Theorem (Power Rule)

$$\left(x^n\right)' =$$

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$$\left(x^n\right)' = nx^{n-1}$$

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Exercise

Find the derivative of $f(x) = x^3$.

A. $3x^2$ B. $3x$ C. $6x$ D. 6

Fact

$(x + h)^2 =$

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$$(x+h)^2 = x^2 + 2xh + h^2$$

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$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

Fact

$$(x+h)^2 = x^2 + 2xh + h^2$$

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$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$x^n$$

Fact

$$(x+h)^2 = x^2 + 2xh + h^2$$

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$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$x^n + nx^{n-1}h$$

Fact

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$x^n + nx^{n-1}h \quad + \quad (\cdots)h^2$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

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$$
\begin{aligned}
& f(x+h) \\
= \quad & x^n + nx^{n-1}h + (\cdots)h^2
\end{aligned}
$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\begin{aligned} & f(x+h) - f(x) \\ = \quad & x^n + nx^{n-1}h + (\cdots)h^2 - x^n \end{aligned}$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$f(x+h) - f(x)$$

$$= \cancel{x^n} + nx^{n-1}h + (\cdots)h^2 - \cancel{x^n}$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\Big(f(x+h) - f(x)\Big)\left(\frac{1}{h}\right)$$

$$= \quad \Big(\cancel{x^n} + nx^{n-1}h + (\cdots)h^2 - \cancel{x^n}\Big)\left(\frac{1}{h}\right)$$

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Fix a positive integer n. Let $f(x) = x^n$. So,

$$
\begin{aligned}
& \left(f(x+h) - f(x)\right)\left(\frac{1}{h}\right) \\
= \quad & \left(\cancel{x^n} + nx^{n-1}\cancel{h} + (\cdots)h^{\cancel{2}} - \cancel{x^n}\right)\left(\frac{1}{\cancel{h}}\right) \\
= \quad & nx^{n-1} + (\cdots)h
\end{aligned}
$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\lim_{h\to 0}\left(f(x+h) - f(x)\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0}\left(\cancel{x^n} + nx^{n-1}\cancel{h} + (\cdots)h^{\cancel{2}} - \cancel{x^n}\right)\left(\frac{1}{\cancel{h}}\right)$$

$$= \lim_{h\to 0} \quad nx^{n-1} + (\cdots)h$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\begin{aligned}
& \lim_{h\to 0} \left(f(x+h) - f(x)\right)\left(\frac{1}{h}\right) \\
= {} & \lim_{h\to 0} \left(\cancel{x^n} + nx^{n-1}\cancel{h} + (\cdots)h^{\cancel{2}} - \cancel{x^n}\right)\left(\frac{1}{\cancel{h}}\right) \\
= {} & \lim_{h\to 0} \quad nx^{n-1} + (\cdots)h \\
= {} & \qquad\qquad nx^{n-1}
\end{aligned}$$