

# H. Lab 8. Newton's Method

H1. Example. Algebraically find the root of  $f(x) = x^3 - 10$  (solve  $f(x) = x^3 - 10 = 0$ ).

$$\begin{aligned} f(x) &= x^3 - 10 = 0 \\ x^3 &= 10 \\ x &= \sqrt[3]{10} \approx 2.1544 \end{aligned}$$

H2. Formula. Newton's Method  $x - f(x)/f'(x)$

H3. Example. Use Newton's Method to estimate the root of  $f(x) = x^3 - 10$ .

Take a guess for the root:  $x = 2$

Apply Newton's Method:  $x - f(x)/f'(x)$

$$\begin{aligned} f(x) &= x^3 - 10 & f(2) &= 8 - 10 = -2 \\ f'(x) &= 3x^2 & f'(2) &= 3(4) = 12 \end{aligned}$$

$$2 - (-2)/12 = \overset{\text{new } x}{2.1667}$$

Apply Newton's Method:  $x - f(x)/f'(x)$

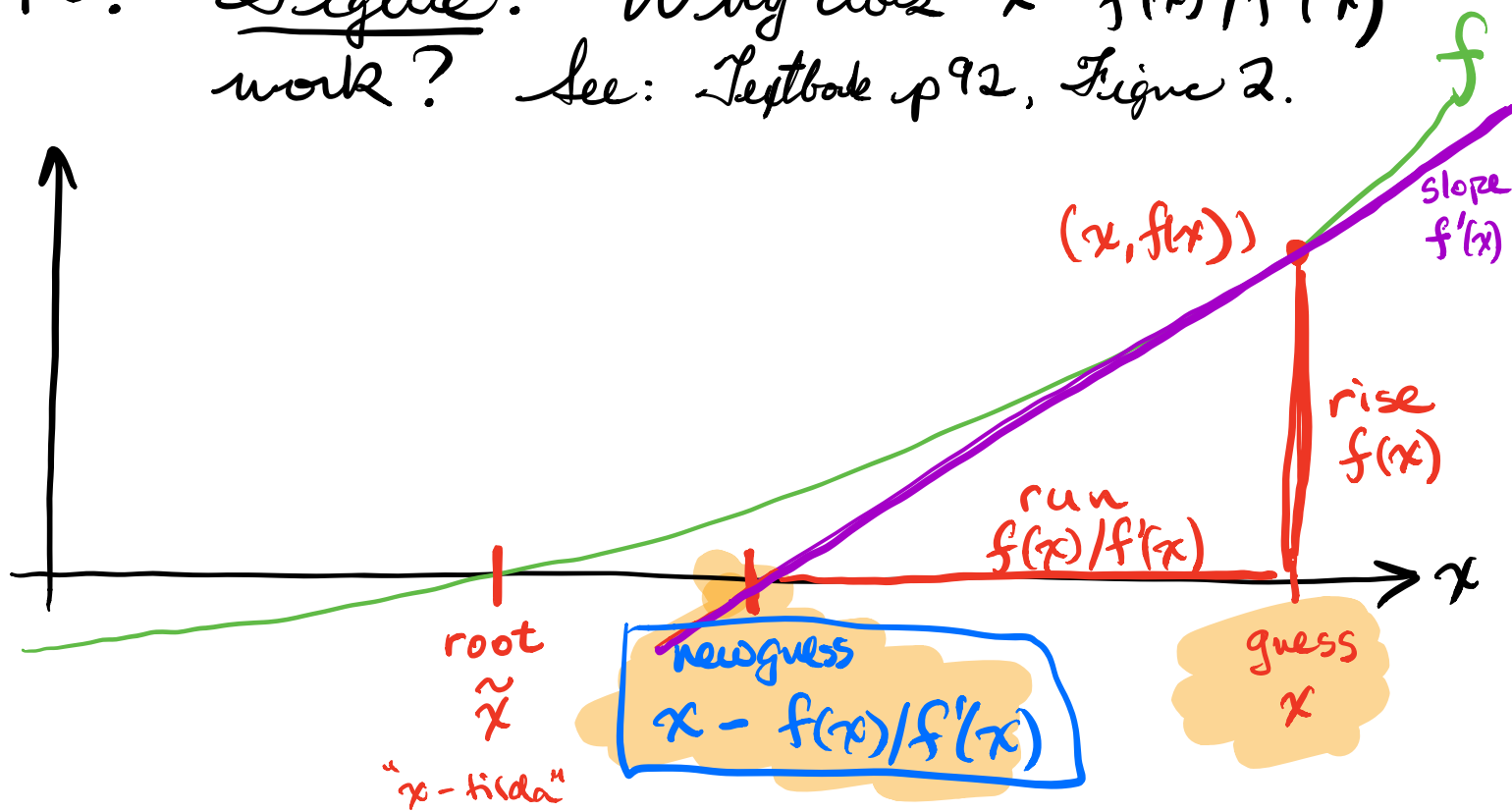
$$\begin{aligned} f(2.1667) &= 2.1667^3 - 10 = 0.1718 \\ f'(2.1667) &= 3(2.1667)^2 = \cancel{14.0833} \end{aligned}$$

$$2.1667 - 0.1718/\cancel{14.0833} = \overset{\text{new } x}{\cancel{2.1544}} = 2.1545$$

H4. Repeat H3 in Matlab.

