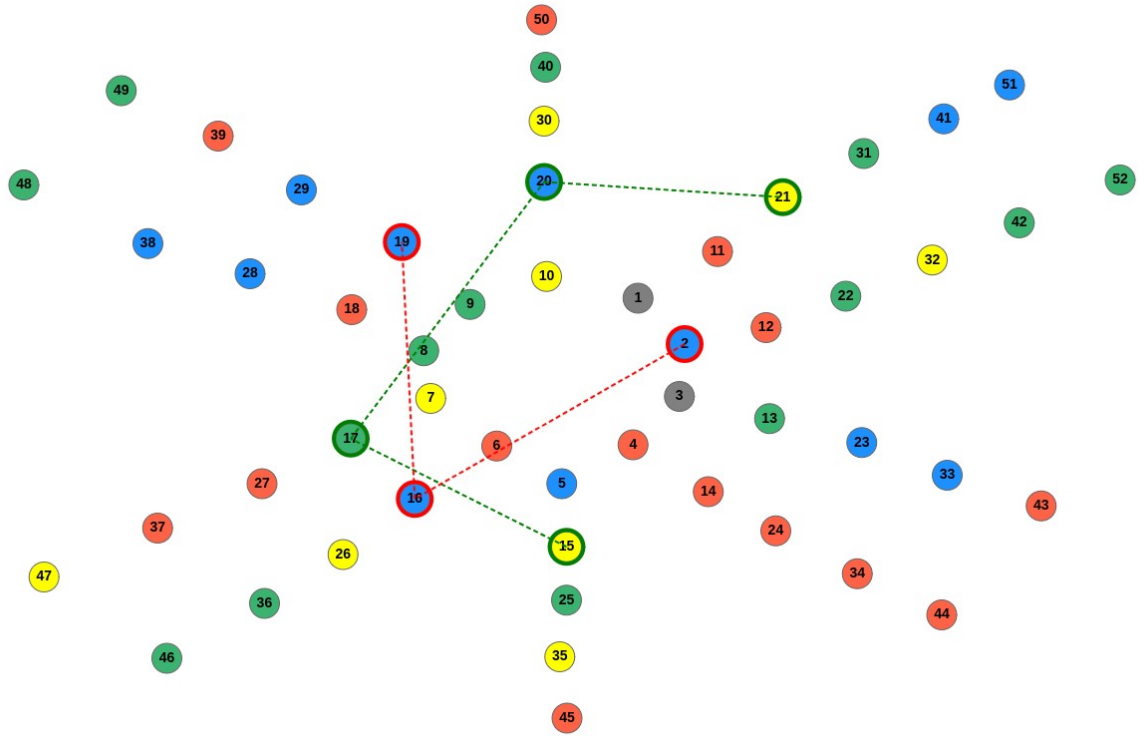
M25= (M15 + M17 + M22 + M24) - (M02 + M13 + M23)



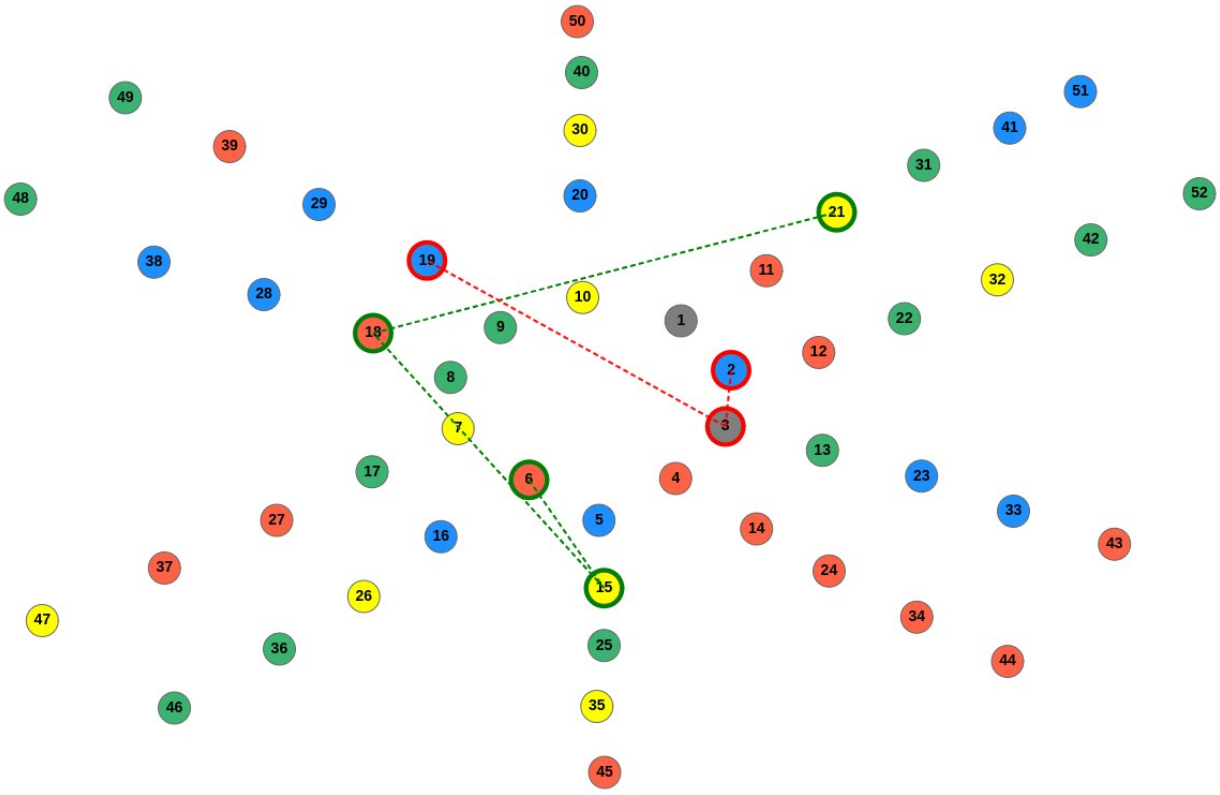
M24= (M15 + M17 + M22 + M23) - (M02 + M13 + M19)



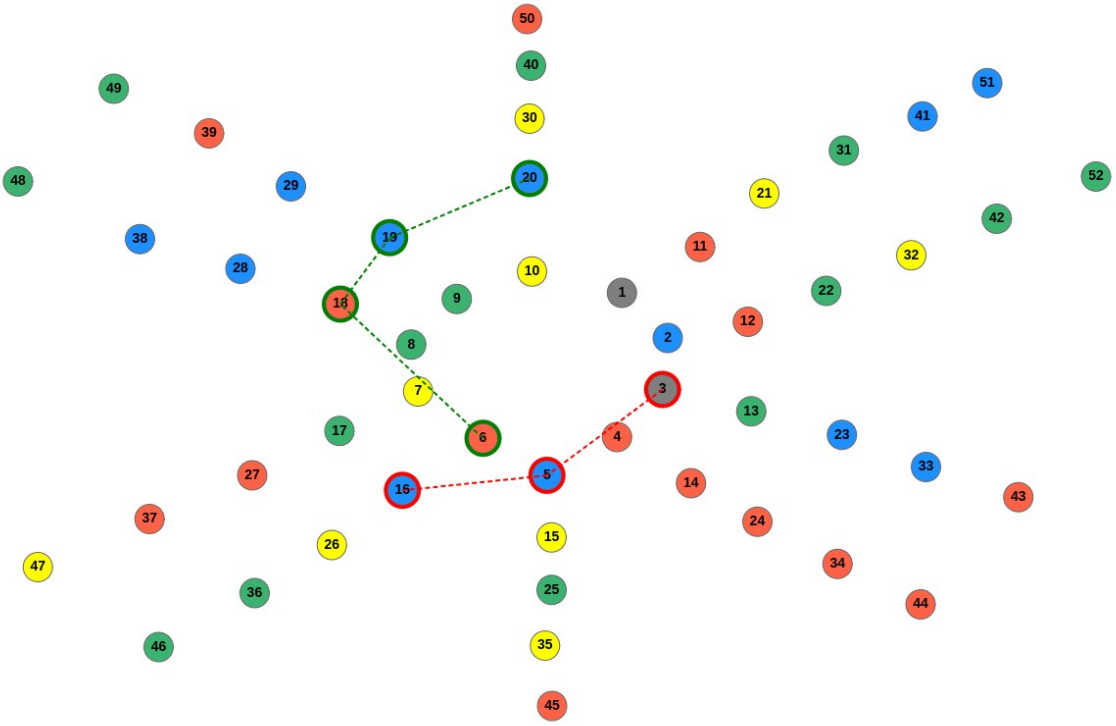
M23= (M15 + M17 + M20 + M21) - (M02 + M16 + M19)



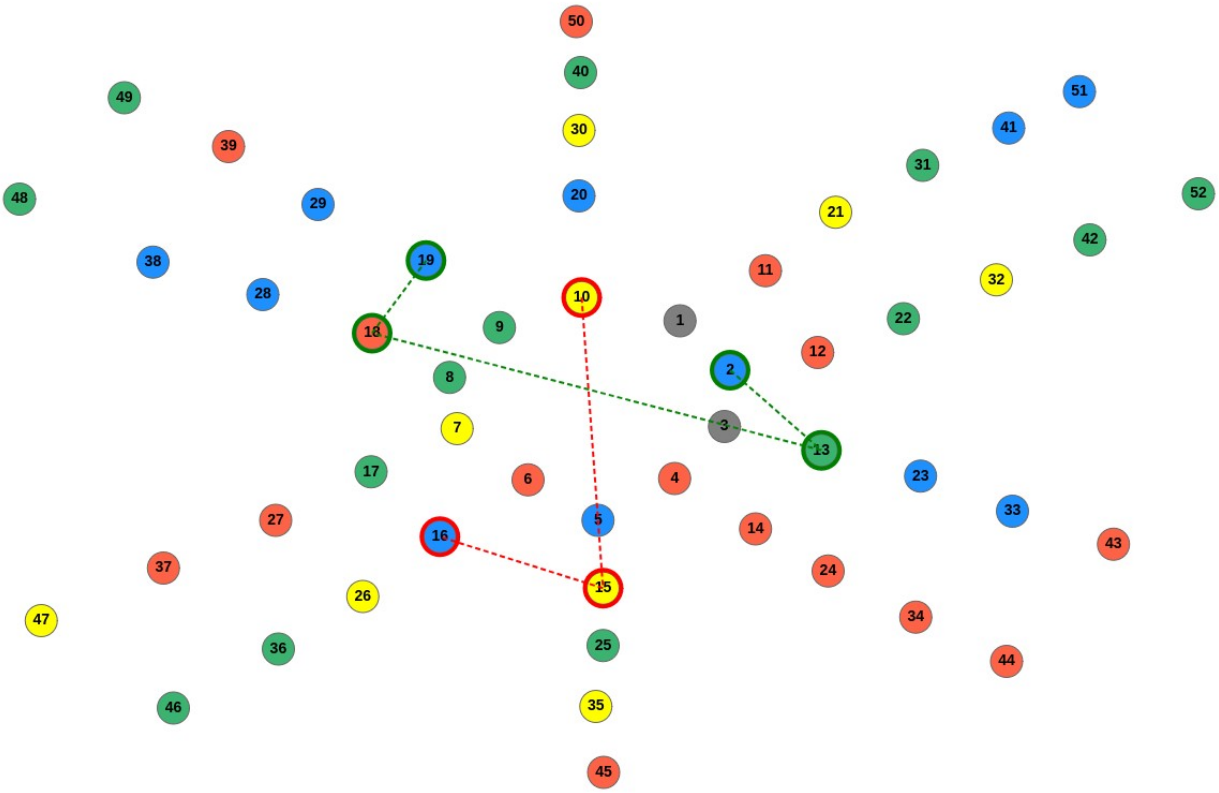
M22= (M06 + M15 + M18 + M21) - (M02 + M03 + M19)



