

# Mersennes Exponents Equations & visuals on Spiral Geometry

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## Abstract

This study is a continuation of previous research in *Geometric Decomposition of Mersenne Exponents: A Novel Additive Representation System* (1).

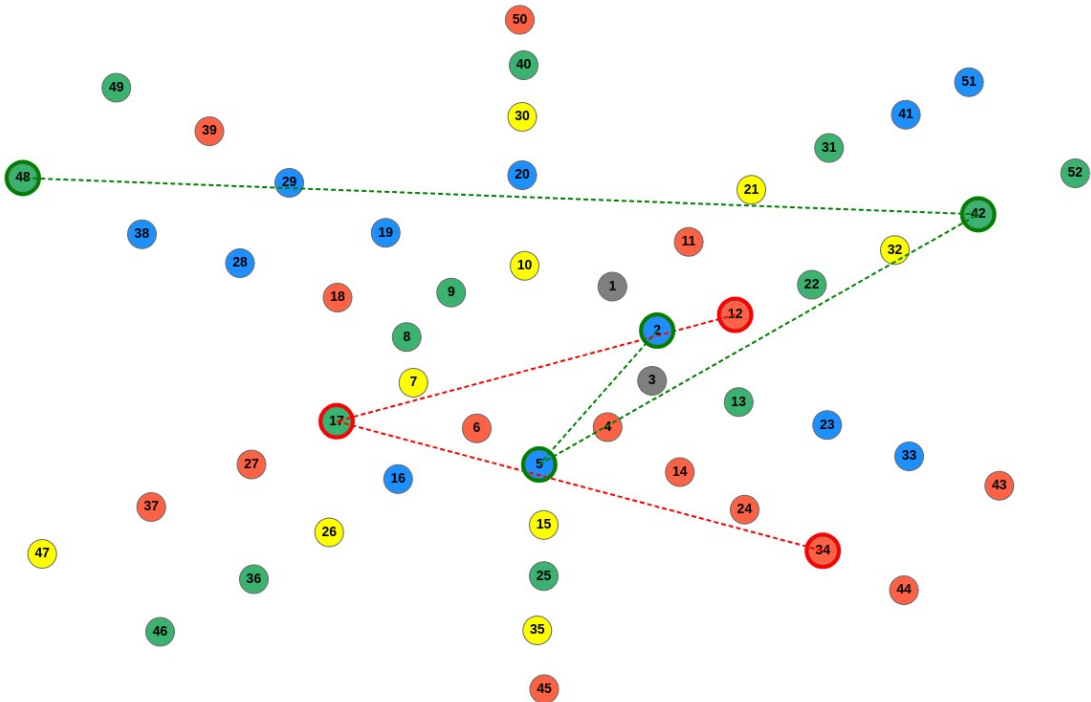
This paper presents an observed geometric pattern in the sequence of known Mersenne prime exponents (M1–M51). When plotted in a two-dimensional spiral arrangement based on index order and last digit, a remarkable structure emerges.

Each Mersenne exponent corresponds to a unique point in the spiral

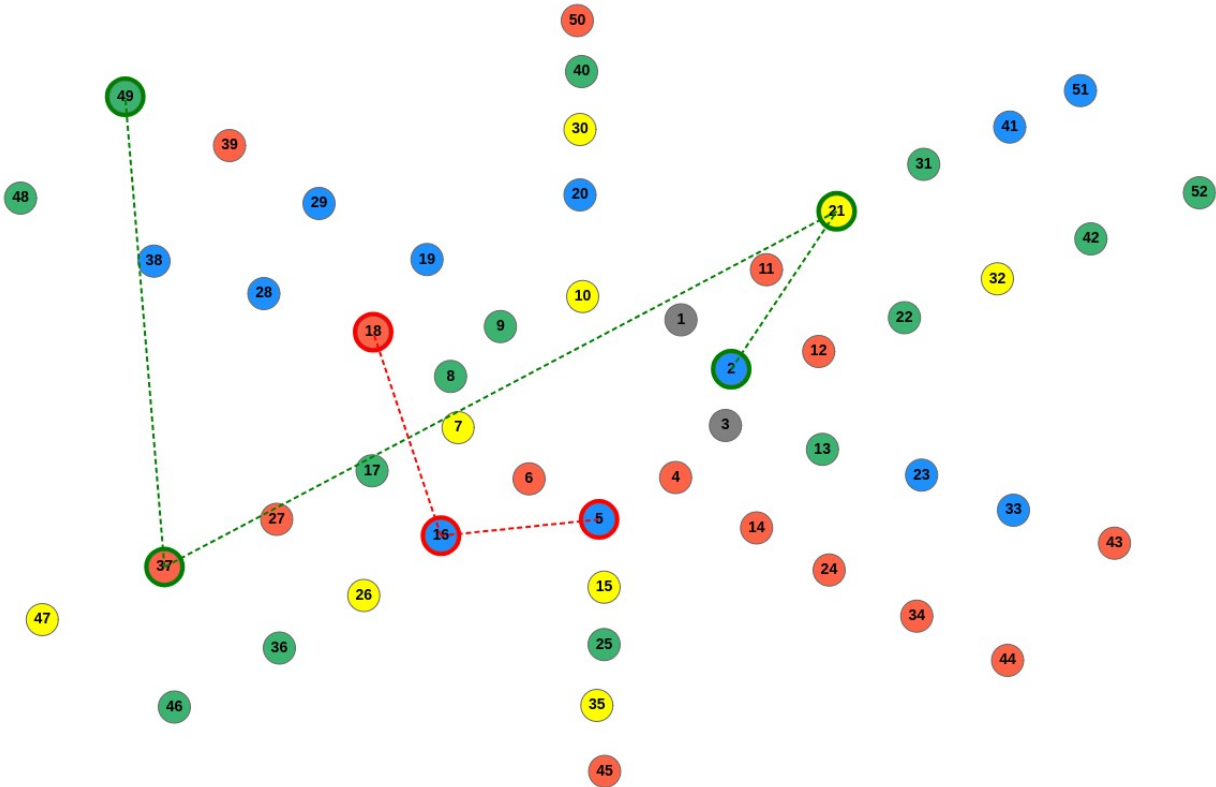
- For every known Mersenne prime up to M51, there exist sets of 4 positive and 3 negative indices whose vector sum (in spiral coordinates) lands precisely on the target point
- These geometric visuals helps to identify patterns while the sums are revealing relationship
- The pattern reveals an unexpected additive structure linking earlier and later Mersenne primes

"For any Mersenne exponent  $M_n$  (with a few exceptions), there exist sets of indices  $P$  and  $N$ , with  $|P|=4$ ,  $|N|=3$ , such that in any ordinal embedding, the points corresponding to  $P$  form a triangle, the points corresponding to  $N$  form another triangle, and these triangles are balanced around  $M_n$ ."

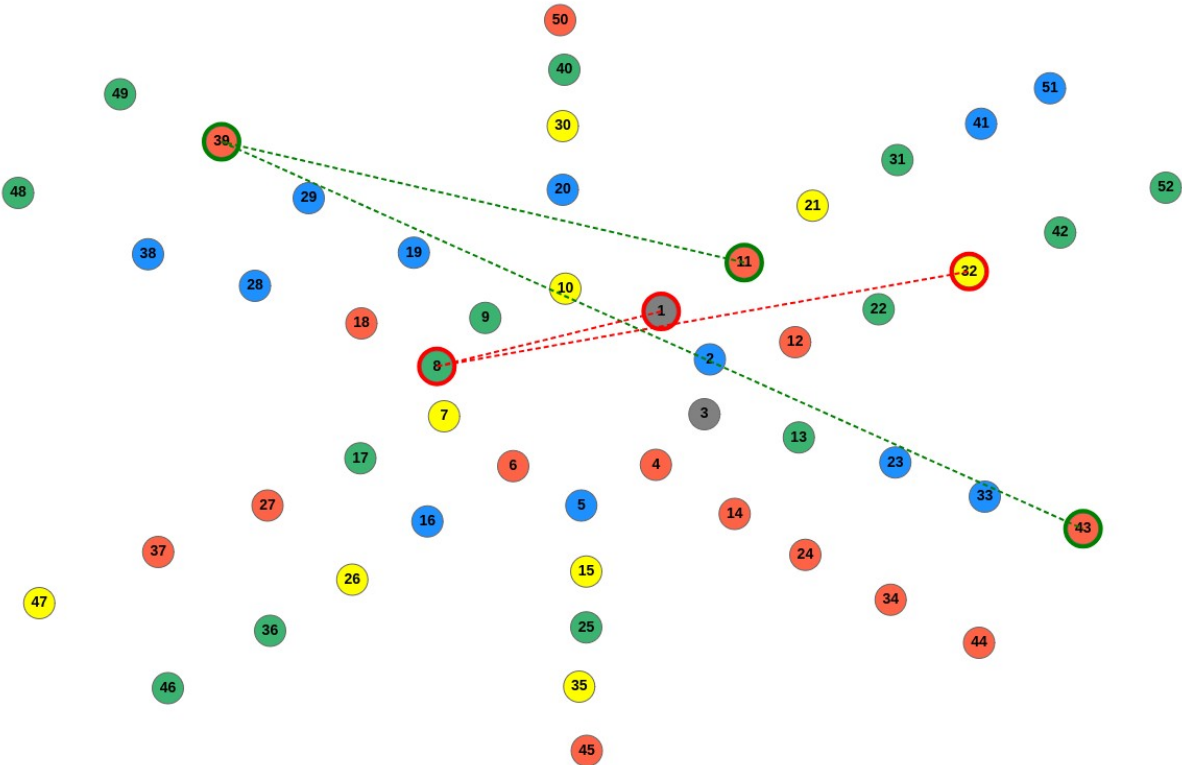
$$M51 = (M02 + M05 + M42 + M48) - (M12 + M17 + M34)$$



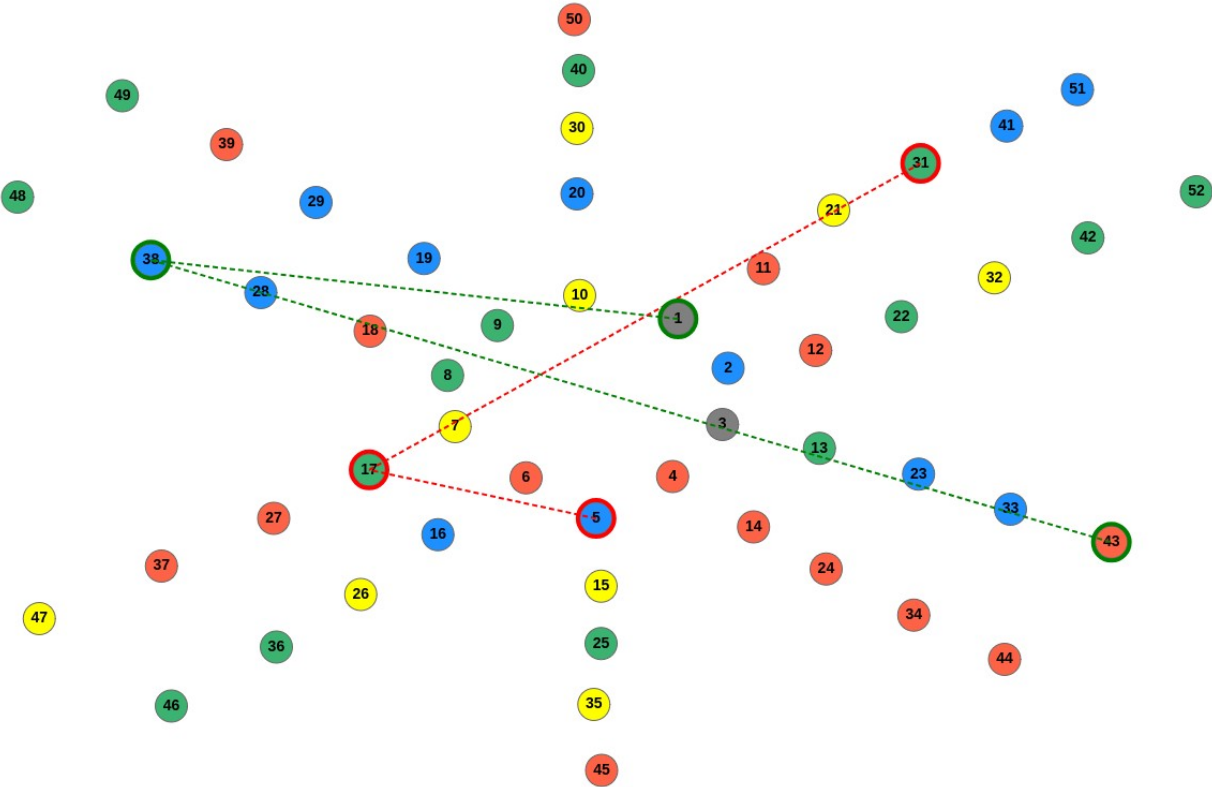
$$M50 = (M02 + M21 + M37 + M49) - (M05 + M16 + M18)$$



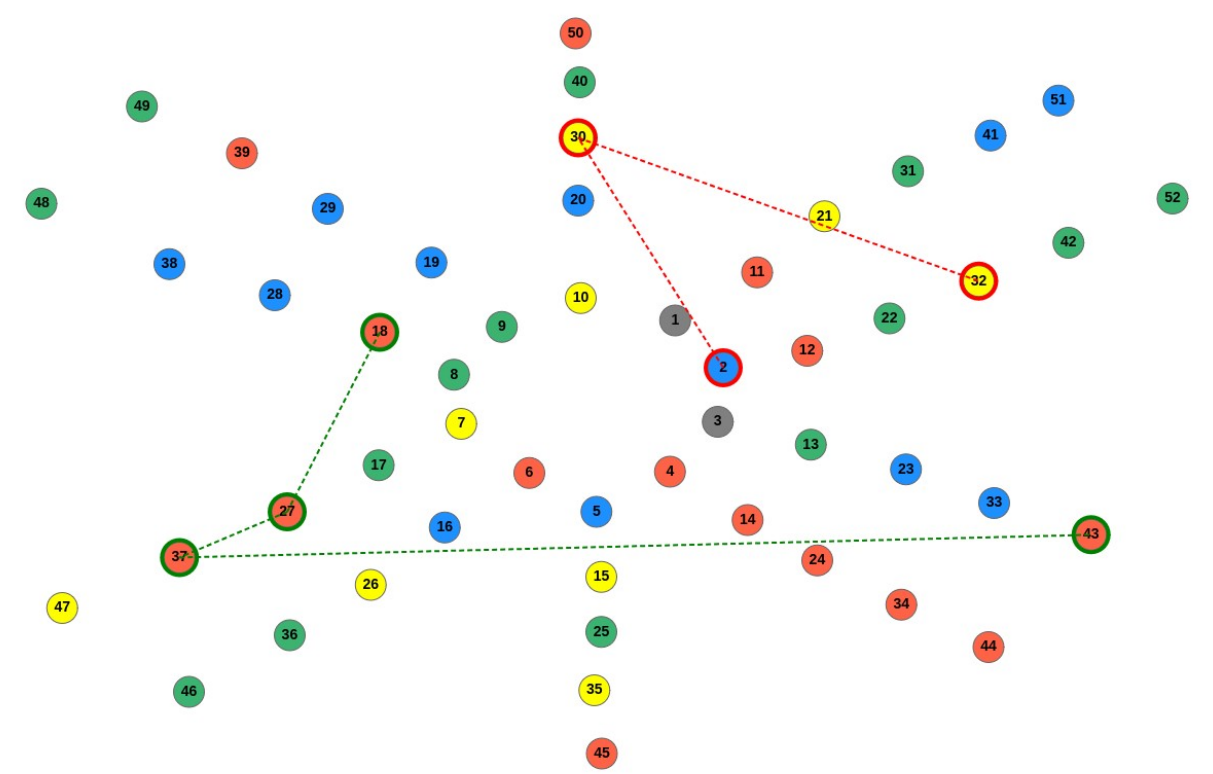
$$M47 = (M11 + M39 + M43) - (M01 + M08 + M32)$$