H5. Discussion. Why was Newton's Method invented? This gives a way to find decimal approx for exact values like  $\sqrt{10}$ . More importantly, not every  $f(x)=0$  has a "closed form solution", so numerical approximation is the best we can do.

H6. Figure. Why does  $x - f(x)/f'(x)$  work? See: Textbook p92, Figure 2.



$$\text{slope} = f'(x) = \text{rise} / \text{run}$$

$$f'(x) = f(x) / \text{run}$$

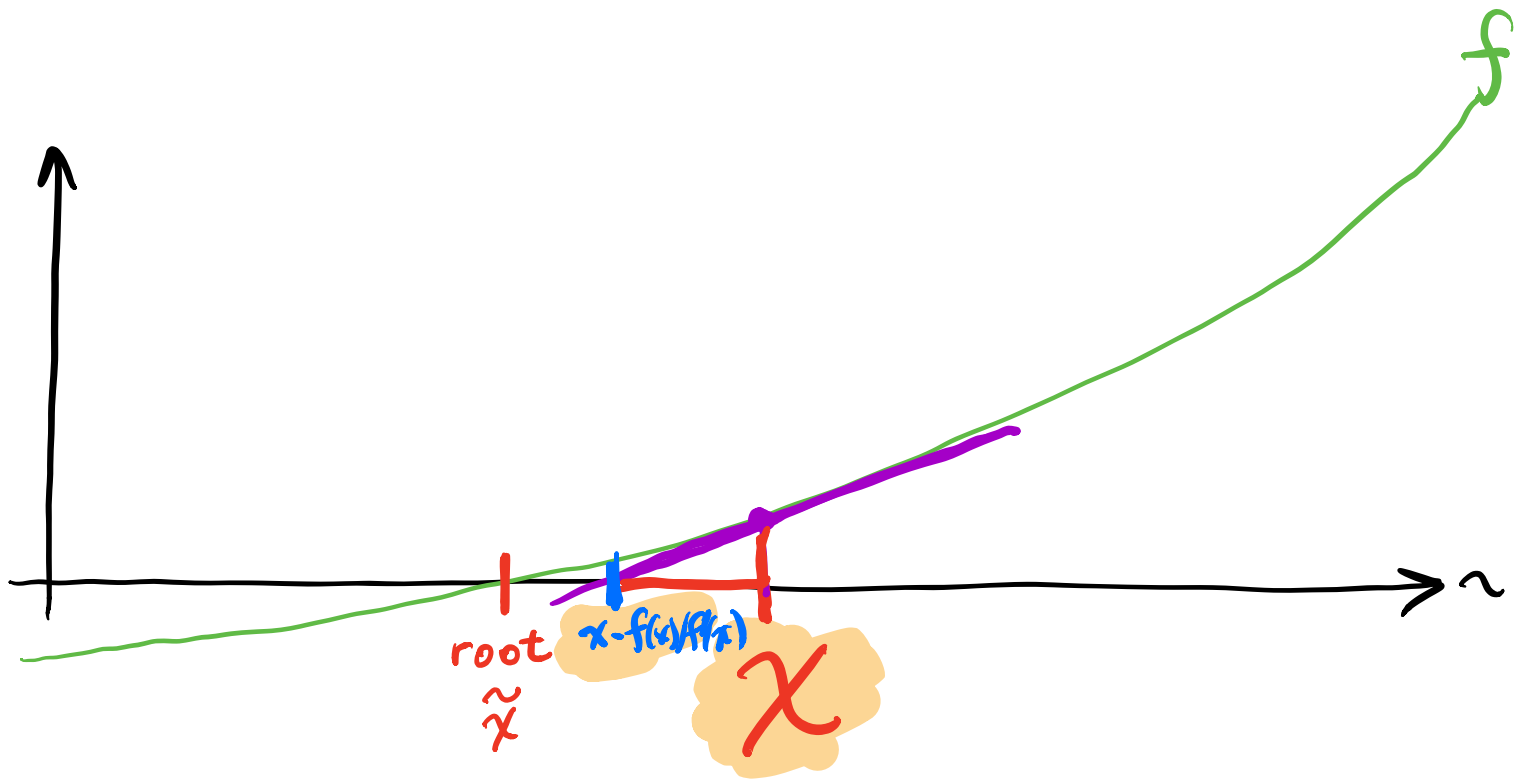
Cross  
multiply

$$\text{run } f'(x) = f(x)$$

Divide  $f'(x)$

$$\text{run} = f(x) / f'(x)$$

That was 1 iteration. Let's repeat.



```
f.m x +
1 function y=f(x)
2 y = x.^3 - 10;
```

```
fp.m x +
1 function y=fp(x)
2 y = 3*x.^2;
```

```
>> x = 2
```

```
x =
```

```
2
```

```
>> x = x - f(x)/fp(x)
```

```
x =
```

```
2.1667
```

```
>> x = x - f(x)/fp(x)
```

```
x =
```

```
2.1545
```

```
>> x = x - f(x)/fp(x)
```

```
x =
```

```
2.1544
```

```
>> x = x - f(x)/fp(x)
```

```
x =
```

```
2.1544
```