- Staff email always the same

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- webwork login

username = CUNY username
password = CUNY username

all lowercase

first.last00

SECTION 9.1 First-Degree Equations and Formulas

Solving First-Degree Equations

Suppose that the fuel economy, in miles per gallon, of a particular car traveling at a speed of v miles per hour can be calculated using the variable expression $-0.02v^2 + 1.6v + 3$, where $10 \le v \le 75$. For example, if the speed of a car is 30 miles per hour, we can calculate the fuel economy by substituting 30 for v in the variable expression and then using the order of operations agreement to evaluate the resulting numerical expression.

$$-0.02v^2 + 1.6v + 3$$
$$\begin{aligned} -0.02(30)^2 + 1.6(30) + 3 &= -0.02(900) + 1.6(30) + 3 \\ &= -18 + 48 + 3 \\ &= 33 \end{aligned}$$

The fuel economy is 33 miles per gallon.

The **terms** of a variable expression are the addends of the expression. The expression $-0.02v^2 + 1.6v + 3$ has three terms. The terms $-0.02v^2$ and $1.6v$ are **variable terms** because each contains a variable. The term 3 is a **constant term**; it does not contain a variable.

Each variable term is composed of a **numerical coefficient** and a **variable part** (the variable or variables and their exponents). For the variable term $-0.02v^2$, -0.02 is the coefficient and v^2 is the variable part.

Like terms of a variable expression are terms with the same variable part. Constant terms are also like terms. Examples of like terms are

$4x$ and $7x$
$9y$ and y
$5x^2y$ and $6x^2y$
8 and -3

An **equation** expresses the equality of two mathematical expressions. Each of the following is an equation.

$$8 + 5 = 13$$
$$4y - 6 = 10$$
$$x^2 - 2x + 1 = 0$$
$$b = 7$$

Each of the equations below is a **first-degree equation in one variable**. *First degree* means that the variable has an exponent of 1.

$$x + 11 = 14$$
$$3z + 5 = 8z$$
$$2(6y - 1) = 34$$

A **solution** of an equation is a number that, when substituted for the variable, results in a true equation.

3 is a solution of the equation $x + 4 = 7$ because $3 + 4 = 7$.
9 is not a solution of the equation $x + 4 = 7$ because $9 + 4 \ne 7$.

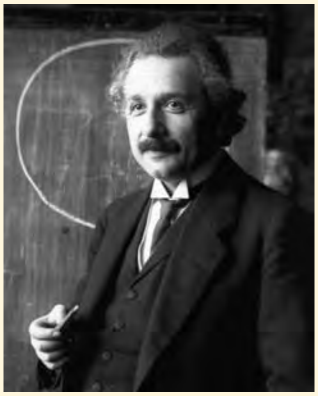
To **solve an equation** means to find all solutions of the equation. The following properties of equations are often used to solve equations.

TAKE NOTE

Recall that the order of operations agreement states that when simplifying a numerical expression you should perform the operations in the following order:

1. Perform operations inside parentheses.
2. Simplify exponential expressions.
3. Do multiplication and division from left to right.
4. Do addition and subtraction from left to right.

POINT OF INTEREST



Albert Einstein Archive, Jerusalem, Ferdinand Schmutzer.

One of the most famous equations is $E = mc^2$. This equation, stated by Albert Einstein (īn′stīn), shows that there is a relationship between mass m and energy E. In this equation, c is the speed of light.

TAKE NOTE

It is important to note the difference between an expression and an equation. An equation contains an equals sign; an expression doesn't.

