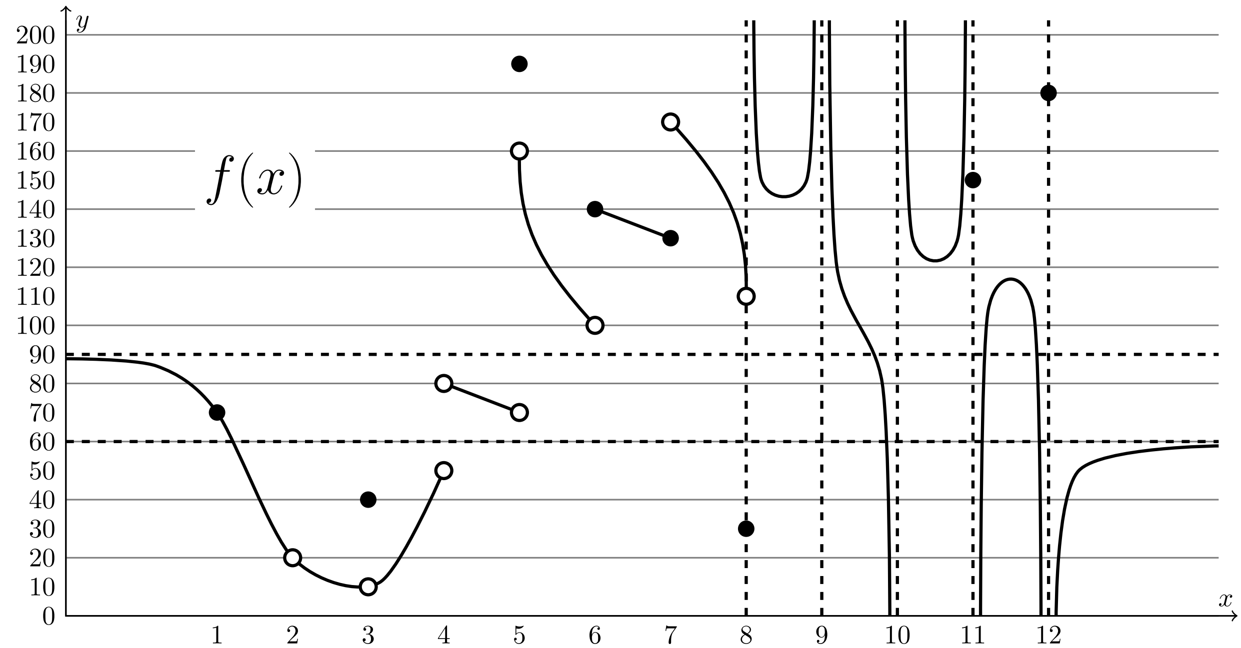



## 1 Find Limits Graphically

1. $\begin{cases} f(1) = & \textbf{70} \\ \lim_{x\to 1^-} f(x) = & \textbf{70} \\ \lim_{x\to 1^+} f(x) = & \textbf{70} \\ \lim_{x\to 1} f(x) = & \textbf{70} \end{cases}$

2. $\begin{cases} f(2) = & \textbf{undefined} \\ \lim_{x\to 2^-} f(x) = & \textbf{20} \\ \lim_{x\to 2^+} f(x) = & \textbf{20} \\ \lim_{x\to 2} f(x) = & \textbf{20} \end{cases}$

3. $\begin{cases} f(3) = & \textbf{40} \\ \lim_{x\to 3^-} f(x) = & \textbf{10} \\ \lim_{x\to 3^+} f(x) = & \textbf{10} \\ \lim_{x\to 3} f(x) = & \textbf{10} \end{cases}$

4. $\begin{cases} f(4) = & \textbf{undefined} \\ \lim_{x\to 4^-} f(x) = & \textbf{50} \\ \lim_{x\to 4^+} f(x) = & \textbf{80} \\ \lim_{x\to 4} f(x) = & \textbf{DNE} \end{cases}$

