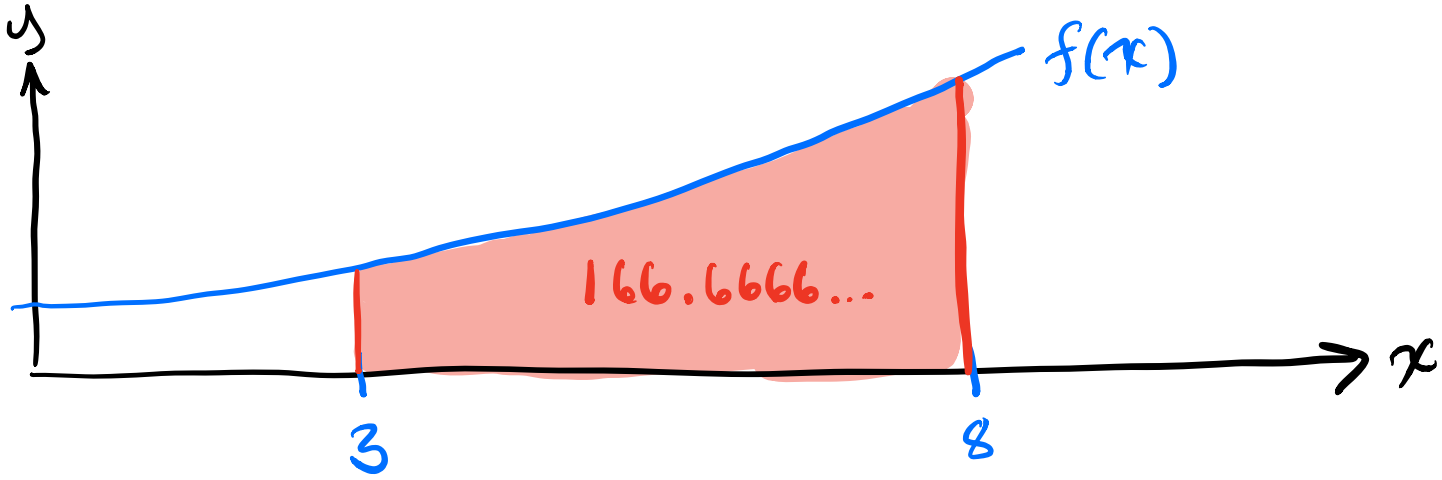
J. Lab 10 – Riemann Sums

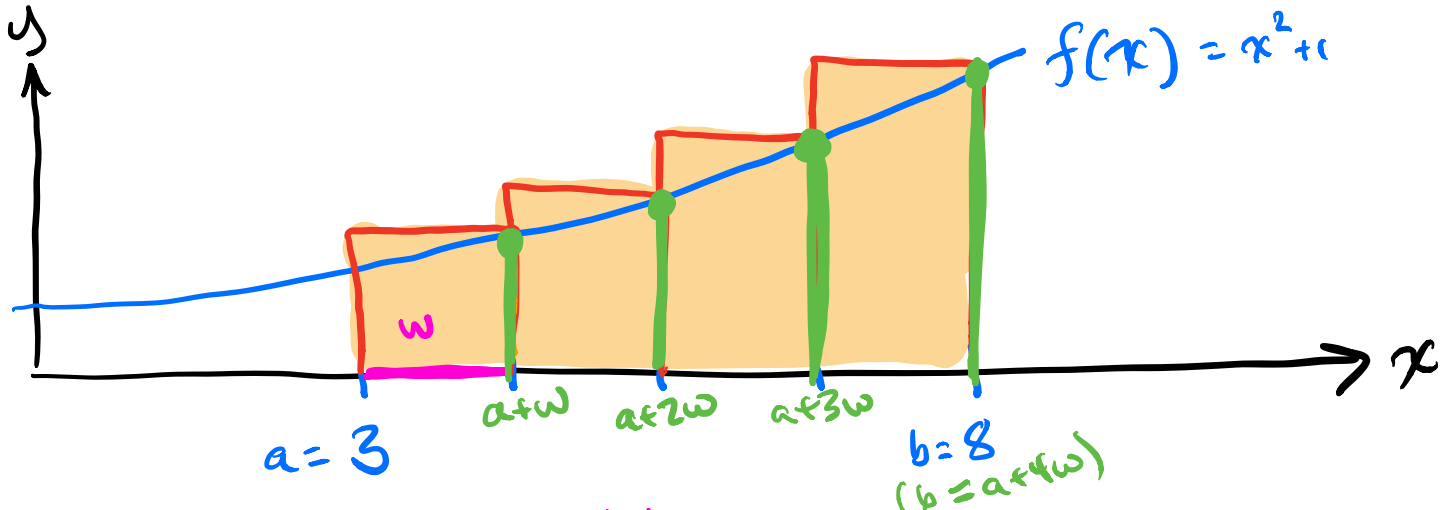
J1. Example. Find $\int_3^8 x^2+1\,dx$.

