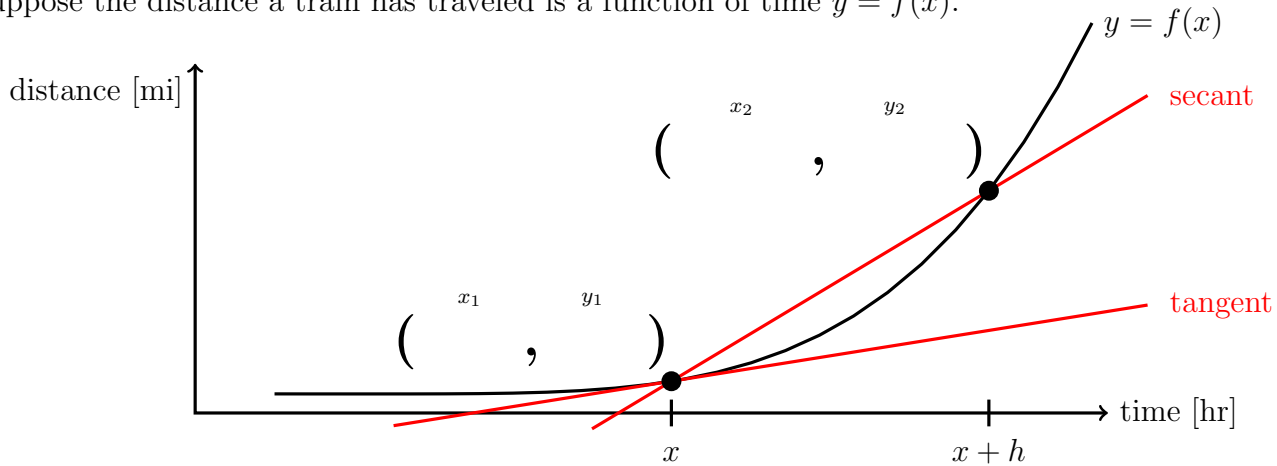


1. Definition. The Limit Definition of Derivative

Suppose the distance a train has traveled is a function of time $y = f(x)$.

