Math 218, 2020-Sep-14, 10.4

Before Wed, read 10.6

How Can We Use Short Lists of Properties to Define Special Quadrilaterals?

What's the difference between the following three categories?

- Quadrilaterals that have 4 right angles
- Quadrilaterals that have 4 right angles and opposite sides parallel
- Quadrilaterals that have 4 right angles and opposite sides of the same length

The surprising answer is that there is no difference. It turns out that each of the three descriptions specifies the exact same collection of shapes—namely, rectangles. If a 4-sided shape has 4 right angles, then *automatically* its opposite sides are parallel and have the same length. How can that be, and why is that so? For now, we leave this as a mystery, to be explored in Chapter 14.

Because we can specify the same category of shapes by listing different collections of properties, we have some choice in how to define shapes. It's worthwhile to pick a short list of properties to define a category of shapes. Why? To decide if a shape is or isn't a rectangle, or some other type of shape, students will check to confirm that all the defining properties hold for the shape. So a short list of properties means less checking.

Look again at the categories in Class Activity 10R, some of which consist of the same shapes. Based on what you found, what *short* lists of properties would you use to define the sets of squares, rectangles, rhombuses, and parallelograms? Compare your definitions with these standard definitions of some special categories of quadrilaterals, which we will use from now on:

square square—quadrilateral with 4 right angles whose sides all have the same length

rectangle rectangle—quadrilateral with 4 right angles

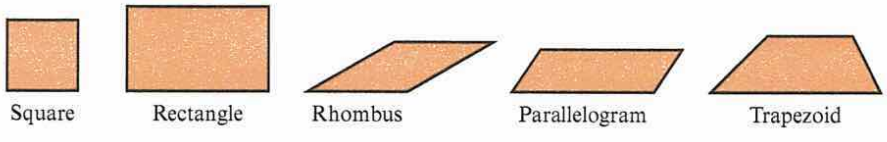
rhombus rhombus—quadrilateral whose sides all have the same length. The name **diamond** is sometimes used instead of rhombus

parallelogram parallelogram—quadrilateral for which opposite sides are parallel

trapezoid trapezoid—quadrilateral for which at least one pair of opposite sides are parallel. (Some books define a trapezoid as a quadrilateral for which *exactly one* pair of opposite sides are parallel.)

Figure 10.67 illustrates some examples.

Figure 10.67
Some special quadrilaterals.



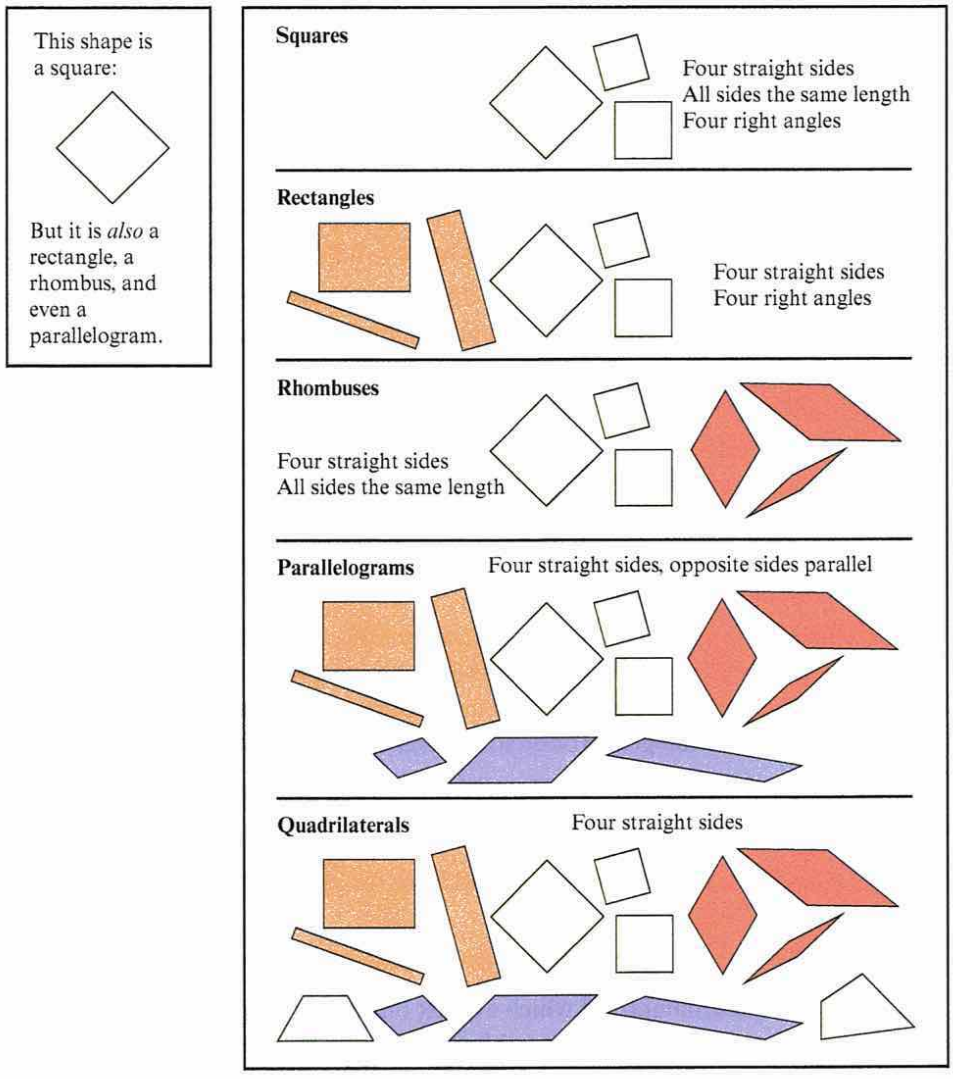
How Can We Classify Special Quadrilaterals in a Hierarchy?