Terminology

$$5y + 4 = 9 - 3(2y+1)$$

there are four terms: $5y$, 4, 9, $-3(2y+1)$

there are only 2 like terms: 4 and 9
(because we haven't opened the parentheses) (2y is "stuck" inside a multiplication for now)

Lets solve

$$
\begin{aligned}
5y + 4 &= 9 - 3(2y+1) \\
5y + 4 &= 9 - 6y - 3 && \text{distribute} \\
5y + 4 &= -6y + 6 && \text{combine like terms} \\
11y + 4 &= 6 && \text{add } 6y \\
11y &= 2 && \text{subtract } 4 \\
y &= \boxed{2/11} && \text{divide } 11
\end{aligned}
$$

HISTORICAL

Finding solutions of equations has been a principal aim of mathematics for thousands of years. However, the equals sign did not occur in any text until 1557, when Robert Recorde (ca. 1510–1558) used the symbol in *The Whetstone of Witte.*

QUESTION Which of the following are first-degree equations in one variable?

a. $5y + 4 = 9 - 3(2y + 1)$ **b.** $\sqrt{x} + 9 = 16$ not in 1st degree because **c.** $p = -14$

d. $2x - 5 = x^2 - 9$ ~~**e.**~~ $3y + 7 = 4z - 10$

not in 1st degree because in 2 variables

all equations : Yes

all in one variable

ANS: a, c

Properties of Equations

Addition Property

The same number can be added to each side of an equation without changing the solution of the equation.

$$\text{If } a = b, \text{ then } a + c = b + c.$$

Subtraction Property

The same number can be subtracted from each side of an equation without changing the solution of the equation.

$$\text{If } a = b, \text{ then } a - c = b - c.$$

Multiplication Property

Each side of an equation can be multiplied by the same *nonzero* number without changing the solution of the equation.

$$\text{If } a = b \text{ and } c \neq 0, \text{ then } ac = bc.$$

Division Property

Each side of an equation can be divided by the same *nonzero* number without changing the solution of the equation.

$$\text{If } a = b \text{ and } c \neq 0, \text{ then } \frac{a}{c} = \frac{b}{c}.$$

TAKE NOTE

In the multiplication property, it is necessary to state $c \neq 0$ so that the solutions of the equation are not changed.

For example, if $\frac{1}{2}x = 4$, then $x = 8$.

But if we multiply each side of the equation by 0, we have

$$0 \cdot \frac{1}{2}x = 0 \cdot 4$$
$$0 = 0$$

The solution $x = 8$ is lost.

In solving a first-degree equation in one variable, the goal is to rewrite the equation with the variable alone on one side of the equation and a constant term on the other side of the equation. The constant term is the solution of the equation.

For example, to solve the equation $t + 9 = -4$, use the subtraction property to subtract the constant term (9) from each side of the equation.

$$t + 9 = -4$$
$$t + 9 - 9 = -4 - 9$$
$$t = -13$$

Now the variable (t) is alone on one side of the equation and a constant term (-13) is on the other side. The solution is -13.

TAKE NOTE

You should always check the solution of an equation. The check for the example at the right is shown below.

