The magnitude distribution of declustered earthquakes in Southern California

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The binned distribution densities of magnitudes in both the complete and the declustered catalogs of earthquakes in the Southern California region have two significantly different branches with crossover magnitude near $M = 4.8$. In the case of declustered earthquakes, the *b*-values on the two branches differ significantly from each other by a factor of about two. The absence of self-similarity across a broad range of magnitudes in the distribution of declustered earthquakes is an argument against the application of an assumption of scale-independence to models of main-shock earthquake occurrence, and in turn to the use of such models to justify the assertion that earthquakes are unpredictable. The presumption of scale-independence for complete local earthquake catalogs is attributable, not to a universal process of self-organization leading to future large earthquakes, but to the universality of the process that produces aftershocks, which dominate complete catalogs.

Because of its reputation of validity over a wide range of magnitudes, the log-linear Gutenberg–Richter (G-R) frequency-magnitude law of earthquake occurrence (1, 2)

$$\log_{10} N = a - bM \qquad [1]$$

has been a simple paradigm for modeling the evolution of earthquake patterns. Eq. **1** is expressed in the power law form

$$N \sim E^{-b/\beta} \qquad [2]$$

through the intermediary of either a logarithmic energy-magnitude relation derived originally by Gutenberg (3) or a logarithmic moment-magnitude relation (4, 5); $\beta \approx 3/2$ for both surface wave and local magnitudes in the magnitude range of this paper (3). Because of the presumed universality of local estimates of the exponent $b \approx 1$, and because of the scale independence implicit in Eq. **2**, the model of self-organized criticality (SOC) to understand earthquake occurrence (6–9) has been proposed and discussed abundantly in recent years. The correspondence between the observation of a power-law relation in nature and in derivations of the power law from the SOC model does not validate the model, since other scale-independent models would also be expected to yield power-law behavior. In this contribution, I show that Eq. **1** does not have a uniform value of b across a broad range of magnitudes of reliable observations of declustered events, and hence Eq. **1** with b constant across a broad range is not a valid paradigm for modeling purposes. I show that aftershocks, which also exhibit power-law behavior, are a statistically preferable alternative to SOC to understand the G-R relation.

There are three points of concern in the interpretation of Eq. **1** as a scale-independent paradigm for modeling earthquakes. The frequency-magnitude law **1** is only one of three local relations for earthquakes presumed to be globally valid. The other two are the Omori decay rate law for the aftershocks of great earthquakes (10, 11),

$$\dot{n} \sim \frac{1}{(t + c)^p} \qquad [3]$$

and the G-R frequency-magnitude law **1**, which is also valid for aftershocks (11); both have often been shown to describe aftershocks of large earthquakes worldwide and both have the same exponents $p \approx 1$, $b \approx 1$, worldwide. It is unexpected that the coefficient $b \approx 1$ should be the same for aftershocks and for the complete catalog. Our first concern is therefore to understand the coincidence or otherwise of the exponents b in the two disparate types of earthquake sequences. Universality of exponents is usually taken to indicate a universality of the physics of earthquake occurrence, and that an explanation that is unique for each local region is not needed. It follows that the observation that Eq. **1** holds for aftershocks suggests that they too are produced by a scale-independent and universal process.

Second, the assumption of universality of Eq. **1** is usually based on the perceived linearity of the cumulative distribution, introduced into the literature of seismicity by Richter (12). Cumulative distributions are integrations and therefore biased; the smoothing perforce obscures fluctuations in the distribution density, if any. The apparent validity of the smoothed G-R law across all magnitudes, except for rolloff at the largest magnitudes, has deterred searches for fluctuations in the distribution density. Indeed, temporal fluctuations in the relation **1** on the decadal scale have been identified (13). We must reassure ourselves that the G-R law **1**, with b a constant across a wide range of magnitudes, is indeed a valid property of the distribution density.

