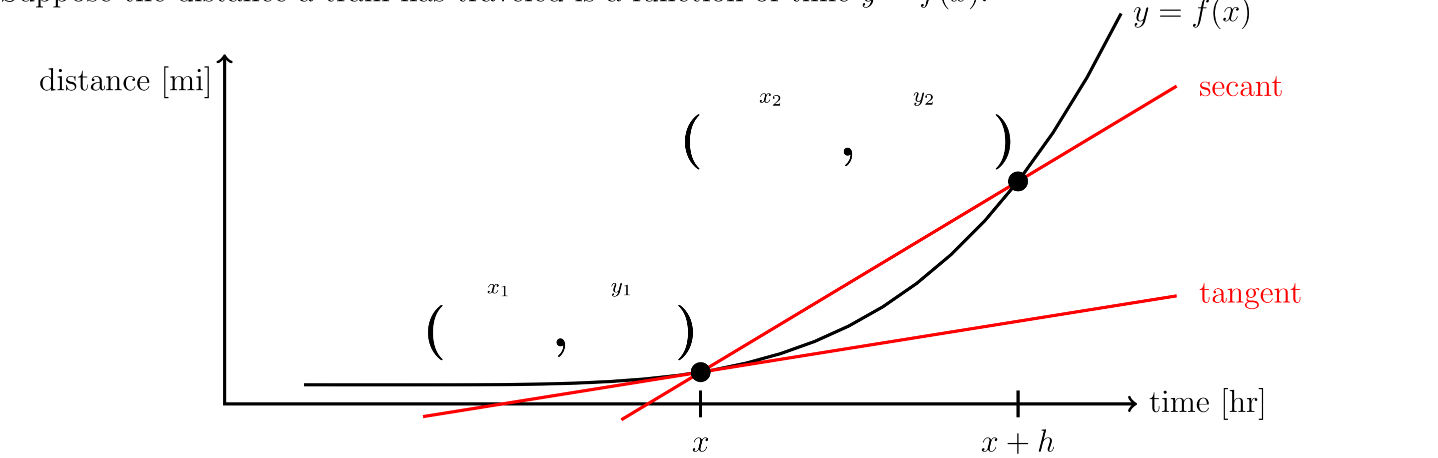
N1. **Definition.** The Limit Definition of Derivative

Suppose the distance a train has traveled is a function of time $y = f(x)$.

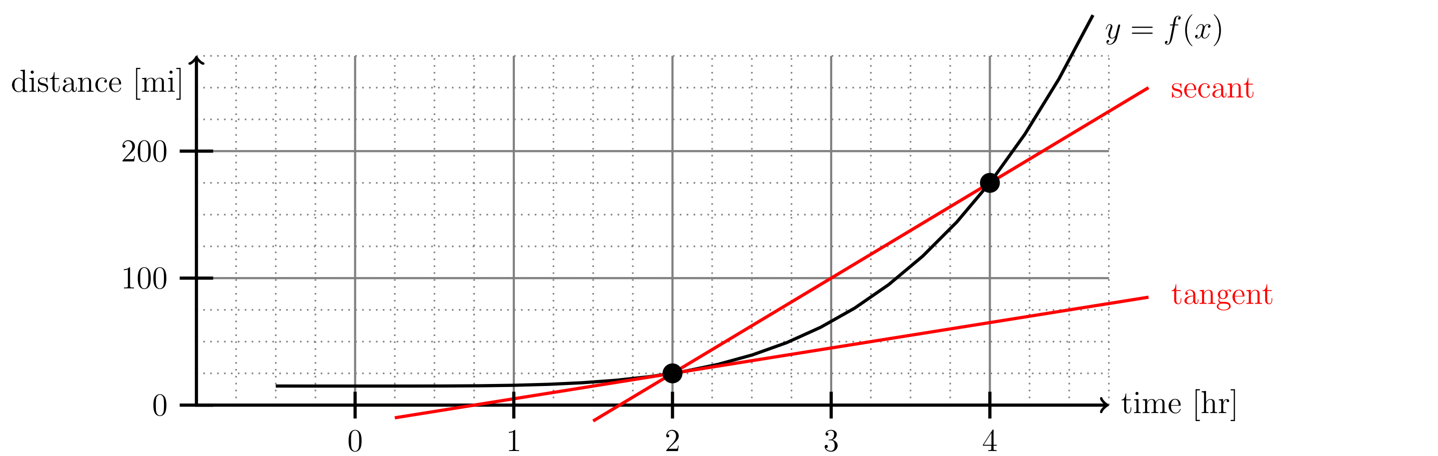


$$\begin{array}{c}\text{average}\\\text{velocity}\\\text{over}\\ [x, x+h]\end{array} = \begin{array}{c}\text{average}\\\text{rate of}\\\text{change}\\\text{over}\\ [x, x+h]\end{array} = \begin{array}{c}\text{slope of}\\\text{secant}\\\text{between}\\ x, x+h\end{array} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \boxed{} = \begin{array}{c}\text{definition of}\\\text{difference}\\\text{quotient}\end{array}$$

$$\begin{array}{c}\text{velocity}\\\text{at } x\end{array} = \begin{array}{c}\text{rate of}\\\text{change}\\\text{at } x\end{array} = \begin{array}{c}\text{slope of}\\\text{tangent}\\\text{at } x\end{array} = \boxed{\lim_{h \to} } = \begin{array}{c}\text{definition of derivative}\\ \text{written } y' \text{ or } \frac{dy}{dx} \text{ or } f'(x) \text{ or } \frac{d}{dx} f(x)\end{array}$$

N2. **Exercise.** Estimate from the diagram: the slope of the **secant** through 2 and 4.