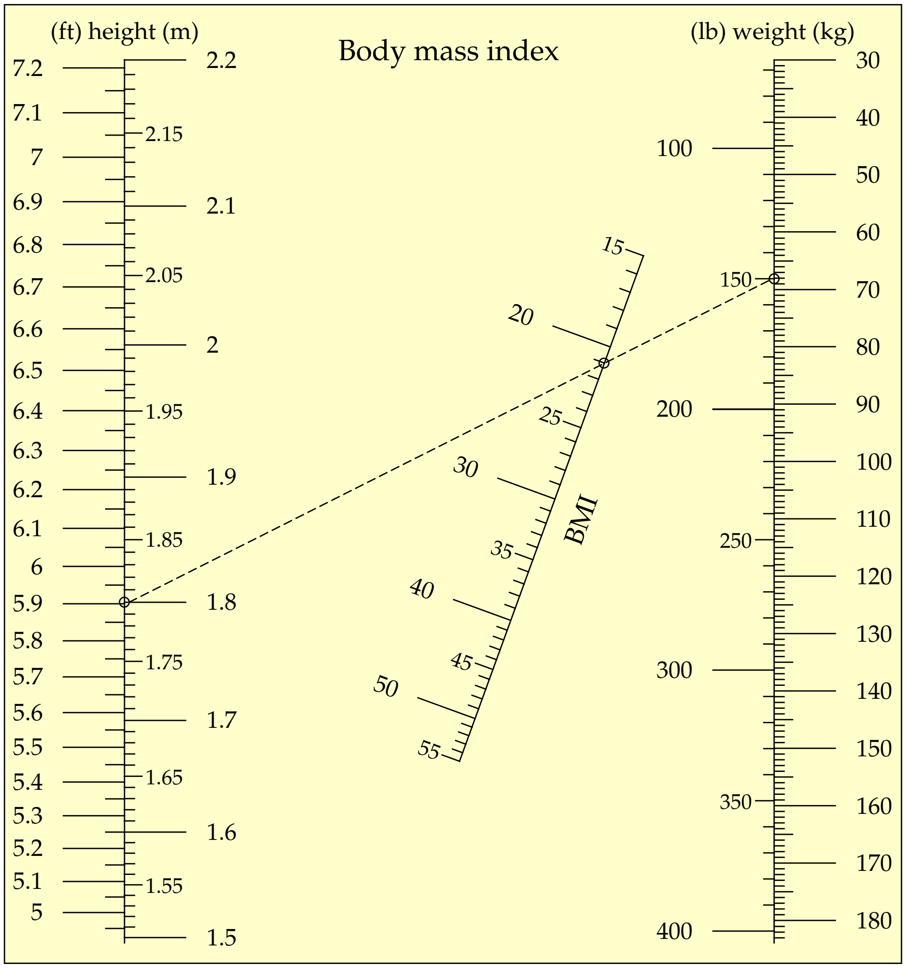
$$\begin{array}{r|l} t + 9 & -4 \\ \hline -13 + 9 & -4 \\ -4 & = -4 \end{array}$$

This is a true equation. The solution -13 checks.

ANSWER The equations in ***a*** and ***c*** are first-degree equations in one variable. The equation in ***b*** is not a first-degree equation in one variable because it contains the square root of a variable. The equation in ***d*** contains a variable with an exponent other than 1. The equation in ***e*** contains two variables.

by using the nomograph compare with the value you obtained by using a calculator?

c. Explain the advantages and disadvantages of using a nomograph to calculate BMI.



EXERCISE SET 9.1

p 506

1. What is the difference between an expression and an equation? Provide an example of each.

2. What is the solution of the equation $x = 8$? Use your answer to explain why the goal in solving an equation is to get the variable alone on one side of the equation.

3. Explain how to check the solution of an equation.

In Exercises 4 to 41, solve the equation.

4. $x + 7 = -5$
5. $9 + b = 21$
6. $-9 = z - 8$
7. $b - 11 = 11$
8. $-3x = 150$
9. $-48 = 6z$
10. $-9a = -108$
11. $-\frac{3}{4}x = 15$
12. $\frac{5}{2}x = -10$
13. $-\frac{x}{4} = -2$
14. $\frac{2x}{5} = -8$
15. $4 - 2b = 2 - 4b$
16. $4y - 10 = 6 + 2y$
17. $5x - 3 = 9x - 7$
18. $10z + 6 = 4 + 5z$
19. $3m + 5 = 2 - 6m$
20. $6a - 1 = 2 + 2a$
21. $5x + 7 = 8x + 5$
22. $2 - 6y = 5 - 7y$
23. $4b + 15 = 3 - 2b$
24. $2(x + 1) + 5x = 23$
25. $9n - 15 = 3(2n - 1)$
26. $7a - (3a - 4) = 12$
27. $5(3 - 2y) = 3 - 4y$

7.

$$b - 11 = 11$$

$$b = \boxed{22}$$ add 11

12.

$$\frac{5}{2}x = -10$$

(we have only 1 term on each side, so it's time to multiply or divide)

$$x = (-\cancel{10}^{2})\left(\frac{2}{\cancel{5}_{1}}\right)$$ multiply $\frac{2}{5}$

$$x = \boxed{-4}$$ simplify

---

10 is the numerator

$$-10 \cdot \frac{2}{5} = \frac{-20}{5} = \boxed{-4}$$

(reduce)

divide top & bottom by same number (here divide top & bottom by 5)

"no denominator" means "denominator is 1"