Third, we note that most simulations of the earthquake evolutionary process, such as those of SOC and others, are limited to modeling main sequence earthquakes or main shocks, which are the response to a steadily increasing stress (14) derived from plate tectonics. On the other hand, real catalogs of earthquakes include not only main shocks, but also clustered bursts of earthquakes that include aftershocks, foreshocks, and swarms; the aftershocks dominate the cluster population. Aftershocks are the response to a suddenly applied stress impulse, usually from a single very large earthquake. Not only do the driving mechanisms differ, but also the two sets of earthquake events occur on different time scales: the cluster sequences occur on a time scale of months to years whereas, except for great earthquakes on the San Andreas fault (SAF), the time intervals between the largest earthquakes on the same fault in Southern California are of the order of several millennia (15). Great earthquakes on the SAF do occur on intervals of the order of a century with large fluctuations (16), but the aftershock sequence of the great 1857 earthquake in this region was also relatively shorter; we assume the aftershock sequences of other great earthquakes on the SAF are similarly short. Most catalogs list earthquakes collected over time scales that are more appropriate to cluster processes than to tectonic processes. To derive an appropriate statistical target for physics-based modeling of the forward organization of main-sequence earthquakes, we must

Abbreviations: G-R, Gutenberg–Richter; SOC, self organized criticality; SAF, San Andreas Fault.

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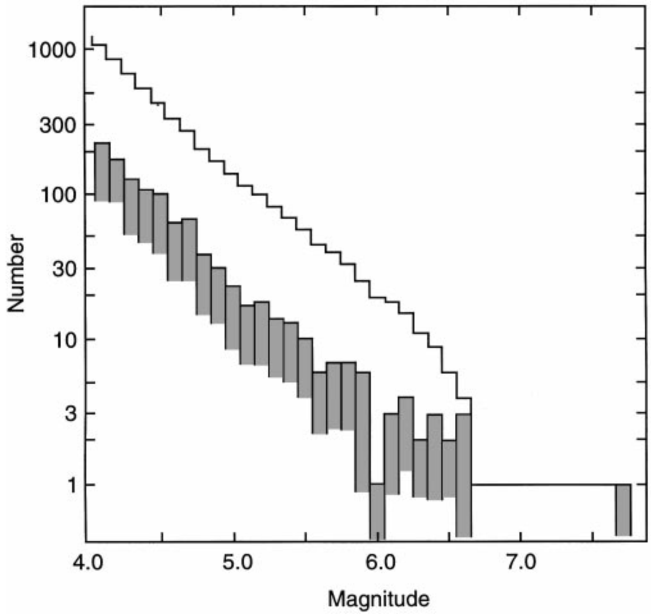

Fig. 1. Cumulative and differential (shaded) distributions of the magnitudes of all earthquakes in the Southern California region from 7-1-44 to 3-1-90.

subtract the clusters from the full catalog to derive the properties of a main shock catalog. Otherwise, we would be guilty of having used the properties of a catalog that includes both clusters and main shocks and having attributed these properties to the main shocks alone.

The Complete Regional Catalog

We analyze the local catalog of the Southern California region, a well-defined 22-sided polygon (17, 18). In addition, we consider only earthquakes with epicenters north of 32.5° N to avoid questions of completeness south of this boundary. We use only earthquakes after July 1, 1944, which was the date of transition from the binning of magnitudes by intervals $\Delta M =$ 0.5 to intervals $\Delta M = 0.1$. We use earthquakes only before March 1, 1990, which is the date of a major re-evaluation of the magnitudes of all earthquakes in the catalog with $M \geq 4.8$ (19), but not the smaller ones; for consistency, we use the unrevised magnitudes. We note below that our conclusions are not altered by the revision. We use magnitudes greater than 4.1, there being a statistically demonstrable rolloff at $M = 4.0$. The largest earthquake in this catalog fragment is the 1952 Kern County earthquake with unrevised magnitude $M = 7.7$. The cumulative distribution and the binned density are shown in Fig. 1. The cumulative distribution seems to have the property of self-similarity across the entire range of magnitudes to about $M = 6.6$. As expected, the distribution density displays larger fluctuations.

