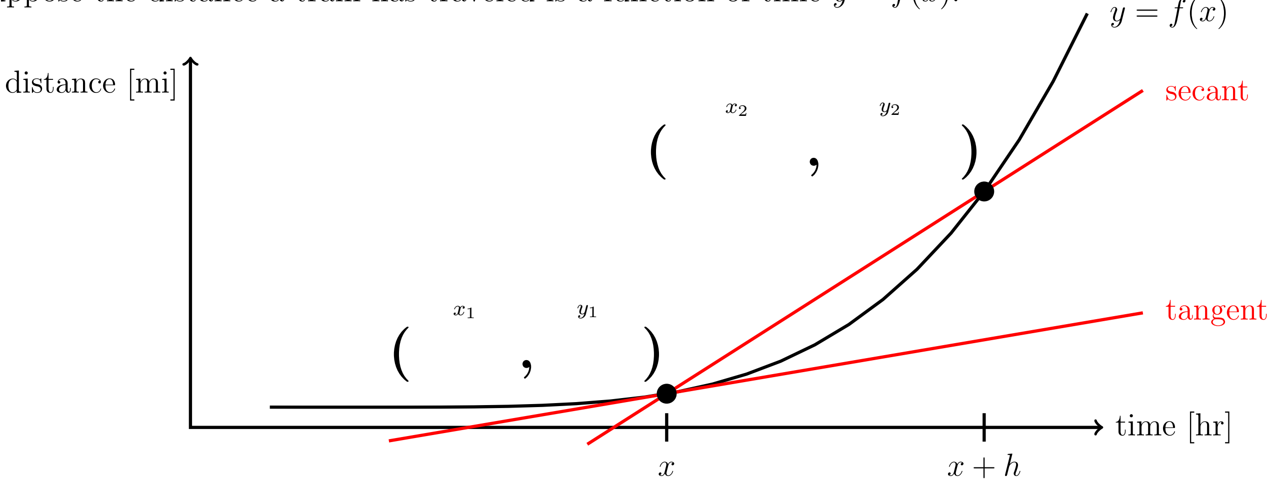
1. **Definition.** The Limit Definition of Derivative

Suppose the distance a train has traveled is a function of time $y = f(x)$.



average velocity over $[x, x{+}h]$ = average rate of change over $[x, x{+}h]$ = slope of secant between $x, x{+}h$ = $\frac{\Delta y}{\Delta x}$ = $\frac{y_2 - y_1}{x_2 - x_1}$ = $\boxed{}$ = definition of difference quotient

velocity at x = rate of change at x = slope of tangent at x = $\boxed{\lim\limits_{h \to}}$ = definition of derivative
written y' or $\frac{dy}{dx}$ or $f'(x)$ or $\frac{d}{dx}f(x)$

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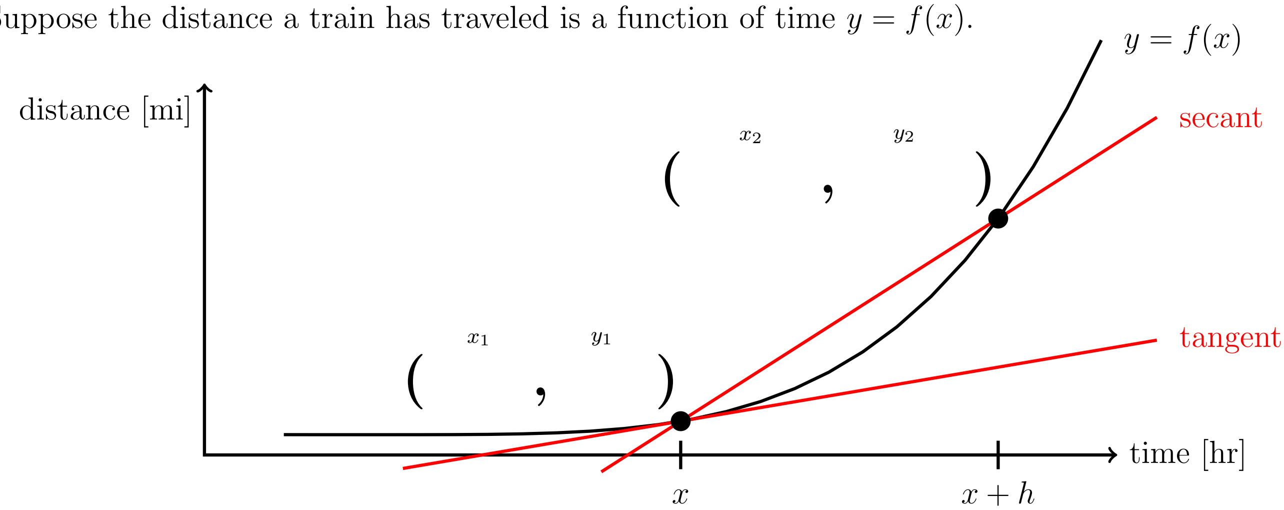