---

15. $4 - 2b = 2 - 4b$

$$4 + 2b = 2$$ add $4b$

$-2b + 4b$

lose 2, gain 4, end of the day: gain 2

$$2b = -2$$ subtract 4

$2 - 4$

gain 2, lose 4 gives you lose 2

$$b = \boxed{-1}$$ divide 2

Note: a "degree 1 equation"

can also be called a "linear equation"

28. $9 - 7x = 4(1 - 3x)$

29. $2(3b + 5) - 1 = 10b + 1$

30. $2z - 2 = 5 - (9 - 6z)$

31. $4a + 3 = 7 - (5 - 8a)$

32. $5(6 - 2x) = 2(5 - 3x)$

33. $4(3y + 1) = 2(y - 8)$

34. $2(3b - 5) = 4(6b - 2)$

35. $3(x - 4) = 1 - (2x - 7)$

36. $\frac{2y}{3} - 4 = \frac{y}{6} - 1$

37. $\frac{x}{8} + 2 = \frac{3x}{4} - 3$

38. $\frac{2x - 3}{3} + \frac{1}{2} = \frac{5}{6}$

39. $\frac{2}{3} + \frac{3x + 1}{4} = \frac{5}{3}$

40. $\frac{1}{2}(x + 4) = \frac{1}{3}(3x - 6)$

41. $\frac{3}{4}(x - 8) = \frac{1}{2}(2x + 4)$

42) Plug in P. (Given) solve for L. (Goal)

$P = 0.018417L$

$(\$300) = 0.018417L$ plug in $P = \$300$

$\frac{\$300}{0.018417} = L$ divide 0.018417

$\boxed{\$16289.30} = L$ use calculator (because they say round to the nearest cent)

■ **Car Payments** The monthly car payment on a 60-month car loan at a 5% rate is calculated by using the formula $P = 0.018417L$, where P is the monthly car payment and L is the loan amount. Use this formula for Exercises 42 and 43.

$P = \$300$

42. If you can afford a maximum monthly car payment of \$300, what is the maximum loan amount you can afford? Round to the nearest cent. $= L$ goal "want to find"

43. If the maximum monthly car payment you can afford is \$350, what is the maximum loan amount you can afford? Round to the nearest cent.

■ **Deep-Sea Diving** The pressure on a diver can be calculated using the formula $P = 15 + \frac{1}{2}D$, where P is the pressure in pounds per square inch and D is the depth in feet. Use this formula for Exercises 44 and 45.

44. Find the depth of a diver when the pressure on the diver is 45 lb/in^2.

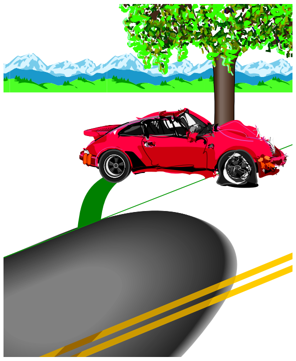
45. Find the depth of a diver when the pressure on the diver is 55 lb/in^2.

■ **Foot Races** The world-record time for a 1-mile race can be approximated by $t = 16.11 - 0.0062y$, where y is the year of the race, $1950 \le y \le 2005$, and t is the time, in minutes, of the race. Use this formula for Exercises 46 and 47.

46. Approximate the year in which the first "4-minute mile" was run. The actual year was 1954.

47. In 1999, the world-record time for a 1-mile race was 3.72 min. For what year does the equation predict this record time?

■ **Black Ice** Black ice is an ice covering on roads that is especially difficult to see and therefore extremely dangerous for motorists. The distance a car traveling at 30 mph will slide after its brakes are applied is related to the outside temperature by the formula $C = \frac{1}{4}D - 45$, where C is the Celsius temperature and D is the distance, in feet, that the car will slide. Use this formula for Exercises 48 and 49.



48. Determine the distance a car will slide on black ice when the outside air temperature is -3°C.

49. How far will a car slide on black ice when the outside air temperature is -11°C?

■ **Crickets** The formula $N = 7C - 30$ approximates N, the number of times per minute a cricket chirps when the air temperature is C degrees Celsius. Use this formula for Exercises 50 and 51.

