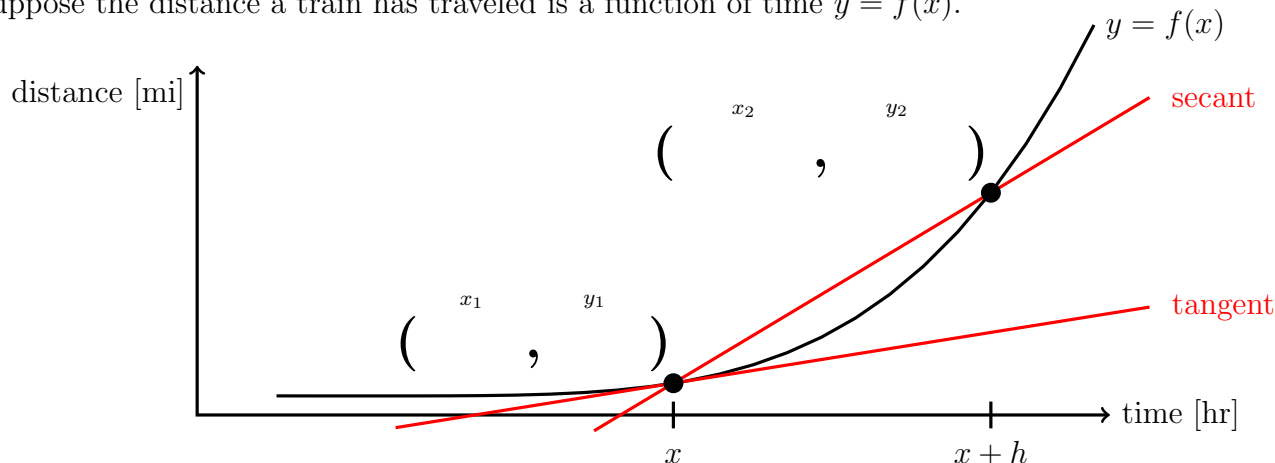


N1. Definition. The Limit Definition of Derivative

Suppose the distance a train has traveled is a function of time $y = f(x)$.

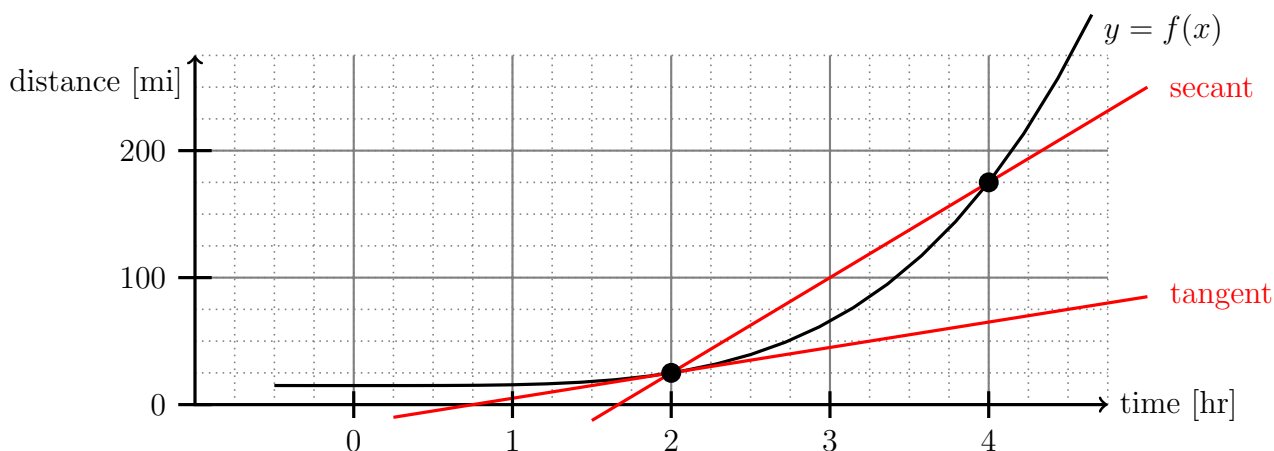


$$\begin{array}{ccccccc} \text{average} & & \text{average} & & \text{slope of} & & \\ \text{velocity} & & \text{rate of} & & \text{secant} & & \\ \text{over} & = & \text{change} & = & \text{between} & = & \\ [x, x+h] & = & \text{over} & = & x, x+h & = & \\ & & [x, x+h] & & & & \end{array} \quad \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \boxed{} = \text{definition of difference quotient}$$

$$\begin{array}{ccccccc} \text{("instantaneous")} & & \text{rate of} & & \text{slope of} & & \\ \text{velocity} & = & \text{change} & = & \text{tangent} & = & \\ \text{at } x & = & \text{at } x & = & \text{at } x & = & \end{array} \quad \boxed{\lim_{h \rightarrow 0} } = \text{definition of derivative written } y' \text{ or } \frac{dy}{dx} \text{ or } f'(x) \text{ or } \frac{d}{dx} f(x)$$

N2. Exercise. Estimate from the diagram: the slope of the **secant** through 2 and 4.

N3. Exercise. Estimate from the diagram: the slope of the **tangent** at 2.



