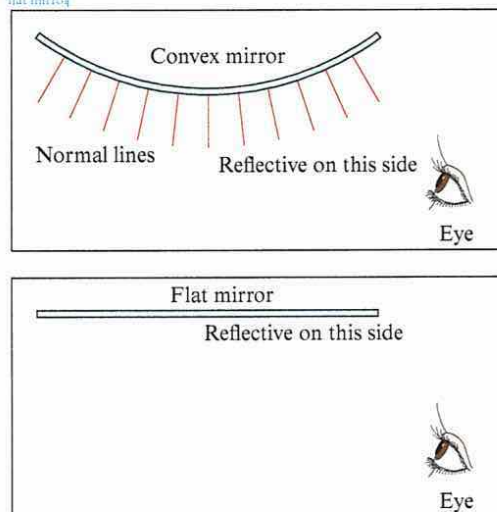


**Figure 10.55** An eye looking in a concave mirror and a flat mirror.

on your drawings, explain why a concave mirror makes a good mirror for applying makeup. (By the way, although a concave mirror is curved in the same direction as the bowl of a spoon, the smaller amount of curvature in a concave mirror prevents it from reflecting your image upside down, as a spoon does.) See figure.

- \*8. A **convex** mirror is a mirror that curves out, so that the normal lines on the reflective side of the mirror point away from each other, as shown at the top of Figure 10.56. Convex mirrors are often used as side-view mirrors on cars and trucks. This problem will help you see why convex mirrors are useful for this purpose.

Figure 10.56 shows a bird's-eye view of a cross-section of a convex mirror, a cross-section of a flat mirror, and eyes looking into these mirrors. Draw a copy of these mirrors and the eyes looking into them. Use the laws of reflection to help you compare how much of the surrounding environment each eye can see, looking into its mirror.

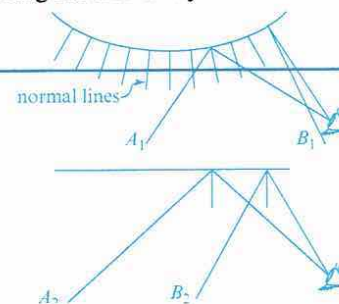


**Figure 10.56** A convex mirror and a flat mirror.

Now explain why convex mirrors are often used as side-view mirrors on cars and trucks.

- \*9. A convex mirror is a mirror that curves out, so that the normal lines on the reflective side of the mirror point away from each other, as shown at the top of Figure 10.56. Convex mirrors are often used as side-view mirrors on cars and trucks, but these mirrors usually carry the warning sign that reads, "Objects are closer than they appear." Explain why objects reflected in a convex mirror appear to be farther away than they actually are. Use the fact that the eye interprets a smaller image as being farther away.

9. See figure to right, ... the convex mirror shows a field of view that is larger than we are used to seeing with flat mirrors in a small space on the mirror, so the image is smaller. Our brain interprets this smaller image as being farther away—based again on comparison to what we are accustomed to seeing in a flat mirror.



## 10.3 Circles and Spheres

**CCSS** Common Core State Standards Grades K, 7

**TEACHING TIP**  
Go to  
MyMathLab for  
an additional  
activity for this  
section: *More on  
Circle Designs.*

What are circles and spheres? To the eye, circles and spheres are distinguished by their perfect roundness. Informally, we might say that a sphere is the surface of a ball. This describes circles and spheres from an informal or artistic point of view, but there is also a mathematical point of view. As we'll see, the mathematical definitions of circles and spheres yield practical applications that cannot be anticipated by considering only the look of these shapes.

### CLASS ACTIVITY

**10L**  Points That Are a Fixed Distance from a Given Point, p. CA-212

So I can use a 1 foot piece of string to ~~make~~ draw a circle with radius 1 foot

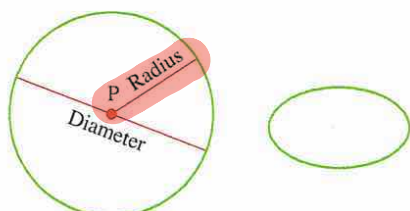
## What Are Mathematical Definitions of Circle and Sphere?

**circle** A circle is the collection of all the points in a plane that are a certain fixed distance away from a certain fixed point in the plane. This fixed point is called the **center** of the circle, and the fixed distance is called the **radius** of the circle (plural: **radii**). So the radius is the distance from the center of the circle to any point on the circle. The **diameter** of a circle is two times its radius. Informally, the diameter is the distance “all the way across” the circle, going through the center.

For example, let's fix a point in a plane, and let's call this point P, as in Figure 10.57. All the points in the plane that are 1 unit away from the point P form a circle of radius 1 unit, centered at the point P, as shown in Figure 10.57. The diameter of this circle is 2 units.

