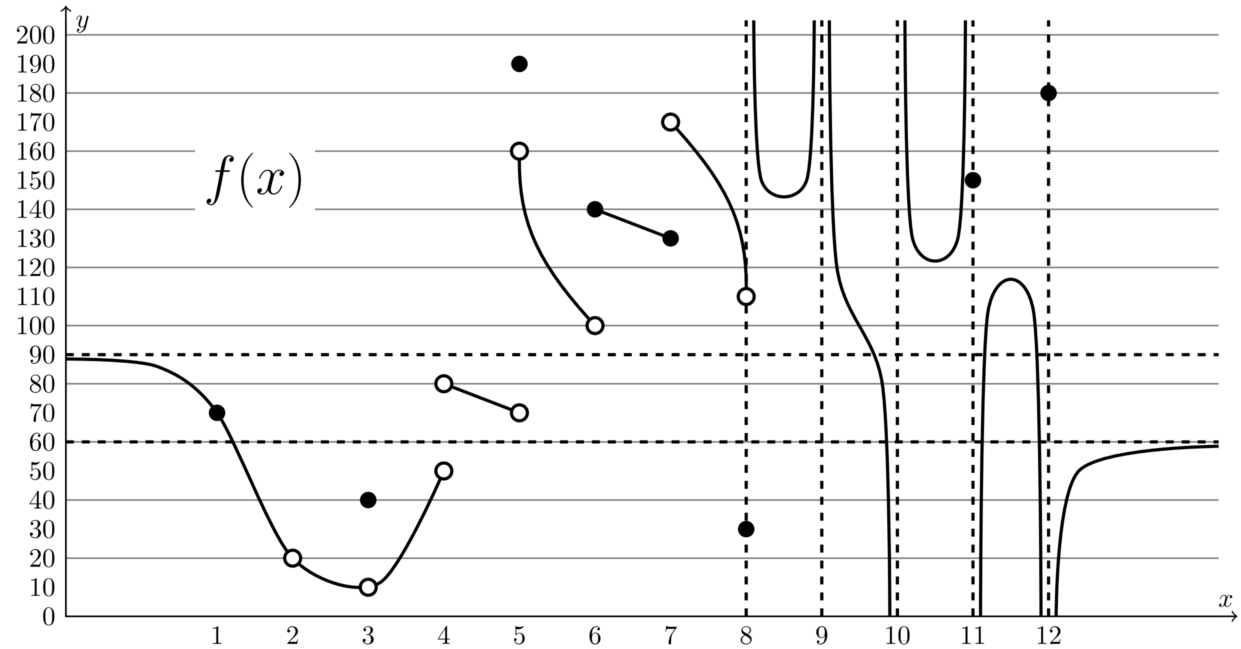


1 Find Limits Graphically

1. $$\begin{cases} f(1) = & \textbf{70} \\ \lim\limits_{x\to 1^-} f(x) = & \textbf{70} \\ \lim\limits_{x\to 1^+} f(x) = & \textbf{70} \\ \lim\limits_{x\to 1} f(x) = & \textbf{70} \end{cases}$$

2. $$\begin{cases} f(2) = & \textbf{undefined} \\ \lim\limits_{x\to 2^-} f(x) = & \textbf{20} \\ \lim\limits_{x\to 2^+} f(x) = & \textbf{20} \\ \lim\limits_{x\to 2} f(x) = & \textbf{20} \end{cases}$$

3. $$\begin{cases} f(3) = & \textbf{40} \\ \lim\limits_{x\to 3^-} f(x) = & \textbf{10} \\ \lim\limits_{x\to 3^+} f(x) = & \textbf{10} \\ \lim\limits_{x\to 3} f(x) = & \textbf{10} \end{cases}$$

4. $$\begin{cases} f(4) = & \textbf{undefined} \\ \lim\limits_{x\to 4^-} f(x) = & \textbf{50} \\ \lim\limits_{x\to 4^+} f(x) = & \textbf{80} \\ \lim\limits_{x\to 4} f(x) = & \textbf{DNE} \end{cases}$$

5. $$\begin{cases} f(5) = & \textbf{190} \\ \lim\limits_{x\to 5^-} f(x) = & \textbf{70} \\ \lim\limits_{x\to 5^+} f(x) = & \textbf{160} \\ \lim\limits_{x\to 5} f(x) = & \textbf{DNE} \end{cases}$$

6. $$\begin{cases} f(6) = & \textbf{140} \\ \lim\limits_{x\to 6^-} f(x) = & \textbf{100} \\ \lim\limits_{x\to 6^+} f(x) = & \textbf{140} \\ \lim\limits_{x\to 6} f(x) = & \textbf{DNE} \end{cases}$$

