Theorem

$$\left(x^n\right)' = nx^{n-1}$$

Plickers Exercise

Find the derivative of $f(x) = x^3$.

A. $3x^2$ B. $3x$ C. $6x$ D. 6

Fact

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$(x+h)^3 = x^3 + 3x^2h + (3xh^2 + h^3)$$

$$(x+h)^4 = x^4 + 4x^3h + (6x^2h^2 + 4xh^3 + h^4)$$

$$x^n \quad nx^{n-1}h \quad (\cdots)h^2$$

Proof.

Fix a positive integer n. Let $f(x) = x^n$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^n} + nx^{n-1}\cancel{h} + (\cdots)h^{\cancel{2}} - \cancel{x^n}}{\cancel{h}}$$

$$= nx^{n-1}$$