Theorem (Power Rule)

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Exercise

Find the derivative of $f(x) = x^3$.

A. $3x^2$ B. $3x$ C. $6x$ D. 6

Fact

$$(x + h)^2 =$$

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$$x^n$$

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$$x^n + nx^{n-1}h$$

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$$x^n + nx^{n-1}h \quad + \quad (\cdots)h^2$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

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\begin{aligned}
& f(x+h) \\
= \quad & x^n + nx^{n-1}h + (\cdots)h^2 \\
= \lim_{h\to 0} \quad & nx^{n-1} + (\cdots)h
\end{aligned}
$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$f(x+h) - f(x)$$

$$= \quad x^n + nx^{n-1}h + (\cdots)h^2 - x^n$$

$$= \lim_{h \to 0} \quad nx^{n-1} + (\cdots)h$$

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\end{aligned}
$$

Proof (Power Rule)

Fix a positive integer n. Let $f(x) = x^n$. So,

$$\begin{aligned} & \Big(f(x+h) - f(x)\Big)\left(\frac{1}{h}\right) \\ = & \left(\cancel{x^n} + nx^{n-1}h + (\cdots)h^2 - \cancel{x^n}\right)\left(\frac{1}{h}\right) \\ = \lim_{h\to 0} & \quad nx^{n-1} + (\cdots)h \end{aligned}$$

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