7. $$\begin{cases} f(7) = & \textbf{130} \\ \lim\limits_{x\to 7^-} f(x) = & \textbf{130} \\ \lim\limits_{x\to 7^+} f(x) = & \textbf{170} \\ \lim\limits_{x\to 7} f(x) = & \textbf{DNE} \end{cases}$$

2 Find Limits Involving Infinity Graphically

8. $$\begin{cases} f(8) = & \textbf{30} \\ \lim\limits_{x\to 8^-} f(x) = & \textbf{110} \\ \lim\limits_{x\to 8^+} f(x) = & +\infty \\ \lim\limits_{x\to 8} f(x) = & \textbf{DNE} \end{cases}$$

9. $$\begin{cases} f(9) = & \textbf{undefined} \\ \lim\limits_{x\to 9^-} f(x) = & +\infty \\ \lim\limits_{x\to 9^+} f(x) = & +\infty \\ \lim\limits_{x\to 9} f(x) = & +\infty \end{cases}$$

10. $$\begin{cases} f(10) = & \textbf{undefined} \\ \lim\limits_{x\to 10^-} f(x) = & -\infty \\ \lim\limits_{x\to 10^+} f(x) = & +\infty \\ \lim\limits_{x\to 10} f(x) = & \textbf{DNE} \end{cases}$$

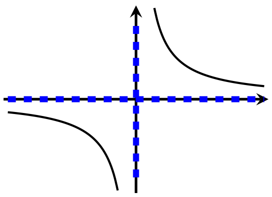
11. $$\begin{cases} f(11) = & \textbf{150} \\ \lim\limits_{x\to 11^-} f(x) = & +\infty \\ \lim\limits_{x\to 11^+} f(x) = & -\infty \\ \lim\limits_{x\to 11} f(x) = & \textbf{DNE} \end{cases}$$

12. $$\begin{cases} f(12) = & \textbf{180} \\ \lim\limits_{x\to 12^-} f(x) = & -\infty \\ \lim\limits_{x\to 12^+} f(x) = & -\infty \\ \lim\limits_{x\to 12} f(x) = & -\infty \end{cases}$$

13. $\lim\limits_{x\to -\infty} f(x) = \textbf{90}$

14. $\lim\limits_{x\to +\infty} f(x) = \textbf{60}$

3 Famous Functions

15. 

$1/0 =$ **undefined**

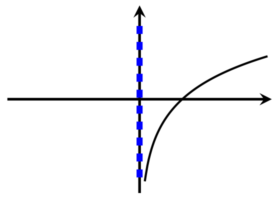
$\lim_{x\to 0^-} 1/x = -\infty$

$\lim_{x\to 0^+} 1/x = +\infty$

$\lim_{x\to 0} 1/x =$ **DNE**

$\lim_{x\to -\infty} 1/x = \mathbf{0}$

$\lim_{x\to +\infty} 1/x = \mathbf{0}$

16. 

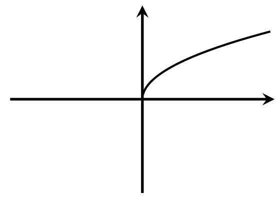
$\ln 0 =$ **undefined**

$\lim_{x\to 0^-} \ln x =$ **DNE**

$\lim_{x\to 0^+} \ln x = -\infty$

$\lim_{x\to -\infty} \ln x =$ **DNE**

$\lim_{x\to +\infty} \ln x = +\infty$

17. 

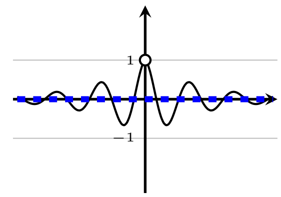
$\sqrt{0} = \mathbf{0}$

$\lim_{x\to 0^-} \sqrt{x} =$ **DNE**

$\lim_{x\to 0^+} \sqrt{x} = \mathbf{0}$

$\lim_{x\to -\infty} \sqrt{x} =$ **DNE**

$\lim_{x\to +\infty} \sqrt{x} = +\infty$

18. 

$\sin(0)/0 =$ **undefined**

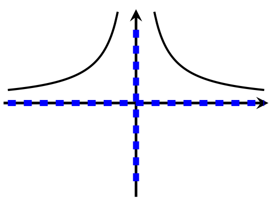
$\lim_{x\to 0^-} \sin(x)/x = \mathbf{1}$

$\lim_{x\to 0^+} \sin(x)/x = \mathbf{1}$

$\lim_{x\to 0} \sin(x)/x = \mathbf{1}$

$\lim_{x\to -\infty} \sin(x)/x = \mathbf{0}$

$\lim_{x\to +\infty} \sin(x)/x = \mathbf{0}$

19. 