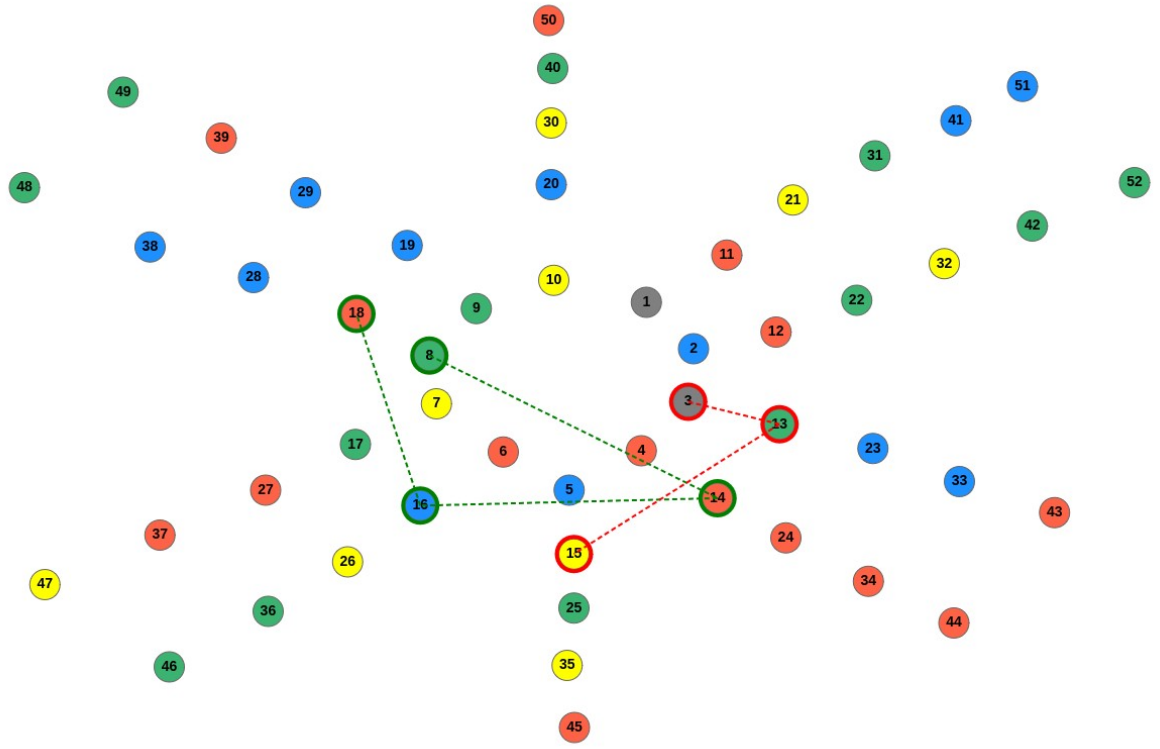
M21= (M06 + M18 + M19 + M20) - (M03 + M05 + M16)



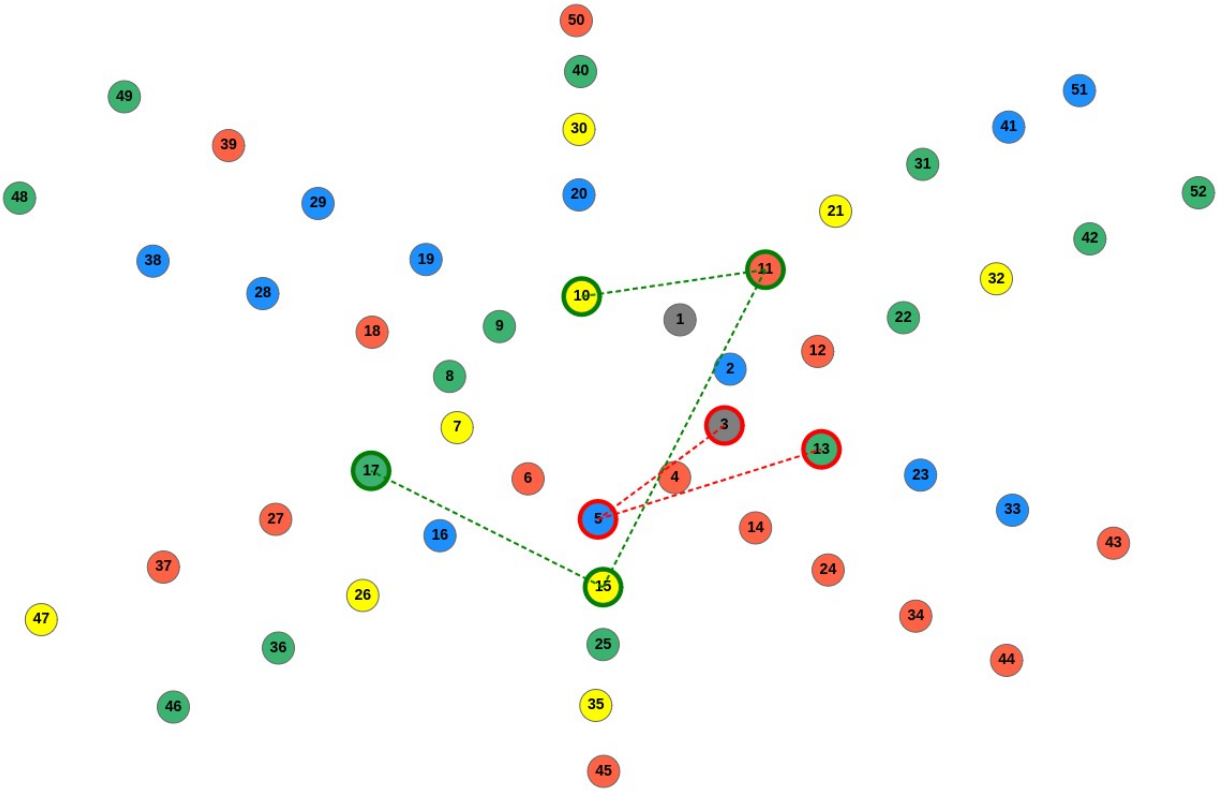
M20= (M02 + M13 + M18 + M19) - (M10 + M15 + M16)



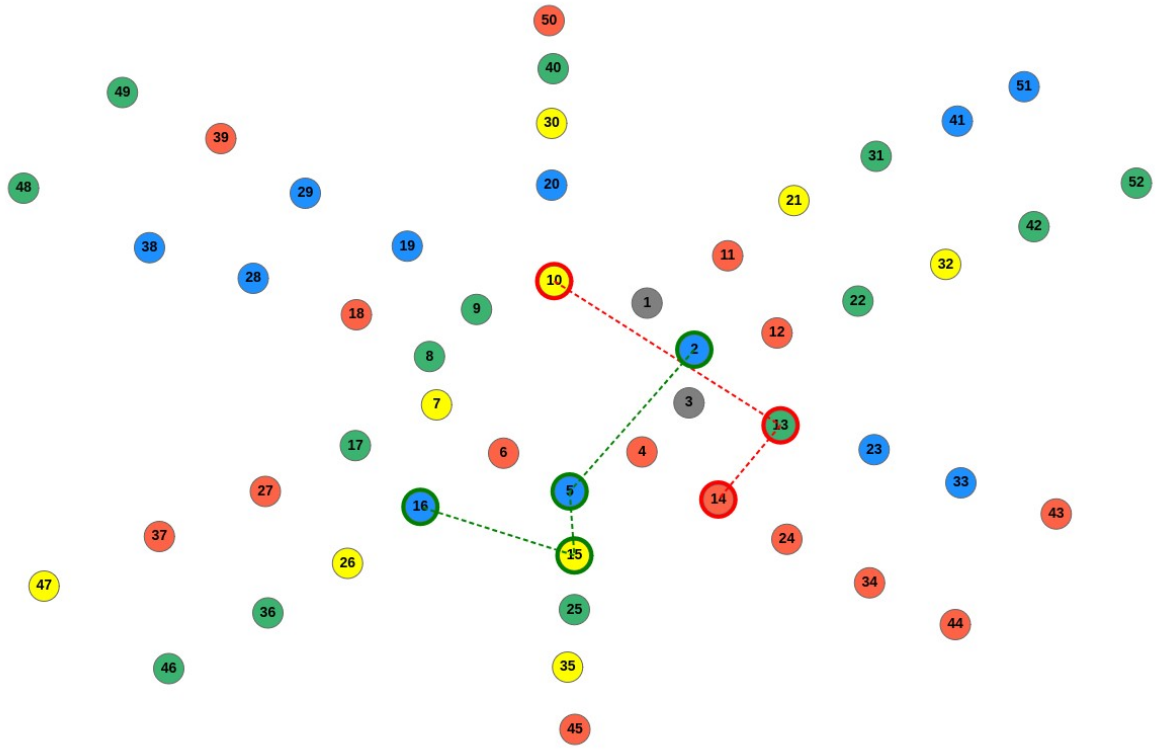
M19= (M08 + M14 + M16 + M18) - (M03 + M13 + M15)