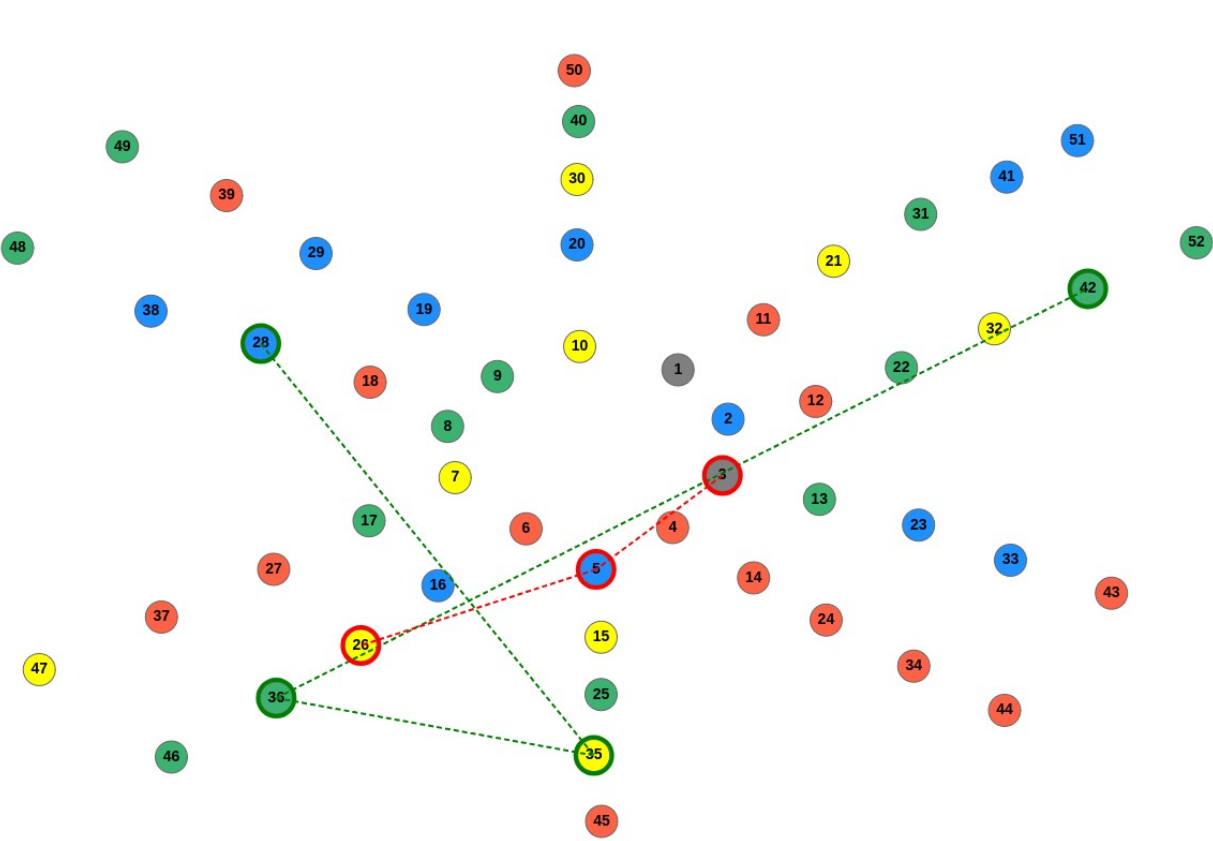
$$M45 = (M01 + M38 + M43) - (M05 + M17 + M31)$$



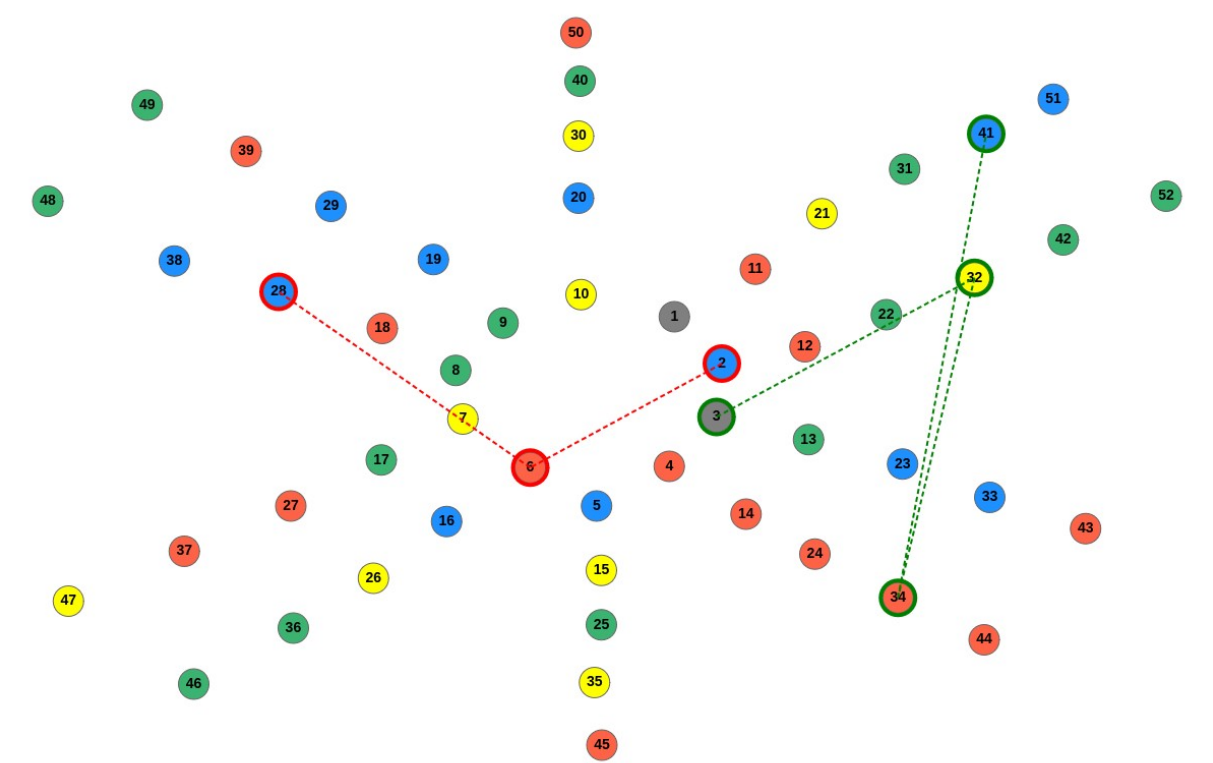
$$M44 = (M18 + M27 + M37 + M43) - (M02 + M30 + M32)$$



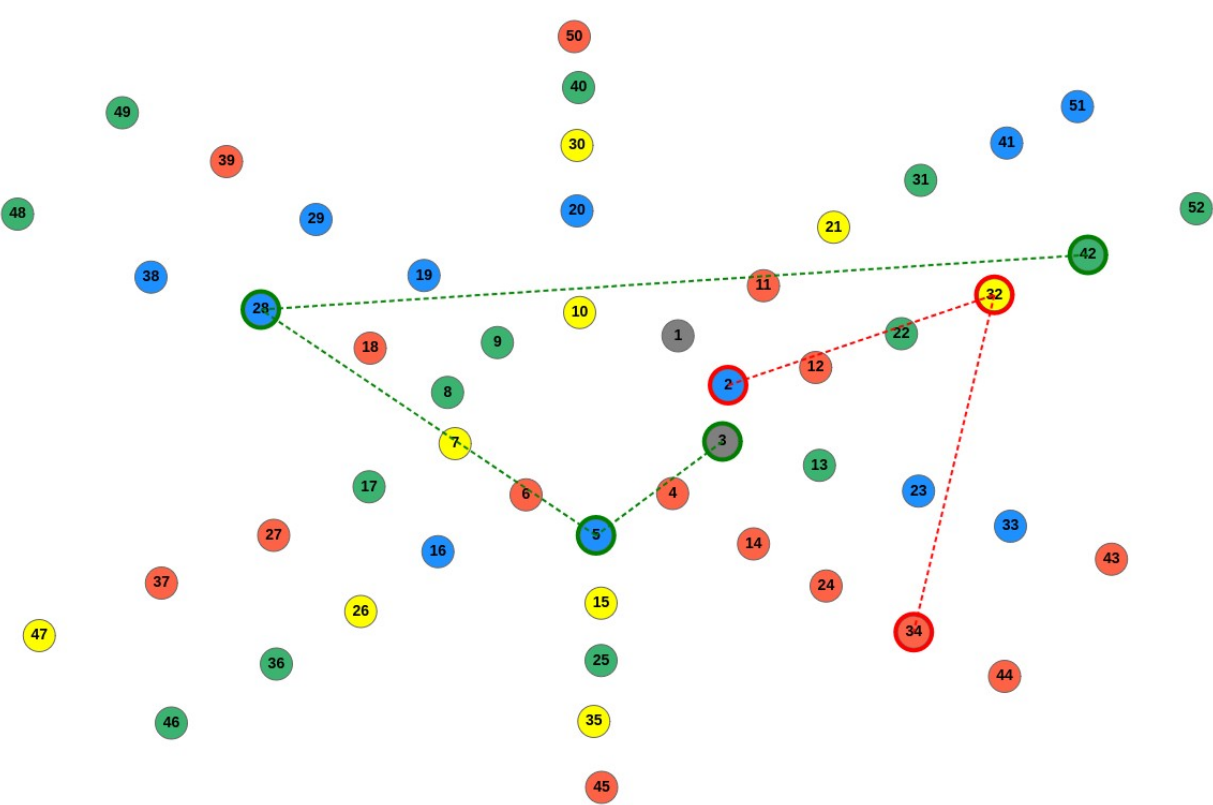
$$M43 = (M28 + M35 + M36 + M42) - (M03 + M05 + M26)$$



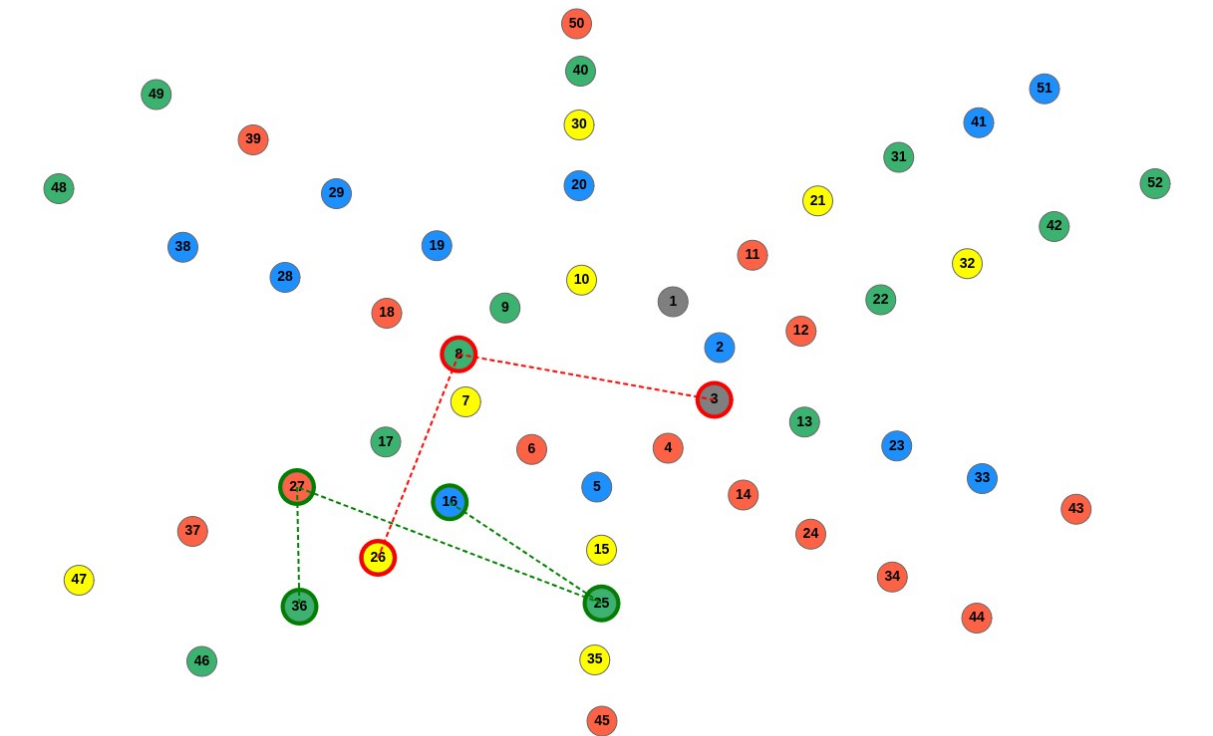
$$M42 = (M03 + M32 + M34 + M41) - (M02 + M06 + M28)$$



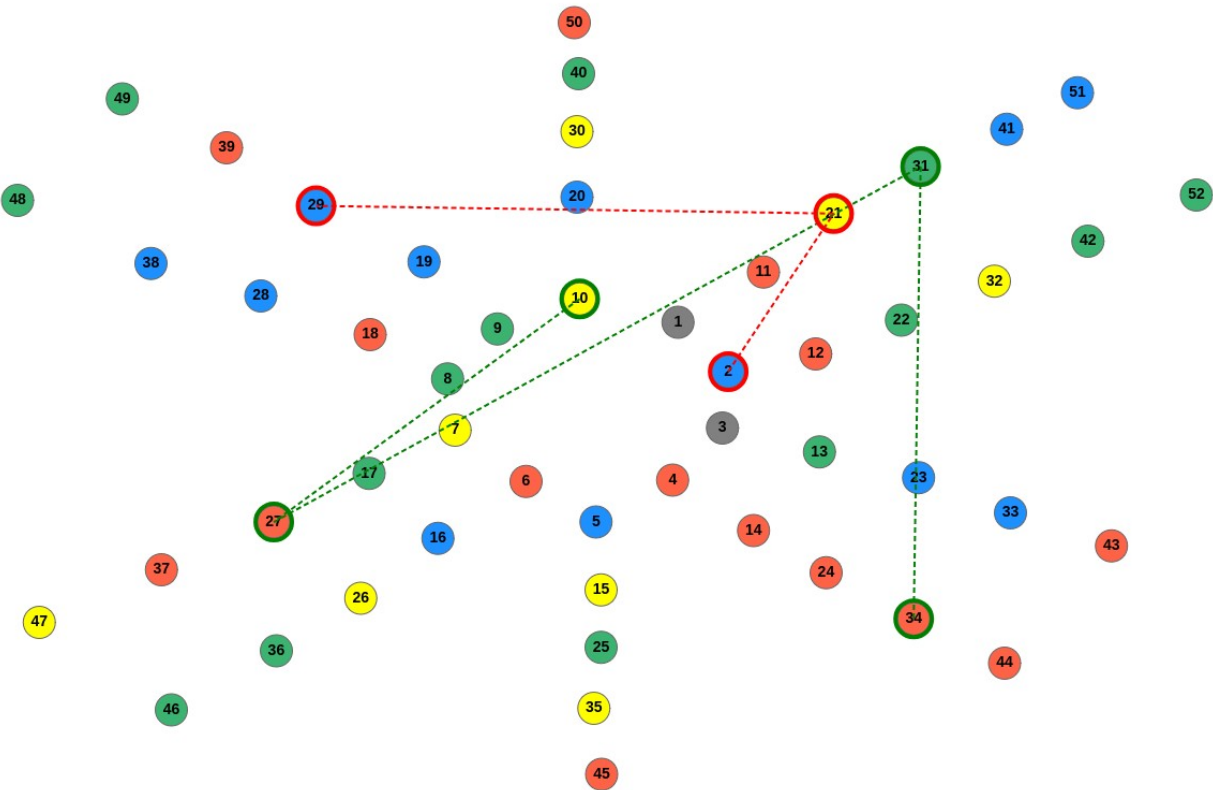
$$M41 = (M03 + M05 + M28 + M42) - (M02 + M32 + M34)$$



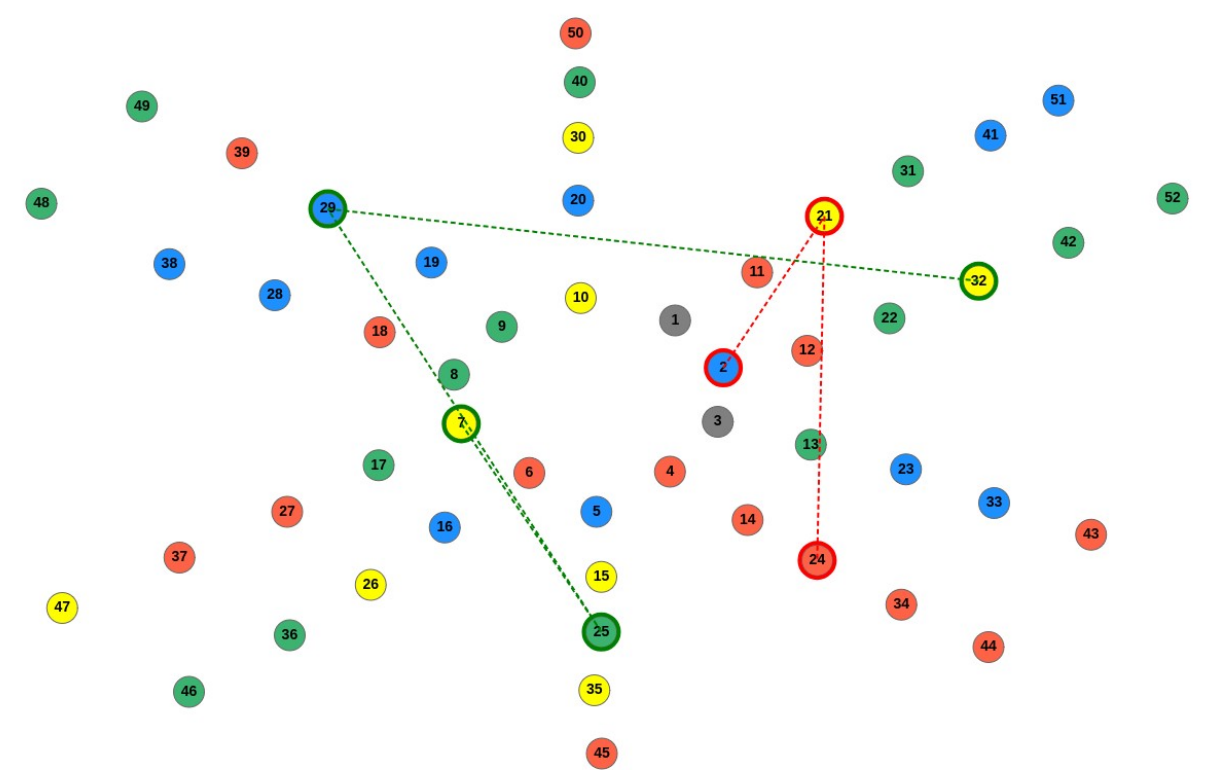
$$M37 = (M16 + M25 + M27 + M36) - (M03 + M08 + M26)$$