$1/0^2 =$ **undefined**

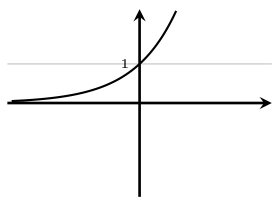
$\lim_{x\to 0^-} 1/x^2 = +\infty$

$\lim_{x\to 0^+} 1/x^2 = +\infty$

$\lim_{x\to 0} 1/x^2 = +\infty$

$\lim_{x\to -\infty} 1/x^2 = \mathbf{0}$

$\lim_{x\to +\infty} 1/x^2 = \mathbf{0}$

20. 

$e^0 = \mathbf{1}$

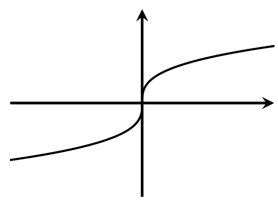
$\lim_{x\to 0^-} e^x = \mathbf{1}$

$\lim_{x\to 0^+} e^x = \mathbf{1}$

$\lim_{x\to 0} e^x = \mathbf{1}$

$\lim_{x\to -\infty} e^x = \mathbf{0}$

$\lim_{x\to +\infty} e^x = +\infty$

21. 

$\sqrt[3]{0} = \mathbf{0}$

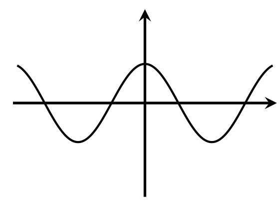
$\lim_{x\to 0^-} \sqrt[3]{x} = \mathbf{0}$

$\lim_{x\to 0^+} \sqrt[3]{x} = \mathbf{0}$

$\lim_{x\to 0} \sqrt[3]{x} = \mathbf{0}$

$\lim_{x\to -\infty} \sqrt[3]{x} = -\infty$

$\lim_{x\to +\infty} \sqrt[3]{x} = +\infty$

22. 

$\cos 0 = \mathbf{1}$

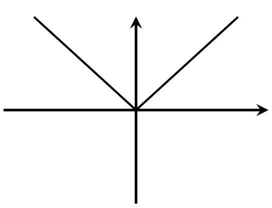
$\lim_{x\to 0^-} \cos x = \mathbf{1}$

$\lim_{x\to 0^+} \cos x = \mathbf{1}$

$\lim_{x\to 0} \cos x = \mathbf{1}$

$\lim_{x\to -\infty} \cos x =$ **DNE**

$\lim_{x\to +\infty} \cos x =$ **DNE**

23. 

$|0| = \mathbf{0}$

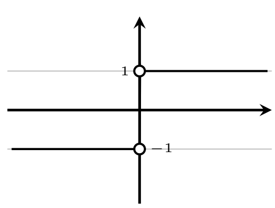
$\lim_{x\to 0^-} |x| = \mathbf{0}$

$\lim_{x\to 0^+} |x| = \mathbf{0}$

$\lim_{x\to 0} |x| = \mathbf{0}$

$\lim_{x\to -\infty} |x| = +\infty$

$\lim_{x\to +\infty} |x| = +\infty$

24. 

$|0|/0 =$ **undefined**

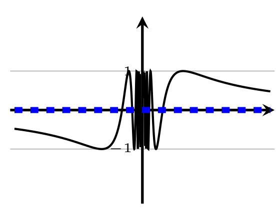
$\lim_{x\to 0^-} |x|/x = -1$

$\lim_{x\to 0^+} |x|/x = \mathbf{1}$

$\lim_{x\to 0} |x|/x =$ **DNE**

$\lim_{x\to -\infty} |x|/x = -1$

$\lim_{x\to +\infty} |x|/x = \mathbf{1}$

25. 

$\sin(1/0) =$ **undefined**

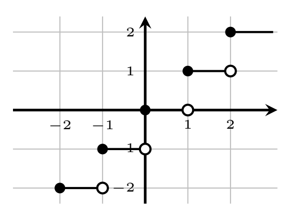
$\lim_{x\to 0^-} \sin(1/x) =$ **DNE**

$\lim_{x\to 0^+} \sin(1/x) =$ **DNE**

$\lim_{x\to 0} \sin(1/x) =$ **DNE**

$\lim_{x\to -\infty} \sin(1/x) = \mathbf{0}$

$\lim_{x\to +\infty} \sin(1/x) = \mathbf{0}$

26. 