CCSS
3.G.1
4.G.2
5.G.3
5.G.4

In Class Activity 10R, you probably noticed that some categories of quadrilaterals are subcategories of other categories. An important consequence of this is that *a given shape can belong to more than one category*. For example, the shape on the left in Figure 10.68 is a square because it has four sides of the same length and four right angles. But it is *also* a rectangle because it has four sides and four right angles. To help young students understand this idea, teachers can point out that squares are special kinds of rectangles, and they can call squares "square rectangles" to make that point.

How do hierarchies of shape categories come about? Think about this question in terms of lists of properties. If the list has only one property, such as "4 straight sides," then many will satisfy just that one property. If we add another property to that list, so it becomes "4 straight sides and 4 right angles," then only some of the shapes in the initial category will satisfy both of those properties. Usually, the more properties we add to the list, the fewer shapes satisfy all those

Figure 10.68 Classifying some special quadrilaterals in a hierarchy.



TEACHING TIP Go to MyMathLab and use the **Categories of Quadrilaterals** applet to show categories and subcategories.

properties. Conversely, if we start with a long list of properties, such as "4 straight sides, all sides the same length, 4 right angles," then as we remove properties from the list we get larger categories of shapes.

Relationships among categories of shapes and hierarchies of shape categories can be described and depicted in various ways—for example, with the diagram in Figure 10.68, with words, and with Venn diagrams as discussed later in this section and in the Practice Exercises.

What Are Triangles?

triangle

How can children decide which figures in Figure 10.69 are triangles and which are not? They must see if the figure has all the characteristics of a triangle. A **triangle** is a closed two-dimensional shape made of 3 line segments.

CCSS 2.G.1

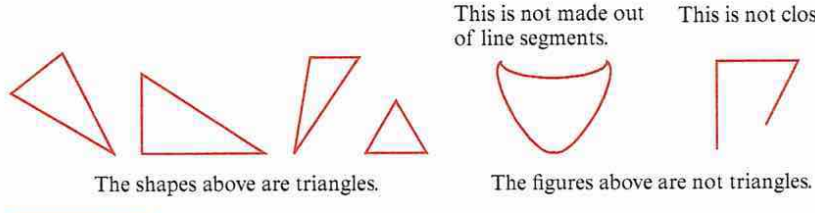


Figure 10.69 Triangles and figures that are not triangles.

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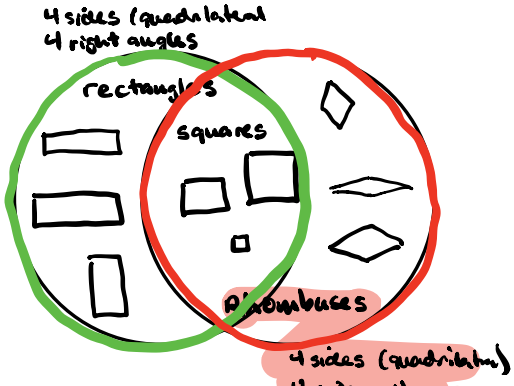
Practice Exercises for Section 10.4

1. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of rectangles and rhombuses. Which shapes are in both categories? Explain.
2. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of parallelograms and trapezoids. Explain.
3. Some books define trapezoids as quadrilaterals that have *exactly one* pair of parallel sides. Draw a Venn diagram (or other clear diagram) showing how the categories of parallelograms and trapezoids are related when this alternate definition of trapezoid is used. Explain.
4. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of rhombuses and parallelograms.



All squares are rectangles, but not all rectangles are squares (some are).

All squares are rhombuses but not all rhombuses squares.

If you're a rectangle & a rhombus then you're a square.

If you're a rectangle but not a square, then you're not a rhombus.

If you're a rhombus but not a square, then, you're not a rectangle.

If you're not a square,

(A) You might be a rectangle, or you might not.