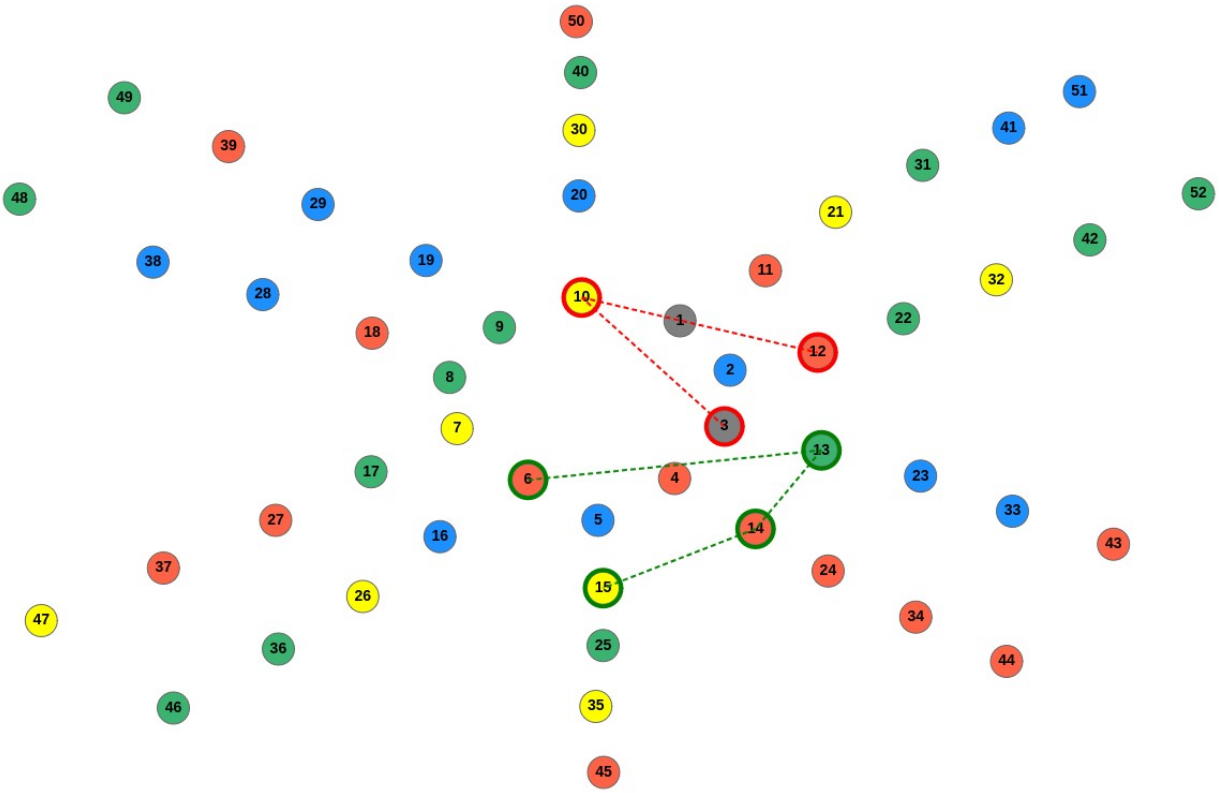
M18= (M10 + M11 + M15 + M17) - (M03 + M05 + M13)



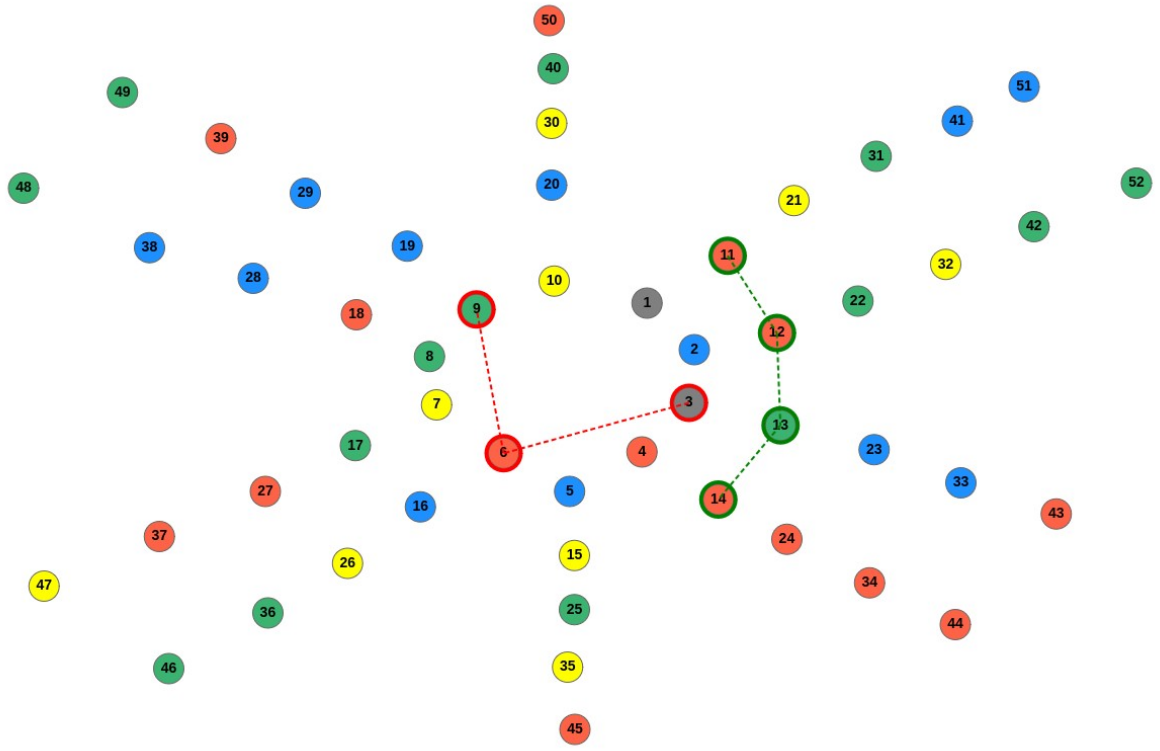
M17= (M02 + M05 + M15 + M16) - (M10 + M13 + M14)



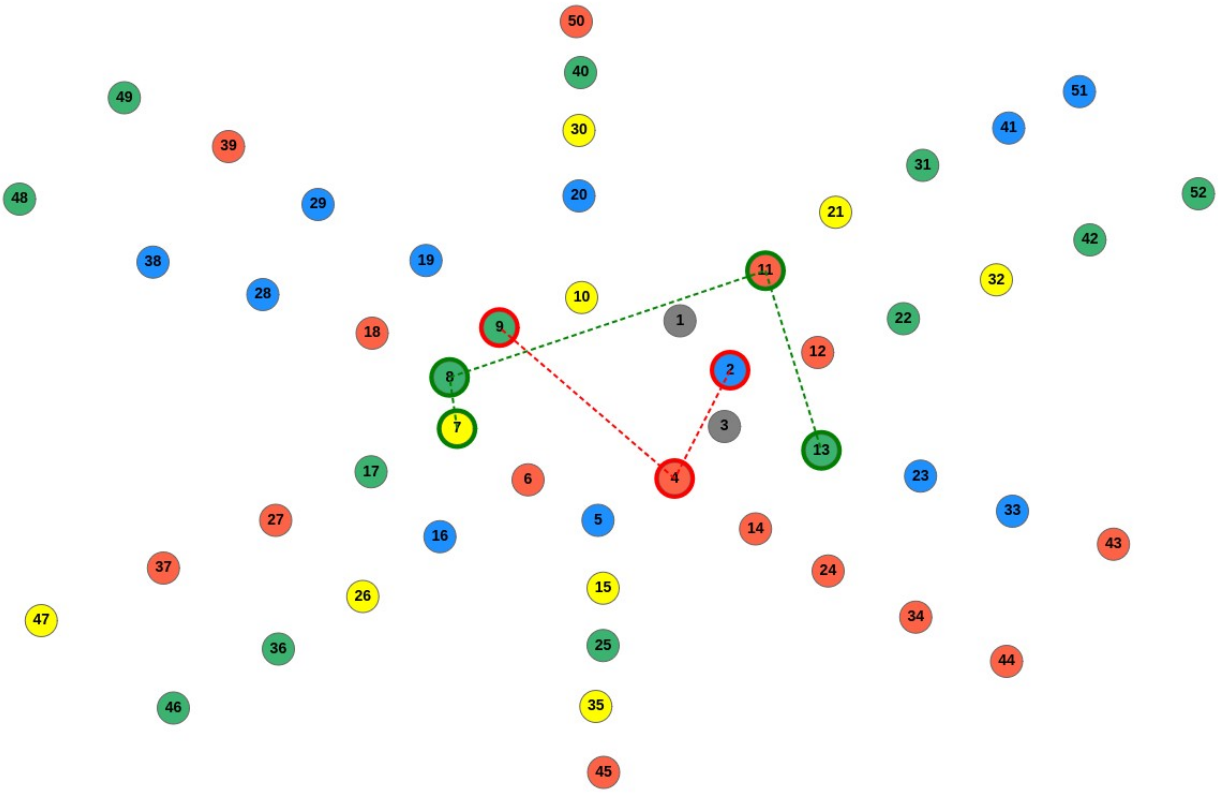
M16= (M06 + M13 + M14 + M15) - (M03 + M10 + M12)



