

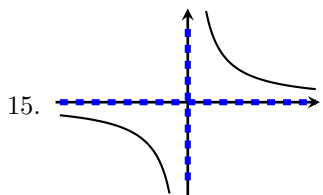
## 1 Find Limits Graphically

1. 
$$\begin{cases} f(1) = 70 \\ \lim_{x \rightarrow 1^-} f(x) = 70 \\ \lim_{x \rightarrow 1^+} f(x) = 70 \\ \lim_{x \rightarrow 1} f(x) = 70 \end{cases}$$
2. 
$$\begin{cases} f(2) = \text{undefined} \\ \lim_{x \rightarrow 2^-} f(x) = 20 \\ \lim_{x \rightarrow 2^+} f(x) = 20 \\ \lim_{x \rightarrow 2} f(x) = 20 \end{cases}$$
3. 
$$\begin{cases} f(3) = 40 \\ \lim_{x \rightarrow 3^-} f(x) = 10 \\ \lim_{x \rightarrow 3^+} f(x) = 10 \\ \lim_{x \rightarrow 3} f(x) = 10 \end{cases}$$
4. 
$$\begin{cases} f(4) = \text{undefined} \\ \lim_{x \rightarrow 4^-} f(x) = 50 \\ \lim_{x \rightarrow 4^+} f(x) = 80 \\ \lim_{x \rightarrow 4} f(x) = \text{DNE} \end{cases}$$
5. 
$$\begin{cases} f(5) = 190 \\ \lim_{x \rightarrow 5^-} f(x) = 70 \\ \lim_{x \rightarrow 5^+} f(x) = 160 \\ \lim_{x \rightarrow 5} f(x) = \text{DNE} \end{cases}$$
6. 
$$\begin{cases} f(6) = 140 \\ \lim_{x \rightarrow 6^-} f(x) = 100 \\ \lim_{x \rightarrow 6^+} f(x) = 140 \\ \lim_{x \rightarrow 6} f(x) = \text{DNE} \end{cases}$$
7. 
$$\begin{cases} f(7) = 130 \\ \lim_{x \rightarrow 7^-} f(x) = 130 \\ \lim_{x \rightarrow 7^+} f(x) = 170 \\ \lim_{x \rightarrow 7} f(x) = \text{DNE} \end{cases}$$

## 2 Find Limits Involving Infinity Graphically

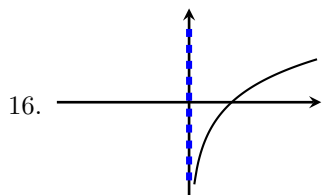
8. 
$$\begin{cases} f(8) = 30 \\ \lim_{x \rightarrow 8^-} f(x) = 110 \\ \lim_{x \rightarrow 8^+} f(x) = +\infty \\ \lim_{x \rightarrow 8} f(x) = \text{DNE} \end{cases}$$
9. 
$$\begin{cases} f(9) = \text{undefined} \\ \lim_{x \rightarrow 9^-} f(x) = +\infty \\ \lim_{x \rightarrow 9^+} f(x) = +\infty \\ \lim_{x \rightarrow 9} f(x) = +\infty \end{cases}$$
10. 
$$\begin{cases} f(10) = \text{undefined} \\ \lim_{x \rightarrow 10^-} f(x) = -\infty \\ \lim_{x \rightarrow 10^+} f(x) = +\infty \\ \lim_{x \rightarrow 10} f(x) = \text{DNE} \end{cases}$$
11. 
$$\begin{cases} f(11) = 150 \\ \lim_{x \rightarrow 11^-} f(x) = +\infty \\ \lim_{x \rightarrow 11^+} f(x) = -\infty \\ \lim_{x \rightarrow 11} f(x) = \text{DNE} \end{cases}$$
12. 
$$\begin{cases} f(12) = 180 \\ \lim_{x \rightarrow 12^-} f(x) = -\infty \\ \lim_{x \rightarrow 12^+} f(x) = -\infty \\ \lim_{x \rightarrow 12} f(x) = -\infty \end{cases}$$
13. 
$$\lim_{x \rightarrow -\infty} f(x) = 90$$
14. 
$$\lim_{x \rightarrow +\infty} f(x) = 60$$