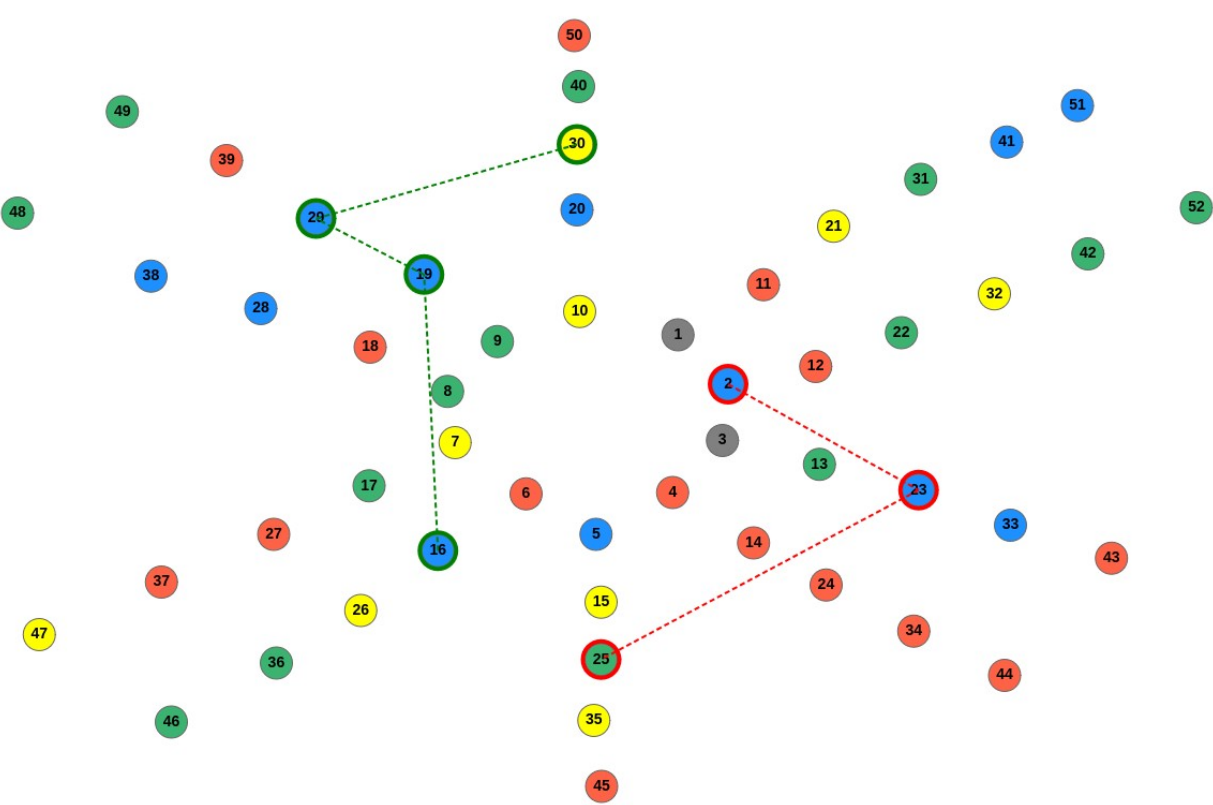
$$M35 = (M10 + M27 + M31 + M34) - (M02 + M21 + M29)$$



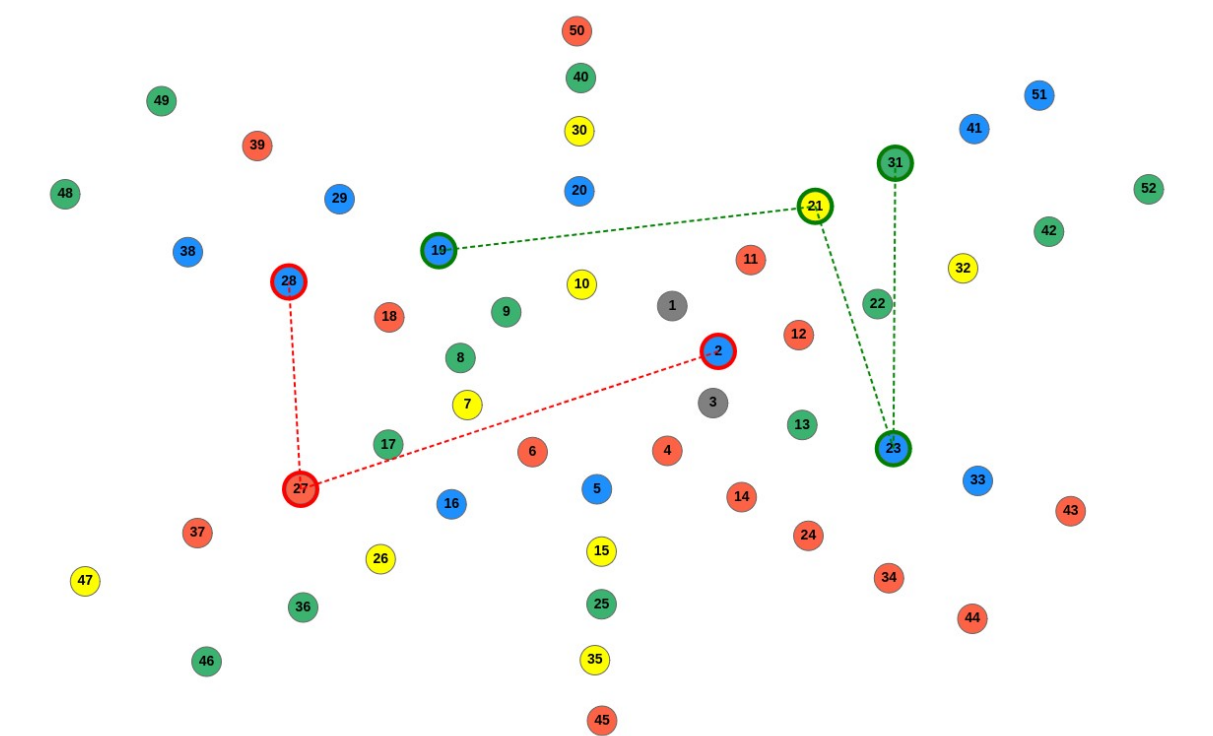
$$M33 = (M07 + M25 + M29 + M32) - (M02 + M21 + M24)$$



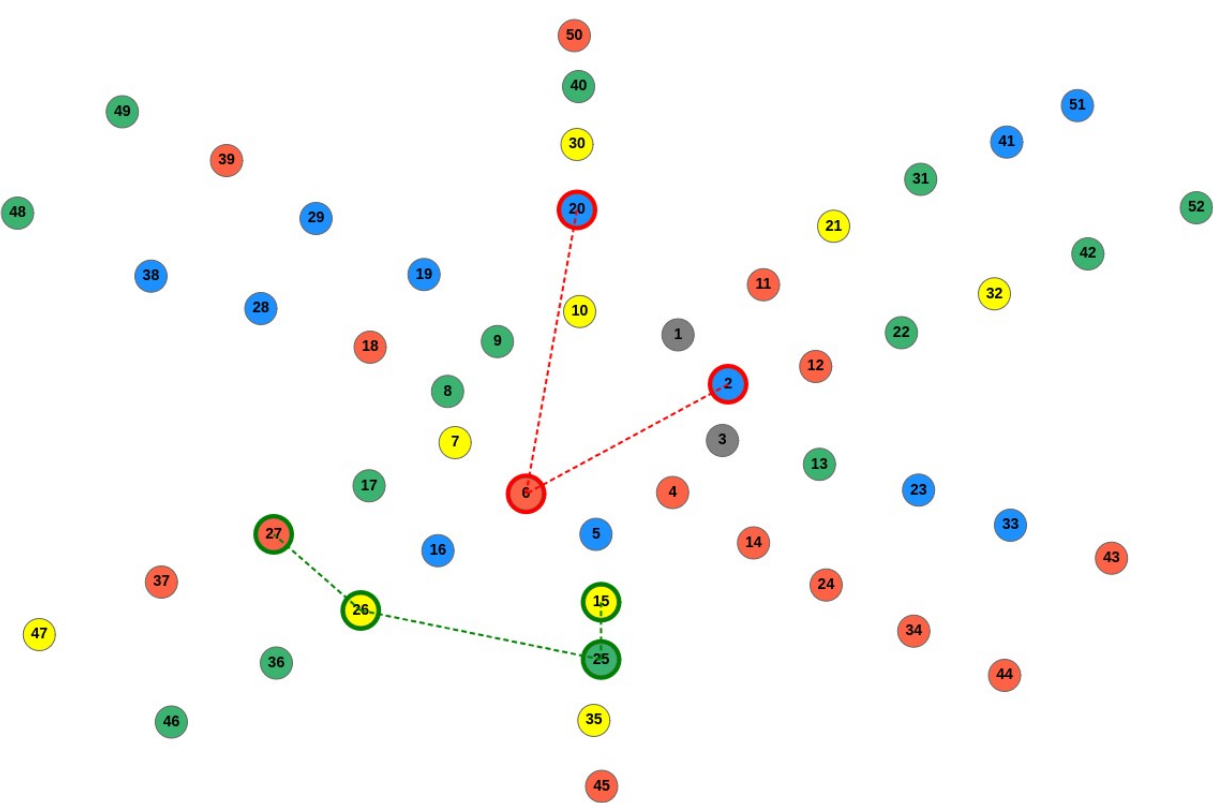
$$M31 = (M16 + M19 + M29 + M30) - (M02 + M23 + M25)$$



$$M29 = (M19 + M21 + M23 + M31) - (M02 + M27 + M28)$$

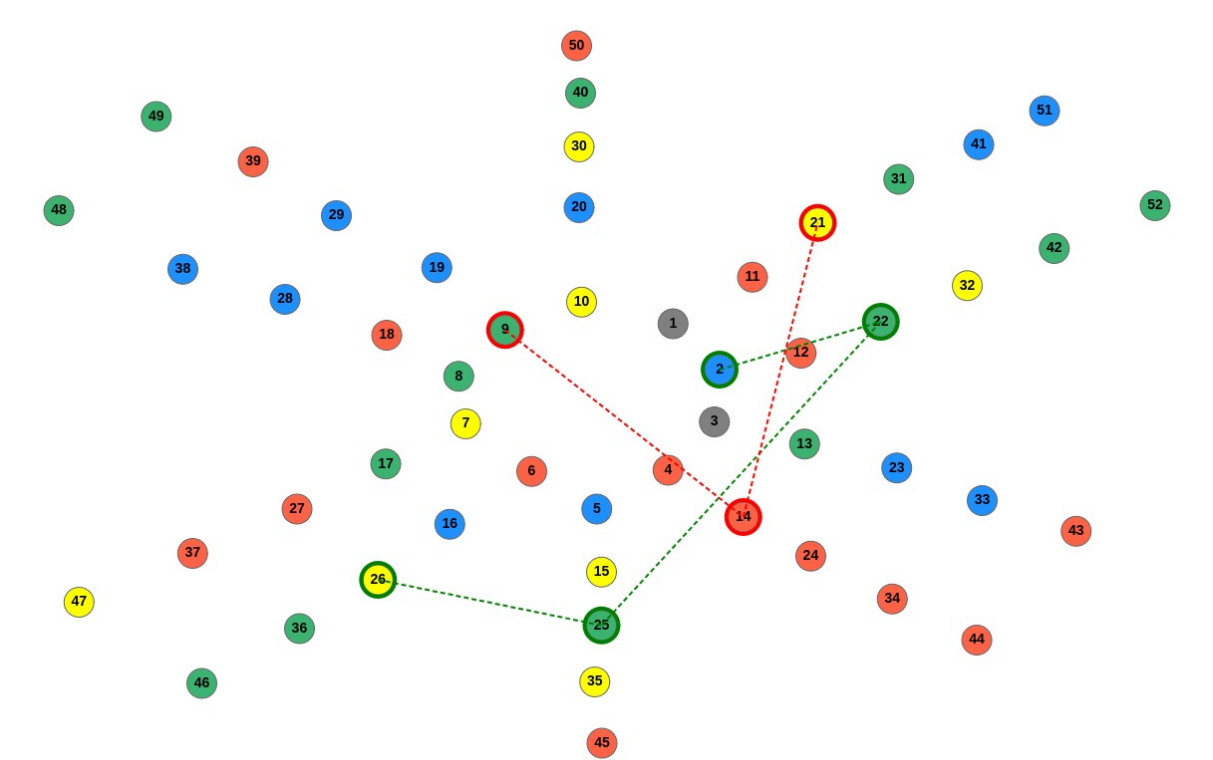


$$M28 = (M15 + M25 + M26 + M27) - (M02 + M06 + M20)$$

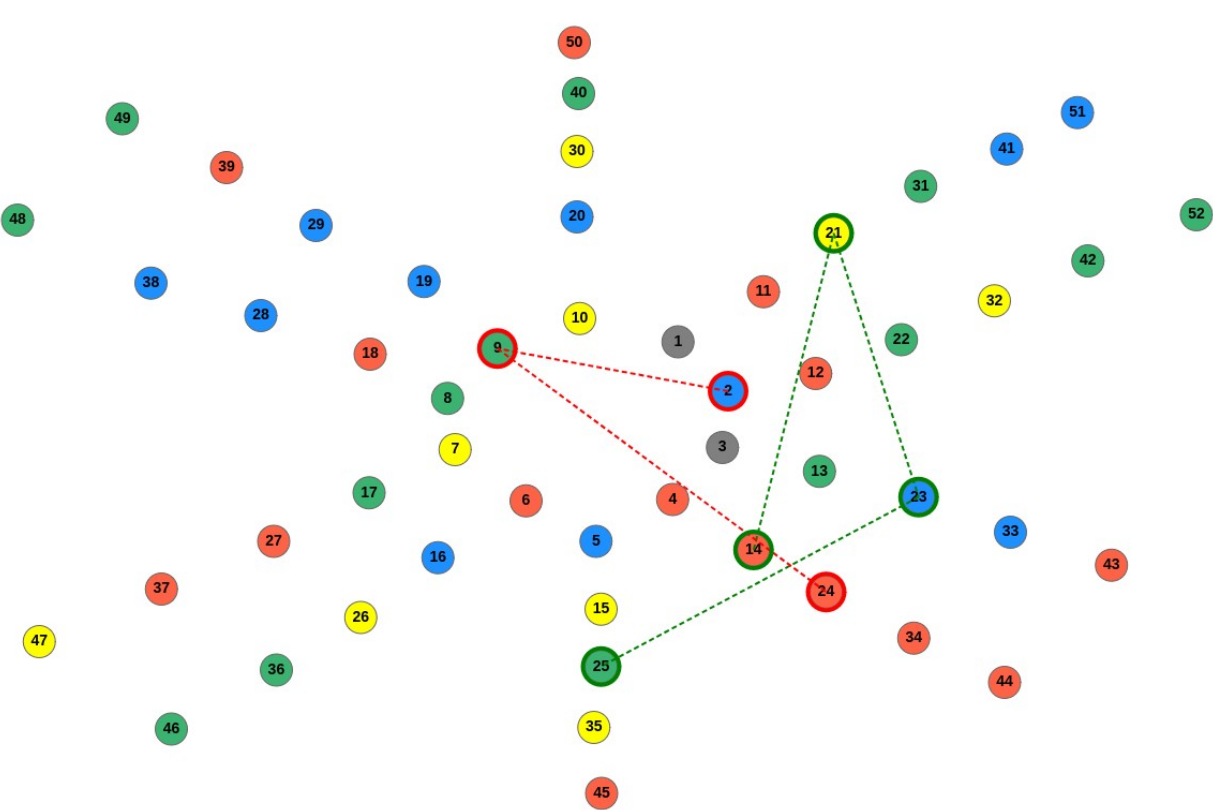




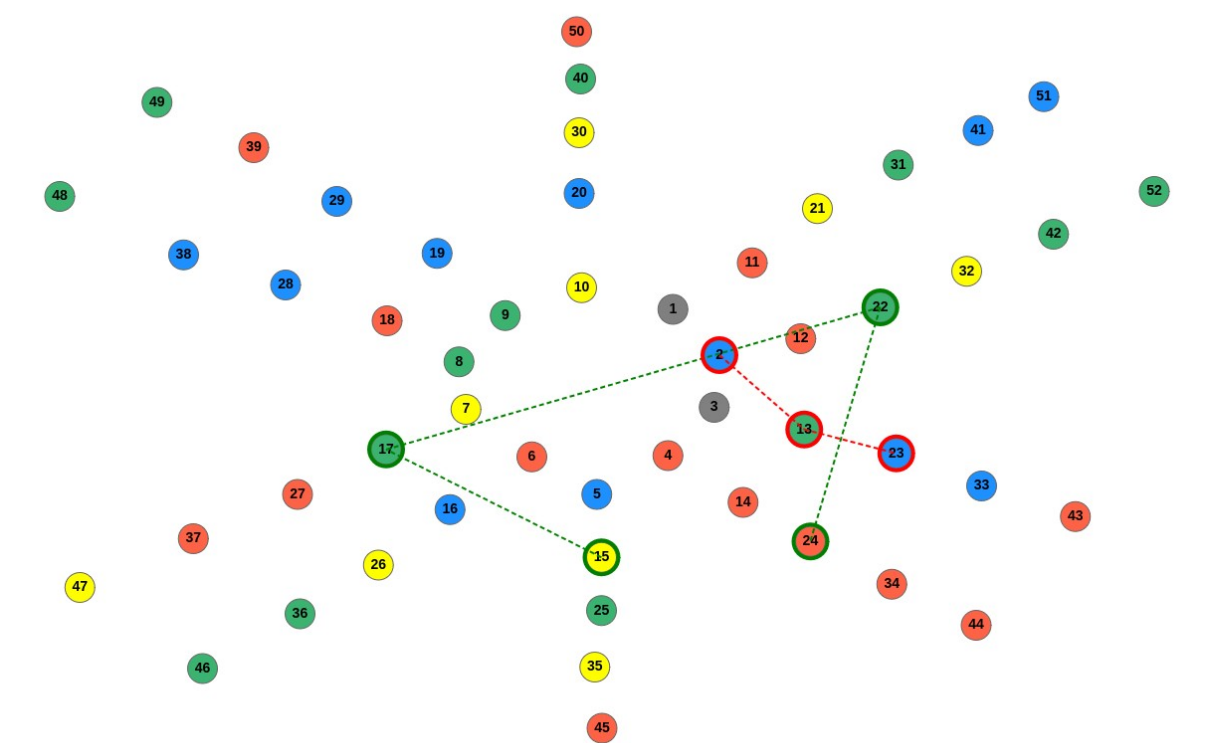
$$M27 = (M02 + M22 + M25 + M26) - (M09 + M14 + M21)$$



