H. Lab 8. Newton's Method

H1. Example. Algebraically

find the root of $f(x) = x^3 - 10$

(solve $f(x) = x^3 - 10 = 0$).

$$f(x) = x^3 - 10 = 0$$
$$x^3 = 10$$
$$x = \sqrt[3]{10} \approx 2.1544$$

H2. Formula. Newton's Method $x - f(x)/f'(x)$

H3. Example. Use Newton's Method to estimate the root of $f(x) = x^3 - 10$.

Take a guess for the root: $x = 2$

Apply Newton's Method: $x - f(x)/f'(x)$

new x

$f(x) = x^3 - 10$ $\quad f(2) = 8 - 10 = -2$ $\qquad 2 - -2/12 = 2.1667$

$f'(x) = 3x^2$ $\quad f'(2) = 3(4) = 12$

Apply Newton's Method: $x - f(x)/f'(x)$

new x

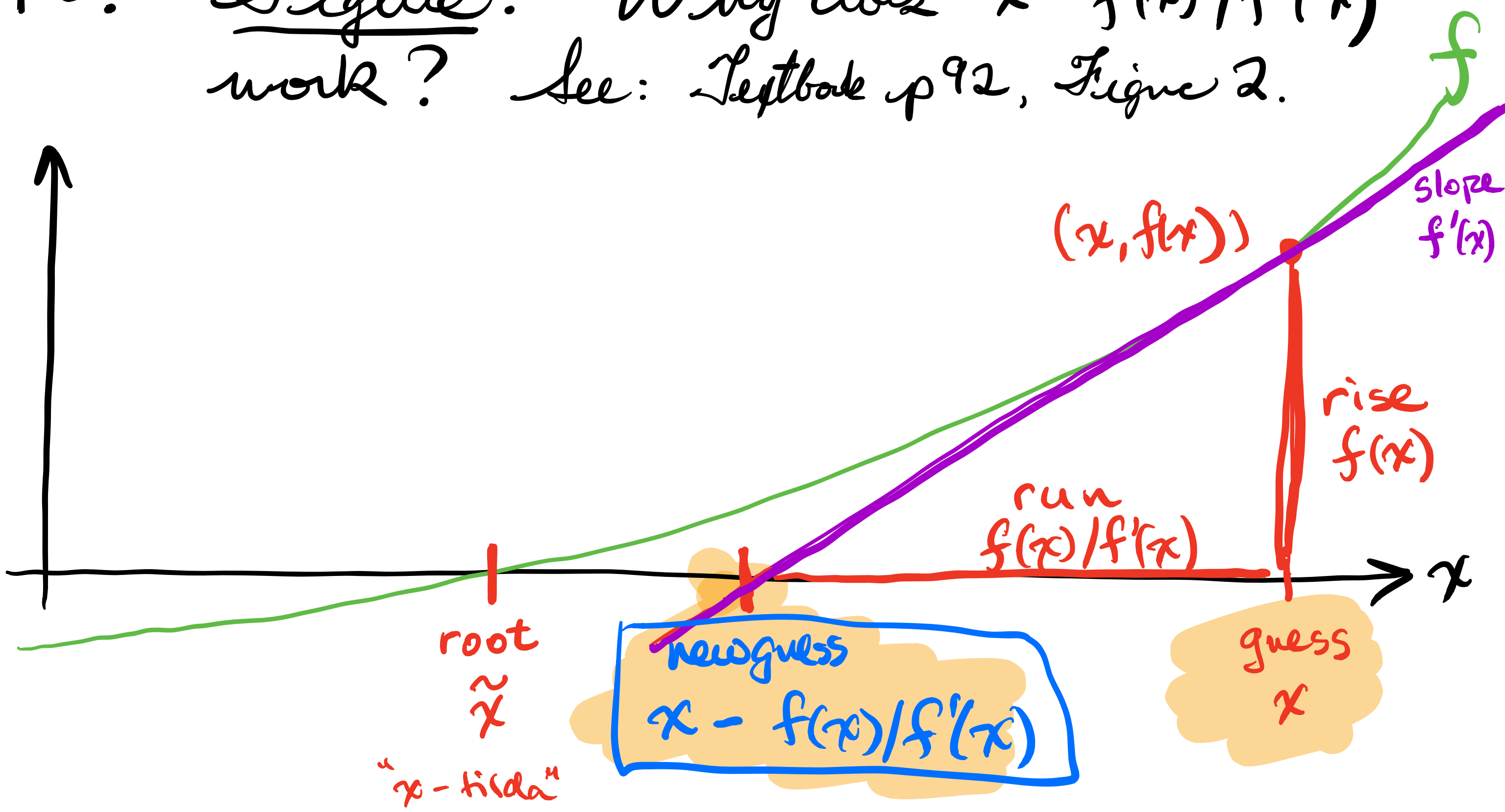
$f(2.1667) = 2.1667^3 - 10 = 0.1718$ $\qquad 2.1667 - 0.1718/\cancel{9.3592}$ 14.0833 $= \cancel{2.1484}$ 2.1545

$f'(2.1667) = 3(2.1667)^2 = \cancel{9.3592}$ 14.0833

H4. Repeat H3 in Matlab.

H5. Discussion. Why was Newton's Method invented? This gives a way to find decimal approx for exact values like $\sqrt[3]{10}$. More importantly, not every $f(x)=0$ has a "closed form solution", so numerical approximation is the best we can do.

H6. Figure. Why does $x - f(x)/f'(x)$ work? See: Textbook p92, Figure 2.



slope $= f'(x) =$ rise/run

$f'(x) = f(x)/\text{run}$

Cross multiply $\quad \text{run}\, f'(x) = f(x)$

Divide $f'(x)$ $\quad \text{run} = f(x)/f'(x)$

That was 1 iteration. Let's repeat.