

Math 113-V1 2020-Sep-2

Today: Histograms 2.2

WeBWorK is posted & due
this Sunday (2 sets)

Reading will be posted today

No class Mon

Reading for Wed 9/9 will
be posted to Canvas today.

District	Registered voter population	Overall city population
3	9.8%	9.0%
4	17.4%	18.5%
5	22.8%	20.7%
6	22.3%	16.8%

Table 2.12

2.2 | Histograms, Frequency Polygons, and Time Series Graphs

For most of the work you do in this book, you will use a histogram to display the data. One advantage of a histogram is that it can readily display large data sets. A rule of thumb is to use a histogram when the data set consists of 100 values or more.

A **histogram** consists of contiguous (adjoining) boxes. It has both a horizontal axis and a vertical axis. The horizontal axis is labeled with what the data represents (for instance, distance from your home to school). The vertical axis is labeled either **frequency** or **relative frequency** (or percent frequency or probability). The graph will have the same shape with either label. The histogram (like the stemplot) can give you the shape of the data, the center, and the spread of the data.

The relative frequency is equal to the frequency for an observed value of the data divided by the total number of data values in the sample. (Remember, frequency is defined as the number of times an answer occurs.) If:

- f = frequency
- n = total number of data values (or the sum of the individual frequencies), and
- RF = relative frequency,

then:

$$RF = \frac{f}{n}$$

For example, if three students in Mr. Ahab's English class of 40 students received from 90% to 100%, then, $f = 3$, $n = 40$, and $RF = \frac{f}{n} = \frac{3}{40} = 0.075$. 7.5% of the students received 90–100%. 90–100% are quantitative measures.

To construct a histogram, first decide how many **bars** or **intervals**, also called classes, represent the data. Many histograms consist of five to 15 bars or classes for clarity. The number of bars needs to be chosen. Choose a starting point for the first interval to be less than the smallest data value. A **convenient starting point** is a lower value carried out to one more decimal place than the value with the most decimal places. For example, if the value with the most decimal places is 6.1 and this is the smallest value, a convenient starting point is 6.05 ($6.1 - 0.05 = 6.05$). We say that 6.05 has more precision. If the value with the most decimal places is 2.23 and the lowest value is 1.5, a convenient starting point is 1.495 ($1.5 - 0.005 = 1.495$). If the value with the most decimal places is 3.234 and the lowest value is 1.0, a convenient starting point is 0.9995 ($1.0 - 0.0005 = 0.9995$). If all the data happen to be integers and the smallest value is two, then a convenient starting point is 1.5 ($2 - 0.5 = 1.5$). Also, when the starting point and other boundaries are carried to one additional decimal place, no data value will fall on a boundary. The next two examples go into detail about how to construct a histogram using continuous data and how to create a histogram using discrete data.

Example 2.7

The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data, since height is measured.
60; 60.5; 61; 61; 61.5

Example 2.7

p77 make a histogram with 8 classes (buckets)

The following data are the heights (in inches to the nearest half inch) of 100 male semiprofessional soccer players. The heights are **continuous** data, since height is measured.

60; 60.5; 61; 61; 61.5

63.5; 63.5; 63.5

64; 64; 64; 64; 64; 64; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5

66; 66; 66; 66; 66; 66; 66; 66; 66; 66; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 67; 67; 67;

67; 67; 67; 67; 67; 67; 67; 67; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5