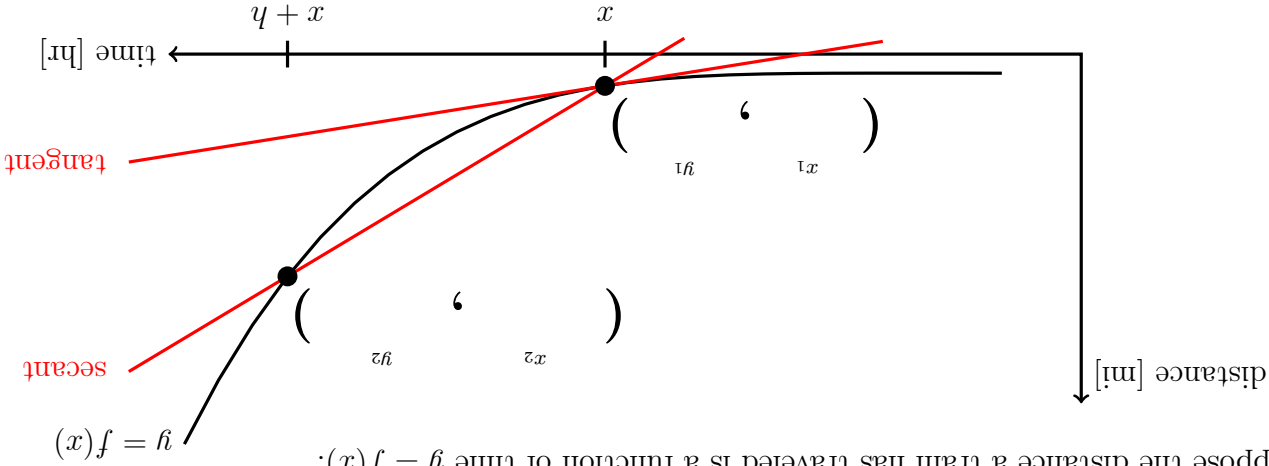
N4. EXAMPLE. Suppose $f(x) = 15 + \frac{5}{8}x^4$. Compute the slope of the **secant** through x and $x + h$.

$$\begin{aligned} f(x+h) &= 15 + \frac{5}{8}(x+h)^4 = 15 + \frac{5}{8}x^4 + \frac{20}{8}x^3h + \frac{30}{8}x^2h^2 + \frac{20}{8}xh^3 + \frac{5}{8}h^4 \\ \frac{f(x+h)-f(x)}{h} &= \end{aligned}$$

N5. Exercise. Suppose $f(x) = 15 + \frac{5}{8}x^4$. Compute the slope of the **tangent** line at x .

N6. Exercise. Use your answer to N5 to compute $f'(2)$.

N1. **Definition.** The Limit Definition of Derivative
 Suppose the distance a train has traveled is a function of time $y = f(x)$.

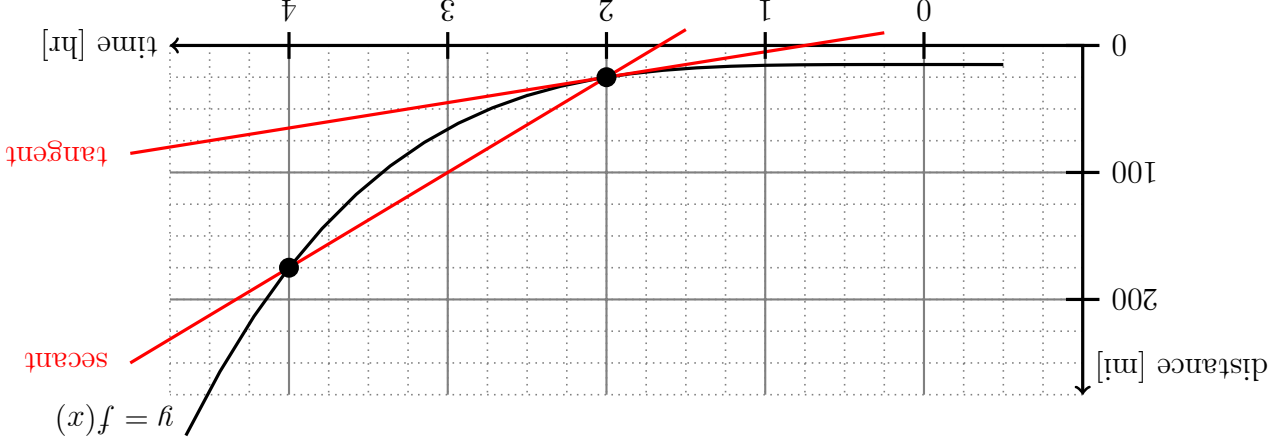


average velocity over $[x, x+h]$ = rate of change over $[x, x+h]$ = slope of secant between $x, x+h$ = $\frac{\Delta y}{\Delta x}$ = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\frac{\text{difference}}{\text{quotient}}$

(“instantaneous”) velocity at x = rate of change at x = slope of tangent at x = $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ = definition of derivative written y' or $\frac{dy}{dx}$ or $f'(x)$ or $\frac{d}{dx}f(x)$

N2. **Exercise.** Estimate from the diagram: the slope of the secant through 2 and 4.

N3. **Exercise.** Estimate from the diagram: the slope of the tangent at 2.



N4. **EXAMPLE.** Suppose $f(x) = 15 + \frac{8}{5}x^4$. Compute the slope of the secant through x and $x + h$.

$$= 15 + \frac{8}{5}(x+h)^4 = 15 + \frac{8}{5}x^4 + \frac{8}{20}x^3h + \frac{8}{30}x^2h^2 + \frac{8}{20}xh^3 + \frac{8}{5}h^4$$

$$= \frac{f(x+h) - f(x)}{h}$$

N5. **Exercise.** Suppose $f(x) = 15 + \frac{8}{5}x^4$. Compute the slope of the tangent line at x .

N6. **Exercise.** Use your answer to N5 to compute $f'(2)$.