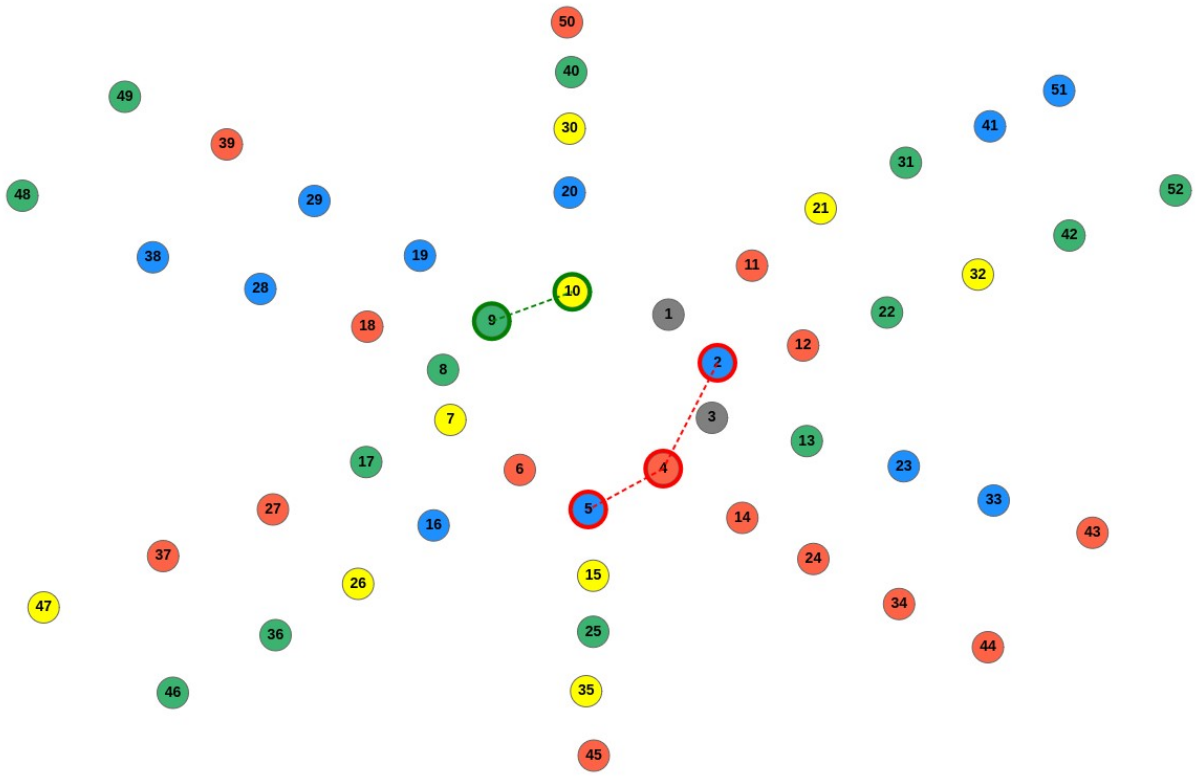
M15= (M11 + M12 + M13 + M14) - (M03 + M06 + M09)



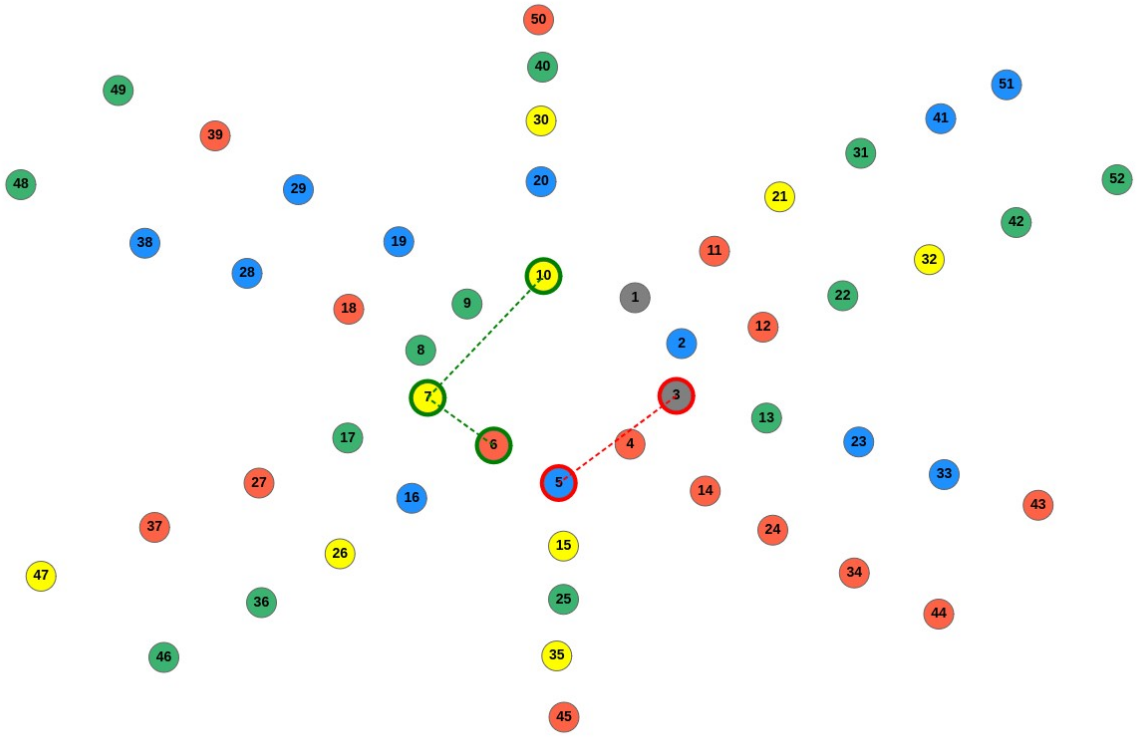
M14= (M07 + M08 + M11 + M13) - (M02 + M04 + M09)



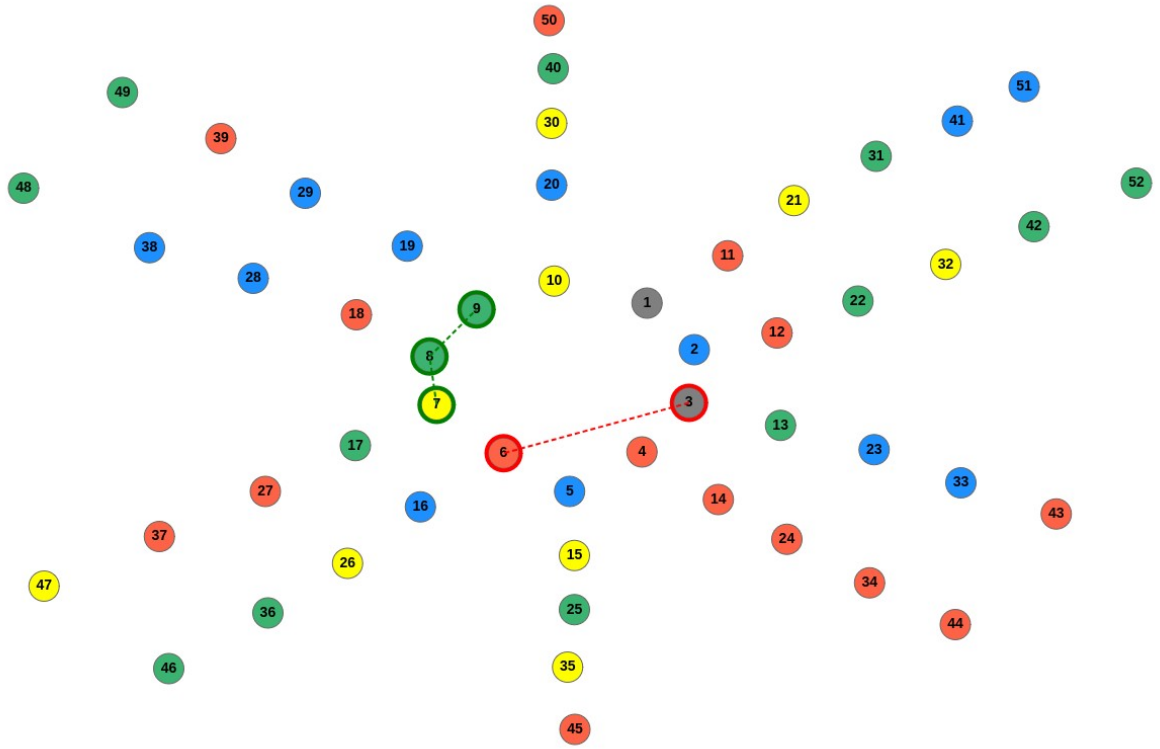
M12= (M09 + M10) - (M02 + M04 + M05)



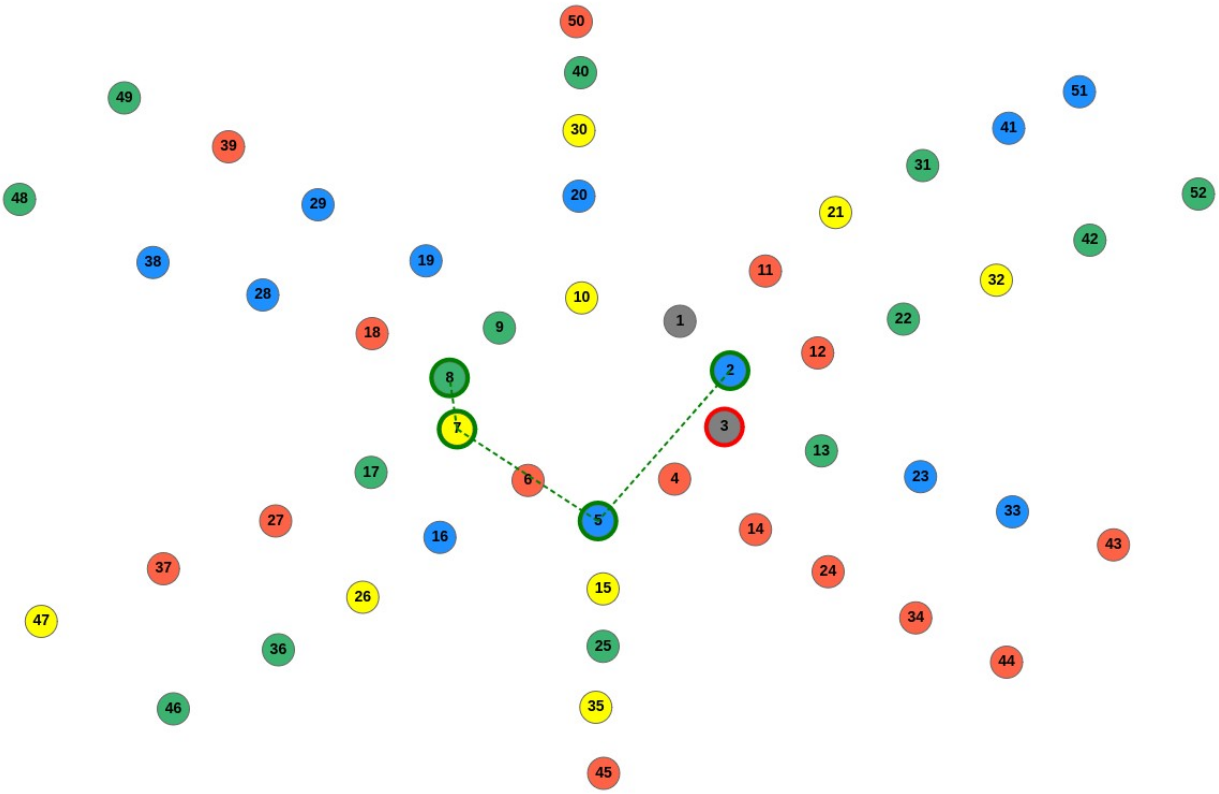
M11= (M06 + M07 + M10) - (M03 + M05)