68; 68; 69; 69; 69; 69; 69; 69; 69; 69; 69.5; 69.5; 69.5; 69.5

70; 70; 70; 70; 70; 70.5; 70.5; 70.5; 71; 71; 71

72; 72; 72.5; 72.5; 73; 73.5

74

min 60

max 74

number of buckets told to use 8

- Make buckets
$$\frac{\text{max} - \text{min}}{8} = \frac{74 - 60}{8} = 1.75$$

 \downarrow round up

→ (class width) bucket width = 2

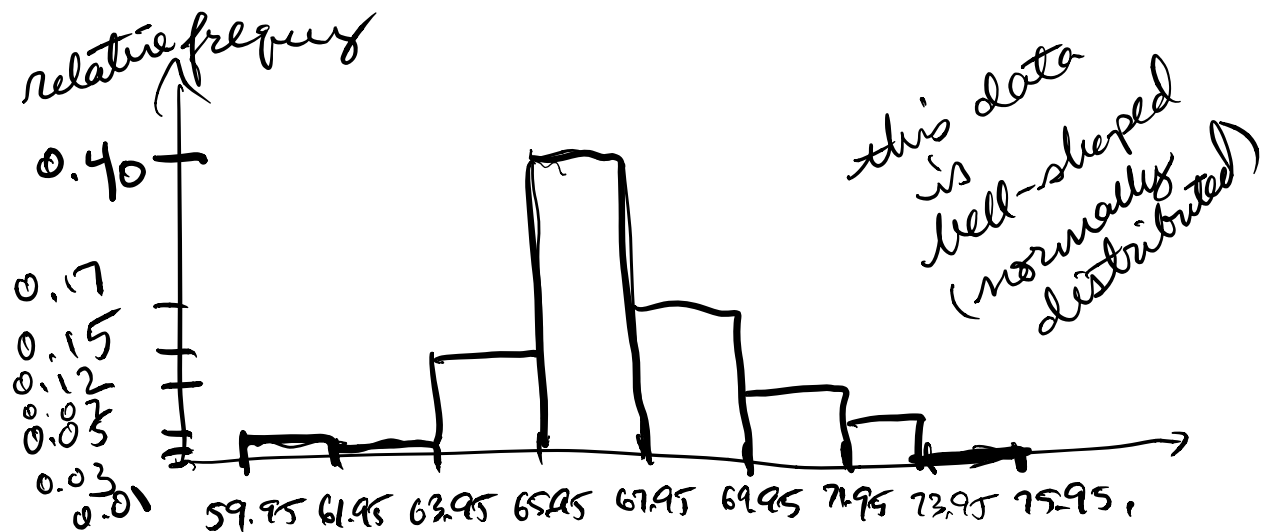
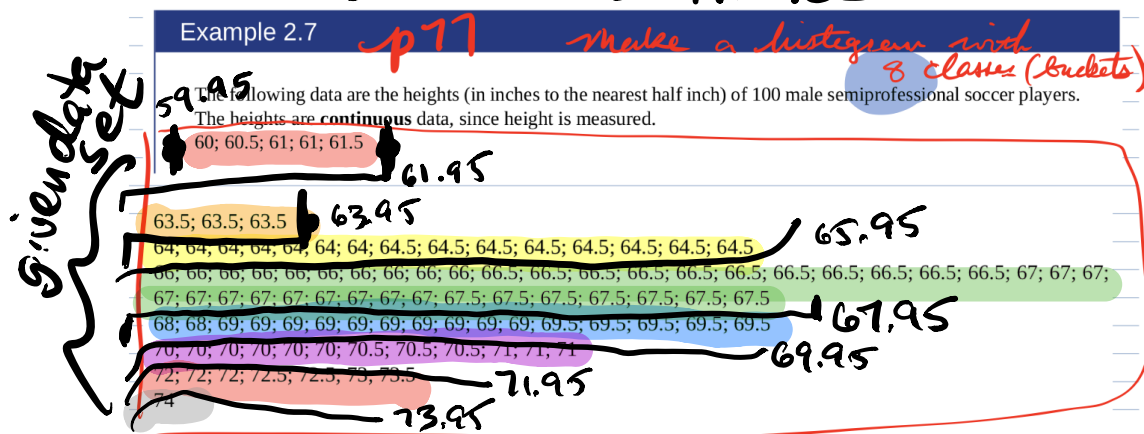
→ starting point?
 Our data is measured to 1 decimal place
 starting point will be 0.05 below half of 1 decimal place min.

$$60 - 0.05 = 59.95$$

data is all whole numbers: start 0.5 below min
 " " to 1 decimal place: start 0.05 " "
 " " to 2 " " : start 0.005 " "

class	frequency	relative frequency (freq/n)
59.95 - 61.95	5	$5/100 = 0.05$
61.95 - 63.95	3	$3/100 = 0.03$
63.95 - 65.95	15	$15/100 = 0.15$
65.95 - 67.95	40	$40/100 = 0.40$
67.95 - 69.95	17	$17/100 = 0.17$
69.95 - 71.95	12	$12/100 = 0.12$
71.95 - 73.95	7	$7/100 = 0.07$
73.95 - 75.95	1	$1/100 = 0.01$