To bypass the problem of the smoothing and the bias in the cumulative distribution, and to study fluctuations within smaller magnitude ranges, we consider the subset of the binned distribution density $n[M_\ell, M_u]$ where M_u and M_ℓ are the greatest and least binned magnitudes. The maximum likelihood estimate for the b-value of the binned, truncated version of Eq. **1** is easily shown to be the solution to the transcendental equation (20)

$$L(b) = \frac{\Delta M}{10^{b\Delta M} - 1} - \frac{\mu}{10^{b\mu} - 1} = \bar{M} - M_\ell, \qquad [4]$$

where $\mu = M_u - M_\ell + \Delta M$, and $\bar{M}$ is the average of the magnitudes of the earthquakes in the sample. The first term corrects for

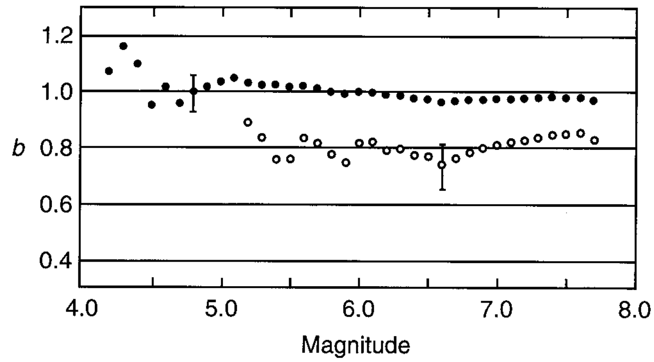

Fig. 2. Maximum likelihood estimates of b-values as a function of variable upper magnitude M_u for the complete Southern California catalog. The lower magnitudes M_ℓ are fixed at 4.1 (solid circles) and 4.9 (open circles) in the two cases. Error bars indicate the range $\pm 1\sigma$ for the independent estimates $b[4.1, 4.8]$ and $b[4.9, 6.6]$.

centroid bias due to binning and the second accounts for truncation at the upper magnitude. Under the assumption that the magnitudes are independent of each other, the variance of the estimate of b is

$$\sigma_b^2 = \frac{1}{(N-1)|\partial L/\partial b|}, \qquad [5]$$

where N is the total number of earthquakes in the sample. In the limit $\Delta M \to 0$, $M_u \to \infty$ Eqs. **4** and **5** give the usual result (21–23),

$$\frac{1}{b \log_e 10} = \bar{M} - M_\ell, \quad \sigma_b = b/\sqrt{N-1}. \qquad [6]$$

Because of the large number of earthquakes involved in the analysis, we are not concerned with uncertainty in the magnitude determinations.

The exponent $b[4.1, M_u]$ as a function of M_u is shown as the solid dots in Fig. 2, and is listed for selected M_u in Table 1. The sequence of values is asymptotic to the standard value $b \approx 1$ for M_u large enough. All standard deviations are given as 1σ. However, the values $b[4.9, M_u]$ are asymptotic to a significantly lower value, $b \approx 0.8$. Values of $b[M_\ell, M_u]$ for other choices of M_ℓ are not shown or given here; the choice $M_I = 4.8$ maximizes the differences between the b-values of the two independent samples $b[4.1, M_I]$ and $b[M_I + 0.1, M_u]$ for M_u large enough, after taking the variances into account; thus $M_I = 4.8$ is not a sharp crossover magnitude. The independent estimates $b[4.1, 4.8]$ and $b[4.9, M_u]$, M_u large enough, differ largely because of the properties of the distributions near the two centroids, which are near $M = 4.5$ and 5.3; the stability of the estimates of b for large M_u in Fig. 2, is due to the small numbers of largest earthquakes. The differences are statistically significant according to standard tests for the difference of two means (24) weighted by $|\partial L/\partial b|$.

We cannot discriminate among values of $b[4.9, M_u]$ for M_u large enough. For example, the differences between the values $b[4.9, 6.6] = 0.73$ and $b[4.9, 7.7] = 0.82$ are not significant; the

Table 1. *b*-values for complete catalog

Mag. range	Total	Clusters	Main shocks
4.1–7.7	0.97 ± .03		0.87 ± .05
4.1–6.6	0.96 ± .03	1.01 ± .04*	0.86 ± .05
4.1–4.8	0.98 ± .07	1.01 ± .08	0.99 ± .12

*M_u = 6.4.