N3. **Exercise.** Estimate from the diagram: the slope of the **tangent** at 2.



N4. Example. Suppose $f(x) = 15 + \frac{5}{8}x^4$. Compute the slope of the **secant** through x and $x + h$.

$f(x+h) \quad = 15 + \frac{5}{8}(x+h)^4 \quad = 15 + \frac{5}{8}x^4 + \frac{20}{8}x^3h + \frac{30}{8}x^2h^2 + \frac{20}{8}xh^3 + \frac{5}{8}h^4$

$\frac{f(x+h) - f(x)}{h} =$

N5. **Exercise.** Suppose $f(x) = 15 + \frac{5}{8}x^4$. Compute the slope of the **tangent** line at x.

N6. **Exercise.** Use your answer to N5 to compute $f'(2)$.

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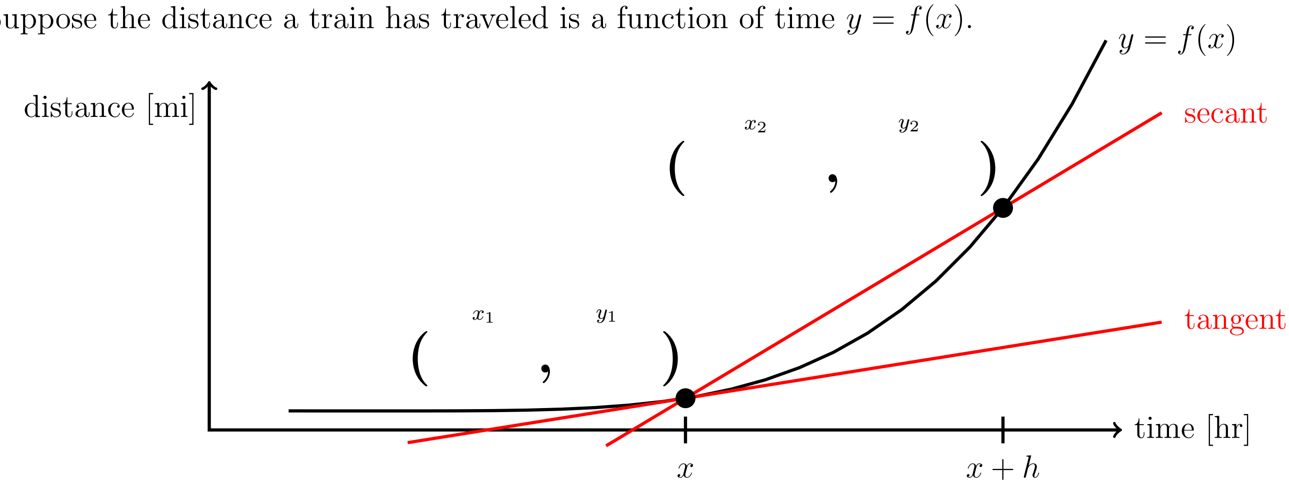

average velocity over $[x, x{+}h]$	=	average rate of change over $[x, x{+}h]$	=	slope of secant between $x, x{+}h$	=	$\frac{\Delta y}{\Delta x}$	=	$\frac{y_2 - y_1}{x_2 - x_1}$	=	$\boxed{}$	=	definition of difference quotient
velocity at x	=	rate of change at x	=	slope of tangent at x	=	$\boxed{\lim\limits_{h\to} }$	=	definition of derivative written y' or $\frac{dy}{dx}$ or $f'(x)$ or $\frac{d}{dx}f(x)$				

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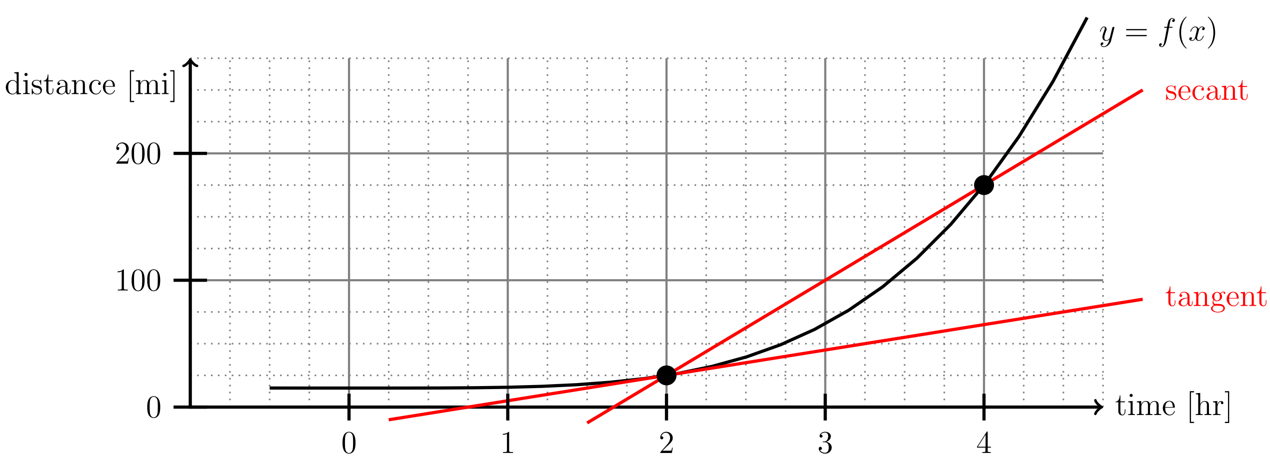

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