$$\int_3^8 x^2+1\,dx = \frac{1}{3}x^3 + x \Big]_3^8 = \left(\frac{1}{3}8^3+8\right) - \left(\frac{1}{3}3^3+3\right)$$

$$= 178.6666\ldots - 12 = \boxed{166.6666\ldots}$$



J2. Example. Illustrate the Regular Right Riemann sum approximation of $\int_3^8 x^2+1\,dx$. $n=4$



(Δx) $w = (b-a)/n$

$= (8-3)/4 = 5/4 = 1.25$

area of rectangles

$R = wh_1 + wh_2 + wh_3 + wh_4$

$= w(h_1 + h_2 + h_3 + h_4)$

$= w(f(a+w) + f(a+2w) + f(a+3w) + f(a+4w))$

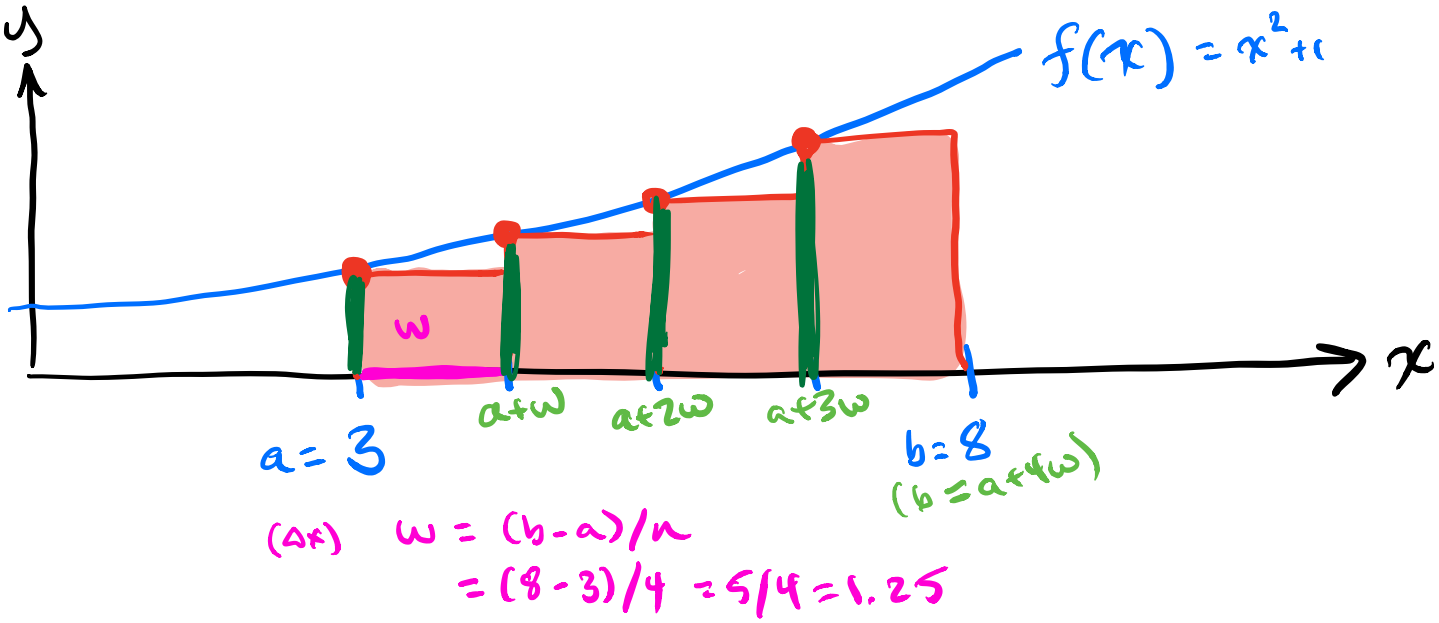
In matlab Regular Right Riemann Sum R =

```
R = w * sum(f(a + [1:4] * w))
```