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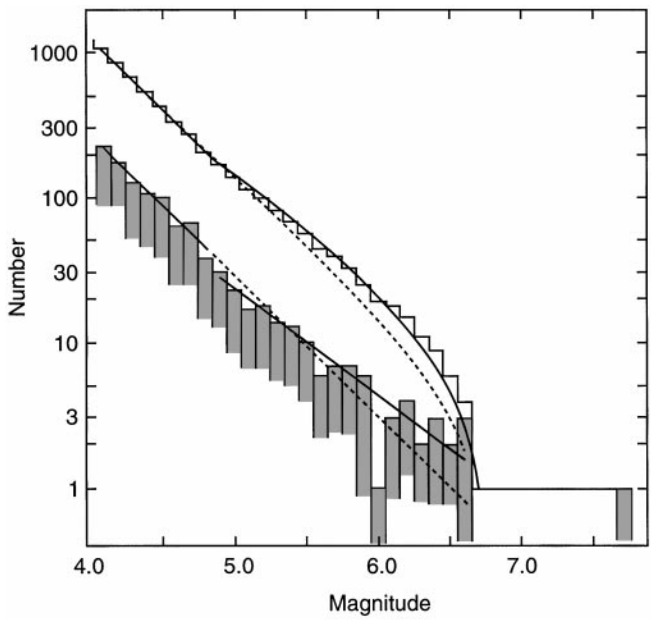

Fig. 3. Maximum likelihood fits to the cumulative and differential (shaded) distributions of the magnitudes of the complete catalog of earthquakes in the Southern California region from 7-1-44 to 3-1-90. The dashed line is the fit $4.1 \leq M \leq 6.6$ under the assumption that there is a single branch to the distribution. The solid curve is the fit for the two independent branches $4.1 \leq M \leq 4.8$ and $4.9 \leq M \leq 6.6$. The curvature in the cumulative fit at large magnitudes is due to the assumption of a finite upper magnitude cutoff.

nominal differences in the two b-values are ascribable to the absence of earthquakes with magnitudes between 6.6 and 7.7. We plot the two branches with values $b[4.1, 4.8] = 0.98$ and $b[4.9, 6.6] = 0.73$ as the fit to the distribution in Fig. 3; we could with equal justification plot the larger-magnitude branch as having $b[4.9, 7.7]$, since as remarked, the differences are not significant. The rolloff in the cumulative distribution in Fig. 3 is due to the finite upper magnitude cutoff, $M_u = 6.6$. Whether the cutoff is taken to be $M = 6.6$ or larger, we conclude that the distribution for the full catalog, which contains both clusters and main shocks, has a characteristic crossover magnitude around $M_I = 4.8$; the crossover corresponds to a characteristic length scale of fractures of roughly 3 km. Even without declustering, these results indicate that there is no justification for the use of fracture models such as SOC and others to model the main shock earthquake process under an assumption of the scale independence of Eq. **1**.

The Catalog of Main Shocks

We construct a catalog of main shocks appropriate for modeling emergent processes. We define the main shock catalog to be the residual catalog after clusters are identified and removed. Our algorithm for the identification of aftershocks also identifies foreshocks and clusters. We take the main shock to be the largest earthquake in a cluster series. Windowing algorithms for cluster identification (25–27) make use of a space-time window around and following each event, whether it is a cluster event or not; any earthquake occurring within the window is deemed a cluster event. The window is opened wider for stronger predecessor events. We use window parameters for smaller parent earthquakes (see Table 3) that are intermediate between the broad values of Gardner and Knopoff and the restrictive values of Reasenberg, the latter derived under the unreasonable assumption that an aftershock site is not in a prestressed state.

Table 2. *b*-values for partitioned catalog

Mag. range	Total	Clusters	Main shocks
4.1–4.8	0.98 ± .07	1.01 ± .08	0.99 ± .12
4.9–7.7	0.82 ± .07		0.64 ± .10
4.9–6.6	0.73 ± .08	0.83 ± .11*	0.52 ± .12
4.9–5.9	0.74 ± .12	0.90 ± .16	0.50 ± .19

*$M_u = 6.4$.