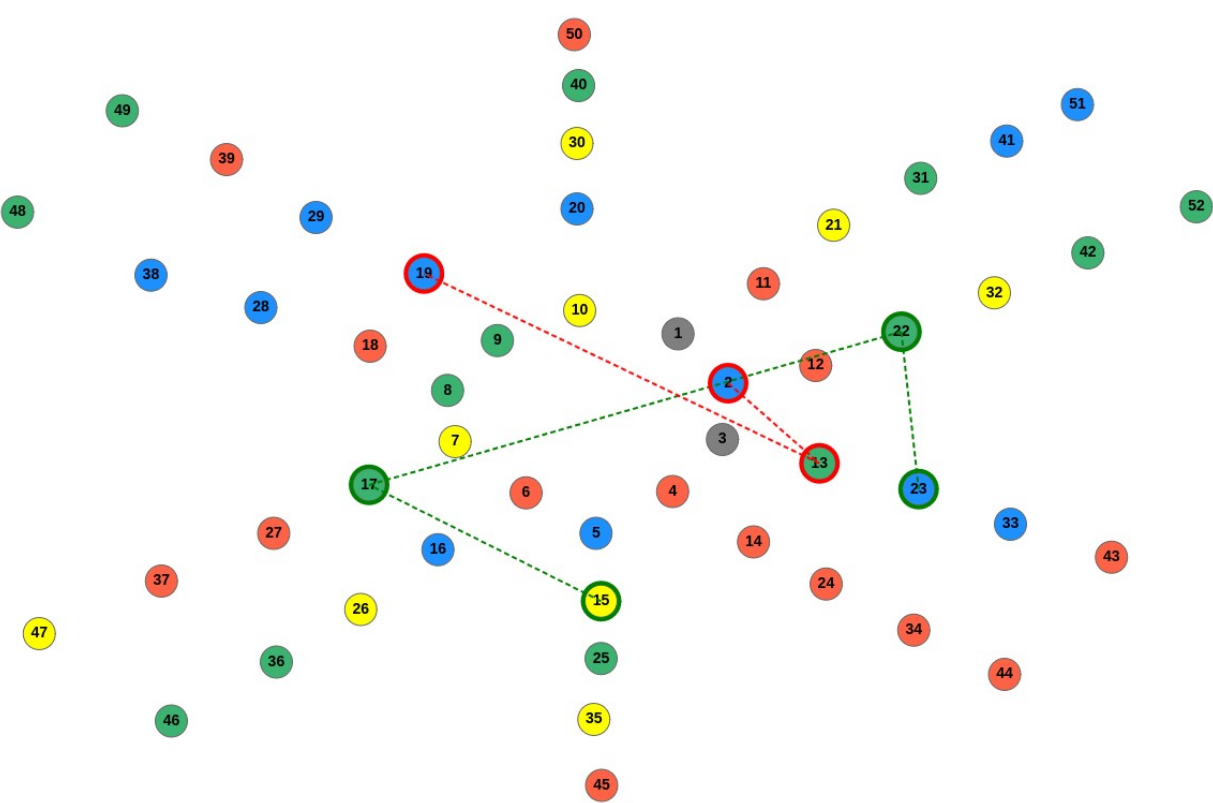
$$M26 = (M14 + M21 + M23 + M25) - (M02 + M09 + M24)$$



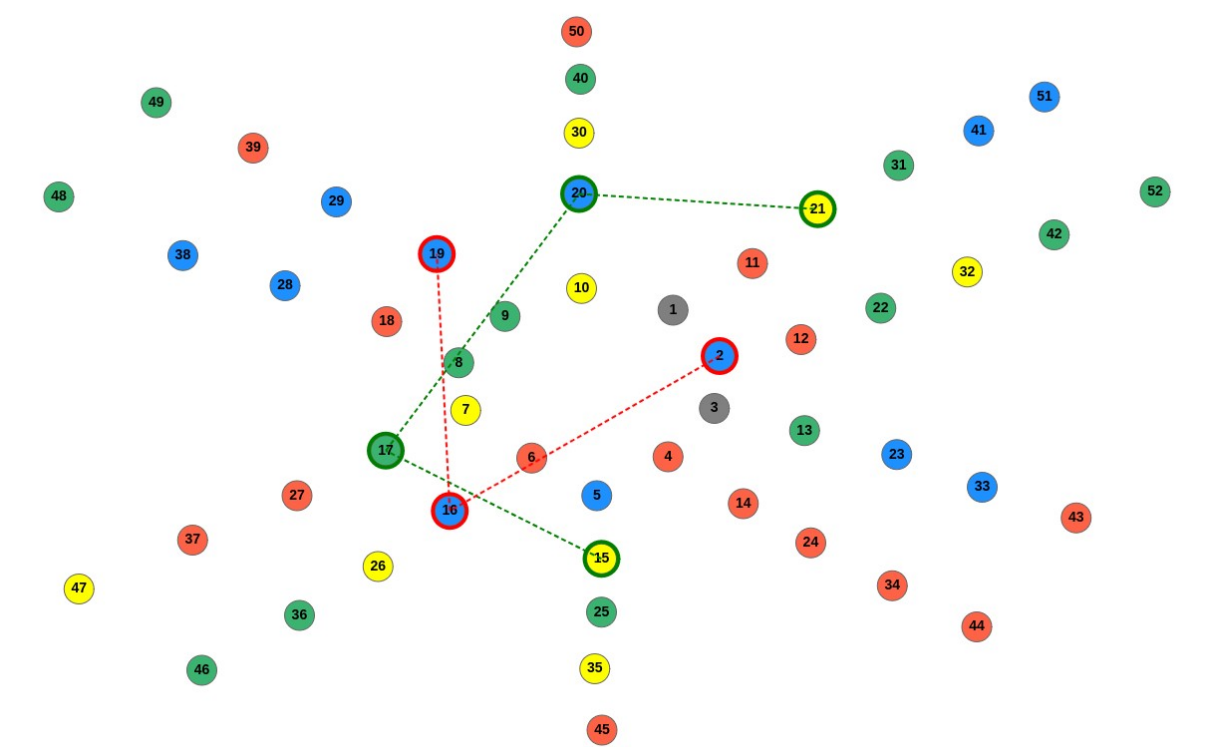
$$M25 = (M15 + M17 + M22 + M24) - (M02 + M13 + M23)$$



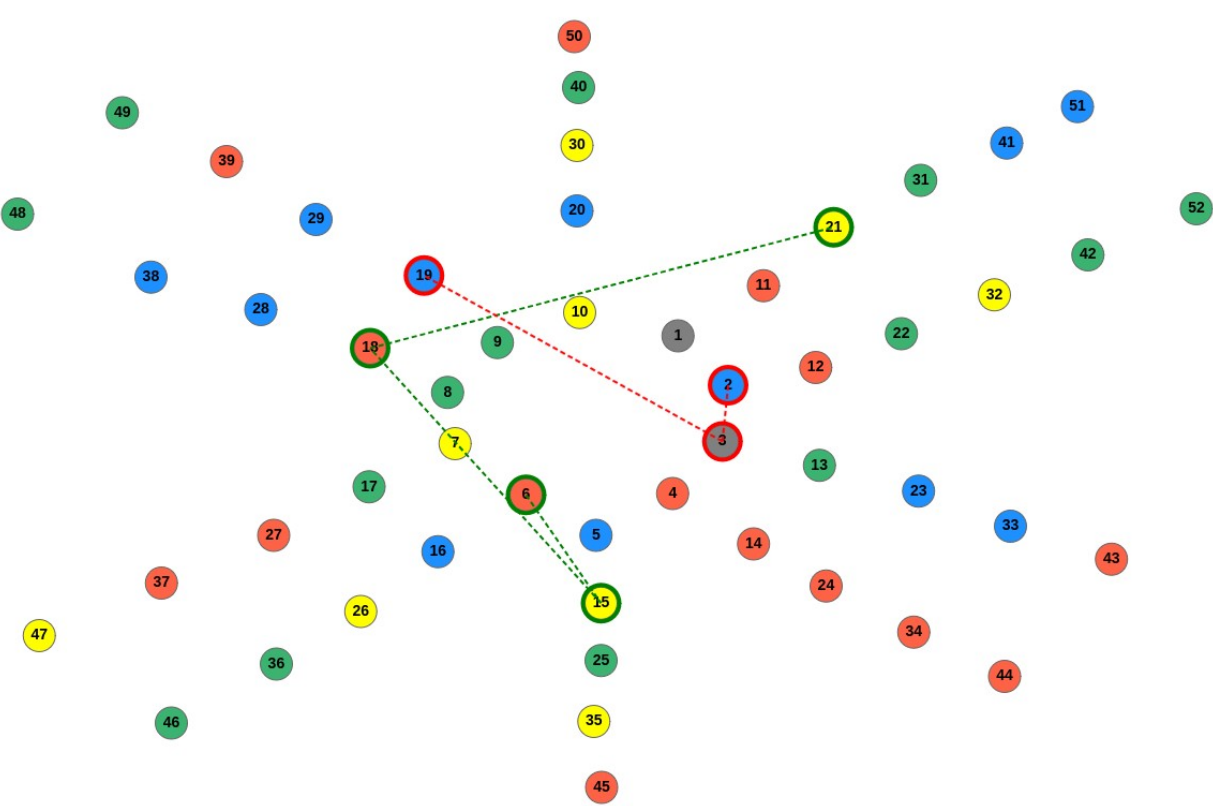
$$M24 = (M15 + M17 + M22 + M23) - (M02 + M13 + M19)$$



$$M23 = (M15 + M17 + M20 + M21) - (M02 + M16 + M19)$$



$$M22 = (M06 + M15 + M18 + M21) - (M02 + M03 + M19)$$



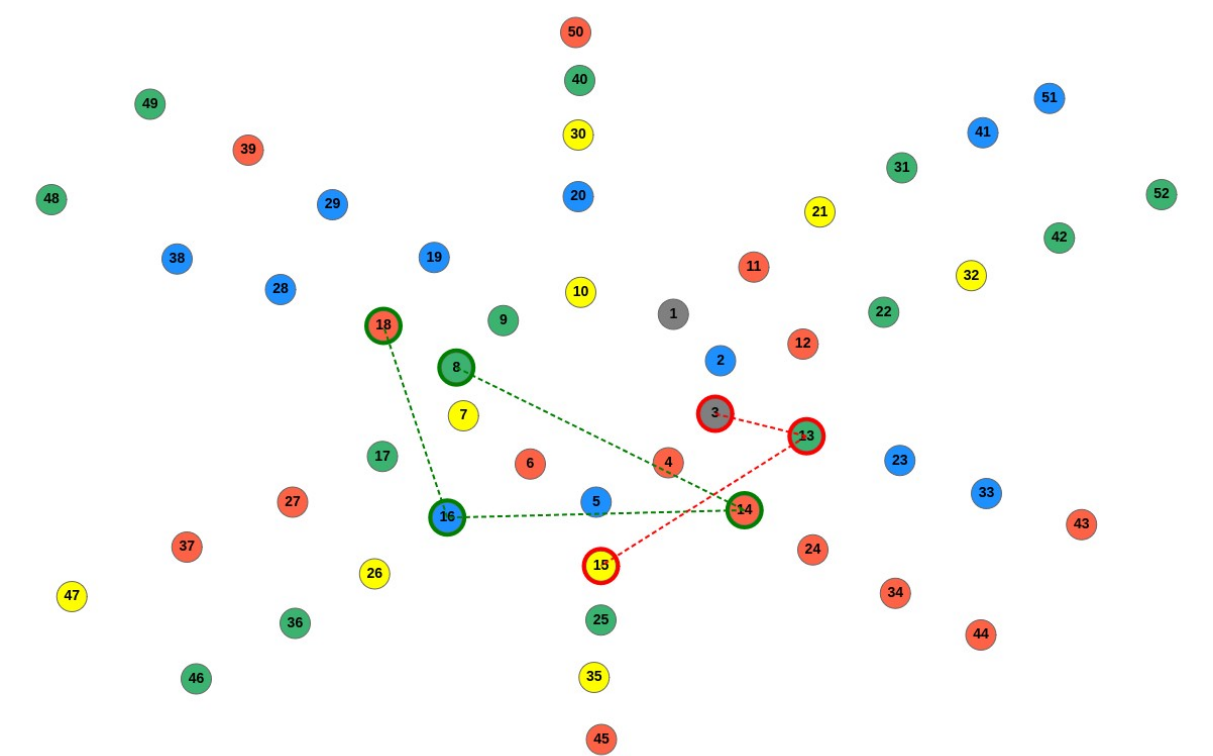
The figure displays a network graph with 52 nodes, each represented by a colored circle with a number. The nodes are distributed across the plot area. A specific path is highlighted with green dashed lines, connecting nodes 16, 18, 19, and 20. Another path is highlighted with red dashed lines, connecting nodes 16, 5, 3, and 4. The nodes are colored in five categories: red (e.g., 1, 3, 4, 5, 11, 12, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), green (e.g., 2, 6, 7, 8, 9, 10, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), blue (e.g., 1, 2, 3, 4, 5, 11, 12, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), yellow (e.g., 1, 2, 3, 4, 5, 11, 12, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), and grey (e.g., 1, 2, 3, 4, 5, 11, 12, 14, 15, 16, 17, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52).

The graph consists of 52 nodes, each represented by a colored circle with a number. The nodes are distributed across the image, with some clusters and many isolated nodes. The connections are as follows:

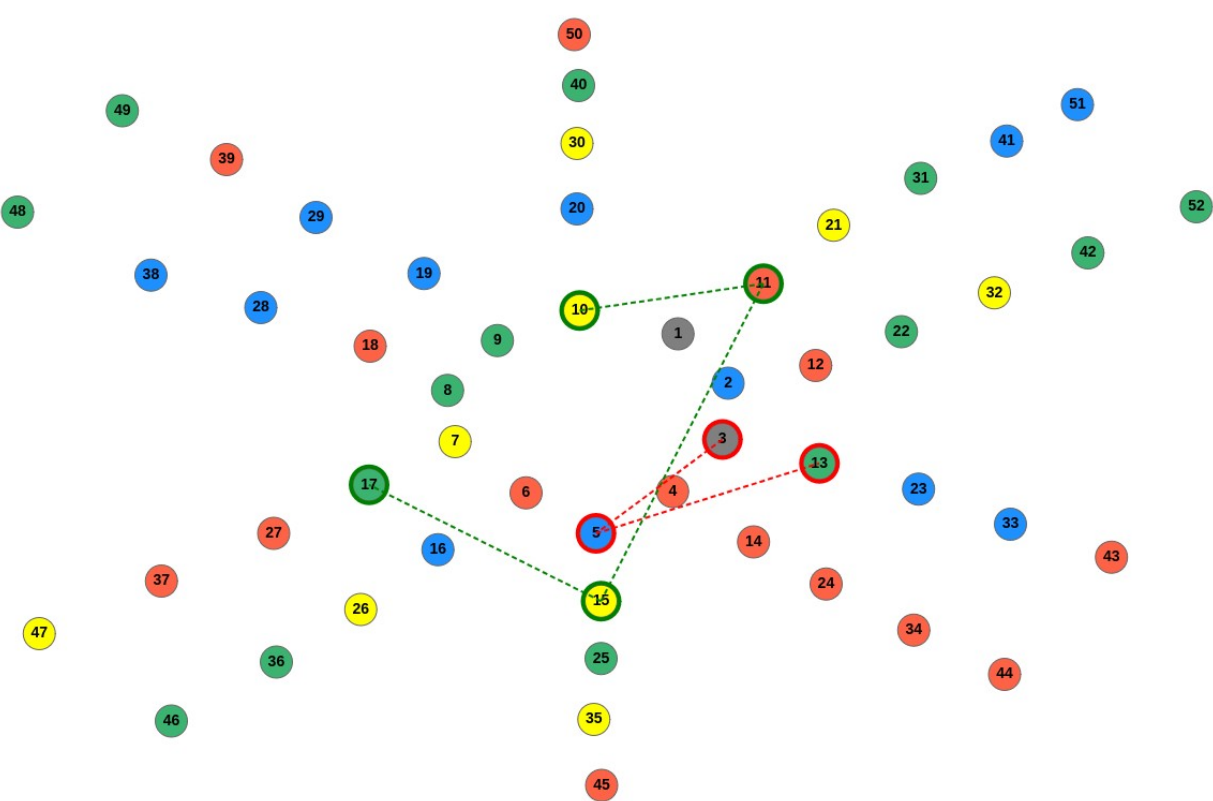
- Red dashed line:** Connects node 10 (yellow with red border) to node 15 (yellow with red border).
- Green dashed lines:**
  - Connects node 10 to node 18 (green with red border).
  - Connects node 18 to node 19 (blue with green border).
  - Connects node 18 to node 13 (green with green border).
  - Connects node 2 (blue with green border) to node 13.
- Grey dashed line:** Connects node 1 (grey) to node 3 (grey).