59.95 + 2
 (2 is the class width)
 class boundaries + $n = 100$



63.5; 63.5; 63.5
 64; 64; 64; 64; 64; 64; 64; 64; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5; 64.5
 66; 66; 66; 66; 66; 66; 66; 66; 66; 66; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 66.5; 67; 67; 67;
 67; 67; 67; 67; 67; 67; 67; 67; 67; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5; 67.5
 68; 68; 69; 69; 69; 69; 69; 69; 69; 69; 69; 69.5; 69.5; 69.5; 69.5
 70; 70; 70; 70; 70; 70; 70.5; 70.5; 70.5; 71; 71; 71
 72; 72; 72; 72.5; 72.5; 73; 73.5
 74

The smallest data value is 60. Since the data with the most decimal places has one decimal (for instance, 61.5), we want our starting point to have two decimal places. Since the numbers 0.5, 0.05, 0.005, etc. are convenient numbers, use 0.05 and subtract it from 60, the smallest value, for the convenient starting point.

$60 - 0.05 = 59.95$ which is more precise than, say, 61.5 by one decimal place. The starting point is, then, 59.95.

The largest value is 74, so $74 + 0.05 = 74.05$ is the ending value.

Next, calculate the width of each bar or class interval. To calculate this width, subtract the starting point from the ending value and divide by the number of bars (you must choose the number of bars you desire). Suppose you choose eight bars.

$$\frac{74.05 - 59.95}{8} = 1.76$$

NOTE

We will round up to two and make each bar or class interval two units wide. Rounding up to two is one way to prevent a value from falling on a boundary. Rounding to the next number is often necessary even if it goes against the standard rules of rounding. For this example, using 1.76 as the width would also work. A guideline that is followed by some for the number of bars or class intervals is to take the square root of the number of data values and then round to the nearest whole number, if necessary. For example, if there are 150 values of data, take the square root of 150 and round to 12 bars or intervals.

The boundaries are:

- 59.95
- $59.95 + 2 = 61.95$
- $61.95 + 2 = 63.95$
- $63.95 + 2 = 65.95$
- $65.95 + 2 = 67.95$
- $67.95 + 2 = 69.95$
- $69.95 + 2 = 71.95$
- $71.95 + 2 = 73.95$
- $73.95 + 2 = 75.95$

The heights 60 through 61.5 inches are in the interval 59.95–61.95. The heights that are 63.5 are in the interval 61.95–63.95. The heights that are 64 through 64.5 are in the interval 63.95–65.95. The heights 66 through 67.5 are in the interval 65.95–67.95. The heights 68 through 69.5 are in the interval 67.95–69.95. The heights 70 through 71 are in the interval 69.95–71.95. The heights 72 through 73.5 are in the interval 71.95–73.95. The height 74 is in the interval 73.95–75.95.

The following histogram displays the heights on the x-axis and relative frequency on the y-axis.

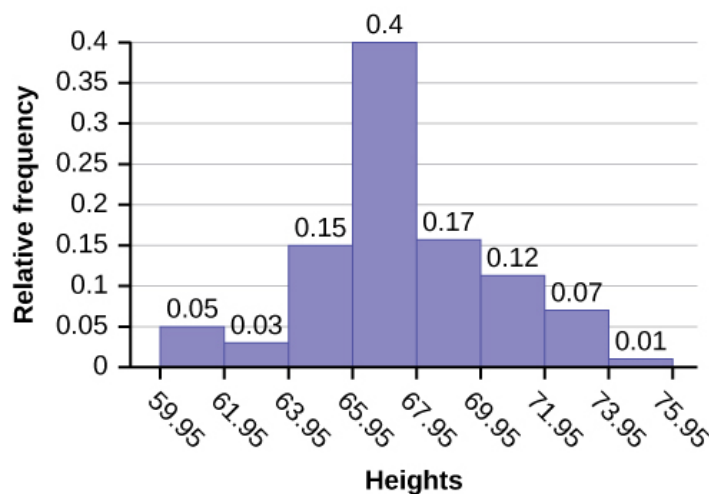


Figure 2.5

Try It Σ

2.7 The following data are the shoe sizes of 50 male students. The sizes are continuous data since shoe size is measured. Construct a histogram and calculate the width of each bar or class interval. Suppose you choose six bars.