50. What is the approximate air temperature when a cricket chirps 100 times per minute? Round to the nearest tenth.

51. Determine the approximate air temperature when a cricket chirps 140 times per minute. Round to the nearest tenth.

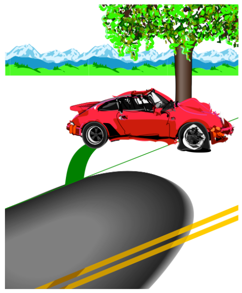
■ **Bowling** In order to equalize all the bowlers' chances of winning, some players in a bowling league are given a handicap, or a bonus of extra points. Some leagues use the formula $H = 0.8(200 - A)$, where H is the handicap and A is the bowler's average score in past games. Use this formula for Exercises 52 and 53.

52. A bowler has a handicap of 20. What is the bowler's average score?

53. Find the average score of a bowler who has a handicap of 25.

Anton Balazh/Shutterstock.com

Black Ice Black ice is an ice covering on roads that is especially difficult to see and therefore extremely dangerous for motorists. The distance a car traveling at 30 mph will slide after its brakes are applied is related to the outside temperature by the formula $C = \frac{1}{4}D - 45$, where C is the Celsius temperature and D is the distance, in feet, that the car will slide. Use this formula for Exercises 48 and 49.



$C = \frac{1}{4}D - 45$

$(-3) = \frac{1}{4}D - 45$ substitute $C=-3$

$42 = \frac{1}{4}D$ add 45

$\boxed{168} = D$ multiply 4

48. Determine the distance a car will slide on black ice when the outside air temperature is $-3°C$.

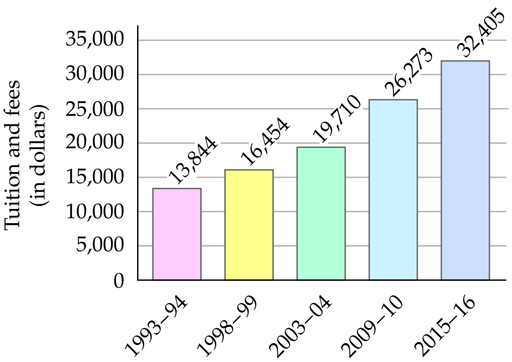
$C = -3$

■ In Exercises 54 to 63, write an equation as part of solving the problem.

54. **College Tuition** The graph below shows average tuition and fees at private 4-year colleges for selected years.

a. For the 2003–04 school year, the average tuition and fees at private 4-year colleges were \$934 more than four times the average tuition and fees at public 4-year colleges. Find the average tuition and fees at public 4-year colleges for the school year 2003–04.

b. For the 2015–16 school year, the average tuition and fees at private 4-year colleges were \$4175 more than three times the average tuition and fees at public 4-year colleges. Determine the average tuition and fees at public 4-year colleges for the 2015–16 school year.

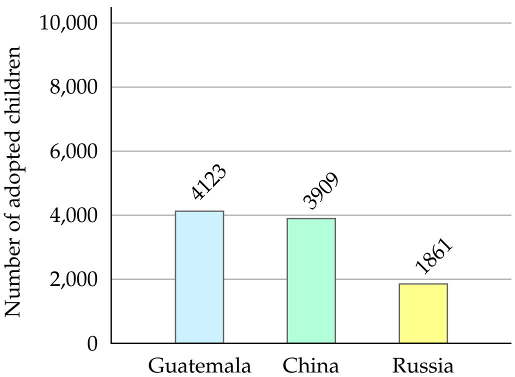


Tuition and Fees at Private 4-Year Colleges
SOURCE: The College Board

55. **Adoption** In a recent year, Americans adopted 17,438 children from foreign countries. In the graph below are the top three countries where the children were born.

a. The number of children adopted from Guatemala was 1052 less than three times the number adopted from Ethiopia. Determine the number of children adopted from Ethiopia that year.

b. The number of children adopted from China was 189 more than eight times the number adopted from Ukraine. Determine the number of children adopted from Ukraine that year.