J3. Compute R in Matlab

R = 202.3438

J4. Example. Illustrate the Regular Left Riemann sum approximation of $\int_3^8 x^2+1\,dx$. ($n=4$)



(Δx) $w = (b-a)/n$

$= (8-3)/4 = 5/4 = 1.25$

$$L = wh_1 + wh_2 + wh_3 + wh_4$$
$$= w(h_1 + h_2 + h_3 + h_4)$$
$$= w(f(a) + f(a+w) + f(a+2w) + f(a+3w))$$

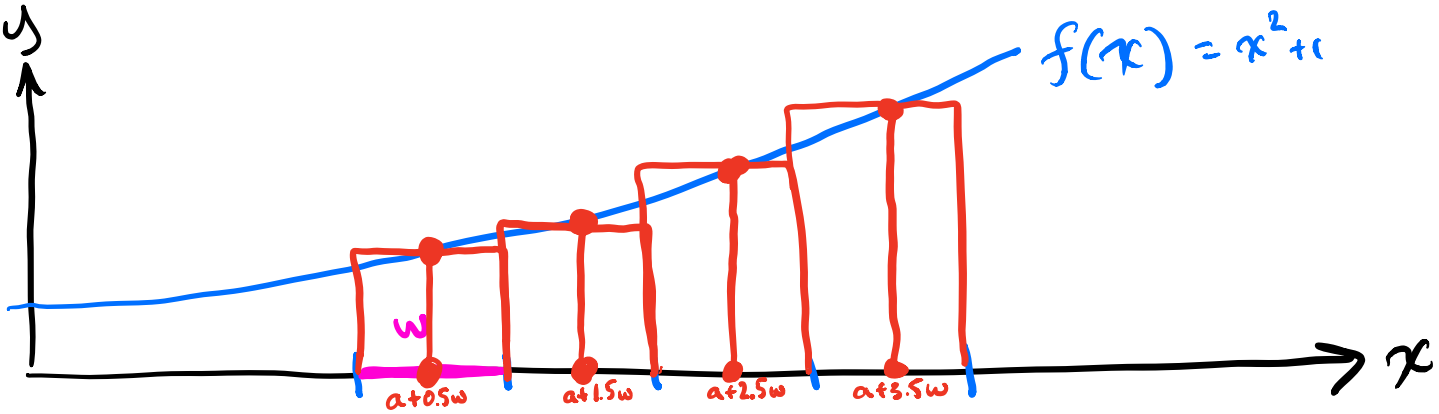
In matlab Regular Left Riemann Sum L =

```
L = w * sum( f( a + [0:n-1] * w ) )
```

J5. Compute L in Matlab.

L = 133.5938

J6. Example. Illustrate the Regular Midpoint Riemann Sum approximation of $\int_3^8 x^2+1\,dx$. ($n=4$)



