[과제1] 다음 주어진 각 함수의 도함수를 도함수의 정의 $f'(x) = \lim_{x \to \infty} \frac{f(x+h) - f(x)}{h}$ 를 사용하여 유도하시오.

(1) $f(x) = c$, (c는 상수)

$f(x+h) = c$, $f(x) = c$ 이므로

$$f'(x) = \lim_{h \to 0} \frac{c - c}{h} = \lim_{h \to 0} \frac{0}{h} = \lim_{h \to 0} 0 = 0$$

※ 상수 미분하면 0임을 확인

(2) $f(x) = x$

$f(x+h) = x+h$, $f(x) = x$

$$f'(x) = \lim_{h \to 0} \frac{x+h-x}{h} = \lim_{h \to 0} \frac{h}{h} = \lim_{h \to 0} 1 = 1$$

(3) $f(x) = x^2$

$f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$

$f(x) = x^2$

$$f'(x) = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = \lim_{h \to 0} 2x + h = 2x$$

※ (2) 에서 $(x)' = 1$, (3) 에서 $(x^2)' = 2x$

(4) $f(x) = x^3$

※ 여기서 $(x^3)' = 3x^2$ 을 예상가능.
(물론 실제 증명은 다른 방법)

$f(x+h) = (x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$

$f(x) = x^3$

$$\therefore f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} = \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3}{h} = \lim_{h \to 0} 3x^2 + 3xh + h^2 = 3x^2$$

(5) $cf(x)$, (c는 상수)

$$\{cf(x)\}' = \lim_{h \to 0} \frac{cf(x+h) - cf(x)}{h} = \lim_{h \to 0} c \cdot \frac{f(x+h) - f(x)}{h} = c \cdot \underbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}_{= f'(x)} = c \cdot f'(x)$$

※ 물론 f(x)가 미분가능이라는 조건이 필요. 귀찮아서 쓰지 않았음...

(6) $f(x) + g(x)$

$$\{f(x) + g(x)\}' = \lim_{h \to 0} \frac{\{f(x+h) + g(x+h)\} - \{f(x) + g(x)\}}{h}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \frac{g(x+h) - g(x)}{h}$$

질문1 왜 이 단계를 굳이 써 주셨을까요?

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= f'(x) + g'(x)$$

(7) $f(x)g(x)$

질문2 다음 중 질문1과 같은 이유로 식을 수정한 순간은?

$$\{f(x)g(x)\}' = \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

①

$$= \lim_{h \to 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) - f(x)g(x) + f(x)g(x+h)}{h}$$

②

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \cdot g(x+h) + f(x) \cdot \frac{g(x+h) - g(x)}{h}$$

③

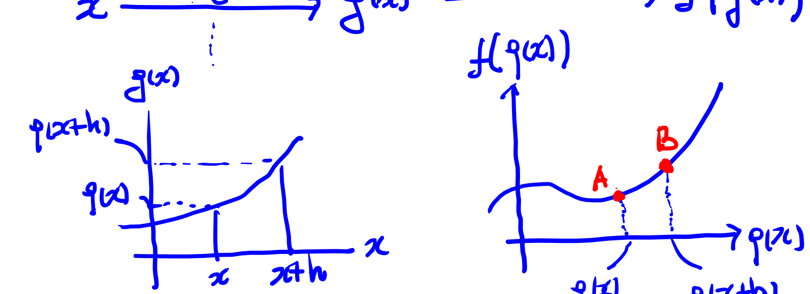
$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \times \lim_{h \to 0} g(x+h) + \lim_{h \to 0} f(x) \times \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

④

$$= f'(x)g(x) + f(x)g'(x)$$

(8) $f(g(x))$

$$x \xrightarrow{g} g(x) \xrightarrow{f} f(g(x))$$



$$\lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h}$$

$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} \cdot \frac{g(x+h) - g(x)}{h}$$

$$= f'(g(x)) \cdot g'(x)$$

$A(g(x),\ f \circ g(x))$

$B(g(x+h),\ f \circ g(x+h))$

$\overline{AB}$ 기울기 $= \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)}$

즉 $\lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h) - g(x)} = f'(g(x))$