9; 9; 9.5; 9.5; 10; 10; 10; 10; 10; 10; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5

11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5

12; 12; 12; 12; 12; 12; 12; 12.5; 12.5; 12.5; 12.5; 14

Example 2.8

Create a histogram for the following data: the number of books bought by 50 part-time college students at ABC College. The number of books bought by 50 part-time college students at ABC College. The number of books is **discrete data**, since books are counted.

1; 1; 1; 1; 1; 1; 1; 1; 1; 1

2; 2; 2; 2; 2; 2; 2; 2; 2

3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3; 3

4; 4; 4; 4; 4

5; 5; 5; 5

6; 6

Eleven students buy one book. Ten students buy two books. Sixteen students buy three books. Six students buy four books. Five students buy five books. Two students buy six books.

Because the data are integers, subtract 0.5 from 1, the smallest data value and add 0.5 to 6, the largest data value. Then the starting point is 0.5 and the ending value is 6.5.

Next, calculate the width of each bar or class interval. If the data are discrete and there are not too many different values, a width that places the data values in the middle of the bar or class interval is the most convenient. Since the data consist of the numbers 1, 2, 3, 4, 5, 6, and the starting point is 0.5, a width of one places the 1 in the middle of the interval from 0.5 to 1.5, the 2 in the middle of the interval from 1.5 to 2.5, the 3 in the middle of the interval from 2.5 to 3.5, the 4 in the middle of the interval from _____ to _____, the 5 in the middle of the interval from _____ to _____, and the _____ in the middle of the interval from _____ to _____.

Try It Σ

2.7 The following data are the shoe sizes of 50 male students. The sizes are continuous data since shoe size is measured. Construct a histogram and calculate the width of each bar or class interval. Suppose you choose six bars.

9; 9; 9.5; 9.5; 10; 10; 10; 10; 10; 10; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 10.5; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5; 11.5; 12; 12; 12; 12; 12; 12; 12; 12.5; 12.5; 12.5; 12.5; 14

given data is rounded to 1 decimal place
data has 1 decimal place
 $0.1 \div 2 = 0.05$

look at min. the next possible data point below would be 8.9

Class width: $\frac{\text{max} - \text{min}}{\# \text{ of bars}} = \frac{14 - 9}{6} = 0.8333... \text{ round up}$

Class width = 1

Data has one decimal place, so start at min - 0.05 = $9 - 0.05 = 8.95$
half of 1 decimal place

class width

class
8.95 - 9.95
9.95 - 10.95
10.95 - 11.95
11.95 - 12.95
12.95 - 13.95
13.95 - 14.95