### 3 Famous Functions



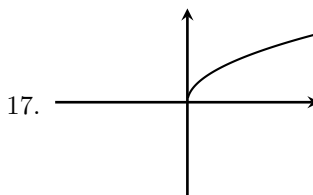
$$1/0 = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} 1/x &= -\infty \\ \lim_{x \rightarrow 0^+} 1/x &= +\infty \\ \lim_{x \rightarrow 0} 1/x &= \text{DNE} \\ \lim_{x \rightarrow -\infty} 1/x &= 0 \\ \lim_{x \rightarrow +\infty} 1/x &= 0\end{aligned}$$



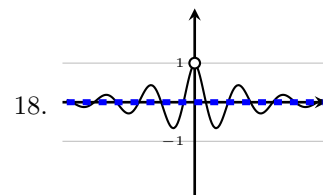
$$\ln 0 = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \ln x &= \text{DNE} \\ \lim_{x \rightarrow 0^+} \ln x &= -\infty \\ \lim_{x \rightarrow 0} \ln x &= \text{DNE} \\ \lim_{x \rightarrow -\infty} \ln x &= \text{DNE} \\ \lim_{x \rightarrow +\infty} \ln x &= +\infty\end{aligned}$$



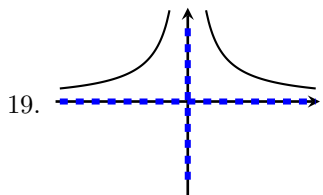
$$\sqrt{0} = 0$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \sqrt{x} &= \text{DNE} \\ \lim_{x \rightarrow 0^+} \sqrt{x} &= 0 \\ \lim_{x \rightarrow -\infty} \sqrt{x} &= \text{DNE} \\ \lim_{x \rightarrow +\infty} \sqrt{x} &= +\infty\end{aligned}$$



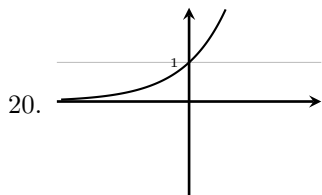
$$\sin(0)/0 = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \sin(x)/x &= 1 \\ \lim_{x \rightarrow 0^+} \sin(x)/x &= 1 \\ \lim_{x \rightarrow 0} \sin(x)/x &= 1 \\ \lim_{x \rightarrow -\infty} \sin(x)/x &= 0 \\ \lim_{x \rightarrow +\infty} \sin(x)/x &= 0\end{aligned}$$



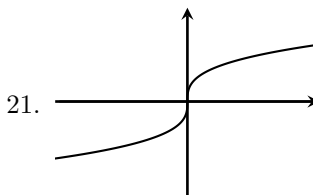
$$1/0^2 = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} 1/x^2 &= +\infty \\ \lim_{x \rightarrow 0^+} 1/x^2 &= +\infty \\ \lim_{x \rightarrow 0} 1/x^2 &= +\infty \\ \lim_{x \rightarrow -\infty} 1/x^2 &= 0 \\ \lim_{x \rightarrow +\infty} 1/x^2 &= 0\end{aligned}$$



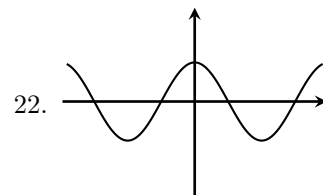
$$e^0 = 1$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} e^x &= 1 \\ \lim_{x \rightarrow 0^+} e^x &= 1 \\ \lim_{x \rightarrow 0} e^x &= 1 \\ \lim_{x \rightarrow -\infty} e^x &= 0 \\ \lim_{x \rightarrow +\infty} e^x &= +\infty\end{aligned}$$



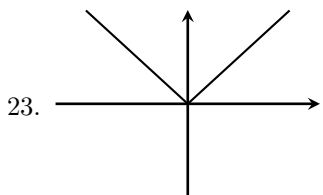
$$\sqrt[3]{0} = 0$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \sqrt[3]{x} &= 0 \\ \lim_{x \rightarrow 0^+} \sqrt[3]{x} &= 0 \\ \lim_{x \rightarrow 0} \sqrt[3]{x} &= 0 \\ \lim_{x \rightarrow -\infty} \sqrt[3]{x} &= -\infty \\ \lim_{x \rightarrow +\infty} \sqrt[3]{x} &= +\infty\end{aligned}$$