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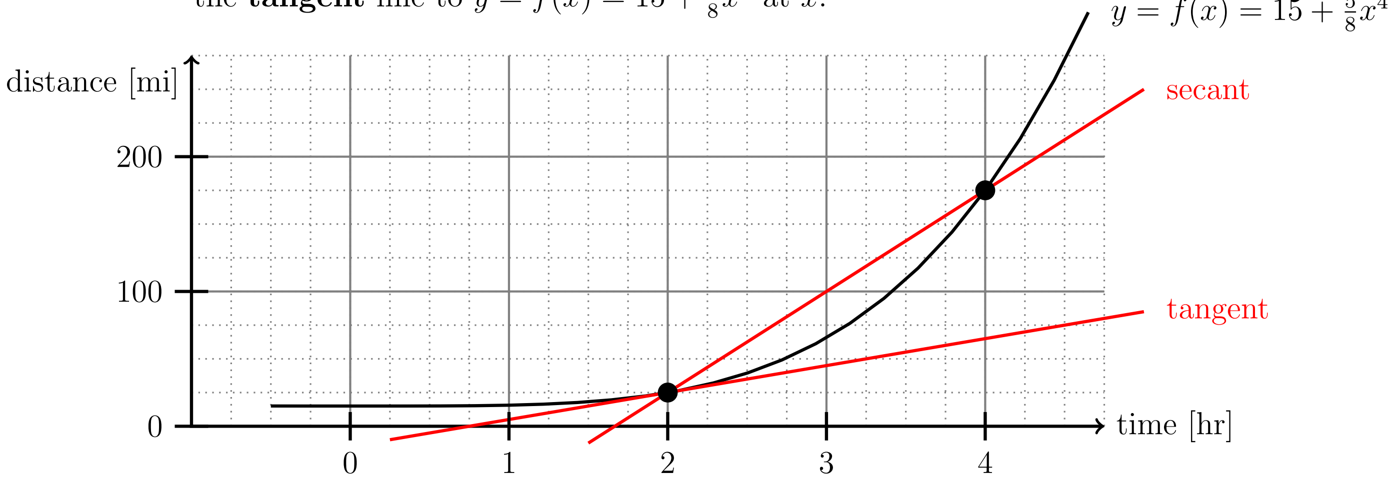
2. **Example.** Use the definition of difference quotient to find the slope of the **secant** line through $y = f(x) = 15 + \frac{5}{8}x^4$ between x and $x + h$.

Hint: $f(x+h) \quad = \quad 15 + \frac{5}{8}x^4 + \frac{20}{8}x^3h + \frac{30}{8}x^2h^2 + \frac{20}{8}xh^3 + \frac{5}{8}h^4$

3. **Exercise.** Use the definition of derivative to find the slope of the **tangent** line to $y = f(x) = 15 + \frac{5}{8}x^4$ at x.



4. **Exercise.** Use the **diagram** to estimate the slope of the **secant** line to $y = f(x) = 15 + \frac{5}{8}x^4$ **between 2 and 4**.

5. **Exercise.** Use your answer to Exercise 3 above to estimate the slope of the **tangent** line to $y = f(x) = 15 + \frac{5}{8}x^4$ **at 2**.

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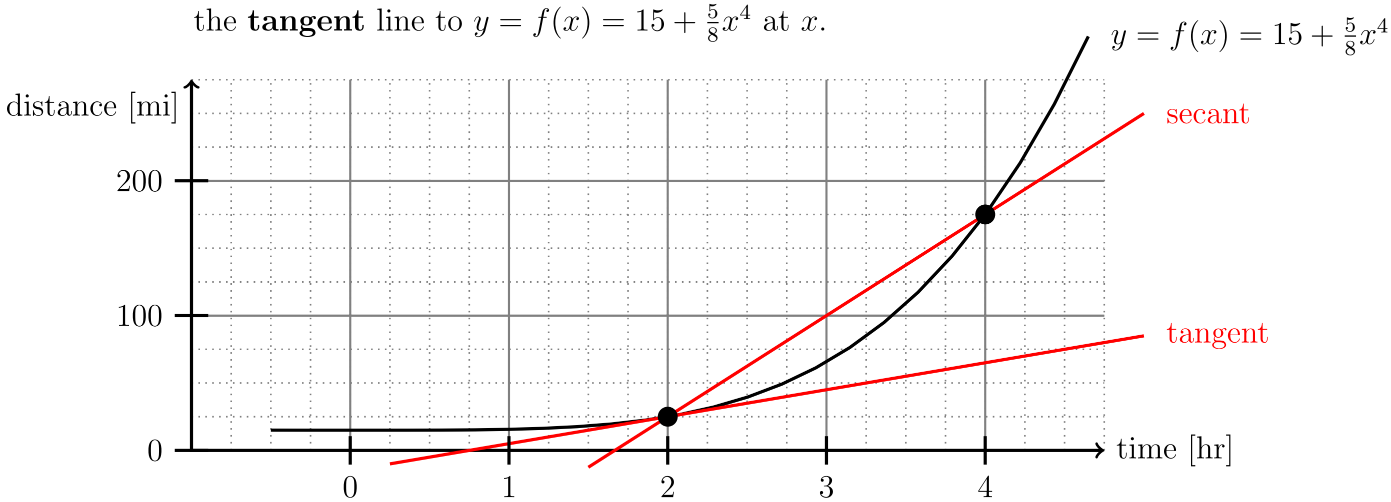

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