Nodes with thick borders (10, 15, 18, 19, 13, 2) are highlighted. The colors of the nodes are: red (e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), blue (e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), green (e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), yellow (e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52), and grey (e.g., 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52).

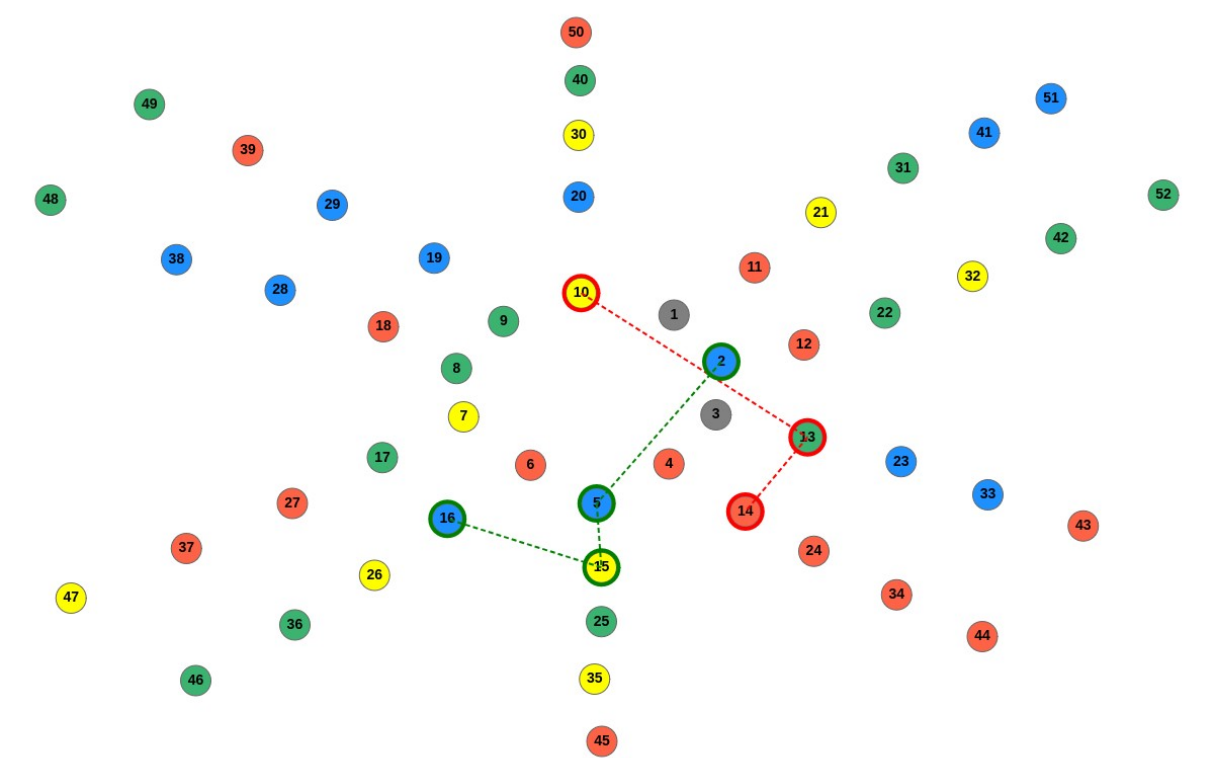
$$M19 = (M08 + M14 + M16 + M18) - (M03 + M13 + M15)$$



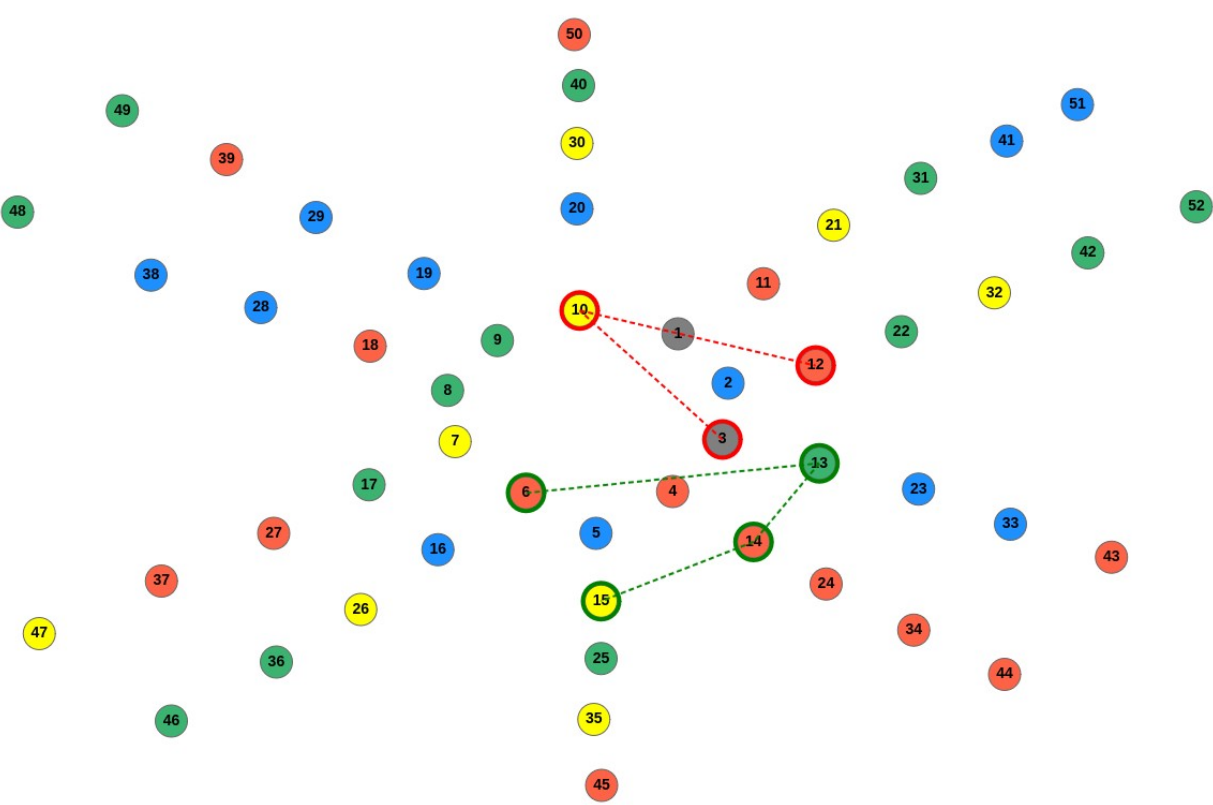
$$M18 = (M10 + M11 + M15 + M17) - (M03 + M05 + M13)$$



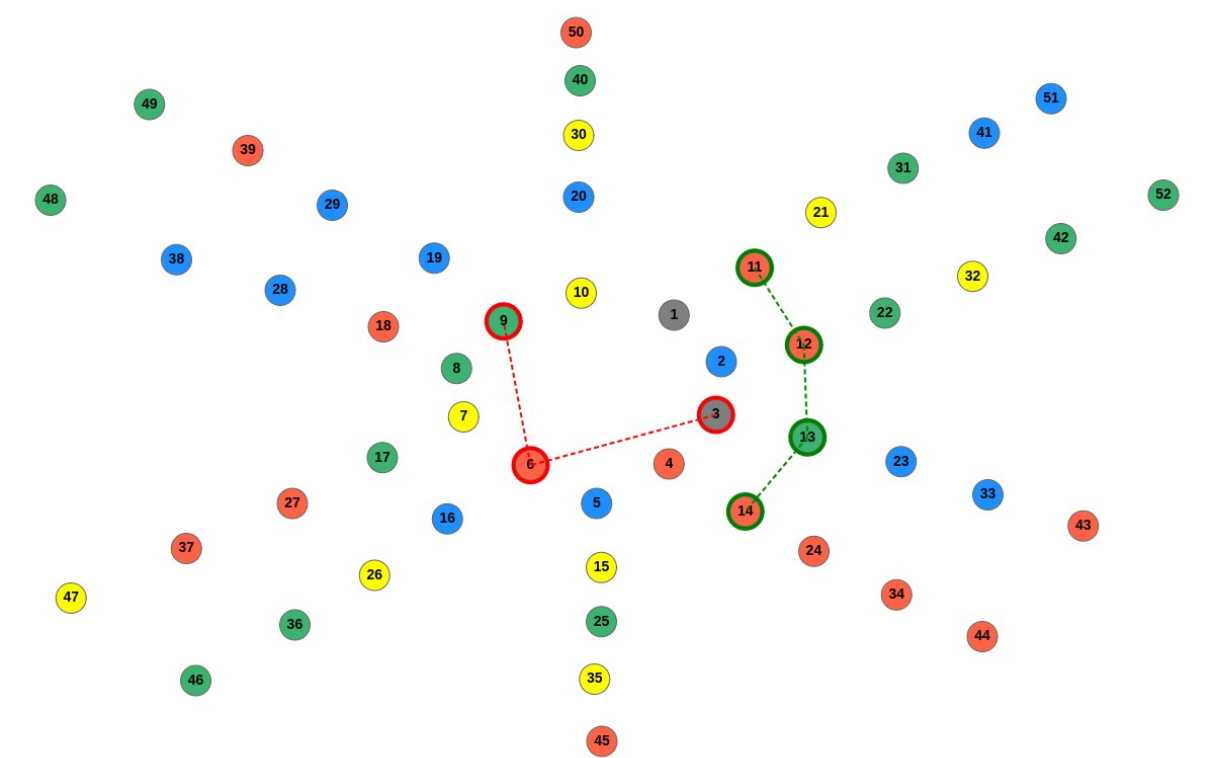
$$M17 = (M02 + M05 + M15 + M16) - (M10 + M13 + M14)$$



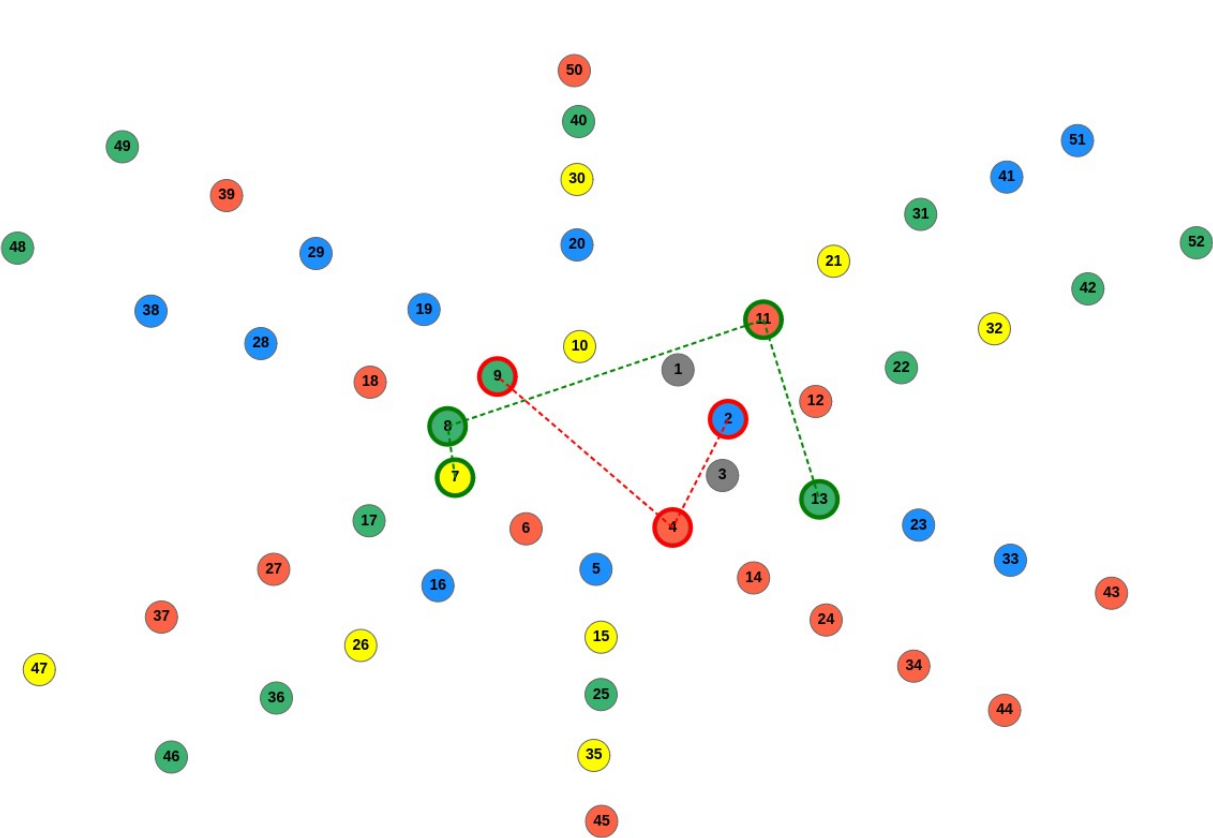
$$M16 = (M06 + M13 + M14 + M15) - (M03 + M10 + M12)$$



$$M15 = (M11 + M12 + M13 + M14) - (M03 + M06 + M09)$$



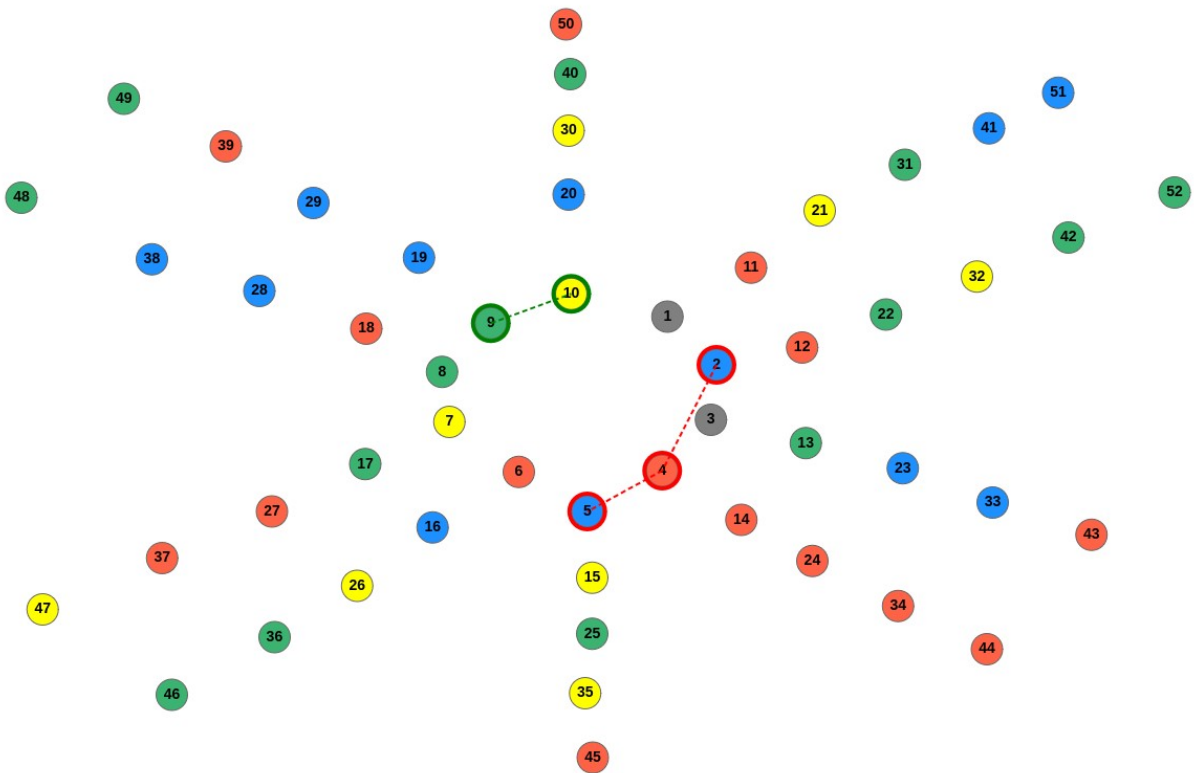
$$M14 = (M07 + M08 + M11 + M13) - (M02 + M04 + M09)$$