$\lfloor 0 \rfloor = \mathbf{0}$

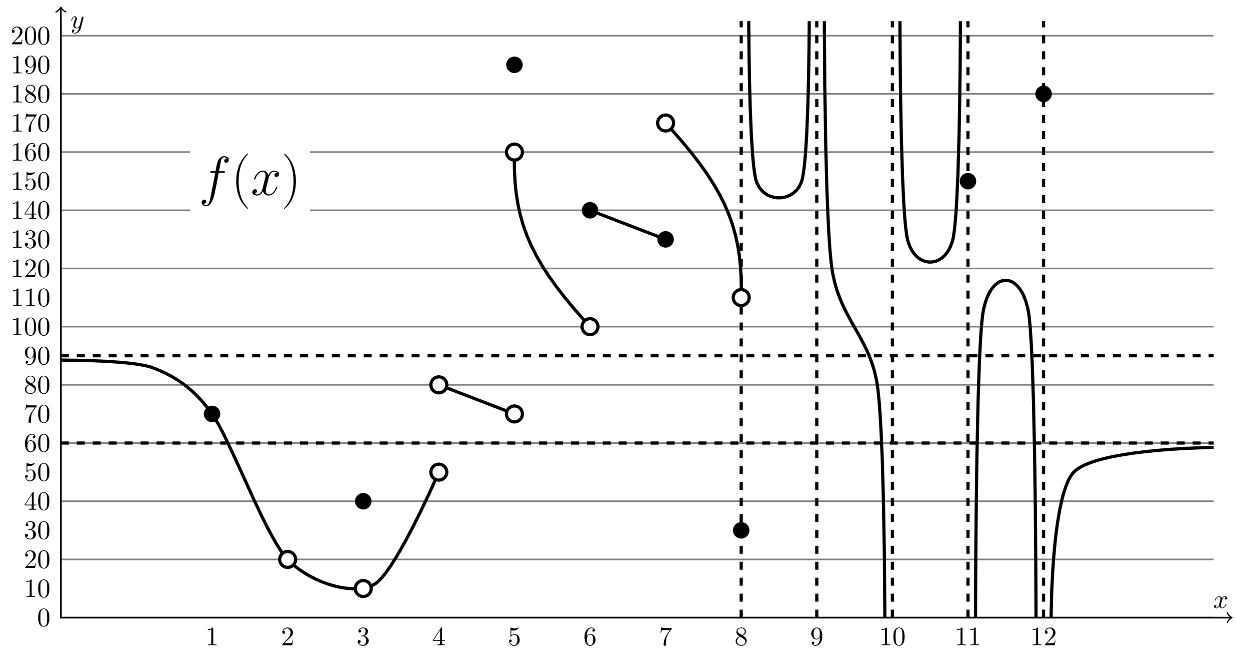
$\lim_{x\to 0^-} \lfloor x \rfloor = -1$

$\lim_{x\to 0^+} \lfloor x \rfloor = \mathbf{0}$

$\lim_{x\to 0} \lfloor x \rfloor =$ **DNE**

$\lim_{x\to -\infty} \lfloor x \rfloor = -\infty$

$\lim_{x\to +\infty} \lfloor x \rfloor = +\infty$



4 Identify Infinite, Jump, Removable Discontinuities Graphically

27. We say f is __continuous__ **at $x = a$** if $f(a) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ and all three exist and are finite.
28. We say f has a __removable__ **discontinuity at $x = a$** if $f(a) \neq \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ and the last two exist and are finite.
29. We say f has a __jump__ **discontinuity at $x = a$** if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ and both exist and are finite.
30. We say f has a __infinite__ **discontinuity at $x = a$** if $\lim_{x\to a^-} f(x)$ or $\lim_{x\to a^+} f(x)$ is infinite and both exist.
31. The function f above is continuous (cts) at the following integers between 1 and 12: __1__.
32. The function f above has removable discontinuities at the following integers: __2, 3__.
33. The function f above has jump discontinuities at the following integers: __4, 5, 6, 7__.
34. The function f above has infinite discontinuities at the following integers: __8, 9, 10, 11, 12__.

5 Continuity on an Interval

35. We say f is **continuous on the open interval (a, b)** if f is continuous at every x in (a, b).
Find the union of all open intervals **on which f is continuous**.

(Set-builder notation) $\{x \mid x \neq 2, 3, 4, \ldots, 12\}$

(Interval notation) $(-\infty, 2) \cup (2, 3) \cup (3, 4) \cup \cdots \cup (11, 12) \cup (12, \infty)$

36. We say f is **continuous everywhere** if f is continuous at every x in __$(-\infty, \infty)$__.
37. We say f is **a continuous function** if f is continuous at every x in its __domain__.

6 Left and Right Continuity

38. We say f is __left continuous__ **at $x = a$** if $f(a) = \lim_{x\to a^-} f(x)$ and both exist and are finite.
39. We say f is __right continuous__ **at $x = a$** if $f(a) = \lim_{x\to a^+} f(x)$ and both exist and are finite.
40. The function f above is left continuous at the following integers between 1 and 12: __1, 7__.
41. The function f above is right continuous at the following integers between 1 and 12: __1, 6__.
42. We say f is **continuous on the closed interval $[a, b]$** if f is continuous at every x in the *open* interval (a, b) and is __right continuous__ at $x = a$ and is __left continuous__ at $x = b$.
43. The function f above is continuous on the closed interval [__6__ , __7__] with integer endpoints between 1 and 12.