CSMM: Lesson 1.4 HW

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For the following questions, use these function definitions:

$$I := \lambda x.x$$

$$S := \lambda wyx.y(wyx)$$

$$+ := \lambda ab.aSb$$

$$\times := \lambda ab.a(+b)0$$

$$E := \lambda ab.ba$$

$$T := \lambda xy.x$$

$$F := \lambda xy.y$$

$$\neg := \lambda x.xFT$$

$$\wedge := \lambda xy.xyF$$

$$\vee := \lambda xy.xTy$$

$$\Phi := \lambda abf.fab$$

$$\oplus := \lambda x.\Phi(xF)(S(xF))$$

$$P := \lambda x.(x \oplus (\Phi 00))T$$

$$- := \lambda ab.bPa$$

$$Z := \lambda a.aF\neg F$$

$$\leq := \lambda ab.Z(-ab)$$

$$\geq := \lambda ab.Z(-ba)$$

$$= := \lambda ab. \wedge (\leq ab)(\geq ab)$$

$$< := \lambda ab. \wedge (\neg(= ab))(\leq ab)$$

$$> := \lambda ab. \wedge (\neg(= ab))(\geq ab)$$

(1-8) For each of the following,

Find the normal form of the following λ-expressions (show work)

1. $(\lambda x. < (aT)(aF))(\Phi 12)$
2. $(\lambda x. > (aT)(aF))(\Phi 12)$
3. $\oplus(\Phi 40)$
4. $(\lambda x. + (xT)(xF))(\Phi 14)$
5. $(\lambda x. - (xT)(xF))(\Phi 14)$
6. $(\lambda x. \times (xT)(xF))(\Phi 14)$
7. $(\lambda x. E(xT)(xF))(\Phi 14)$
8. $(\lambda xy. \wedge (\vee xy)(\neg(\wedge xy)))TT$

ANSWERS

Derivations vary, just be careful with parens and variables.

1. T
2. F
3. $\Phi 01$
4. 5
5. 0
6. 4
7. 1
8. F