5. $\begin{cases} f(5) = & \textbf{190} \\ \lim_{x\to 5^-} f(x) = & \textbf{70} \\ \lim_{x\to 5^+} f(x) = & \textbf{160} \\ \lim_{x\to 5} f(x) = & \textbf{DNE} \end{cases}$

6. $\begin{cases} f(6) = & \textbf{140} \\ \lim_{x\to 6^-} f(x) = & \textbf{100} \\ \lim_{x\to 6^+} f(x) = & \textbf{140} \\ \lim_{x\to 6} f(x) = & \textbf{DNE} \end{cases}$

7. $\begin{cases} f(7) = & \textbf{130} \\ \lim_{x\to 7^-} f(x) = & \textbf{130} \\ \lim_{x\to 7^+} f(x) = & \textbf{170} \\ \lim_{x\to 7} f(x) = & \textbf{DNE} \end{cases}$

## 2 Find Limits Involving Infinity Graphically

8. $\begin{cases} f(8) = & \textbf{30} \\ \lim_{x\to 8^-} f(x) = & \textbf{110} \\ \lim_{x\to 8^+} f(x) = & +\infty \\ \lim_{x\to 8} f(x) = & \textbf{DNE} \end{cases}$

9. $\begin{cases} f(9) = & \textbf{undefined} \\ \lim_{x\to 9^-} f(x) = & +\infty \\ \lim_{x\to 9^+} f(x) = & +\infty \\ \lim_{x\to 9} f(x) = & +\infty \end{cases}$

10. $\begin{cases} f(10) = & \textbf{undefined} \\ \lim_{x\to 10^-} f(x) = & -\infty \\ \lim_{x\to 10^+} f(x) = & +\infty \\ \lim_{x\to 10} f(x) = & \textbf{DNE} \end{cases}$

11. $\begin{cases} f(11) = & \textbf{150} \\ \lim_{x\to 11^-} f(x) = & +\infty \\ \lim_{x\to 11^+} f(x) = & -\infty \\ \lim_{x\to 11} f(x) = & \textbf{DNE} \end{cases}$

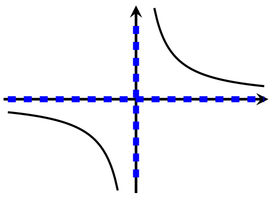
12. $\begin{cases} f(12) = & \textbf{180} \\ \lim_{x\to 12^-} f(x) = & -\infty \\ \lim_{x\to 12^+} f(x) = & -\infty \\ \lim_{x\to 12} f(x) = & -\infty \end{cases}$

13. $\lim_{x\to -\infty} f(x) = \textbf{90}$

14. $\lim_{x\to +\infty} f(x) = \textbf{60}$

# 3 Famous Functions

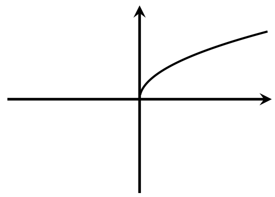
15.



$1/0 =$ **undefined**

$$\lim_{x\to 0^-} 1/x = -\infty$$
$$\lim_{x\to 0^+} 1/x = +\infty$$
$$\lim_{x\to 0} 1/x = \textbf{DNE}$$
$$\lim_{x\to -\infty} 1/x = \mathbf{0}$$
$$\lim_{x\to +\infty} 1/x = \mathbf{0}$$

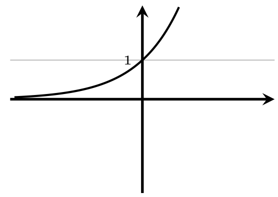
16.