frequency

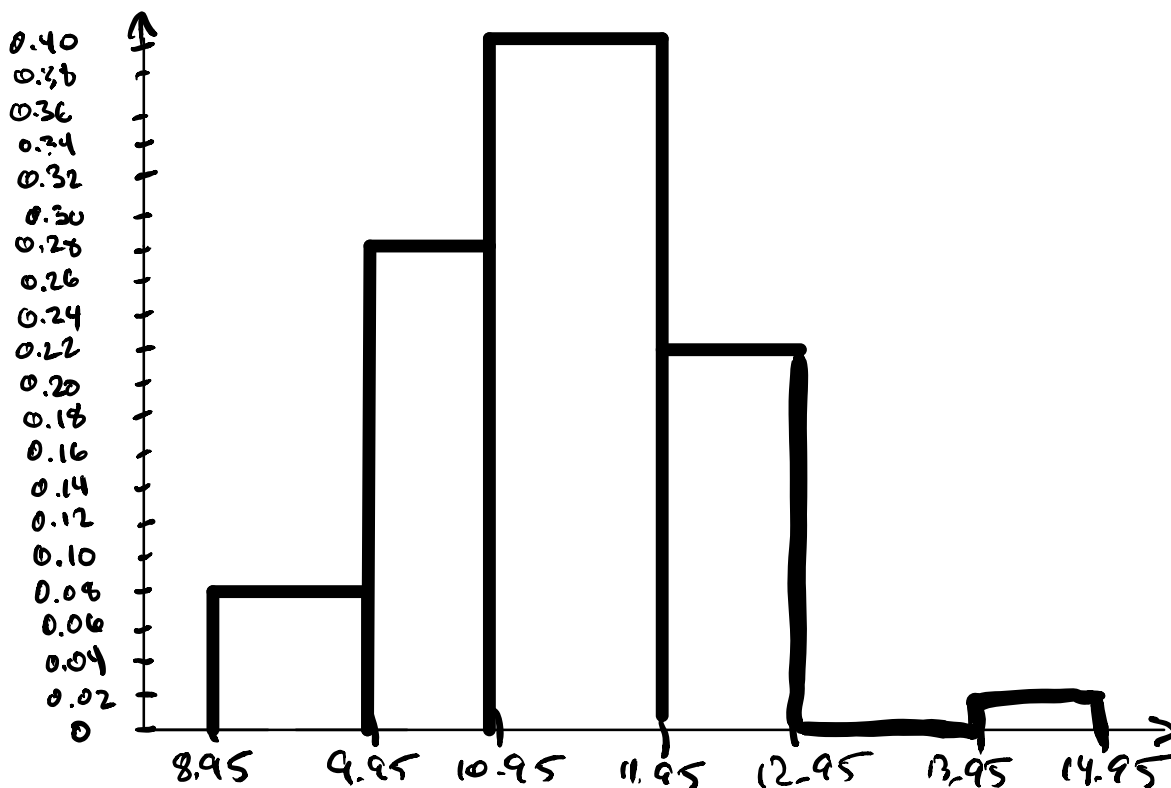
4
14
20
11
0
1

n = 50

relative frequency

4 / 50 = 0.08
14 / 50 = 0.28
20 / 50 = 0.4
11 / 50 = 0.22
0 / 50 = 0
1 / 50 = 0.02

- check data is ~~sorted~~ sorted
- put class barriers in over data



Try It Σ

2.8 The following data are the number of sports played by 50 student athletes. The number of sports is discrete data since sports are counted.

1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1; 1

2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2; 2

3; 3; 3; 3; 3; 3; 3; 3

20 student athletes play one sport. 22 student athletes play two sports. Eight student athletes play three sports.

Fill in the blanks for the following sentence. Since the data consist of the numbers 1, 2, 3, and the starting point is 0.5, a width of one places the 1 in the middle of the interval 0.5 to _____, the 2 in the middle of the interval from _____ to _____, and the 3 in the middle of the interval from _____ to _____.

Example 2.9

Using this data set, construct a histogram.

Number of Hours My Classmates Spent Playing Video Games on Weekends				
9.95	10	2.25	16.75	0
19.5	22.5	7.5	15	12.75
5.5	11	10	20.75	17.5
23	21.9	24	23.75	18
20	15	22.9	18.8	20.5

Table 2.13

Finding the starting point

② data is rounded ~~to the nearest~~ (to the hundredth)

③ $\text{min} - \frac{\text{a hundredth}}{2} = 0 - \frac{0.01}{2} = \boxed{-0.005}$
always \rightarrow ②

Solution 2.9



Figure 2.7

Some values in this data set fall on boundaries for the class intervals. A value is counted in a class interval if it falls on the left boundary, but not if it falls on the right boundary. Different researchers may set up histograms for the same data in different ways. There is more than one correct way to set up a histogram.

Try It Σ

2.9 The following data represent the number of employees at various restaurants in New York City. Using this data, create a histogram.

22; 35; 15; 26; 40; 28; 18; 20; 34; 39; 42; 24; 22; 19; 27; 22; 34; 40; 20; 38; and 28

Use 10–19 as the first interval.



Collaborative Exercise

Count the money (bills and change) in your pocket or purse. Your instructor will record the amounts. As a class, construct a histogram displaying the data. Discuss how many intervals you think is appropriate. You may want to experiment with the number of intervals.

Frequency Polygons

Frequency polygons are analogous to line graphs, and just as line graphs make continuous data visually easy to interpret, so too do frequency polygons.

To construct a frequency polygon, first examine the data and decide on the number of intervals, or class intervals, to use on the x-axis and y-axis. After choosing the appropriate ranges, begin plotting the data points. After all the points are plotted,