Figure 10.57

A circle and a noncircle.



A circle centered at P This is NOT a circle.

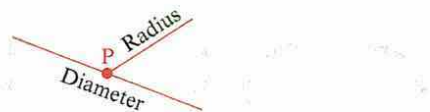
distance of points on circle from center

**sphere** A sphere is defined in almost the same way as a circle, but in space rather than in a plane. A sphere is the collection of all the points in space that are a certain fixed distance away from a certain fixed point in space. This fixed point is called the **center** of the sphere, and this distance is called the **radius** of the sphere. So the radius is the distance from the center of the sphere to any point on the sphere. The **diameter** of a sphere is two times its radius. Informally, the diameter is the distance “all the way across” the sphere, through the center.

For example, fix a point P in space. You might want to think of this point P as located in front of you, a few feet away. Then all the points in space that are 1 foot away from the point P form a sphere of radius 1 foot, centered at P. Try to visualize this. What does it look like? It looks like the surface of a very large ball, as illustrated in Figure 10.58. The diameter of this sphere is 2 feet.

Figure 10.58

A sphere and a nonsphere.



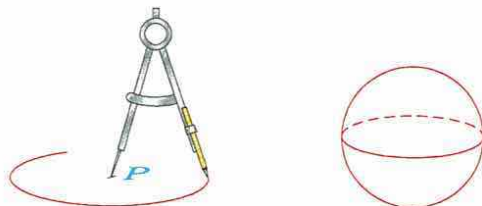
A sphere centered at P This is NOT a sphere.

we can use a piece of string

**compass** It's easy to draw almost perfect circles with the use of a common drawing tool called a **compass**, pictured in Figure 10.59. One side of a compass has a sharp point that you can stick into a point

Figure 10.59

Drawing circles and spheres.



Drawing a circle with a compass A simple drawing of a sphere

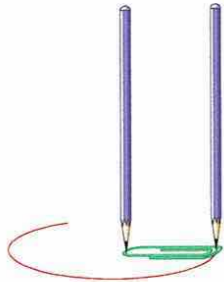


on a piece of paper. The other side of a compass has a pencil attached. To draw a circle centered at a point  $P$  and having a given radius, open the compass to the desired radius, stick the point of the compass in the point  $P$ , and spin the pencil side of the compass in a full revolution around  $P$ , all the while keeping the point of the compass at  $P$ .

It is also possible to draw good circles with simple homemade tools. For example, you can use 2 pencils and a paper clip to draw a circle, as shown in Figure 10.60. Put a pencil inside one end of a paper clip and keep this pencil fixed at one point on a piece of paper. Put another pencil at the other end of the paper clip and use that pencil to draw a circle.

Figure 10.60

Drawing a circle with a paper clip.



Drawing a circle with a paper clip

Because a sphere is an object in space, and not in a plane, it's harder to draw a picture of a sphere. Unless you are an exceptional artist, something like the simple drawing of a sphere in Figure 10.59 will do.

In Class Activity 10M, you will apply the mathematical definition of circles.

## CLASS ACTIVITY

10M Using Circles, p. CA-212

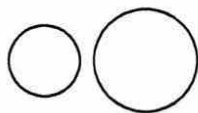
### How Can Circles or Spheres Meet?

What happens when 2 circles meet, or 2 spheres meet? Although this may seem like a topic of purely theoretical interest, it actually has practical applications.

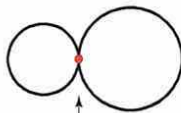
As illustrated in Figure 10.61, there are only three possible arrangements of 2 distinct circles: The circles may not meet at all, the circles may meet at a single point, or the circles may meet at 2 distinct points.

Figure 10.61

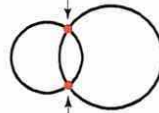
Three ways that two circles can meet.



These 2 circles don't meet.



These 2 circles meet at a single point.



These 2 circles meet at 2 points.

2 circles cannot meet at exactly 3 points

(Aside: 2 identical circles that overlap exactly meet at infinitely many points.)

Now what if you have 2 spheres—how do they meet? This is difficult, but try to visualize 2 spheres meeting. It might help to think about soap bubbles. When you blow soap bubbles you occasionally see a “double bubble” that is similar (although not identical) to 2 spheres meeting. As with circles, 2 distinct spheres might not meet at all, or they might barely touch, meeting at a single point. The only other possibility is that the 2 spheres meet along a circle, as shown in Figure 10.62. This is a

In Class Activity 10N, you will see a practical application of spheres meeting.



Figure 10.62

Three ways  
that two  
spheres can  
meet.

These 2 spheres  
don't meet.

These 2 spheres meet  
at a single point.

These 2 spheres meet  
along a circle.