Birth Countries of Adopted American Children
SOURCE: U.S. State Department

56. **Installment Purchases** The purchase price of a large 4K LED TV, including finance charges, was \$1425. A down payment of \$300 was made, and the remainder was paid in 18 equal monthly installments. Find the monthly payment.

57. **Auto Repair** The cost to replace a water pump in a Corvette was \$355. This included \$115 for the water pump plus \$80/h for labor. How many hours of labor were required to replace the water pump?

58. **Robots** Kiva Systems, Inc., builds robots that companies can use to streamline order fulfillment operations in their warehouses. Salary and other benefits for one human warehouse worker can cost a company about \$64,000 a year, an amount that is 103 times the company's yearly maintenance and operation costs for one robot. Find the yearly costs for a robot. Round to the nearest hundred. (*Source: Boston Globe*)

59. **College Staffing** A university employs a total of 600 teaching assistants and research assistants. There are three times as many teaching assistants as research assistants. Find the number of research assistants employed by the university.

60. **Wages** A service station attendant is paid time-and-a-half for working over 40 hours per week. Last week the attendant worked 47 h and earned \$631.25. Find the attendant's regular hourly wage.

61. **Investments** An investor deposited \$5000 in two accounts. Two times the smaller deposit is \$1000 more than the larger deposit. Find the amount deposited in each account.

62. **Shipping** An overnight mail service charges \$5.60 for the first 6 oz and \$0.85 for each additional ounce or fraction of an ounce. Find the weight, in ounces, of a package that cost \$10.70 to deliver.

63. **Telecommunications** A cellular phone company charges \$59.95 per month for a service plan that includes 6 GB of data. In addition to the basic monthly rate, the company charges \$7.95 for each additional GB of data over 6 GB. A customer on this plan receives a bill for \$115.60. How much data did this customer send and receive during the month?

made-of-letters

■ In Exercises 64 to 81, solve the formula for the indicated variable.

Section: Literal equations.

69) Solve I=Prt for r.

64. $A = \frac{1}{2}bh$; h (geometry)

65. $P = a + b + c$; b (geometry)

66. $d = rt$; t (physics)

67. $E = IR$; R (physics)

68. $PV = nRT$; R (chemistry)

69. $I = Prt$; r (business)

70. $P = 2L + 2W$; W (geometry)

71. $F = \frac{9}{5}C + 32$; C (temperature

$I = Prt$

~~$I - t = Prt - t$~~ ~~subtract t~~

$\frac{I}{t} = Pr$ divide t

~~$\frac{PI}{t} = P^2 r$~~ ~~multiply P~~

$\boxed{\frac{I}{Pt}} = r$ divide P (multiply $\frac{1}{P}$)

$\frac{1}{P} \cdot \left(\frac{I}{t}\right)$

71) solve for C.

$$F = \frac{9}{5}C + 32$$

Do we have only 1 term on each side?
No, on right we have 2 terms.
So not ready to multiply/divide.
Gotta add/subtract.

$$F - 32 = \frac{9}{5}C \qquad \text{Subtract } 32$$

$$\boxed{\frac{5}{9}(F-32)} = C \qquad \text{Multiply } \frac{5}{9}$$

This answer is perfect.
You can distribute if you want to get another perfect answer

$$\frac{5}{9}(F-32)$$

$$\boxed{\frac{5F}{9} - \frac{160}{9}} = C$$

$$\boxed{\frac{5}{9}F - \frac{160}{9}} = C$$

9:07 AM

73) Solve for t

$$A = P + Prt$$

(isolate the term with t)
Subtract P

$$A - P = Prt$$

(isolate the t)
Divide Pr

$$\boxed{\frac{A-P}{Pr}} = t$$

77) Solve for S

$$R = \frac{C-S}{t}$$

$$Rt = C - S \qquad \text{Multiply } t$$

$$Rt - C = -S \qquad \text{Subtract } C$$

$$\boxed{-Rt + C} = S \qquad \text{Multiply } -1$$

$$(-1)(Rt - C)$$

$$-Rt + C$$

77| Something goes wrong if we try multiplying $\frac{t}{c}$ immediately

$$R = \frac{c-s}{t}$$

$$\frac{Rt}{c} = \frac{(c-s)\cancel{t}}{\cancel{t}c} \qquad \text{multiply } \frac{t}{c}$$