$\sqrt{0} =$ **0**

$$\lim_{x\to 0^-} \sqrt{x} = \textbf{DNE}$$
$$\lim_{x\to 0^+} \sqrt{x} = \mathbf{0}$$
$$\lim_{x\to -\infty} \sqrt{x} = \textbf{DNE}$$
$$\lim_{x\to +\infty} \sqrt{x} = +\infty$$

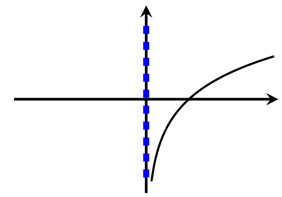
17.




$e^0 =$ **1**

$$\lim_{x\to 0^-} e^x = \mathbf{1}$$
$$\lim_{x\to 0^+} e^x = \mathbf{1}$$
$$\lim_{x\to 0} e^x = \mathbf{1}$$
$$\lim_{x\to -\infty} e^x = \mathbf{0}$$
$$\lim_{x\to +\infty} e^x = +\infty$$

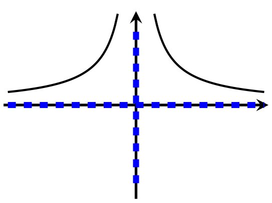
18.



$\ln 0 =$ **undefined**

$$\lim_{x\to 0^-} \ln x = \textbf{DNE}$$
$$\lim_{x\to 0^+} \ln x = -\infty$$
$$\lim_{x\to -\infty} \ln x = \textbf{DNE}$$
$$\lim_{x\to +\infty} \ln x = +\infty$$

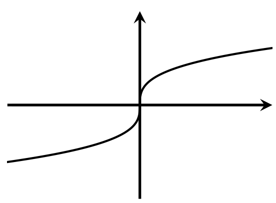
19.



$1/0^2 =$ **undefined**

$$\lim_{x\to 0^-} 1/x^2 = +\infty$$
$$\lim_{x\to 0^+} 1/x^2 = +\infty$$
$$\lim_{x\to 0} 1/x^2 = +\infty$$
$$\lim_{x\to -\infty} 1/x^2 = \mathbf{0}$$
$$\lim_{x\to +\infty} 1/x^2 = \mathbf{0}$$

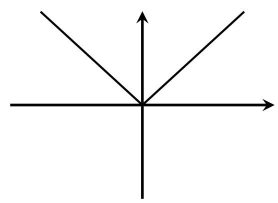
20.



$\sqrt[3]{0} =$ **0**

$$\lim_{x\to 0^-} \sqrt[3]{x} = \mathbf{0}$$
$$\lim_{x\to 0^+} \sqrt[3]{x} = \mathbf{0}$$
$$\lim_{x\to 0} \sqrt[3]{x} = \mathbf{0}$$
$$\lim_{x\to -\infty} \sqrt[3]{x} = -\infty$$
$$\lim_{x\to +\infty} \sqrt[3]{x} = +\infty$$

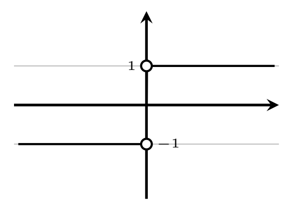
21.



$|0| =$ **0**

$$\lim_{x\to 0^-} |x| = \mathbf{0}$$
$$\lim_{x\to 0^+} |x| = \mathbf{0}$$
$$\lim_{x\to 0} |x| = \mathbf{0}$$
$$\lim_{x\to -\infty} |x| = +\infty$$
$$\lim_{x\to +\infty} |x| = +\infty$$

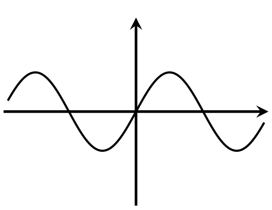
22.




$|0|/0 =$ **undefined**

$$\lim_{x\to 0^-} |x|/x = -1$$
$$\lim_{x\to 0^+} |x|/x = \mathbf{1}$$
$$\lim_{x\to 0} |x|/x = \textbf{DNE}$$
$$\lim_{x\to -\infty} |x|/x = -1$$
$$\lim_{x\to +\infty} |x|/x = \mathbf{1}$$

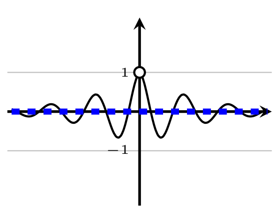
23.