The above procedure satisfactorily identifies cluster events belonging to small predecessor earthquakes, i.e., for small windows. However, spatially circular windows are inadequate for the identification of aftershocks of the largest earthquakes, which form elongated patterns along the length of the fracture of the earthquake; these fractures extend as much as 75 km in the case of the Kern County earthquake. To identify aftershocks associated with events with $M \geq 6.4$, we divide the region around the fracture of large earthquakes into overlapping $1/4° \times 1/4°$ rectangles and identify all contiguous rectangles that have at least one event with $M \geq 4.0$ within 10 days of the main shock. We then plot the cumulative number of events in these rectangles vs. log t to the end of the catalog; the aftershock sequence is defined to terminate when significant deviation from the Omori law (Eq. **3**) for $t \gg c$ with $p \equiv 1$ is identified; cases $p \approx 1$ are similarly treated. In every case, an individual sequence becomes dormant, except in the case of the Kern County sequence for which the aftershocks continue to the end of the catalog; in the latter case, the prolongation of the sequence is consistent with the observation that felt aftershocks of the great Mino-Owari (1891) earthquake that inspired Omori's statistical rate model, have persisted at a rate consistent with the Omori law for 100 years (28).

The aftershocks of the largest earthquakes taken individually, fit the Omori and G-R laws well with the usual exponents. The stacking of the magnitudes of all cluster events also fits the G-R relation well across all values of the magnitude (Table 1); there may be a decrease in b-value for the larger cluster events (Table 2), but it is not significant in this analysis. We show elsewhere by an independent analysis that there are indeed two populations of aftershocks, also divided at the critical value $M \approx 4.8$. The cluster events form about 2/3 of the full catalog, and about 2/3 of all the cluster events are aftershocks of the three largest earthquakes in the catalog. Hence the correspondence between the exponents in the full catalog and in clusters is due to the dominance of the catalog by clusters.

The distributions for the main shock catalog are shown in Fig. 4. The value $b[4.1, 7.7] = 0.87$ for the complete range of the main shock catalog is significantly lower than that for the complete catalog, a result due to a large reduction in the b-value for the larger earthquakes. As before, we divide the main shock catalog into two independent parts at $M = 4.8$, at which again the greatest significant differences between the two parts are displayed, under the *assumption* that the distribution for the larger magnitudes can be fit by Eq. **1**. The b-value for the smaller magnitude branch $b[4.1, 4.8] = 0.99 \pm 0.12$ is almost identical with that for the full catalog. But the b-value for the larger magnitude branch of declustered events $b[4.9, 6.6]$ is now lowered to $b = 0.52$ (Table 2), a number that is significantly different from the value in this range for the full catalog, $b = 0.73$, and from that for the small magnitude branch, $b = 0.99$. The fit is shown in Fig. 4; the discontinuity in b-values is manifested as a kink in the cumulative distribution. If we reduce the upper limit of the larger-magnitude branch to 5.9 to avoid the irregularities beginning around $M = 6.0$, our results are not changed significantly.

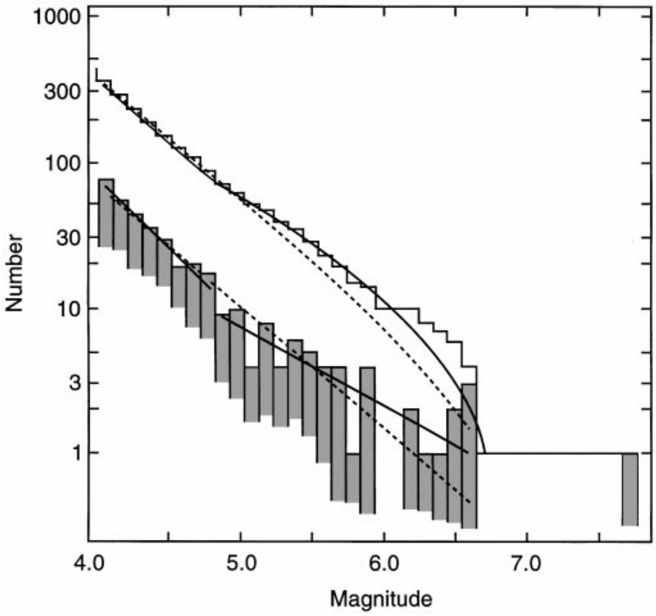

Fig. 4. Cumulative and differential (shaded) magnitude distribution of main shocks in the Southern California region. The distributions are fit by a single segment that spans the range $4.1 \leq M \leq 6.6$ (dashed), and by two segments that span the ranges $4.1 \leq M \leq 4.8$ and $4.9 \leq M \leq 6.6$ (solid). There is a kink in the fit to the cumulative distribution at $M = 4.8$ and a rolloff at large magnitudes because of the assumption of a finite upper magnitude cutoff.