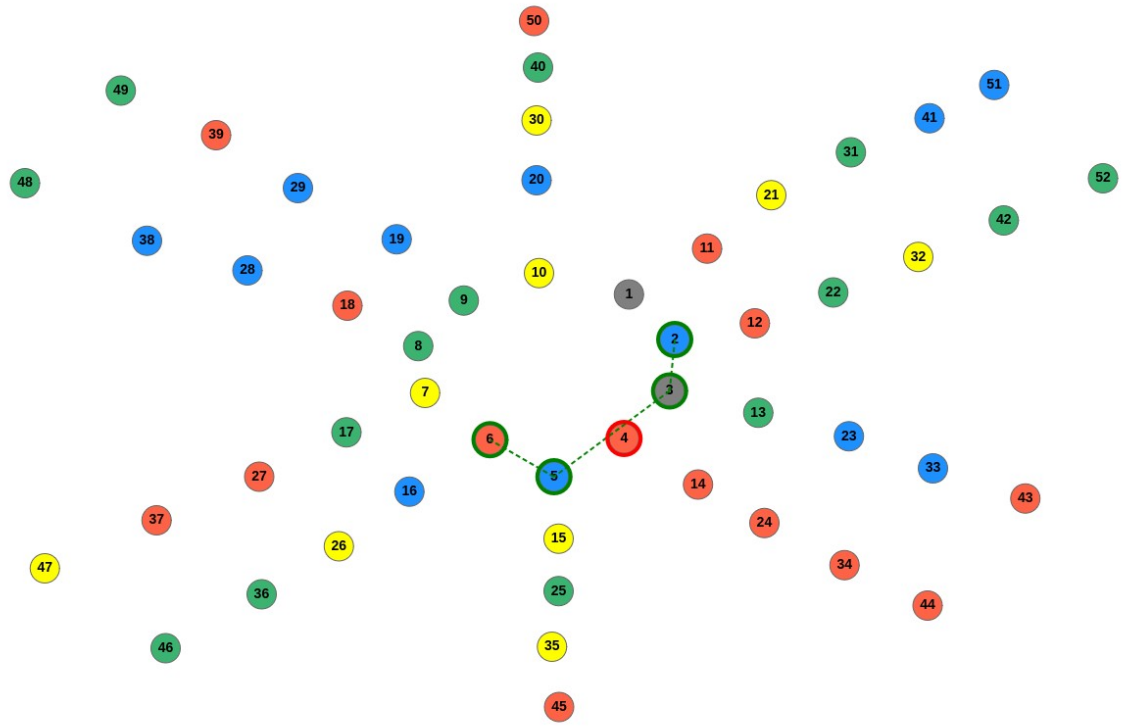
M10= (M07 + M08 + M09) - (M03 + M06)



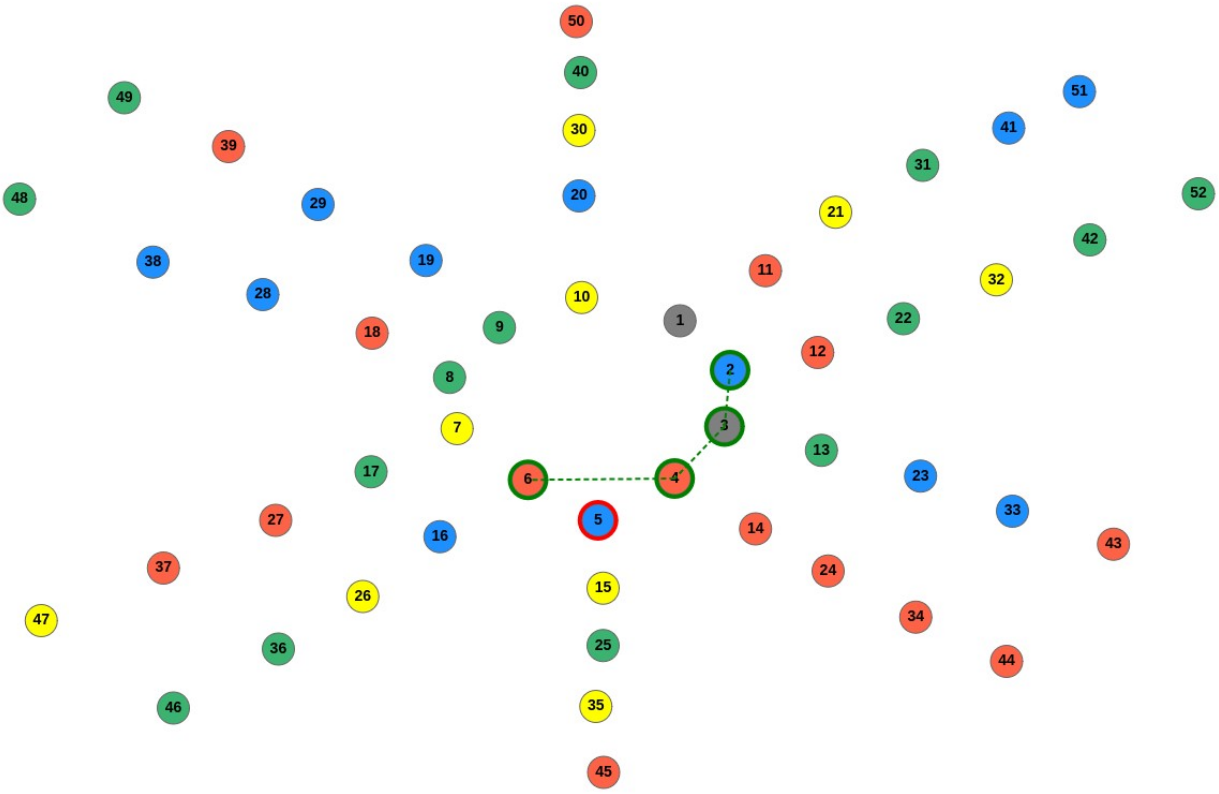
M09= (M02 + M05 + M07 + M08) – (M03)



M08= (M02 + M03 + M05 + M06) – (M04)

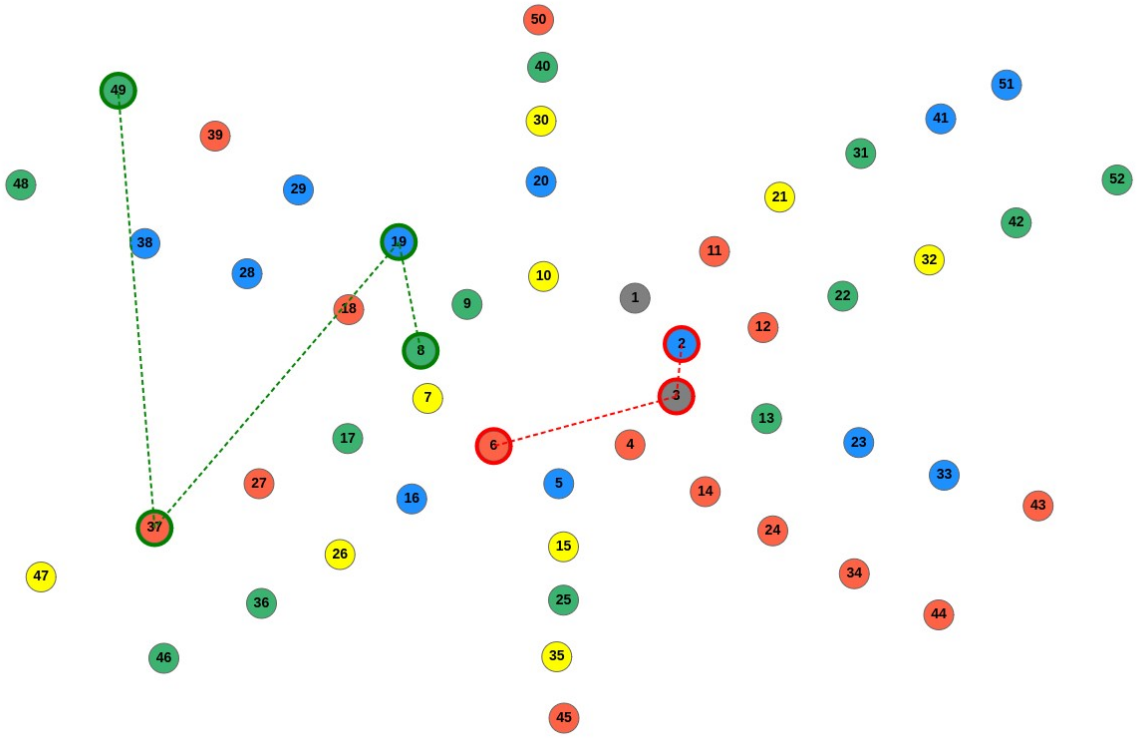


M07= (M02 + M03 + M04 + M06) – (M05)

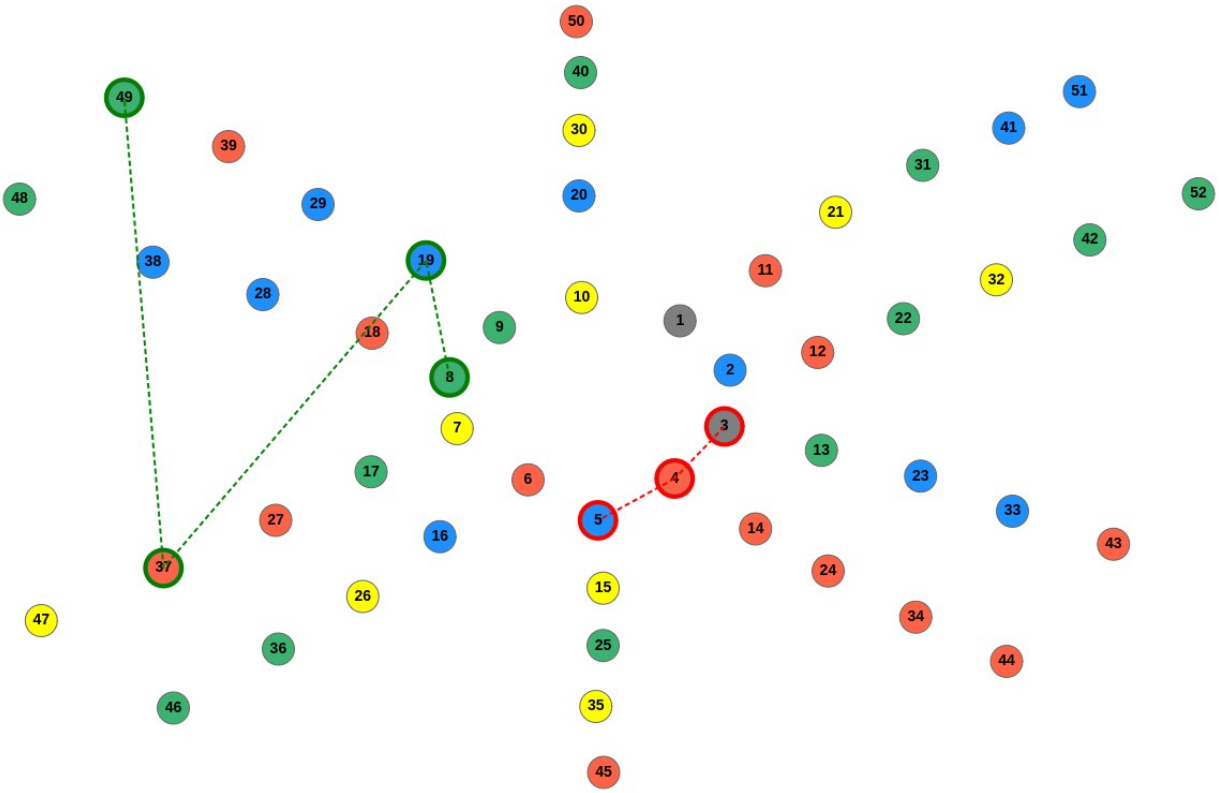


Here are some example for a same number, different decompositions:

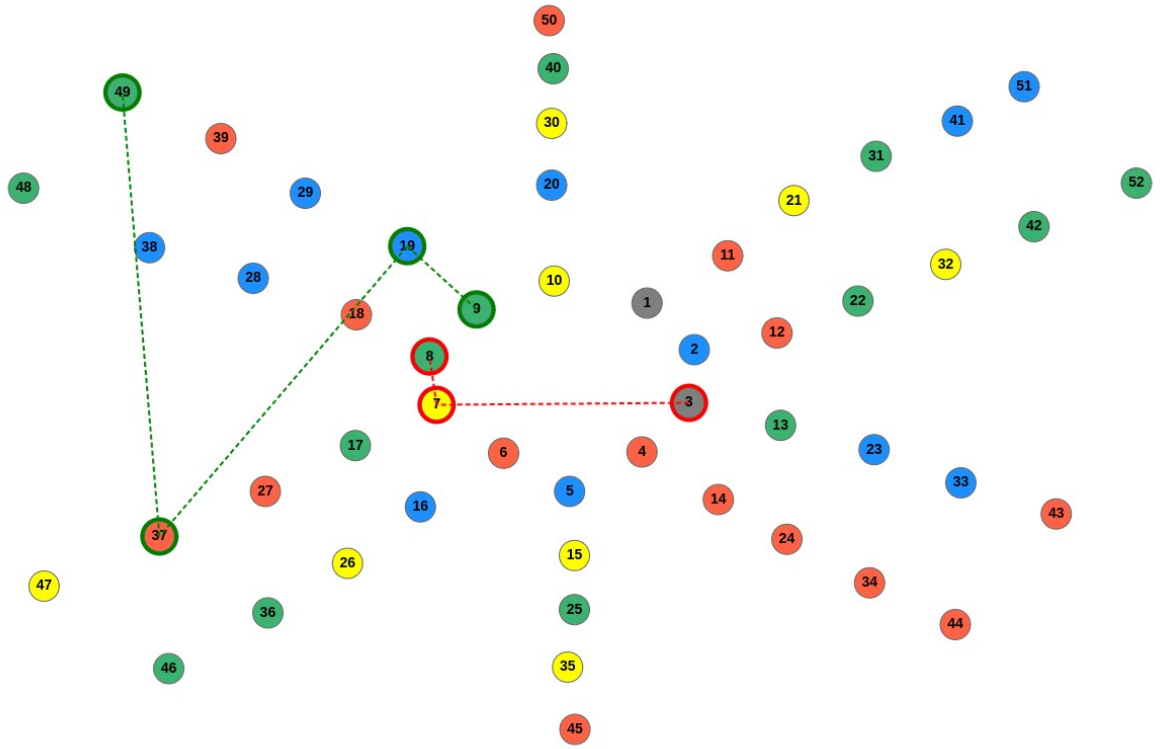
M50= (M08 + M19 + M37 + M49) - (M02 + M03 + M06)



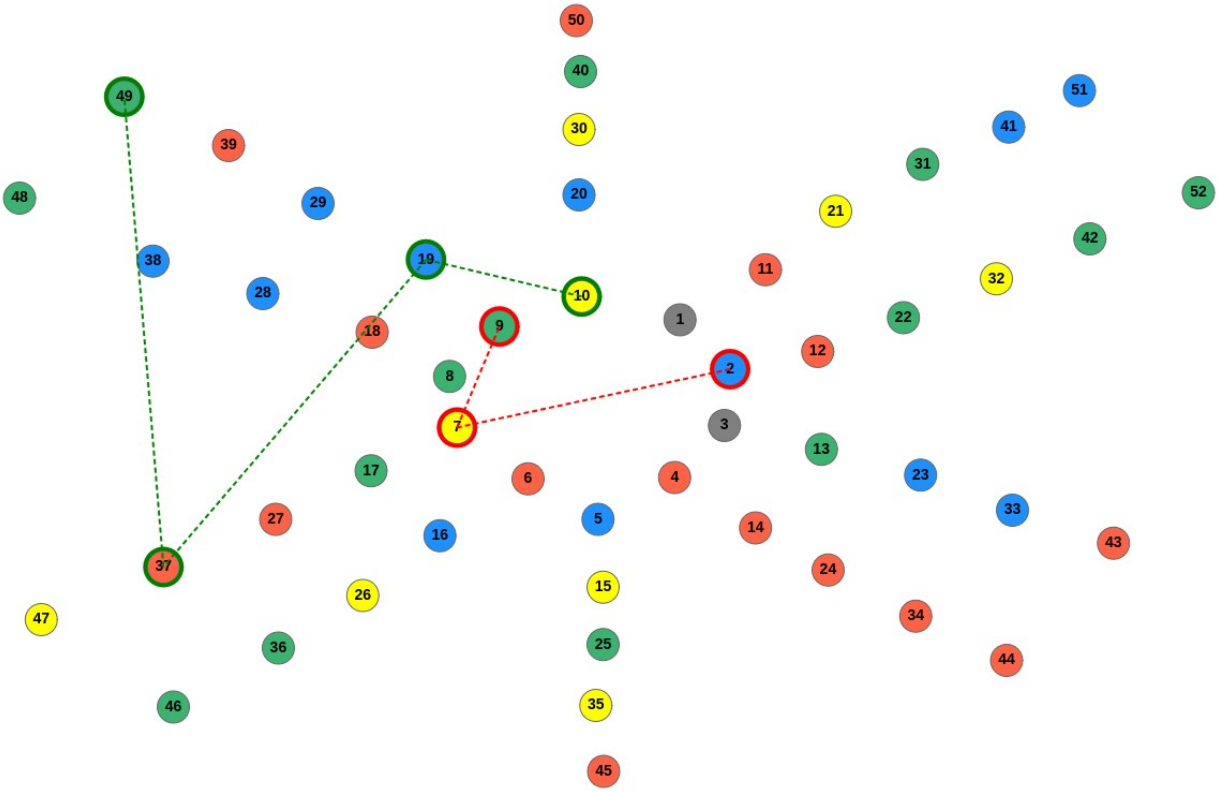
M50= (M08 + M19 + M37 + M49) - (M03 + M04 + M05)



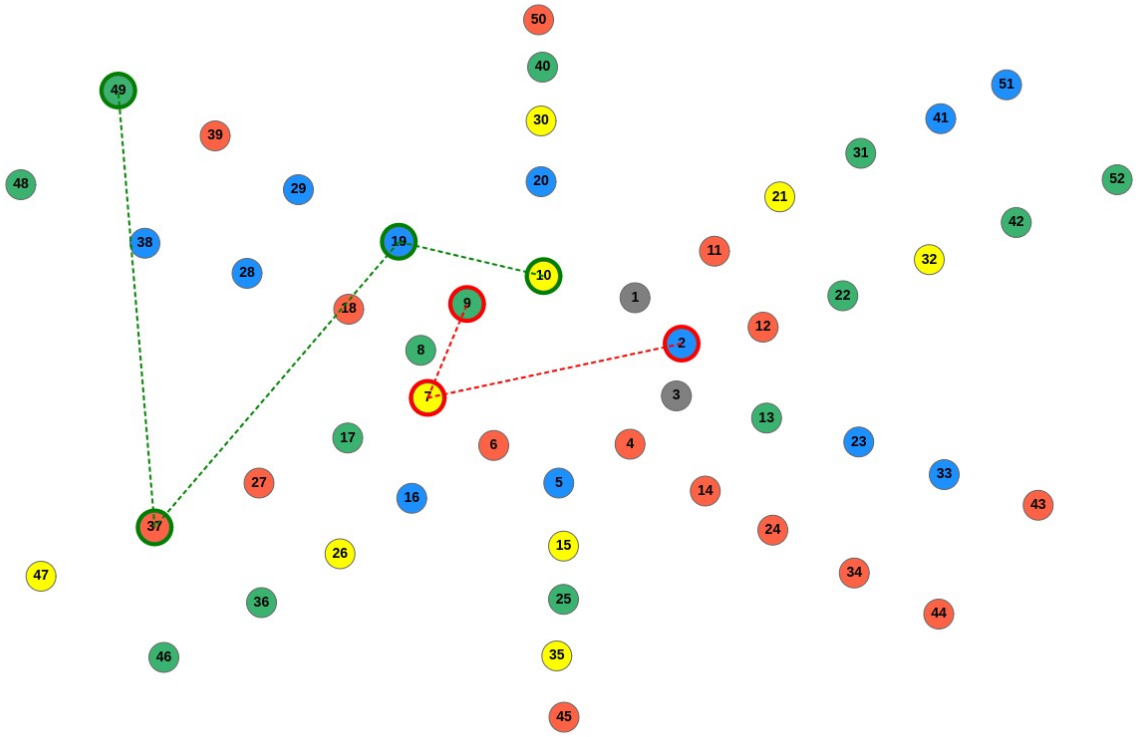
M50= (M09 + M19 + M37 + M49)- (M03 + M07 + M08)