$\sin 0 =$ **0**

$$\lim_{x\to 0^-} \sin x = \mathbf{0}$$
$$\lim_{x\to 0^+} \sin x = \mathbf{0}$$
$$\lim_{x\to 0} \sin x = \mathbf{0}$$
$$\lim_{x\to -\infty} \sin x = \textbf{DNE}$$
$$\lim_{x\to +\infty} \sin x = \textbf{DNE}$$

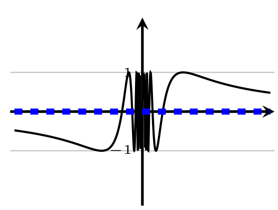
24.




$\sin(0)/0 =$ **undefined**

$$\lim_{x\to 0^-} \sin(x)/x = \mathbf{1}$$
$$\lim_{x\to 0^+} \sin(x)/x = \mathbf{1}$$
$$\lim_{x\to 0} \sin(x)/x = \mathbf{1}$$
$$\lim_{x\to -\infty} \sin(x)/x = \mathbf{0}$$
$$\lim_{x\to +\infty} \sin(x)/x = \mathbf{0}$$

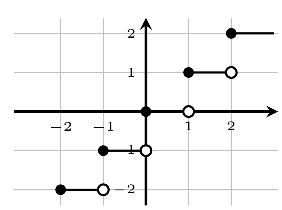
25.




$\sin(1/0) =$ **undefined**

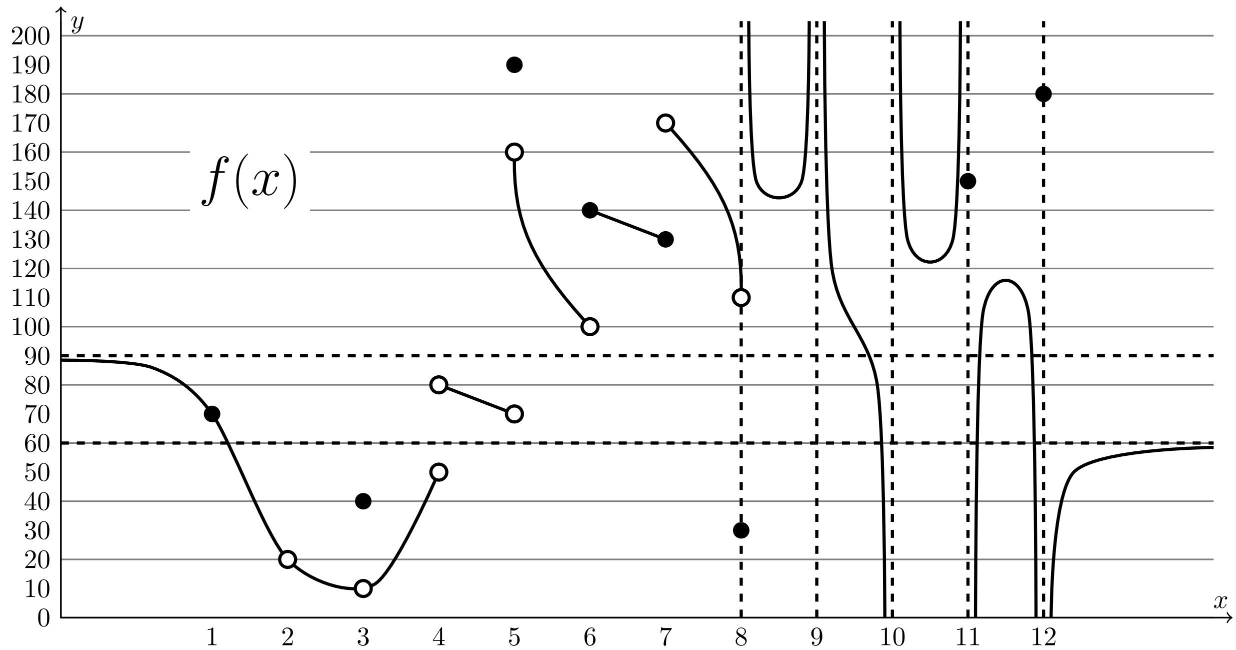
$$\lim_{x\to 0^-} \sin(1/x) = \textbf{DNE}$$
$$\lim_{x\to 0^+} \sin(1/x) = \textbf{DNE}$$
$$\lim_{x\to 0} \sin(1/x) = \textbf{DNE}$$
$$\lim_{x\to -\infty} \sin(1/x) = \mathbf{0}$$
$$\lim_{x\to +\infty} \sin(1/x) = \mathbf{0}$$