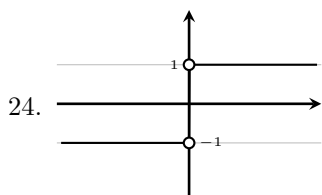
$$\cos 0 = 1$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \cos x &= 1 \\ \lim_{x \rightarrow 0^+} \cos x &= 1 \\ \lim_{x \rightarrow 0} \cos x &= 1 \\ \lim_{x \rightarrow -\infty} \cos x &= \text{DNE} \\ \lim_{x \rightarrow +\infty} \cos x &= \text{DNE}\end{aligned}$$



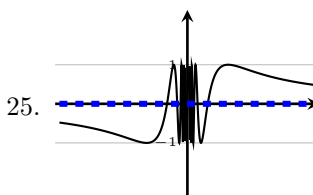
$$|0| = 0$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} |x| &= 0 \\ \lim_{x \rightarrow 0^+} |x| &= 0 \\ \lim_{x \rightarrow 0} |x| &= 0 \\ \lim_{x \rightarrow -\infty} |x| &= +\infty \\ \lim_{x \rightarrow +\infty} |x| &= +\infty\end{aligned}$$



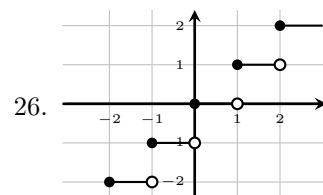
$$|0|/0 = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} |x|/x &= -1 \\ \lim_{x \rightarrow 0^+} |x|/x &= 1 \\ \lim_{x \rightarrow 0} |x|/x &= \text{DNE} \\ \lim_{x \rightarrow -\infty} |x|/x &= -1 \\ \lim_{x \rightarrow +\infty} |x|/x &= 1\end{aligned}$$



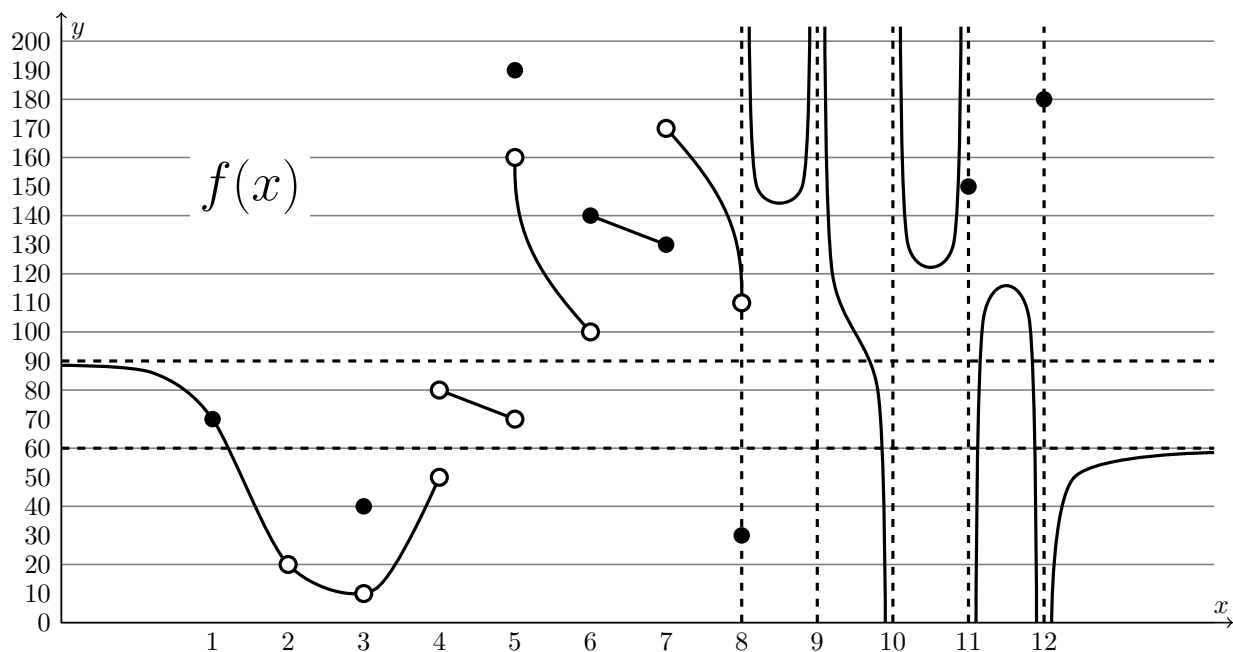
$$\sin(1/0) = \text{undefined}$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \sin(1/x) &= \text{DNE} \\ \lim_{x \rightarrow 0^+} \sin(1/x) &= \text{DNE} \\ \lim_{x \rightarrow 0} \sin(1/x) &= \text{DNE} \\ \lim_{x \rightarrow -\infty} \sin(1/x) &= 0 \\ \lim_{x \rightarrow +\infty} \sin(1/x) &= 0\end{aligned}$$