the t cancel
but the c don't cancel

$$\frac{c-s}{c}$$

no cancelling here

To remember that cancelling doesn't make sense
try an example with numbers

$$\frac{\cancel{2} - 10}{\cancel{2}} = \frac{-8}{2} = -4$$

$$= -10 \ ?$$

For Wed 9/2, read 9.2

For Sun 9/6, will be webwork due, I'll announce when I post it (most likely tomorrow)

72. $P = R - C$; C (business)

73. $A = P + Prt$; t (business)

74. $S = V_0t - 16t^2$; V_0 (physics)

75. $T = fm - gm$; f (engineering)

76. $P = \dfrac{R - C}{n}$; R (business)

77. $R = \dfrac{C - S}{t}$; S (business)

78. $V = \dfrac{1}{3}\pi r^2 h$; h (geometry)

79. $A = \dfrac{1}{2}h(b_1 + b_2)$; b_2 (geometry)

80. $a_n = a_1 + (n - 1)d$; d (mathematics)

81. $S = 2\pi r^2 + 2\pi rh$; h (geometry)

■ In Exercises 82 and 83, solve the equation for y.

82. $2x - y = 4$

83. $4x + 3y = 6$

■ In Exercises 84 and 85, solve the equation for x.

84. $ax + by + c = 0$

85. $y - y_1 = m(x - x_1)$

EXTENSIONS

■ In Exercises 86 and 87, solve the equation.

86. $3(4x + 2) = 7 - 4(1 - 3x)$

87. $4(x + 5) = 30 - (10 - 4x)$

88. Use the numbers 5, 10, and 15 to make equations by filling in the boxes: $x + \square = \square - \square$. Each equation must use all three numbers.

a. What is the largest possible solution of these equations?

b. What is the smallest possible solution of these equations?

89. Solve the equation $ax + b = cx + d$ for x. Is your solution valid for all numbers a, b, c, and d? Explain.

■ **Writing Formulas** When we know there is an explicit relationship between two quantities, often we can write a formula to express the relationship.

For example, suppose that a toll of \$3.75 is collected from each vehicle that crosses a particular bridge. Let A be the total amount of money collected, and let c be the number of vehicles that cross the bridge on a given day. Then,

$$A = \$3.75c$$

is a formula that expresses the total amount of money collected from vehicles on any given day.

In Exercises 90 to 93, write a formula for the situation. Include as part of your answer a list of variables that were used, and state what each variable represents.

90. Write a formula to represent the total cost to rent a copier from a company that charges \$325 per month plus \$0.08 per copy made.

91. Suppose you buy a used car with 30,000 mi on it. You expect to drive the car about 750 mi per month. Write a formula to represent the total number of miles the car has been driven after you have owned it for m months.

92. A parking garage charges \$7.50 for the first hour and \$5.25 for each additional hour. Write a formula to represent the parking charge for parking in this garage for h hours. Assume h is a counting number greater than 1.

93. Write a formula to represent the total cost to rent a car from a company that rents cars for \$29.95 per day plus 50¢ for every mile driven over 100 mi. Assume the car will be driven more than 100 mi.

SECTION 9.2 Rate, Ratio, and Proportion

Rates

The word *rate* is used frequently in our everyday lives. It is used in such contexts as unemployment rate, tax rate, interest rate, hourly rate, infant mortality rate, school dropout rate, inflation rate, and postage rate.

A **rate** is a comparison of two quantities and can be written as a fraction. For instance, if a car travels 135 mi on 6 gal of gas, then the miles-to-gallons rate is written

$$\frac{135 \text{ mi}}{6 \text{ gal}}$$

Note that the units (miles and gallons) are written as part of the rate.