26.




$\lfloor 0 \rfloor =$ **0**

$$\lim_{x\to 0^-} \lfloor x \rfloor = -1$$
$$\lim_{x\to 0^+} \lfloor x \rfloor = \mathbf{0}$$
$$\lim_{x\to 0} \lfloor x \rfloor = \textbf{DNE}$$
$$\lim_{x\to -\infty} \lfloor x \rfloor = -\infty$$
$$\lim_{x\to +\infty} \lfloor x \rfloor = +\infty$$




## 4 Identify Infinite, Jump, Removable Discontinuities Graphically

27. We say $f$ is **continuous** **at $x = a$** if $f(a) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ and all three exist and are finite.
28. We say $f$ has a **removable** **discontinuity at $x = a$** if $f(a) \neq \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$ and the last two exist and are finite.
29. We say $f$ has a **jump** **discontinuity at $x = a$** if $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$ and both exist and are finite.
30. We say $f$ has a **infinite** **discontinuity at $x = a$** if $\lim_{x\to a^-} f(x)$ or $\lim_{x\to a^+} f(x)$ is infinite and both exist.
31. The function $f$ above is continuous (cts) at the following integers between 1 and 12: **1**.
32. The function $f$ above has removable discontinuities at the following integers: **2, 3**.
33. The function $f$ above has jump discontinuities at the following integers: **4, 5, 6, 7**.
34. The function $f$ above has infinite discontinuities at the following integers: **8, 9, 10, 11, 12**.

## 5 Continuity on an Interval

35. We say $f$ is **continuous on the open interval $(a, b)$** if $f$ is continuous at every $x$ in $(a, b)$.
Find the union of all open intervals **on which $f$ is continuous**.

(Set-builder notation) $\{x \mid x \neq 2, 3, 4, \ldots, 12\}$

(Interval notation) $(-\infty, 2) \cup (2, 3) \cup (3, 4) \cup \cdots \cup (11, 12) \cup (12, \infty)$

36. We say $f$ is **continuous everywhere** if $f$ is continuous at every $x$ in $(-\infty, \infty)$.
37. We say $f$ is **a continuous function** if $f$ is continuous at every $x$ in its **domain**.

## 6 Left and Right Continuity

38. We say $f$ is **left continuous** **at $x = a$** if $f(a) = \lim_{x\to a^-} f(x)$ and both exist and are finite.
39. We say $f$ is **right continuous** **at $x = a$** if $f(a) = \lim_{x\to a^+} f(x)$ and both exist and are finite.
40. The function $f$ above is left continuous at the following integers between 1 and 12: **1, 7**.
41. The function $f$ above is right continuous at the following integers between 1 and 12: **1, 6**.
42. We say $f$ is **continuous on the closed interval $[a, b]$** if $f$ is continuous at every $x$ in the *open* interval $(a, b)$ and is **right continuous** at $x = a$ and is **left continuous** at $x = b$.
43. The function $f$ above is continuous on the closed interval [ **6** , **7** ] with integer endpoints between 1 and 12.