$a = 3$ $\quad b = 8$ $\quad (b = a+4w)$

(Δx) $\quad w = (b-a)/n$

$= (8-3)/4 = 5/4 = 1.25$

$$M = wh_1 + wh_2 + wh_3 + wh_4$$

$$= w(h_1 + h_2 + h_3 + h_4)$$

$$= w\left(f(a+0.5w) + f(a+1.5w) + f(a+2.5w) + f(a+3.5w)\right)$$

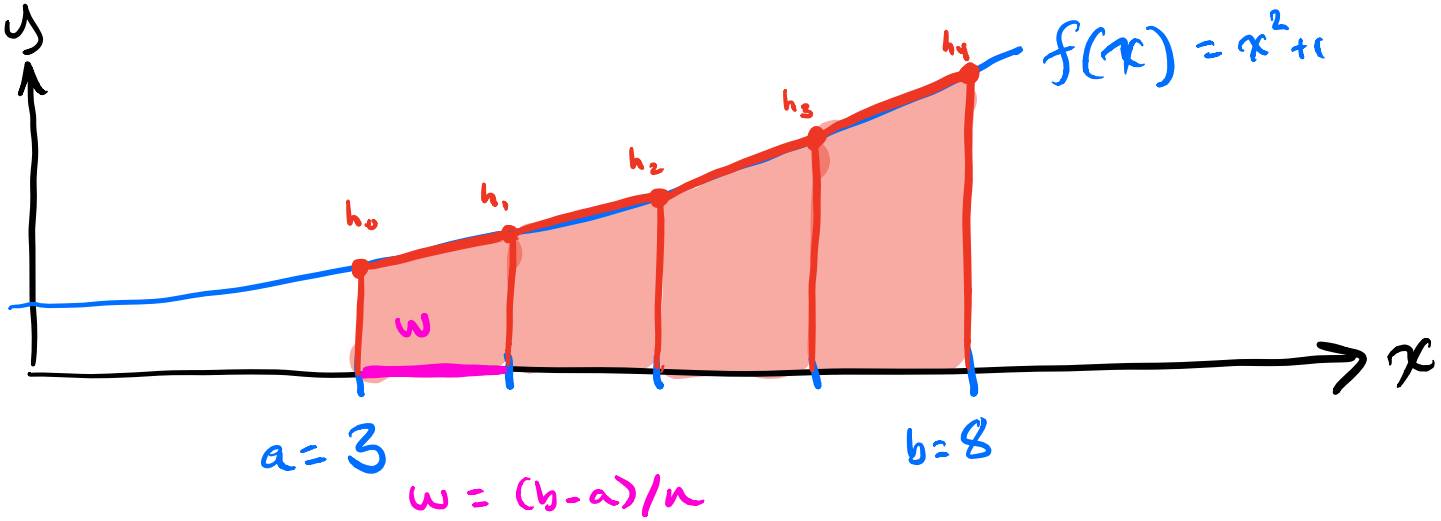
In matlab Regular Riemann Sum =

```
= w * sum( f( a + [0.5:n] * w ) )
```

J7. Compute M in matlab.

M = 166.0156

J6. Example. Illustrate the Regular Trapezoid Rule for $n = 4$ approximation of $\int_3^8 x^2+1 \, dx$.



$\omega = (b-a)/n$

$$T = \omega \frac{h_0+h_1}{2} + \omega \frac{h_1+h_2}{2} + \omega \frac{h_2+h_3}{2} + \omega \frac{h_3+h_4}{2}$$

$$= \frac{\omega}{2}\left(h_0+h_1 + h_1+h_2 + h_2+h_3 + h_3+h_4\right)$$

$$= \frac{\omega}{2}\left((h_0+h_1+h_2+h_3) + (h_1+h_2+h_3+h_4)\right)$$

$$= \frac{\omega}{2}\left(h_0+h_1+h_2+h_3\right) + \frac{\omega}{2}\left(h_1+h_2+h_3+h_4\right)$$

$$= \frac{\omega\left(h_0+h_1+h_2+h_3\right) + \omega\left(h_1+h_2+h_3+h_4\right)}{2}$$

$$\boxed{T = (L+R)/2}$$

J9. Compute T using Matlab.

$$T = 167.9688$$

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f.m

```
1   function y = f(x)
2 -   y = x.^2 + 1;
```

Command Window

```
>> a = 3;
>> b = 8;
>> n = 4;
>>
>> w = (b-a)/n;
>>
>> R = w*sum(f(a+[1:n]*w))

R =

   202.3438

>> L = w*sum(f(a+[0:n-1]*w))

L =

   133.5938

>> M = w*sum(f(a+[0.5:n]*w))

M =

   166.0156

>> T = (L+R)/2

T =

   167.9688
```