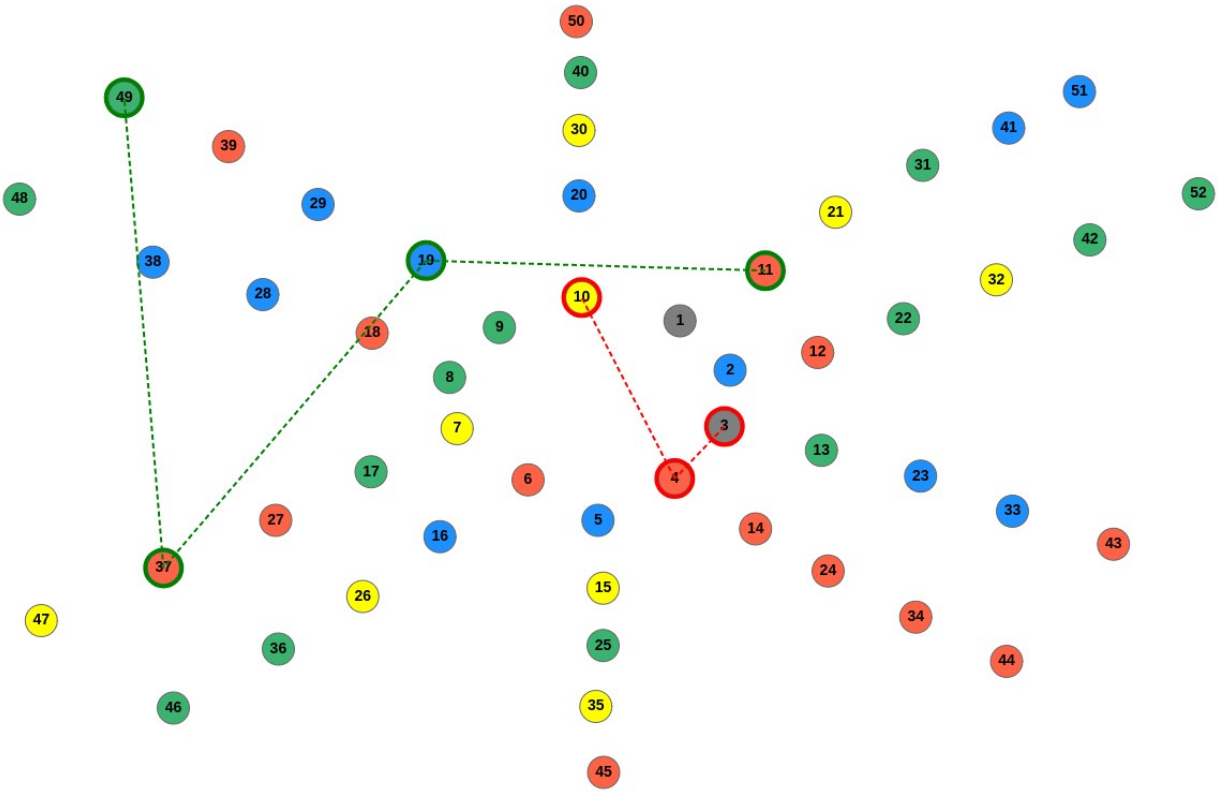
M50= (M10 + M19 + M37 + M49) - (M02 + M07 + M09)



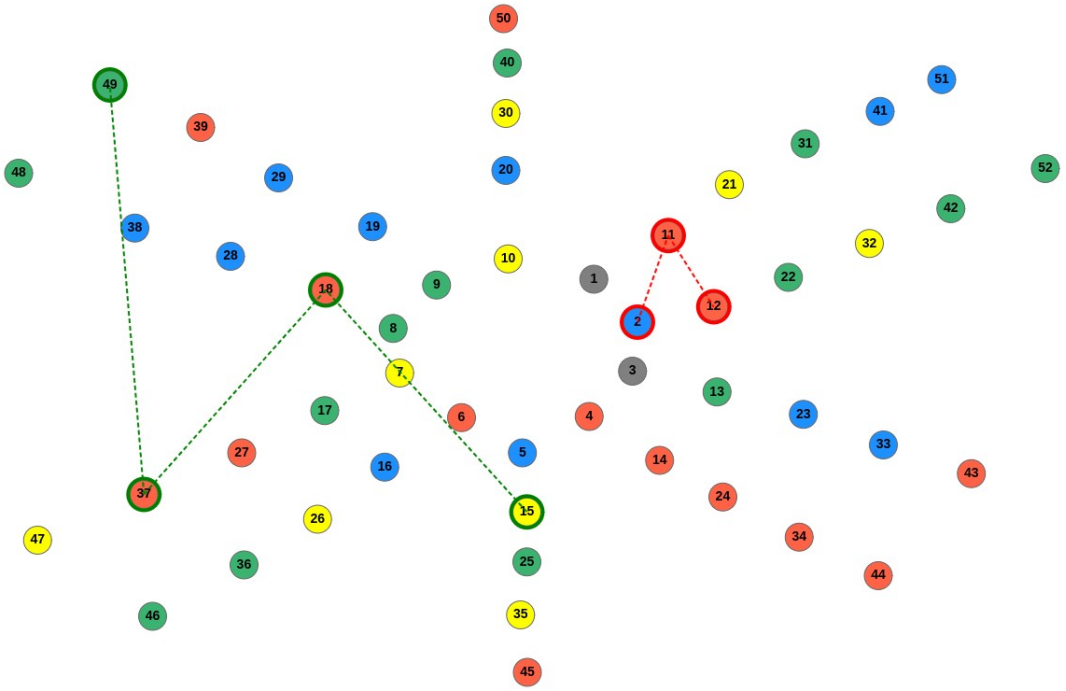
M50= (M10 + M19 + M37 + M49)- (M02 + M07 + M09)



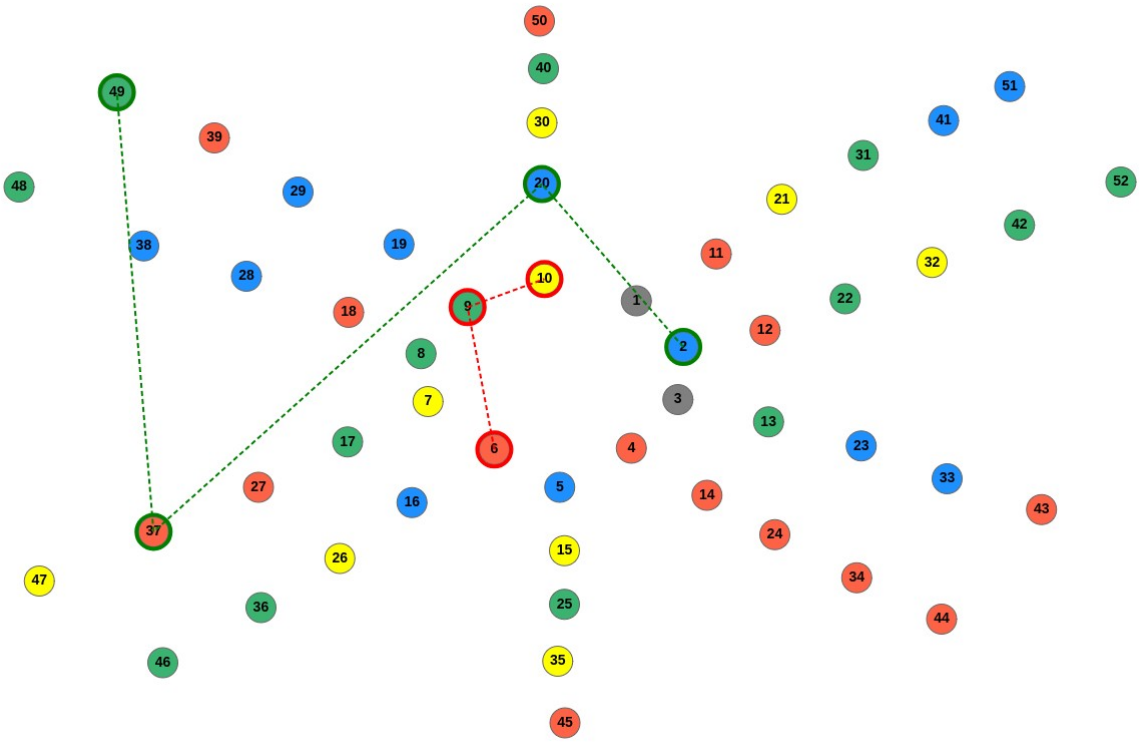
M50= (M11 + M19 + M37 + M49) - (M03 + M04 + M10)



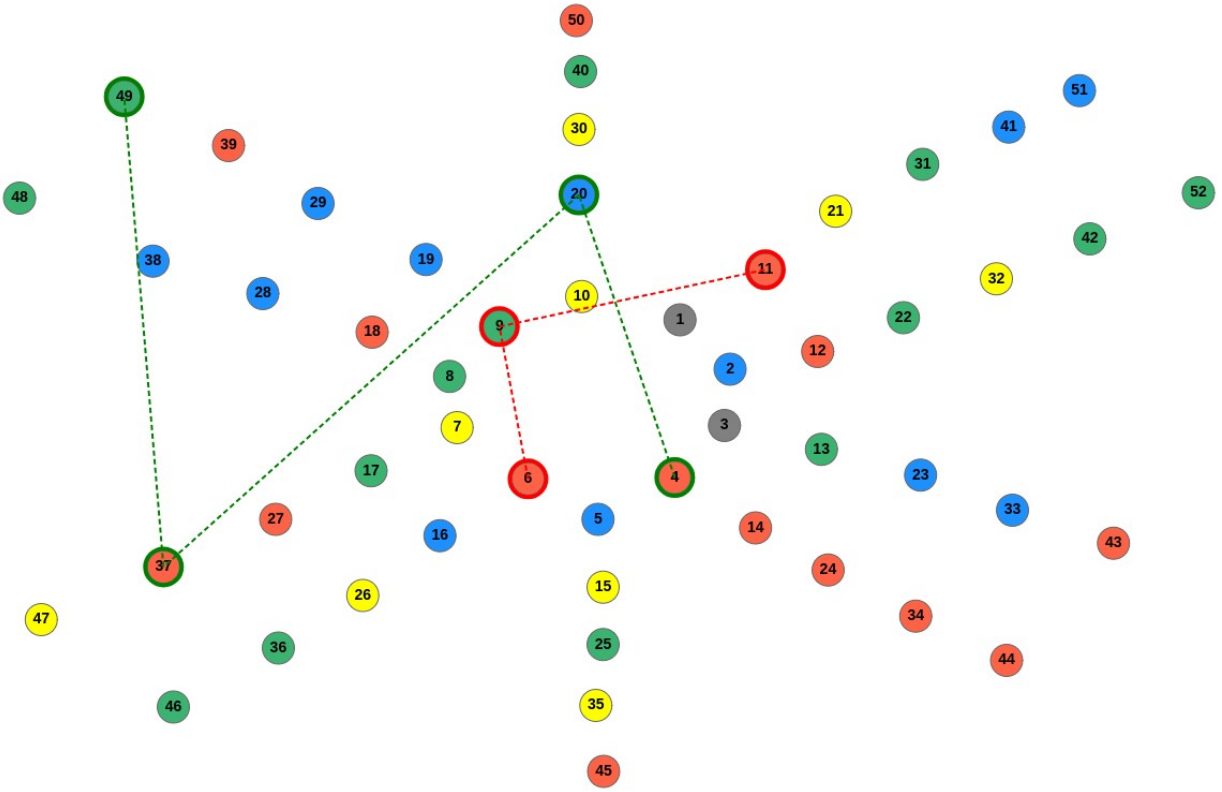
M50= (M15 + M18 + M37 + M49) - (M02 + M11 + M12)