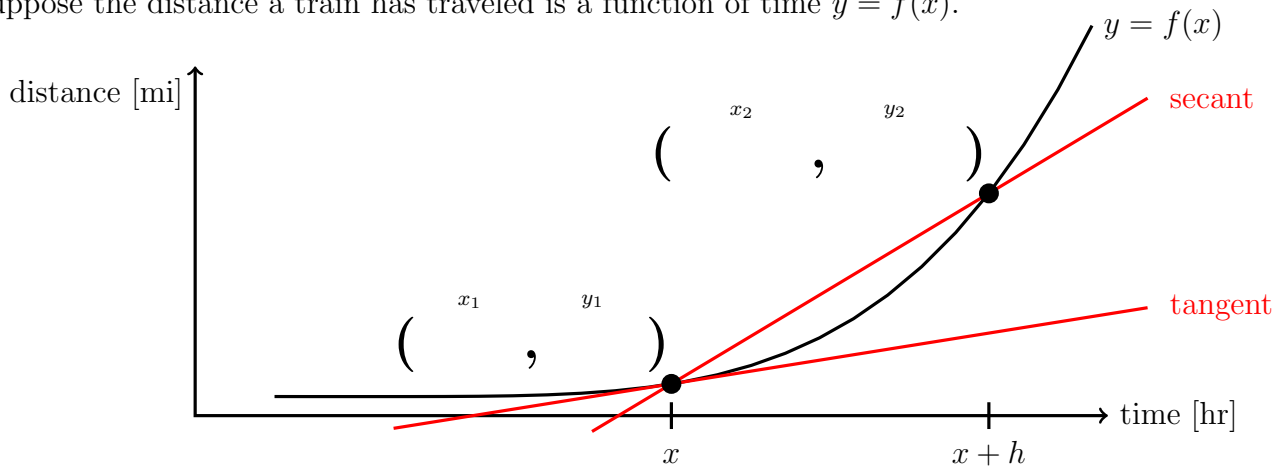


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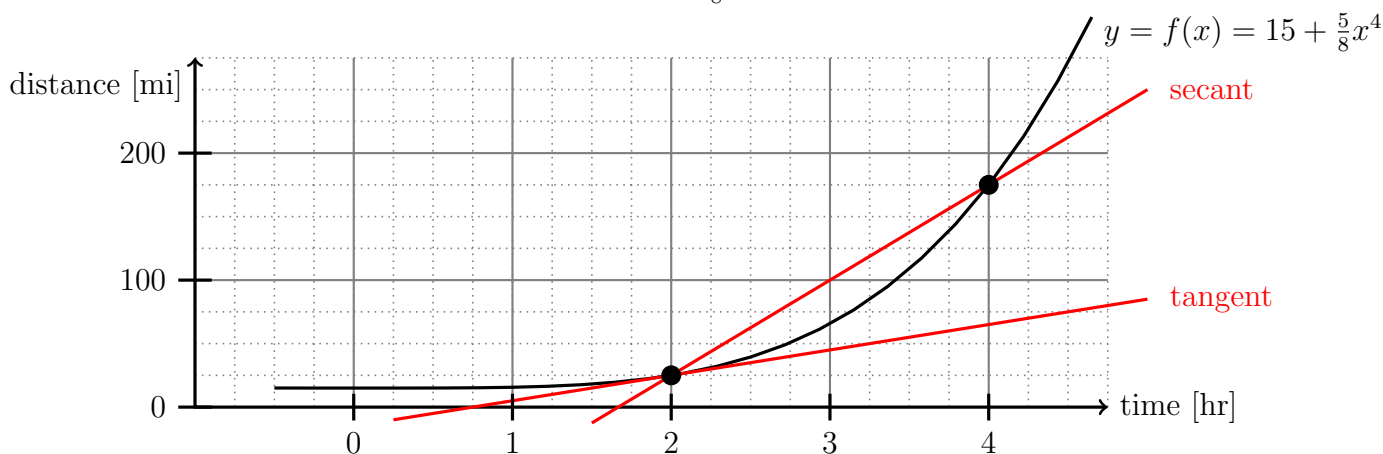


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2. **Example.** Use the definition of difference quotient to find the slope of the **secant** line through $y = f(x) = 15 + \frac{5}{8}x^4$ between x and $x + h$.
Hint: $f(x + h) = 15 + \frac{5}{8}x^4 + \frac{20}{8}x^3h + \frac{30}{8}x^2h^2 + \frac{20}{8}xh^3 + \frac{5}{8}h^4$

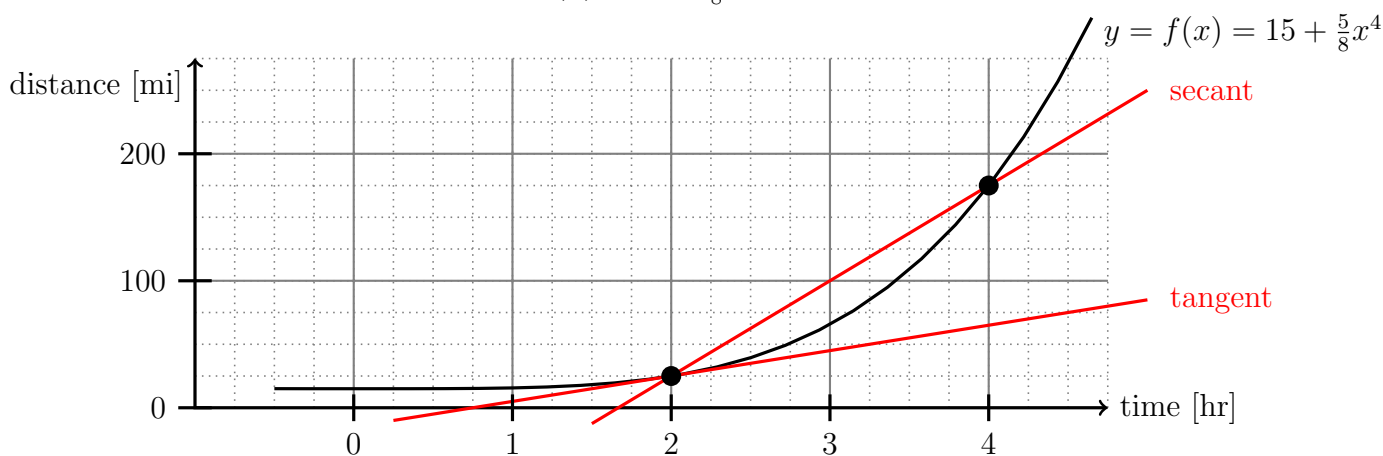
3. **Exercise.** Use the definition of derivative and your answer to **Example 2** to find the slope of the **tangent** line to $y = f(x) = 15 + \frac{5}{8}x^4$ at x .



4. **Exercise.** Use the **diagram** to estimate the slope of the **secant** through $y = 15 + \frac{5}{8}x^4$ **between 2 and 4**.
 5. **Exercise.** Use the **diagram** to estimate the slope of the **tangent** line to $y = f(x) = 15 + \frac{5}{8}x^4$ **at 2**.
 6. **Exercise.** Use your answer to **Exercise 3** to confirm your answer to **Exercise 5**.

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