The gamma distribution has been proposed (29–33) to describe the occurrence of the complete suite of earthquakes by a uniform mechanism extending from the smallest to the largest, culminating in a rolloff at the largest magnitudes. The gamma distribution is inappropriate to describe the properties of main shocks which have a b-value for larger magnitude earthquakes that is smaller than that for the smaller ones by a factor of about two. Whether it is an appropriate description of the distribution of clusters has not been determined.

If the revised magnitudes (19) are used, the two branches have b-values that are not changed significantly from those described above, with the same statistical uncertainties quoted above. The principal difference between the analyses of the two catalogs arises in the form of a large jump in the distribution at the crossover magnitude that gives the illusion of being significant statistically, but can be shown to be due to the revision of the magnitudes. The b-values for the two branches are not changed significantly. If we use the revised magnitudes M^* in the catalog that extends to the present date, well beyond the 1990 cutoff, and which includes the three very large earthquakes with $M \geq 6.7$ since 1994, these conclusions, including the misfit at the crossover, are also unchanged: we now obtain $b[4.9^*, 7.5^*] = 0.57$ for the main shock catalog, which is a reduction from the value 0.64 (see Table 2) for the truncated catalog that excludes these large events. Whether we use the revised or unrevised magnitudes, we reach the same conclusion regarding the existence of two branches to both the complete and main shock distributions, with b-values for the larger magnitude branch that are significantly lower than the traditional value; $b \approx 0.5$ in the case of the larger-magnitude branch of the main-shock distribution.

The Low Magnitude Branch

We invert in the first of the triptych of questions presented at the outset. If the usual value $b \approx 1$ is a characteristic of clusters, then is it a coincidence that the low-magnitude branch of the declustered distribution has the same b-value? These declustered earthquakes are not merely clustered earthquakes that have been misidentified as main shocks. If we unreasonably double the spatial widths of the windows in Table 3, we identify less than 2% of the number of smaller earthquakes needed additionally to reduce the b-value on the low-magnitude branch to 0.5. We also reject the possibility that the excess might be a residue of aftershocks of very large earthquakes that occurred before 1944, under an unsubstantiable assumption of the validity of the Omori law to times of the order of thousands of years after the strongest earthquakes; a simple calculation for presumed triggering of aftershocks by earthquakes of Kern County size at a rate of occurrence of 5 earthquakes per century and a rate of recurrence of 5,000 years for non-SAF earthquakes on the same fault (15), shows that the p-value would have to be lowered to unreasonably low values to account for the excess number of smaller earthquakes. As indicated above, we need not be concerned with the spillover from long-term tails of earlier great SAF earthquakes since these aftershock series are short.

Consider the aftershock series for the 11 strong earthquakes with magnitudes $M \geq 6.4$ in the Southern California region from 1933 to 1990. In all cases, except for the Kern County earthquake as noted above, aftershocks with $M \geq 4.1$ became completely quiescent after times ranging from 13 days to 17 years. In 9 of the 10 cases of quiescence, activity in the magnitude interval $4.1 \leq M \leq 4.8$ resumed after the quiescent episode; there has as yet been no resumption of activity in the most recent case, but it can be expected to take place. The ratios of the times for the resumption of activity to those for the onset of quiescence, measured from the times of the parent shocks, have a geometric mean of about 30.

The following model is suggested. Assume that the magnitude distribution for aftershocks and hence for the complete catalog is a property of a universal geometry associated with faulting in the main shock; we demonstrate the plausibility of this assumption in a separate paper. We assume that most aftershocks with $4.1 \leq M \leq 4.8$ are concentrated in a locally weakened core zone near the fracture surface of very large earthquakes having a scale size of the order of 3 km; most aftershocks that followed the $M = 7.3$ Landers (1992) earthquake have been well-located in a zone astride the main fault trace having this width (34). (Not all aftershocks are located within the presumed core zone; most stronger aftershocks and some smaller ones as well are located outside it. We show elsewhere that the two subsets have different fracture mechanisms.) Aftershocks will embark on a prolonged episode of quiescence when either the stress derived from the strong main shock parent has relaxed sufficiently or the aftershock zone has strengthened sufficiently. Later, when the tectonic stress will have increased sufficiently, activity will resume at a very low rate on the zone adjoining the main fault, since this zone is weaker than sites outside the core. Since the earthquakes