$$\lfloor 0 \rfloor = 0$$

$$\begin{aligned}\lim_{x \rightarrow 0^-} \lfloor x \rfloor &= -1 \\ \lim_{x \rightarrow 0^+} \lfloor x \rfloor &= 0 \\ \lim_{x \rightarrow 0} \lfloor x \rfloor &= \text{DNE} \\ \lim_{x \rightarrow -\infty} \lfloor x \rfloor &= -\infty \\ \lim_{x \rightarrow +\infty} \lfloor x \rfloor &= +\infty\end{aligned}$$



## 4 Identify Infinite, Jump, Removable Discontinuities Graphically

27. We say  $f$  is **continuous** (cts) at  $x = a$  if  $\lim_{x \rightarrow a} f(x)$  is finite and equals  $f(a)$ .

The function  $f$  above is continuous at integers  $x =$  \_\_\_\_\_.

28. We say  $f$  has a **removable** discontinuity at  $x = a$  if  $\lim_{x \rightarrow a} f(x)$  is finite and *unequal* to  $f(a)$ .

The function  $f$  above has this type of discontinuity at integers  $x =$  \_\_\_\_\_.

29. We say  $f$  has a **jump** discontinuity at  $x = a$  if  $\lim_{x \rightarrow a^+} f(x)$  and  $\lim_{x \rightarrow a^-} f(x)$  are finite but *unequal*.

The function  $f$  above has this type of discontinuity at integers  $x =$  \_\_\_\_\_.

30. We say  $f$  has an **infinite** discontinuity at  $x = a$  if  $\lim_{x \rightarrow a^+} f(x)$  or  $\lim_{x \rightarrow a^-} f(x)$  is infinite.

The function  $f$  above has this type of discontinuity at integers  $x =$  \_\_\_\_\_.

## 5 Continuity on an Interval

31. We say  $f$  is **continuous on the open interval**  $(a, b)$  if  $f$  is continuous at every  $x$  in  $(a, b)$ .

Find the union of all open intervals **on which  $f$  is continuous**.

(Set-builder notation) \_\_\_\_\_

(Interval notation) \_\_\_\_\_

32. We say  $f$  is **continuous everywhere** if  $f$  is continuous at every  $x$  in \_\_\_\_\_.

33. We say  $f$  is a **continuous function** if  $f$  is continuous at every  $x$  in its \_\_\_\_\_.

## 6 Left and Right Continuity

34. We say  $f$  is **left** continuous at  $x = a$  if  $\lim_{x \rightarrow a^-} f(x)$  is finite and equals  $f(a)$ .

The function  $f$  above has this type of continuity at integers  $x =$  \_\_\_\_\_.

35. We say  $f$  is **right** continuous at  $x = a$  if  $\lim_{x \rightarrow a^+} f(x)$  is finite and equals  $f(a)$ .

The function  $f$  above has this type of continuity at integers  $x =$  \_\_\_\_\_.

36. We say  $f$  is **continuous on the closed interval**  $[a, b]$  if  $f$  is continuous at every  $x$  in the *open* interval  $(a, b)$

and is **right continuous** at  $x = a$  and is **left continuous** at  $x = b$ .