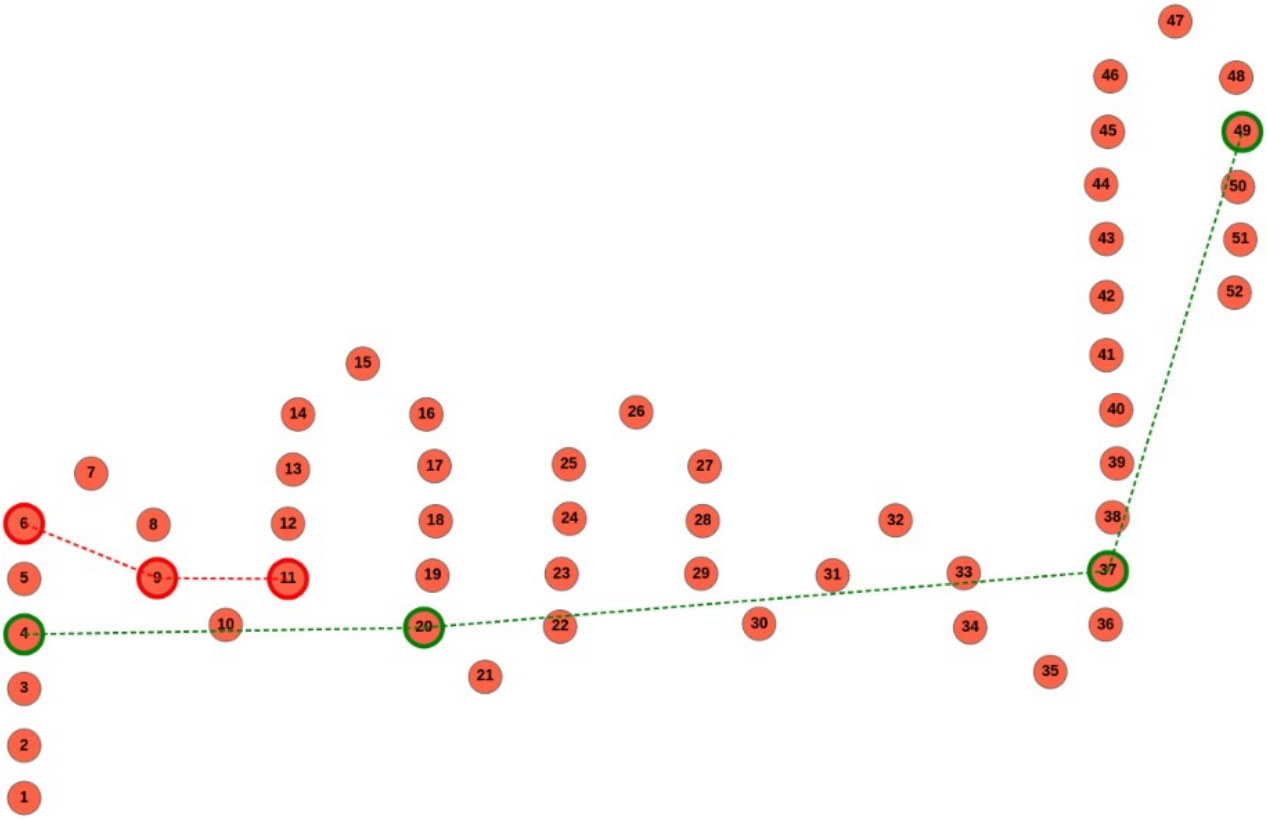
M50= (M02 + M20 + M37 + M49) - (M06 + M09 + M10)



M50= (M04 + M20 + M37 + M49) - (M06 + M09 + M11)



M50= (M04 + M20 + M37 + M49) - (M06 + M09 + M11) – different layer



Apendix:

Labels: M...
[{label:"M52",value:136279841},{label:"M51",value:82589933},{label:"M50",value:77232917},
{label:"M49",value:74207281},{label:"M48",value:57885161},{label:"M47",value:43112609},
{label:"M46",value:42643801},{label:"M45",value:37156667},{label:"M44",value:32582657},
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{label:"M40",value:20996011},{label:"M39",value:13466917},{label:"M38",value:6972593},
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{label:"M28",value:86243},{label:"M27",value:44497},{label:"M26",value:23209},
{label:"M25",value:21701},{label:"M24",value:19937},{label:"M23",value:11213},
{label:"M22",value:9941},{label:"M21",value:9689},{label:"M20",value:4423}, {label:"M19",value:4253},
{label:"M18",value:3217},{label:"M17",value:2281}, {label:"M16",value:2203},{label:"M15",value:1279},
{label:"M14",value:607}, {label:"M13",value:521},{label:"M12",value:127},{label:"M11",value:107},
{label:"M10",value:89},{label:"M09",value:61},{label:"M08",value:31}, {label:"M07",value:19},
{label:"M06",value:17},{label:"M05",value:13}, {label:"M04",value:7},{label:"M03",value:5},
{label:"M02",value:3}, {label:"M01",value:2}]

End digit 9: color yellow
3: color DodgerBlue
7: color Tomato
1 : color MediumSeaGreen
2 or 5: color grey

Positive (black border): example: (02+12 + 26)
Negative (red border): example: - (4 + 7 +8)

Sums used above:

M51 = (M02 + M05 + M42 + M48) - (M12 + M17 + M34)
M50= (M02 + M21 + M37 + M49) - (M05 + M16 + M18)
M47= (M11 + M39 + M43) - (M01 + M08 + M32)
M45= (M01 + M38 + M43) - (M05 + M17 + M31)
M44= (M18 + M27 + M37 + M43) - (M02 + M30 + M32)
M43= (M28 + M35 + M36 + M42) - (M03 + M05 + M26)
M42= (M03 + M32 + M34 + M41) - (M02 + M06 + M28)
M41= (M03 + M05 + M28 + M42) - (M02 + M32 + M34)
M37= (M16 + M25 + M27 + M36) - (M03 + M08 + M26)
M35= (M10 + M27 + M31 + M34) - (M02 + M21 + M29)
M33= (M07 + M25 + M29 + M32) - (M02 + M21 + M24)
M31= (M16 + M19 + M29 + M30) - (M02 + M23 + M25)
M29= (M19 + M21 + M23 + M31) - (M02 + M27 + M28)
M28= (M15 + M25 + M26 + M27) - (M02 + M06 + M20)
M27= (M02 + M22 + M25 + M26) - (M09 + M14 + M21)
M26= (M14 + M21 + M23 + M25) - (M02 + M09 + M24)
M25= (M15 + M17 + M22 + M24) - (M02 + M13 + M23)
M24= (M15 + M17 + M22 + M23) - (M02 + M13 + M19)
M23= (M15 + M17 + M20 + M21) - (M02 + M16 + M19)
M22= (M06 + M15 + M18 + M21) - (M02 + M03 + M19)

M21= (M06 + M18 + M19 + M20) - (M03 + M05 + M16)
M20= (M02 + M13 + M18 + M19) - (M10 + M15 + M16)
M19= (M08 + M14 + M16 + M18) - (M03 + M13 + M15)
M18= (M10 + M11 + M15 + M17) - (M03 + M05 + M13)
M17= (M02 + M05 + M15 + M16) - (M10 + M13 + M14)
M16= (M06 + M13 + M14 + M15) - (M03 + M10 + M12)
M15= (M11 + M12 + M13 + M14) - (M03 + M06 + M09)
M14= (M07 + M08 + M11 + M13) - (M02 + M04 + M09)
M12= (M09 + M10) - (M02 + M04 + M05)
M11= (M06 + M07 + M10) - (M03 + M05)
M10= (M07 + M08 + M09) - (M03 + M06)
M09= (M02 + M05 + M07 + M08) – (M03)
M08= (M02 + M03 + M05 + M06) – (M04)
M07= (M02 + M03 + M04 + M06) – (M05)
M50= (M08 + M19 + M37 + M49) - (M02 + M03 + M06)
M50= (M08 + M19 + M37 + M49) - (M03 + M04 + M05)
M50= (M09 + M19 + M37 + M49)- (M03 + M07 + M08)
M50= (M10 + M19 + M37 + M49) - (M02 + M07 + M09)
M50= (M10 + M19 + M37 + M49)- (M02 + M07 + M09)
M50= (M11 + M19 + M37 + M49) - (M03 + M04 + M10)
M50= (M15 + M18 + M37 + M49) - (M02 + M11 + M12)
M50= (M02 + M20 + M37 + M49) - (M06 + M09 + M10)
M50= (M04 + M20 + M37 + M49) - (M06 + M09 + M11)

Example calculus:

M50= (M04 + M20 + M37 + M49) - (M06 + M09 + M11)

(M04 + M20 + M37 + M49) = 7 + 4,423 + 3,021,377 + 74,207,281= 77,233,088
(M06 + M09 + M11)= 17 + 61 + 107= 185
$77233088 - 185 = 7.7232903 \times 10^7 =$ **M50**

Available in Github: https://github.com/gatanegro/MERSENNE-COLLATZ
App to calculate sums using labels : **Mersenne Sum Explorer Logos Theory.html**
App to calculate equations : **Mersenne Equation Explorer.html**
App to Research Geometry visuals: **Mersenne Spiral – Group Geometry with Import Layer.html**
App to predict possible candidate : **Mersenne Spiral – Sum Calculator with Placeholders**..html
App to Research Geometry visuals importing new layers:**Mersenne Spiral – Layer Switcher.html**

Reference :
(1)Martin, D. (2026). Geometric Decomposition of Mersenne Exponents: A Novel Additive Representation System. Zenodo. https://doi.org/10.5281/zenodo.18256256