Table 3. Window parameters

M	ΔT, days	ΔR, km	M	ΔT, days	ΔR, km
4.2	10	10	5.2	65	20
4.3	12	10	5.3	75	20
4.4	15	10	5.4	87	20
4.5	18	15	5.5	100	20
4.6	21	15	5.6	115	20
4.7	25	15	5.7	130	20
4.8	30	15	5.8	150	20
4.9	36	15	5.9	170	20
5.0	45	20	6.0	200	20
5.1	55	20			

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involved in the recrudescence occur long after the conclusion of the aftershock sequence, they will be classified as main shocks with $M \leq 4.8$; since they are located on the same spatial structures as earlier aftershocks, they will have the same magnitude distribution. Thus, we suppose that much of present-day activity in the smaller magnitudes is due to reactivation of the aftershock zones astride all the faults of the region that have ruptured in earlier major shocks. Although most of the aftershocks and the later localized main shocks are found on the same geometrical features, the time dependences of the two series are expected to be significantly different: the aftershock series obeys the Omori law, while the long-term reactivation is that of a "background" that should occur at a more or less steady state before a future large earthquake, except for precursory earthquakes. Activity on reactivated fault neighborhoods might contribute to the number and magnitude distribution of main shock earthquakes with $M \leq 4.8$.

Comment

The characteristic length of 3 km in the magnitude distribution is a crossover between two different mechanisms in the physics of earthquake occurrence, and is the first identification of a scale size from analysis of the magnitude distribution in Southern California. Two other characteristic scale sizes in the geometry of the earthquake environment in Southern California have been identified. The first is the thickness of the seismogenic zone in Southern California of about 15 km (35–37), corresponding to $M = 6.3$ or 6.4. To date, no correspondence in the magnitude distribution at this threshold has been identified due to the small number of strong earthquakes with larger magnitudes in the short time interval of this earthquake catalog (38, 39); we make the same conclusion in this paper. There are suggestions that a crossover at very large magnitudes might be more apparent in the distributions for other parts of the world (40, 41). The other critical dimension is of the order of 200 m (42) and corresponds to magnitudes that are well below the range of completeness of catalog observations to be able to discern any influence on the distribution. Other scale sizes such as those associated with granularity, irregular geometry of fault surfaces, or the geometry of fault networks can be suggested.

The demonstration that at least one crossover scale size is present in the distribution of mainshock earthquakes indicates that at least two mechanisms control the evolution of mainshock seismicity locally. Thus models such as SOC that depend on the absence of scale and on the absence of multiplicity of mechanism cannot be appropriate to a local description of mainshock earthquakes as defined here. As a consequence, assertions that future earthquakes are unpredictable (43, 44) based on an assumption of the applicability of SOC to the earthquake problem, are not tenable. The presumed universality of the scale independence of the complete suite of local earthquakes is attributable to the universality of the properties of aftershocks.