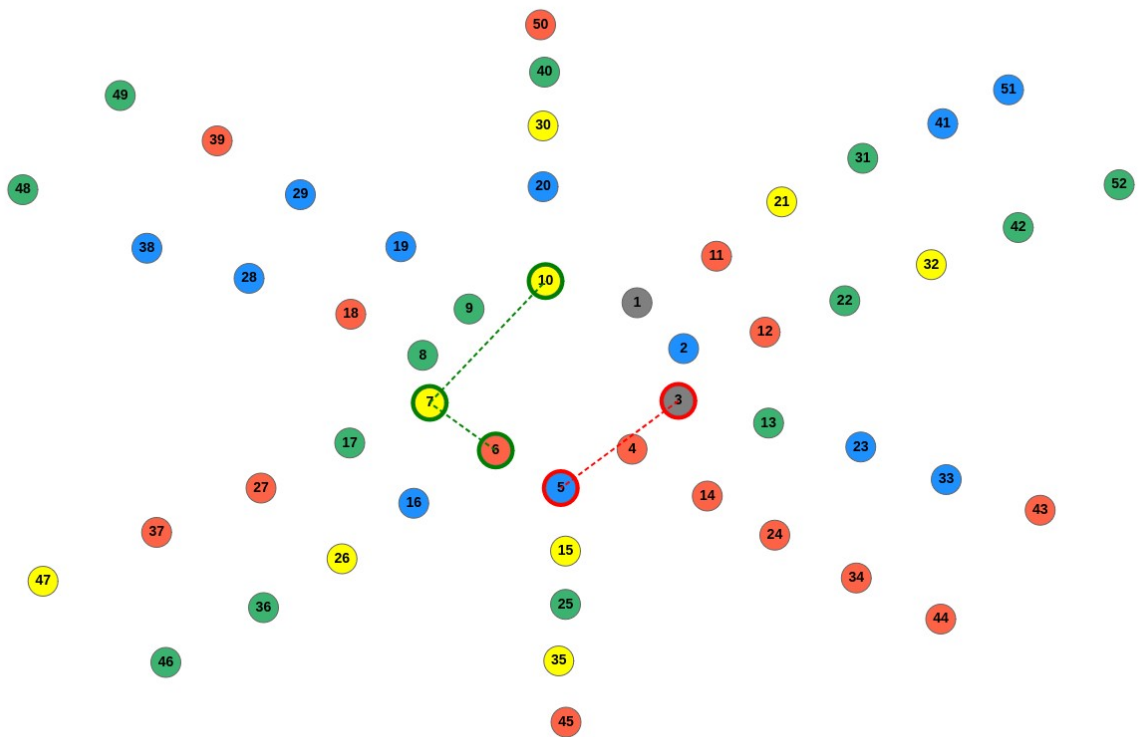
$$M12 = (M09 + M10) - (M02 + M04 + M05)$$



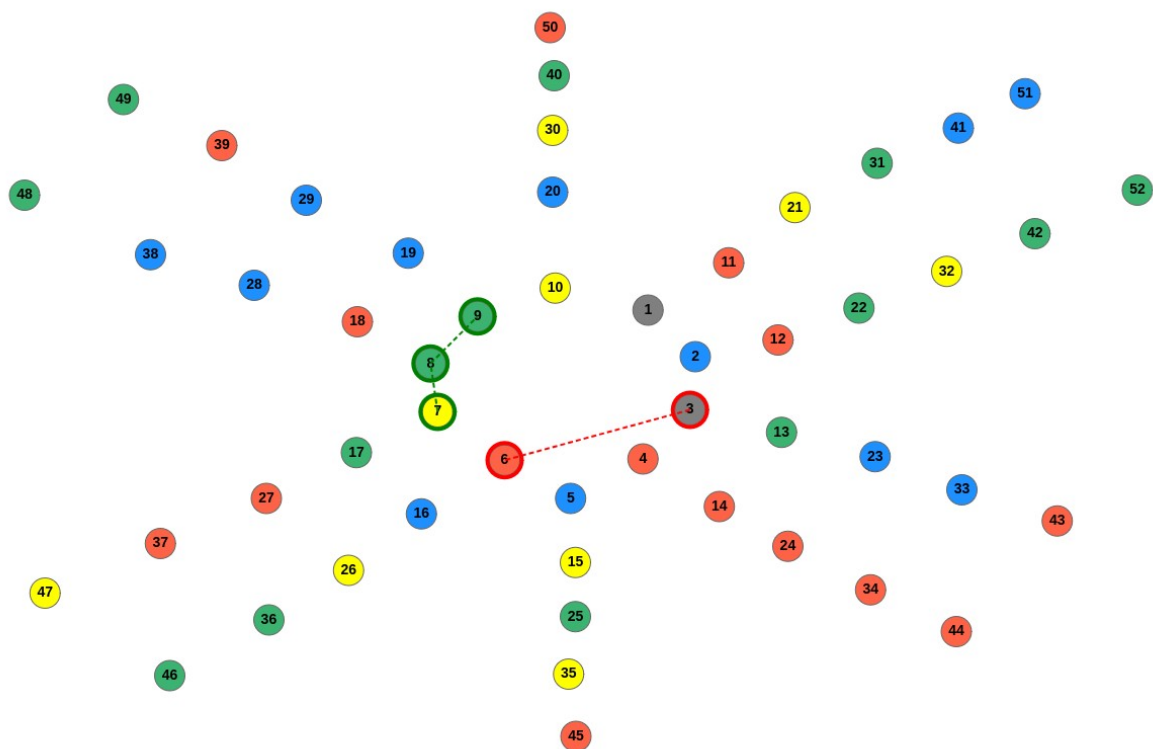




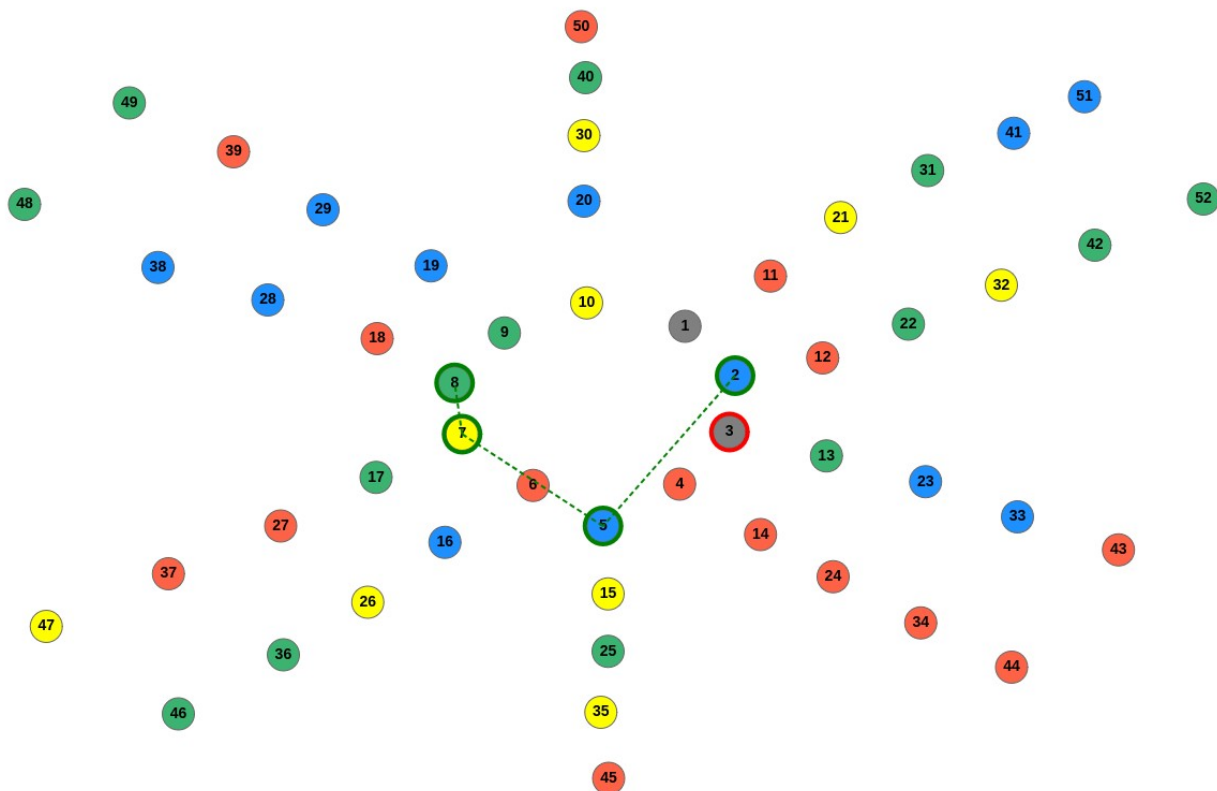
$$M11 = (M06 + M07 + M10) - (M03 + M05)$$



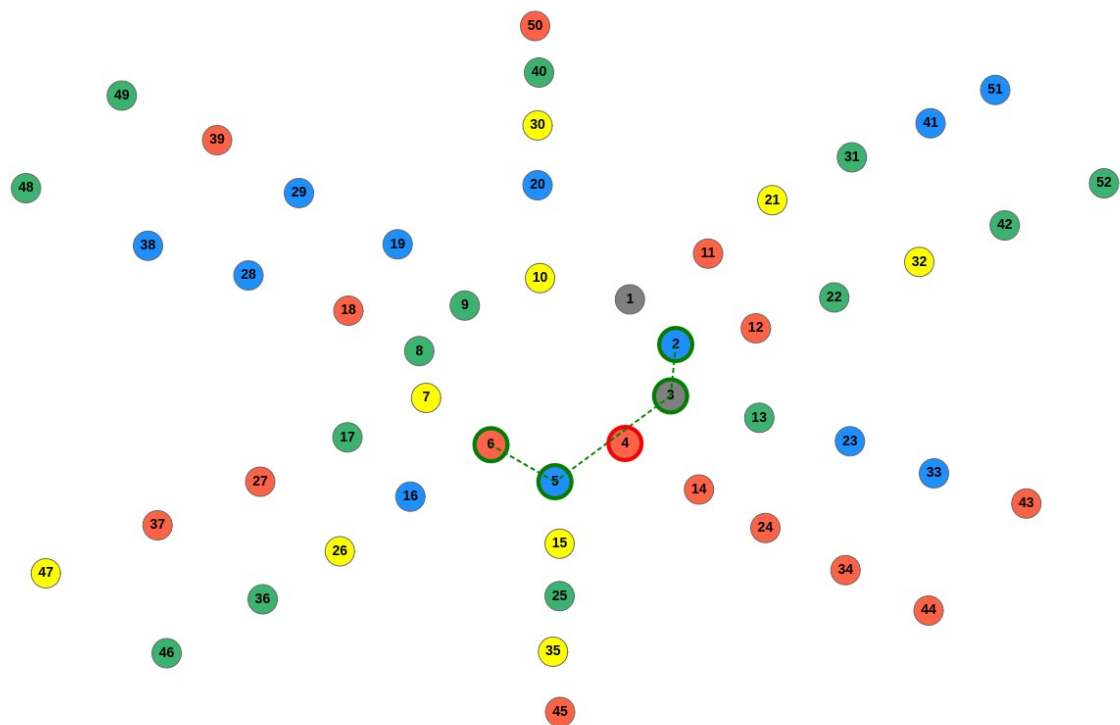
$$M10 = (M07 + M08 + M09) - (M03 + M06)$$



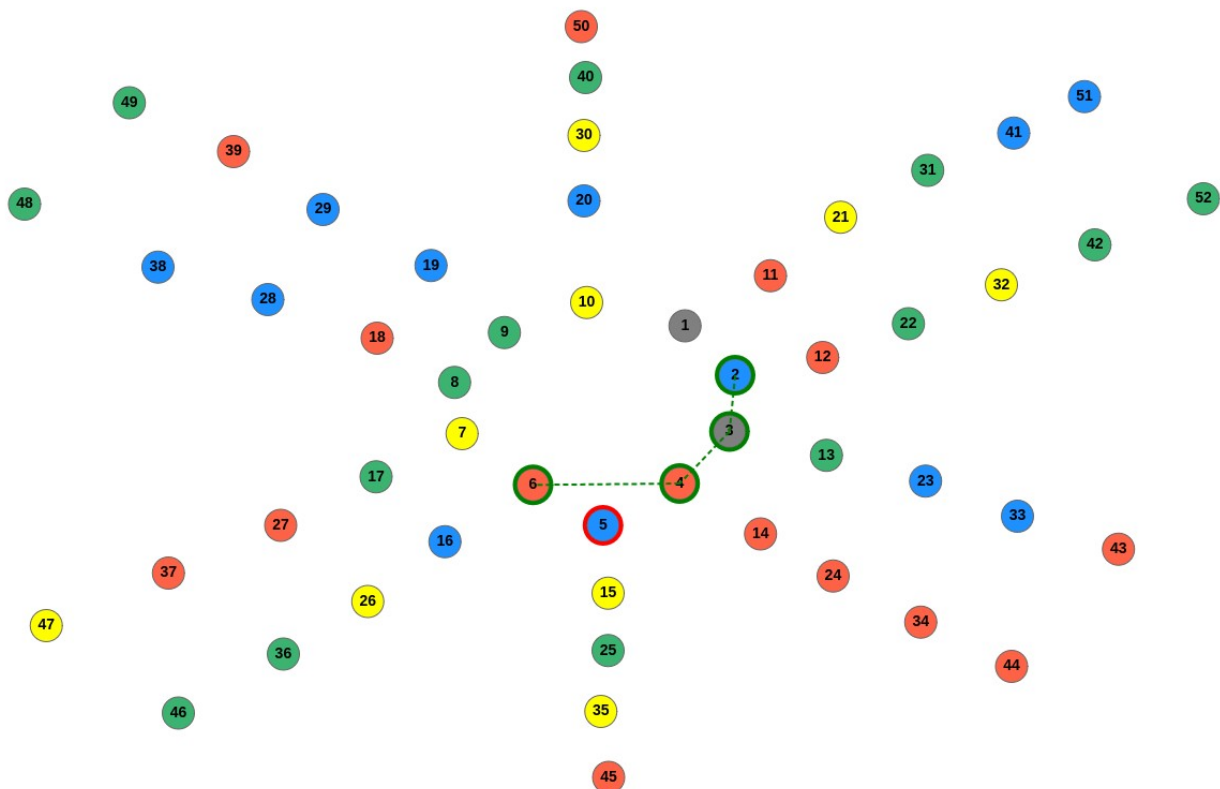
$$M09 = (M02 + M05 + M07 + M08) - (M03)$$



