Theorem (Power Rule)

$$\left(x^n\right)' = nx^{n-1}$$

Plickers Exercise

Find the derivative of $f(x) = x^3$.

A. $3x^2$ B. $3x$ C. $6x$ D. 6

Fact

$$(x+h)^2 = x^2 + 2xh + h^2$$

$$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$(x+h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$x^n + nx^{n-1}h \quad + \quad (\cdots)h^2$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\lim_{h\to 0} \Big(f(x+h) - f(x)\Big)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \left(x^n + nx^{n-1}h + (\cdots)h^2 - x^n\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \quad nx^{n-1} + (\cdots)h$$

$$= nx^{n-1}$$