1. Gutenberg, B. & Richter, C. F. (1944) *Bull. Seismol. Soc. Am.* **34,** 185–188.
2. Gutenberg, B. & Richter, C. F. (1954) *Seismicity of the Earth* (Princeton Univ. Press, Princeton), 2nd Ed.
3. Gutenberg, B. (1956) *Q. J. Geol. Soc. London* **112,** 1–14.
4. Aki, K. (1966) *Bull. Earthquake Res. Inst. Univ. Tokyo* **44,** 73–88.
5. Kanamori, H. (1978) *Nature (London)* **271,** 411–414.
6. Bak, P. & Tang, C. (1989) *J. Geophys. Res.* **94,** 15635–15637.
7. Sornette, A. & Sornette, D. (1989) *Europhys. Lett.* **9,** 197–202.
8. Ito, K. & Matsuzaki, M. (1990) *J. Geophys. Res.* **85,** 6853–6860.
9. Jensen, H. F. (1998) *Self-Organized Criticality* (Cambridge Univ. Press, Cambridge, U.K.).
10. Omori, F. (1894) *J. Coll. Sci. Imp. Univ. Japan* **7,** 111–200.
11. Utsu, T. (1961) *Geophys. Mag.* **30,** 521–605.
12. Richter, C. F. (1958) *Elementary Seismology* (Freeman, San Francisco).
13. Knopoff, L. (1996) *Proc. Natl. Acad. Sci. USA* **93,** 3756–3763.
14. Reid, H. F. (1910) *The California Earthquake of April 19, 1906, Report of the State Earthquake Investigation Commission* (Carnegie Institution of Washington), Vol. 2, pp. 16–28.
15. Sieh, K. (1996) *Proc. Natl. Acad. Sci. USA* **93,** 3764–3771.
16. Sieh, K., Stuiver, M. & Brillinger, D. (1989) *J. Geophys. Res.* **94,** 603–623.
17. Nordquist, J. M. (1964) *Bull. Seismol. Soc. Am.* **54,** 1003–1011.
18. Hileman, J. A., Allen, C. R. & Nordquist, J. M. (1973) *Seismicity of the Southern California Region, 1 January 1932 to 31 December 1972* (Seismol. Lab., Calif. Inst. Technol.).
19. Hutton, L. K. & Jones, L. M. (1993) *Bull. Seismol. Soc. Am.* **83,** 313–329.
20. Kulldorff, G. (1961) *Estimation from Grouped and Partially Grouped Samples* (Wiley, New York) p. 56.
21. Deemer, W. L. & Votaw, D. F. (1955) *Ann. Math. Stat.* **26,** 498–504.
22. Aki, K. (1965) *Bull. Earthquake Res. Inst. Univ. Tokyo* **43,** 237–239.
23. Utsu, T. (1965) *Geophys. Bull. Hokkaido Univ.* **13,** 99–103.
24. Hald, A. (1952) *Statistical Theory with Engineering Applications* (Wiley, New York), p. 238.
25. Knopoff, L. & Gardner, J. K. (1972) *Geophys. J. R. Astron. Soc.* **28,** 311–313.
26. Gardner, J. K. & Knopoff, L. (1974) *Bull. Seismol. Soc. Am.* **64,** 1363–1367.
27. Reasenberg, P. (1985) *J. Geophys. Res.* **90,** 5479–5495.
28. Utsu, T., Ogata, Y. & Matsu'ura, R. (1995) *J. Phys. Earth* **43,** 1–33.
29. Kagan, Y. Y. (1993) *Bull. Seismol. Soc. Am.* **83,** 7–24.
30. Kagan, Y. Y. (1994) *Physica D* **77,** 160–192.
31. Kagan, Y. Y. (1997) *J. Geophys. Res.* **102,** 2835–2852.
32. Main, I. (1996) *Rev. Geophys.* **34,** 433–462.
33. Sornette, D. & Sornette, A. (1999) *Bull. Seismol. Soc. Am.* **89,** 1121–1130.
34. Hauksson, E., Jones, L. M., Hutton, K. & Eberhart-Phillips, D. (1993) *J. Geophys. Res.* **98,** 19835–19858.
35. Kanamori, H. & Anderson, D. L. (1975) *Bull. Seismol. Soc. Am.* **65,** 1073–1095.
36. Romanowicz, B. & Rundle, J. B. (1993) *Bull. Seismol. Soc. Am.* **83,** 1294–1297.
37. Okal, R. & Romanowicz, B. A. (1994) *Phys. Earth Planet. Inter.* **87,** 55–76.
38. Pacheco, J. F., Scholz, C. H. & Sykes, L. R. (1992) *Nature (London)* **355,** 71–73.
39. Sornette, D., Knopoff, L., Kagan, Y. & Vanneste, C. (1996) *J. Geophys. Res.* **101,** 13883–13983.
40. Pacheco, J. F. & Sykes, L. R. (1992) *Bull. Seismol. Soc. Am.* **82,** 1306–1349.
41. Triep, E. G. & Sykes, L. R. (1997) *J. Geophys. Res.* **102,** 9923–9948.
42. Li, Y. G., Aki, K., Adams, D., Hasemi, A. & Lee, W. H. K. (1994) *J. Geophys. Res.* **99,** 11705–11722.
43. Geller, R. J., Jackson, D. D., Kagan, Y. Y. & Mulargia, F. (1997) *Science* **275,** 1616–1617.
44. Evans, R. (Dec. 1997) *Astron. Geophys.* **38,** 4.