### CLASS ACTIVITY

10N The Global Positioning System (GPS), p. CA-214

100 Circle Designs, p. CA-215

If you did Class Activity 10M, you saw some applications of circles. We will use circles again in the next section to construct special triangles and 4-sided figures. The mathematical definition of circle will allow us to explain why our constructions work.

## SECTION SUMMARY AND STUDY ITEMS

### Section 10.3 Circles and Spheres

A circle is the collection of all the points in a plane that are a fixed distance away from a fixed point in the plane. A sphere is the collection of all the points in space that are a fixed distance away from a fixed point in space. Two circles in a plane can either not meet, meet at a single point, or meet at two points. Two spheres in space can either not meet, meet at a single point, or meet along a circle.

#### Key Skills and Understandings

1. Give the definitions of circles and spheres.
2. Given a situation that involves distances from one or more points, use circles or spheres to describe the relevant locations.

### Practice Exercises for Section 10.3

1. Give the (mathematical) definitions of the terms *circle* and *sphere*.
2. Points P and Q are 4 centimeters apart. Point R is 2 cm from P and 3 cm from Q. Use a ruler and a compass to draw a precise picture of how P, Q, and R are located relative to each other.
3. A radio beacon indicates that a certain whale is less than 1 kilometer away from boat A and less than 1 kilometer away from boat B. Boat A and boat B are 1 kilometer apart. Assuming that the whale is swimming near the surface of the water, draw a map showing all the places where the whale could be located.
4. Suppose that during a flight an airplane pilot realizes that another airplane is 1000 feet away. Describe the shape formed by all possible locations of the other airplane at that moment.
 

*the first plane is the center radius*
5. An airplane is in radio contact with two control towers. The airplane is 20 miles from one control tower and 30 miles from another control tower. The control towers are 40 miles apart. Is this information enough to pinpoint the exact location of the airplane? Why or why not? What if you know the altitude of the airplane—do you have enough information to pinpoint the location of the airplane?
 

*sphere (because planes can move in 3 dimensions)*

*the shape is the sphere of radius 1000 ft centered at the first plane.*

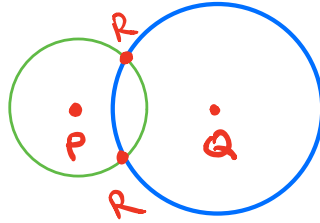
ANS Q4.

### Practice Exercises for Section 10.3

p475


1. Give the (mathematical) definitions of the terms *circle* and *sphere*.
2. Points P and Q are 4 centimeters apart. Point R is 2 cm from P and 3 cm from Q. Use a ruler and a compass to draw a precise picture of how P, Q, and R are located relative to each other.

make a 2cm radius circle around P  
make a 3cm radius circle about Q

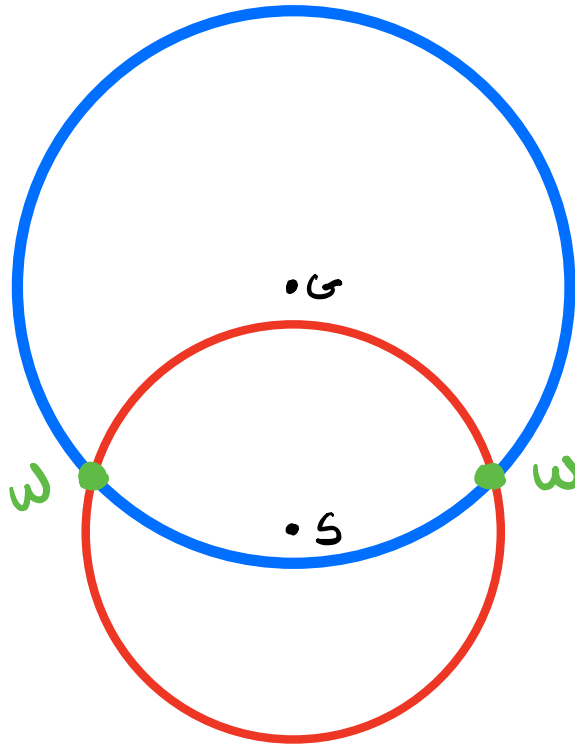


p476 #1

## PROBLEMS FOR SECTION 10.3

1.  Smallville is 7 miles south of Gotham. Will is 8 miles from Gotham and 6 miles from Smallville. Draw a map showing where Will could be. Be sure to show a scale for your map. Explain why you draw your map the way you do. Can you pinpoint Will to one location, or not? No, see figure above. You can only narrow it down to two possible points.

No, there are  
2 possible locations  
for Will.



Gradescope 1tw due Sun 9/20  
do Q2 on p475 on your own.



2. Points P and Q are 4 centimeters apart. Point R is 2 cm from P and 3 cm from Q. Use a ruler and a compass to draw a precise picture of how P, Q, and R are located relative to each other.