$$M08 = (M02 + M03 + M05 + M06) - (M04)$$

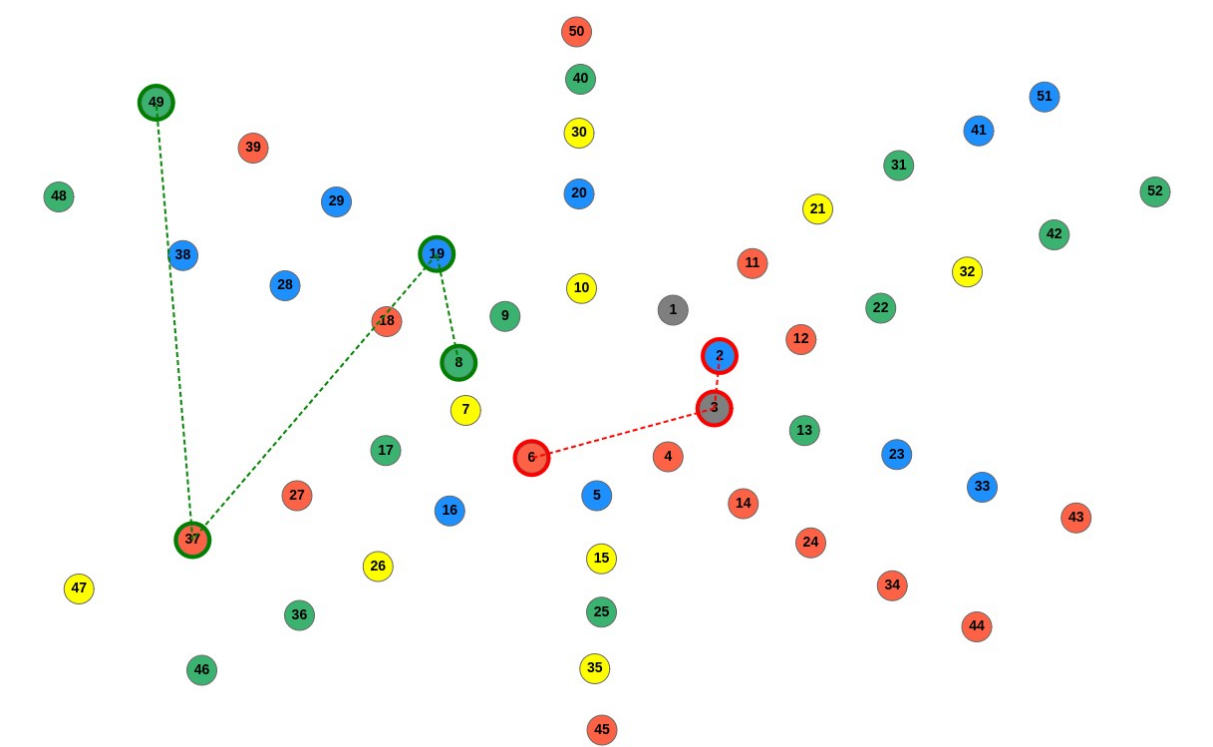


$$M7 = (M02 + M03 + M04 + M06) - (M05)$$

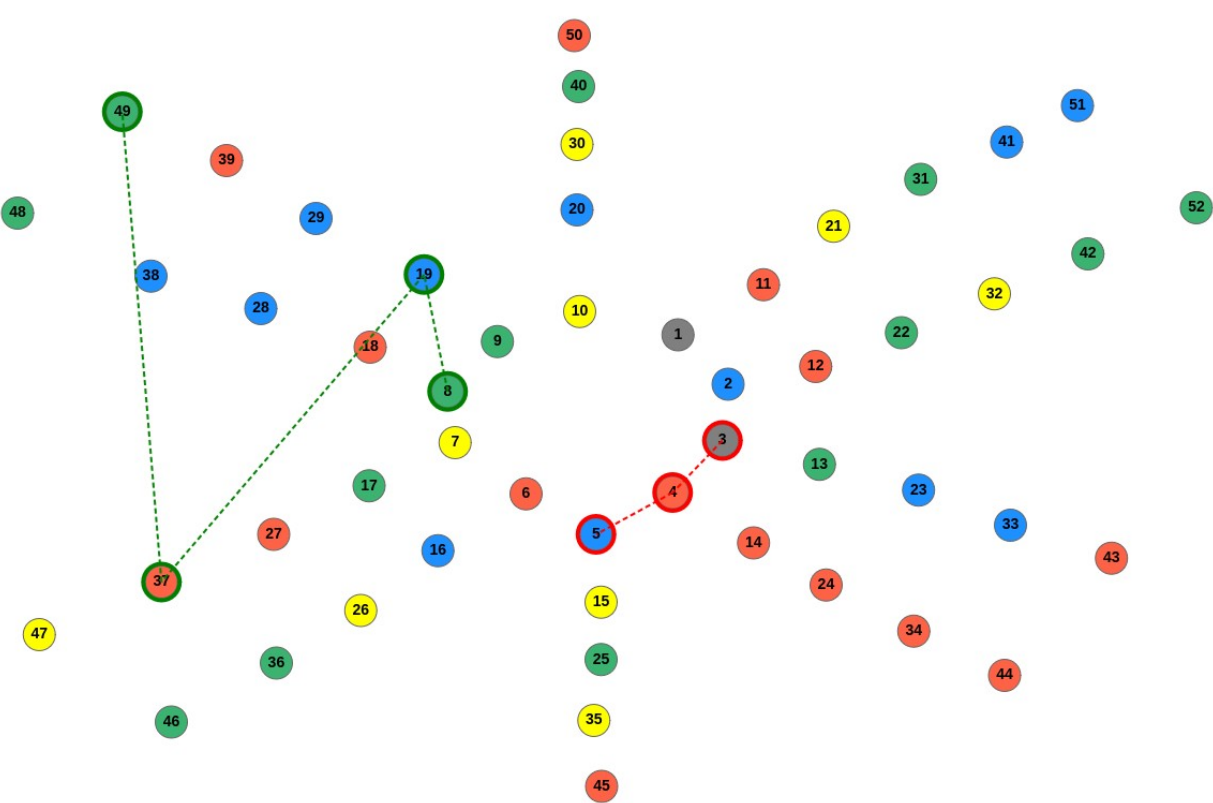


Here are some example for a same number, different decompositions:

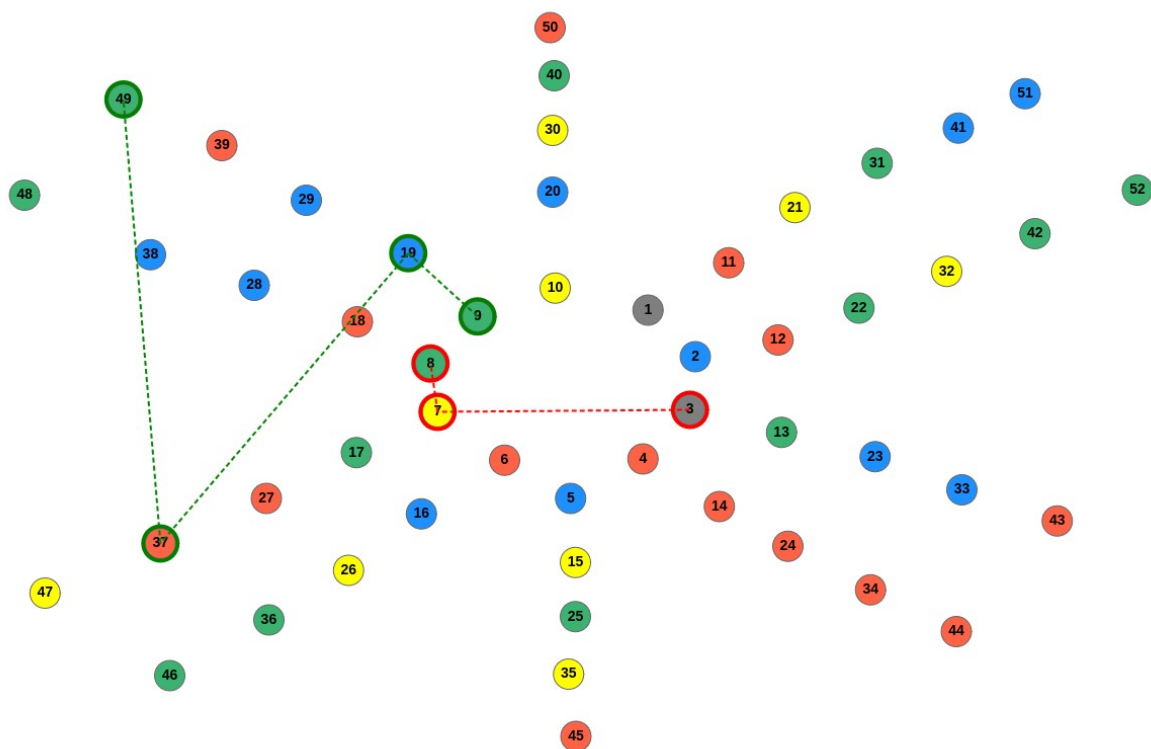
$$M50 = (M08 + M19 + M37 + M49) - (M02 + M03 + M06)$$



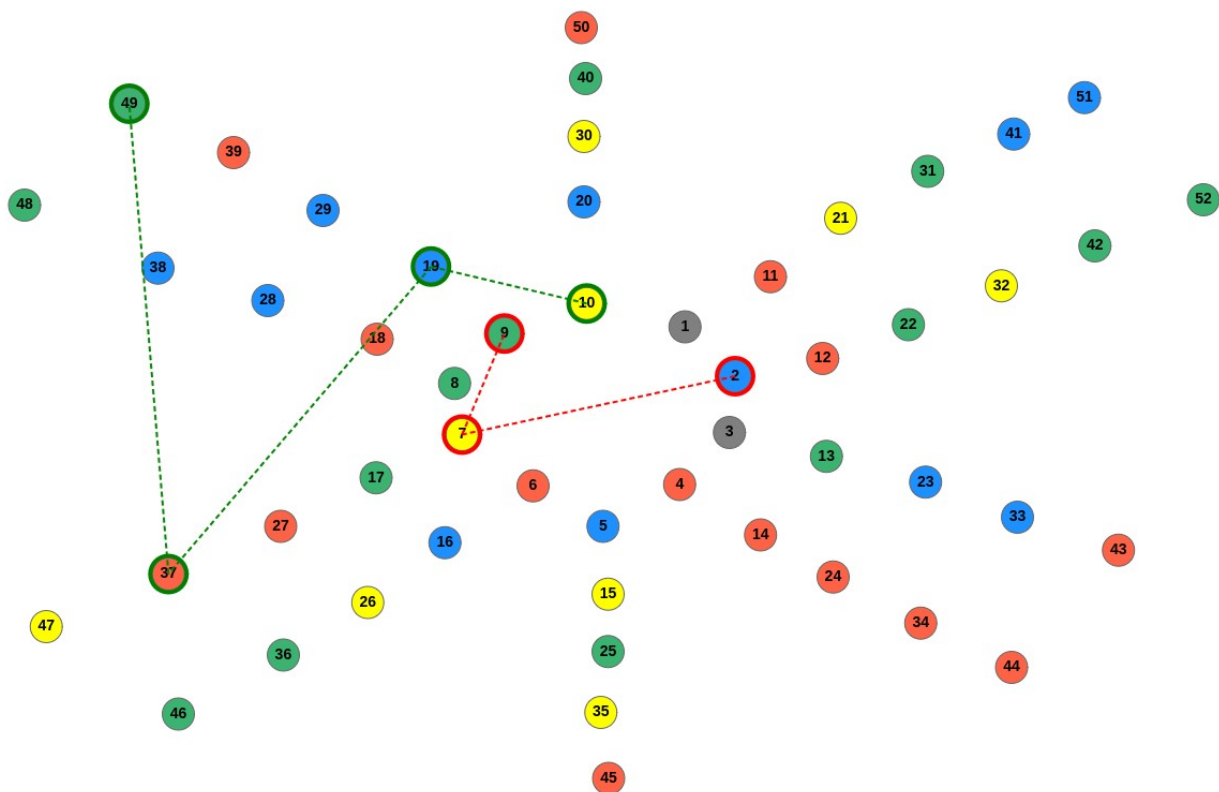
$$M50 = (M08 + M19 + M37 + M49) - (M03 + M04 + M05)$$



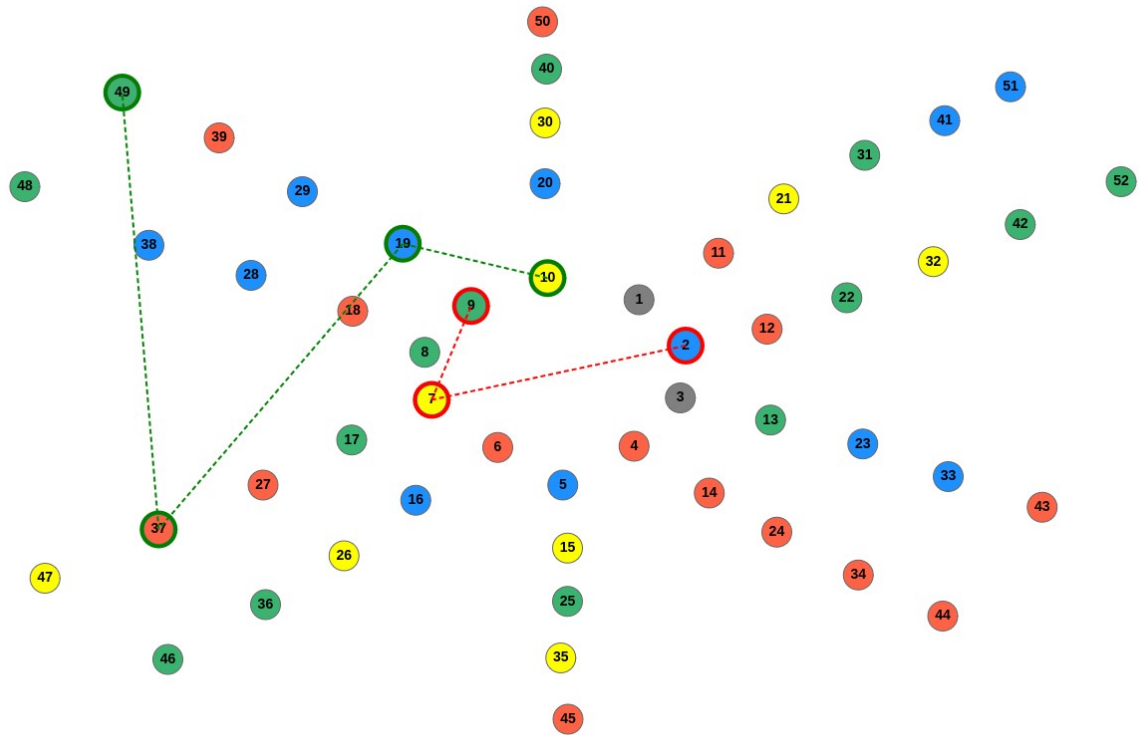
$$M50 = (M09 + M19 + M37 + M49) - (M03 + M07 + M08)$$



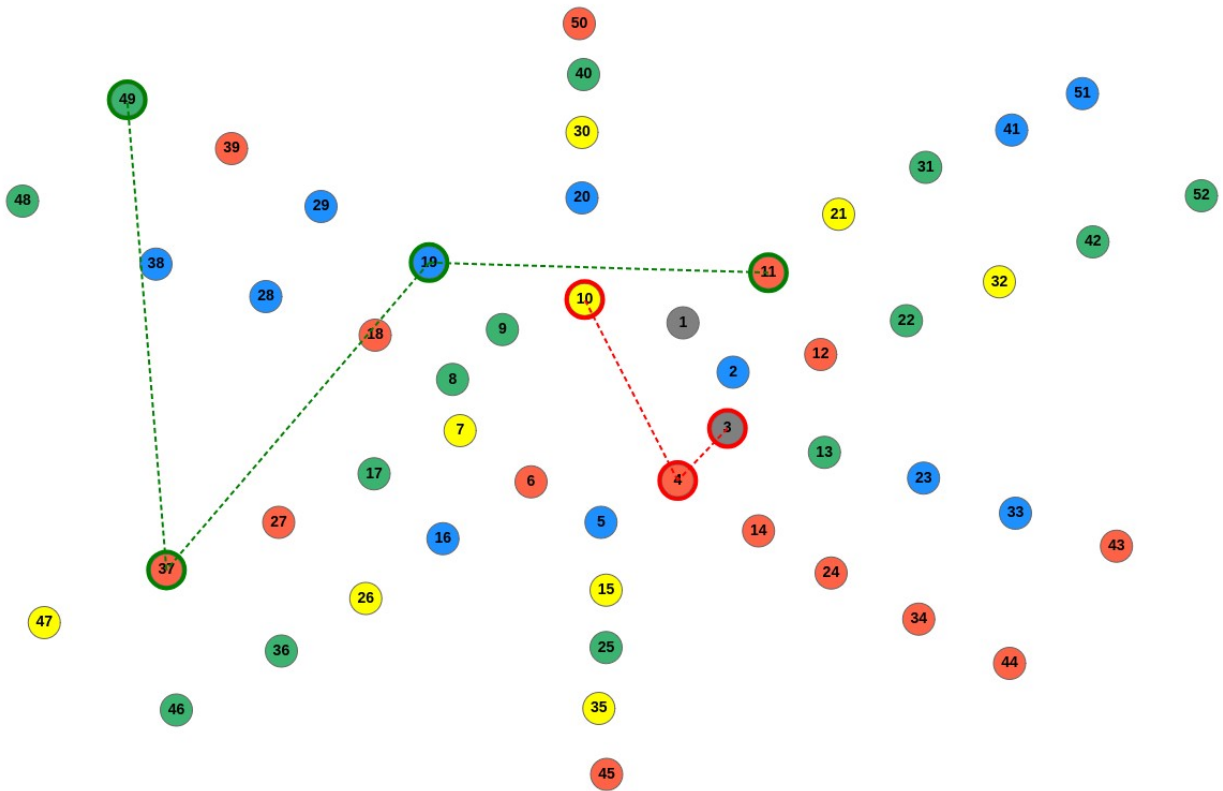
$$M50 = (M10 + M19 + M37 + M49) - (M02 + M07 + M09)$$



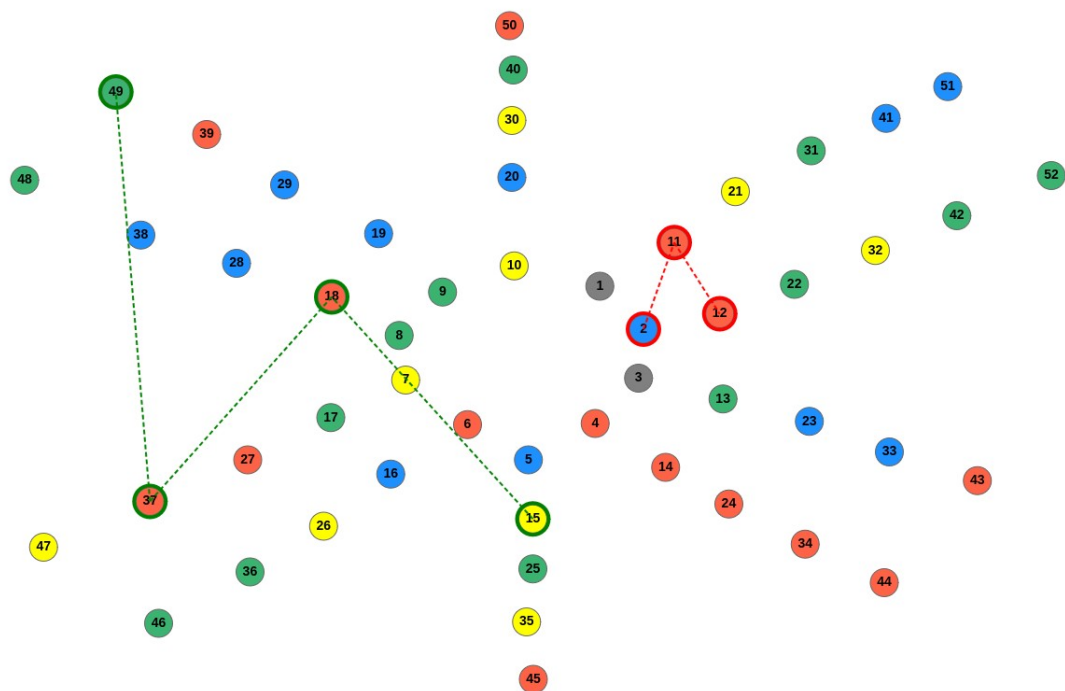
$$M50 = (M10 + M19 + M37 + M49) - (M02 + M07 + M09)$$



