

Theorem (Power Rule)

$$\left(x^n\right)' = nx^{n-1}$$

Exercise

Find the derivative of $f(x) = x^3$.

A. $3x^2$

B. $3x$

C. $6x$

D. 6

Fact

$$(x + h)^2 = x^2 + 2xh + h^2$$

$$(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$$

$$(x + h)^4 = x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$$

$$x^n + nx^{n-1}h + (\dots)h^2$$

Proof (Power Rule)

Fix a positive integer n . Let $f(x) = x^n$. So,

$$\begin{aligned} & \lim_{h \rightarrow 0} \left(f(x+h) - f(x) \right) \left(\frac{1}{h} \right) \\ &= \lim_{h \rightarrow 0} \left(x^n + nx^{n-1}h + (\dots)h^2 - x^n \right) \left(\frac{1}{h} \right) \\ &= \lim_{h \rightarrow 0} \quad nx^{n-1} + (\dots)h \\ &= \quad \quad \quad nx^{n-1} \end{aligned}$$