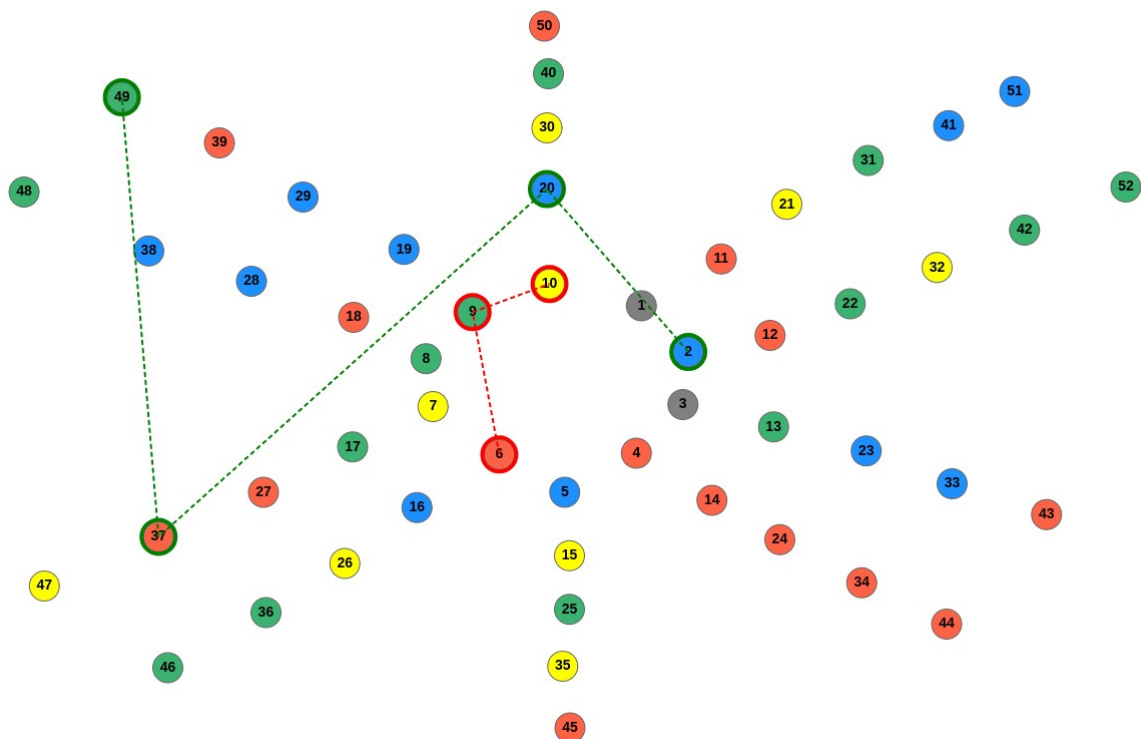
$$M50 = (M11 + M19 + M37 + M49) - (M03 + M04 + M10)$$



$$M50 = (M15 + M18 + M37 + M49) - (M02 + M11 + M12)$$

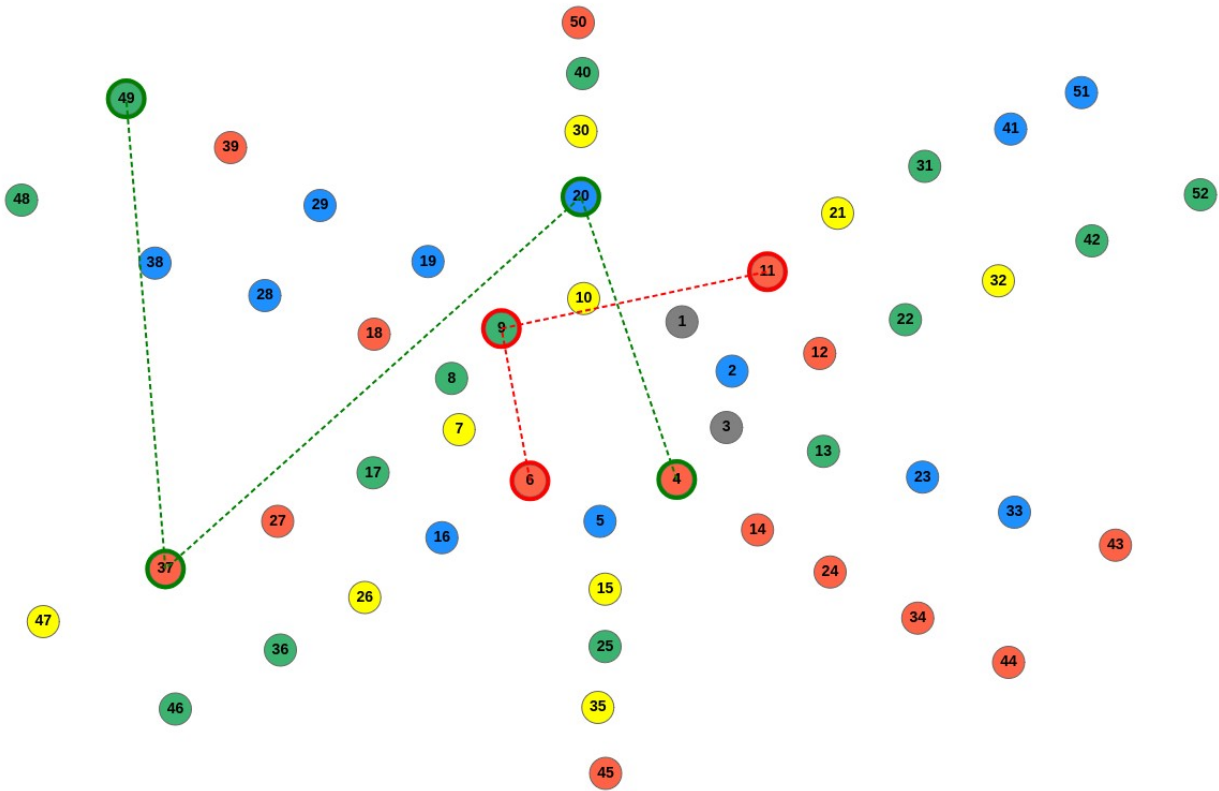


$$M50 = (M02 + M20 + M37 + M49) - (M06 + M09 + M10)$$

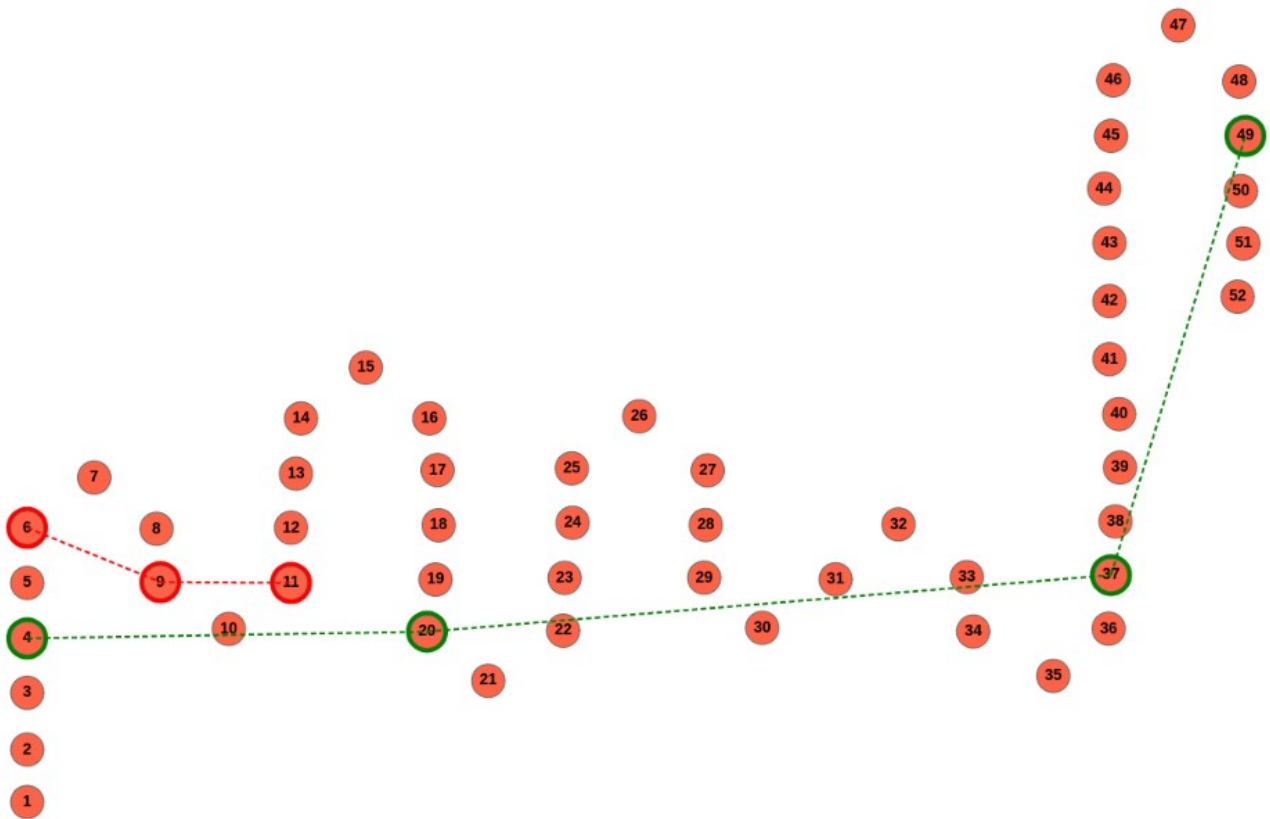


$$M50 = (M04 + M20 + M37 + M49) - (M06 + M09 + M11)$$





$$M50 = (M04 + M20 + M37 + M49) - (M06 + M09 + M11) - \text{different layer}$$



## Appendix:

### Labels: M...

```
[ {label:"M52",value:136279841},{label:"M51",value:82589933},{label:"M50",value:77232917},
{label:"M49",value:74207281},{label:"M48",value:57885161},{label:"M47",value:43112609},
{label:"M46",value:42643801},{label:"M45",value:37156667},{label:"M44",value:32582657},
{label:"M43",value:30402457},{label:"M42",value:25964951},{label:"M41",value:24036583},
{label:"M40",value:20996011},{label:"M39",value:13466917},{label:"M38",value:6972593},
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{label:"M34",value:1257787},{label:"M33",value:859433},{label:"M32",value:756839},
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{label:"M25",value:21701},{label:"M24",value:19937},{label:"M23",value:11213},
{label:"M22",value:9941},{label:"M21",value:9689},{label:"M20",value:4423}, {label:"M19",value:4253},
{label:"M18",value:3217},{label:"M17",value:2281}, {label:"M16",value:2203},{label:"M15",value:1279},
{label:"M14",value:607}, {label:"M13",value:521},{label:"M12",value:127},{label:"M11",value:107},
{label:"M10",value:89},{label:"M09",value:61},{label:"M08",value:31}, {label:"M07",value:19},
{label:"M06",value:17},{label:"M05",value:13}, {label:"M04",value:7},{label:"M03",value:5},
{label:"M02",value:3}, {label:"M01",value:2} ]
```

**End digit 9:** color yellow

3: color DodgerBlue

7: color Tomato

1 : color MediumSeaGreen

2 or 5: color grey

**Positive (black border):** example: (02+12 + 26)

**Negative (red border):** example: - (4 + 7 +8)

## Sums used above:

$$M51 = (M02 + M05 + M42 + M48) - (M12 + M17 + M34)$$

$$M50 = (M02 + M21 + M37 + M49) - (M05 + M16 + M18)$$

$$M47 = (M11 + M39 + M43) - (M01 + M08 + M32)$$

$$M45 = (M01 + M38 + M43) - (M05 + M17 + M31)$$

$$M44 = (M18 + M27 + M37 + M43) - (M02 + M30 + M32)$$

$$M43 = (M28 + M35 + M36 + M42) - (M03 + M05 + M26)$$

$$M42 = (M03 + M32 + M34 + M41) - (M02 + M06 + M28)$$

$$M41 = (M03 + M05 + M28 + M42) - (M02 + M32 + M34)$$

$$M37 = (M16 + M25 + M27 + M36) - (M03 + M08 + M26)$$

$$M35 = (M10 + M27 + M31 + M34) - (M02 + M21 + M29)$$

$$M33 = (M07 + M25 + M29 + M32) - (M02 + M21 + M24)$$

$$M31 = (M16 + M19 + M29 + M30) - (M02 + M23 + M25)$$

$$M29 = (M19 + M21 + M23 + M31) - (M02 + M27 + M28)$$

$$M28 = (M15 + M25 + M26 + M27) - (M02 + M06 + M20)$$

$$M27 = (M02 + M22 + M25 + M26) - (M09 + M14 + M21)$$

$$M26 = (M14 + M21 + M23 + M25) - (M02 + M09 + M24)$$

$$M25 = (M15 + M17 + M22 + M24) - (M02 + M13 + M23)$$

$$M24 = (M15 + M17 + M22 + M23) - (M02 + M13 + M19)$$

$$M23 = (M15 + M17 + M20 + M21) - (M02 + M16 + M19)$$

$$M22 = (M06 + M15 + M18 + M21) - (M02 + M03 + M19)$$

$$\begin{aligned}
M_{21} &= (M_{06} + M_{18} + M_{19} + M_{20}) - (M_{03} + M_{05} + M_{16}) \\
M_{20} &= (M_{02} + M_{13} + M_{18} + M_{19}) - (M_{10} + M_{15} + M_{16}) \\
M_{19} &= (M_{08} + M_{14} + M_{16} + M_{18}) - (M_{03} + M_{13} + M_{15}) \\
M_{18} &= (M_{10} + M_{11} + M_{15} + M_{17}) - (M_{03} + M_{05} + M_{13}) \\
M_{17} &= (M_{02} + M_{05} + M_{15} + M_{16}) - (M_{10} + M_{13} + M_{14}) \\
M_{16} &= (M_{06} + M_{13} + M_{14} + M_{15}) - (M_{03} + M_{10} + M_{12}) \\
M_{15} &= (M_{11} + M_{12} + M_{13} + M_{14}) - (M_{03} + M_{06} + M_{09}) \\
M_{14} &= (M_{07} + M_{08} + M_{11} + M_{13}) - (M_{02} + M_{04} + M_{09}) \\
M_{12} &= (M_{09} + M_{10}) - (M_{02} + M_{04} + M_{05}) \\
M_{11} &= (M_{06} + M_{07} + M_{10}) - (M_{03} + M_{05}) \\
M_{10} &= (M_{07} + M_{08} + M_{09}) - (M_{03} + M_{06}) \\
M_{09} &= (M_{02} + M_{05} + M_{07} + M_{08}) - (M_{03}) \\
M_{08} &= (M_{02} + M_{03} + M_{05} + M_{06}) - (M_{04}) \\
M_{07} &= (M_{02} + M_{03} + M_{04} + M_{06}) - (M_{05}) \\
M_{50} &= (M_{08} + M_{19} + M_{37} + M_{49}) - (M_{02} + M_{03} + M_{06}) \\
M_{50} &= (M_{08} + M_{19} + M_{37} + M_{49}) - (M_{03} + M_{04} + M_{05}) \\
M_{50} &= (M_{09} + M_{19} + M_{37} + M_{49}) - (M_{03} + M_{07} + M_{08}) \\
M_{50} &= (M_{10} + M_{19} + M_{37} + M_{49}) - (M_{02} + M_{07} + M_{09}) \\
M_{50} &= (M_{10} + M_{19} + M_{37} + M_{49}) - (M_{02} + M_{07} + M_{09}) \\
M_{50} &= (M_{11} + M_{19} + M_{37} + M_{49}) - (M_{03} + M_{04} + M_{10}) \\
M_{50} &= (M_{15} + M_{18} + M_{37} + M_{49}) - (M_{02} + M_{11} + M_{12}) \\
M_{50} &= (M_{02} + M_{20} + M_{37} + M_{49}) - (M_{06} + M_{09} + M_{10}) \\
M_{50} &= (M_{04} + M_{20} + M_{37} + M_{49}) - (M_{06} + M_{09} + M_{11})
\end{aligned}$$

#### Example calculus:

$$M_{50} = (M_{04} + M_{20} + M_{37} + M_{49}) - (M_{06} + M_{09} + M_{11})$$

$$(M_{04} + M_{20} + M_{37} + M_{49}) = 7 + 4,423 + 3,021,377 + 74,207,281 = 77,233,088$$

$$(M_{06} + M_{09} + M_{11}) = 17 + 61 + 107 = 185$$

$$77233088 - 185 = 7.7232903 \times 10^7 = M_{50}$$

**Available in Github:** <https://github.com/gatanegro/MERSENNE-COLLATZ>

App to calculate sums using labels : **Mersenne Sum Explorer Logos Theory.html**

App to calculate equations : **Mersenne Equation Explorer.html**

App to Research Geometry visuals: **Mersenne Spiral – Group Geometry with Import Layer.html**

App to predict possible candidate : **Mersenne Spiral – Sum Calculator with Placeholders..html**

App to Research Geometry visuals importing new layers: **Mersenne Spiral – Layer Switcher.html**

Reference :

(1)Martin, D. (2026). Geometric Decomposition of Mersenne Exponents: A Novel Additive Representation System. Zenodo. <https://doi.org/10.5281/zenodo.18256256>