B. You must be a rectangle

C. You must not be a rectangle

Practice Exercises for Section 10.4

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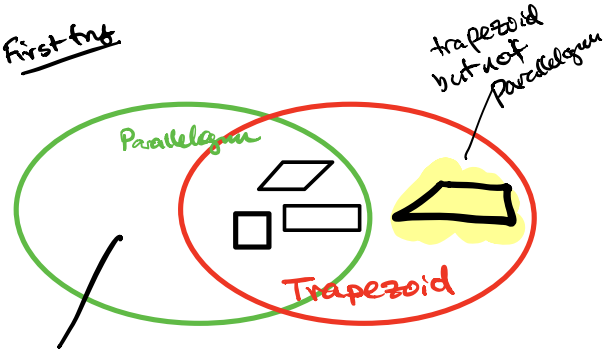
4. Draw a Venn diagram (or other clear diagram) showing the relationship between the categories of rhombuses and parallelograms.

as defined on p479

First try

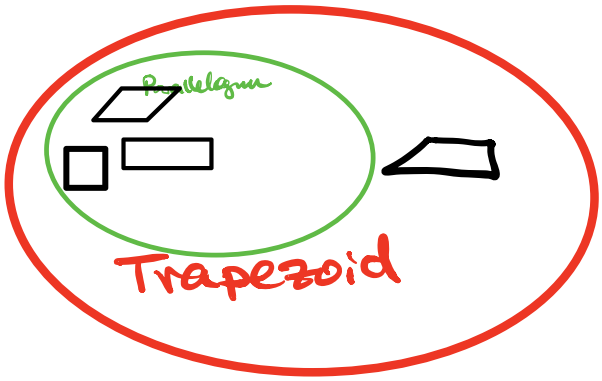


there are no parallelograms that are not also trapezoids

Parallelogram: Quadrilateral with opposite sides parallel

Trapezoid: Quadrilateral with at least one side parallel

ANS



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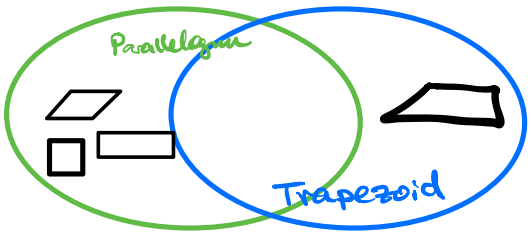
first try



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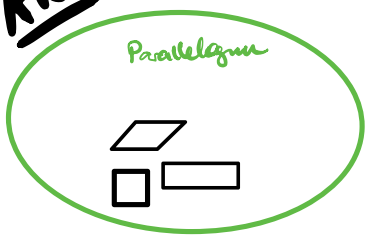

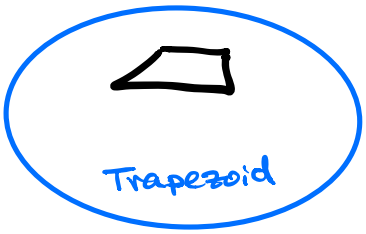

Practice Exercises for Section 10.4

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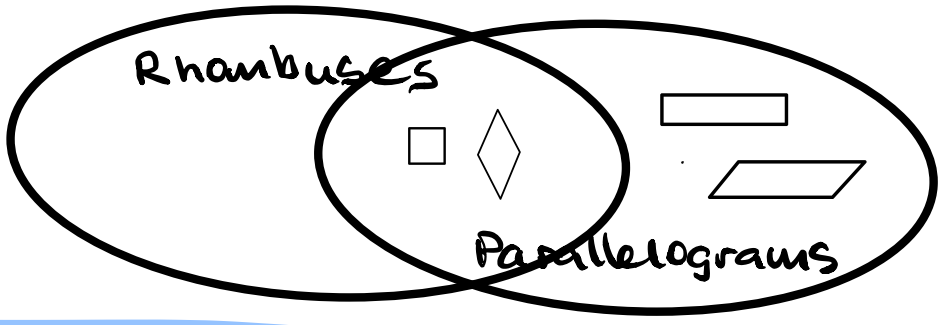
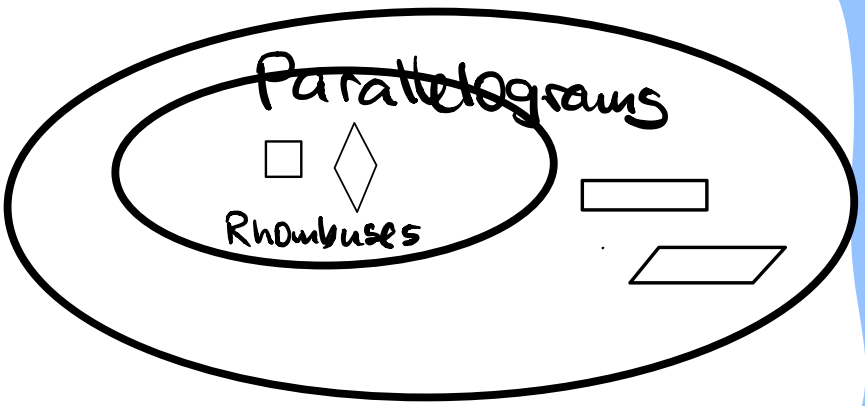
First try



Ans



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PROBLEMS FOR SECTION 10.4

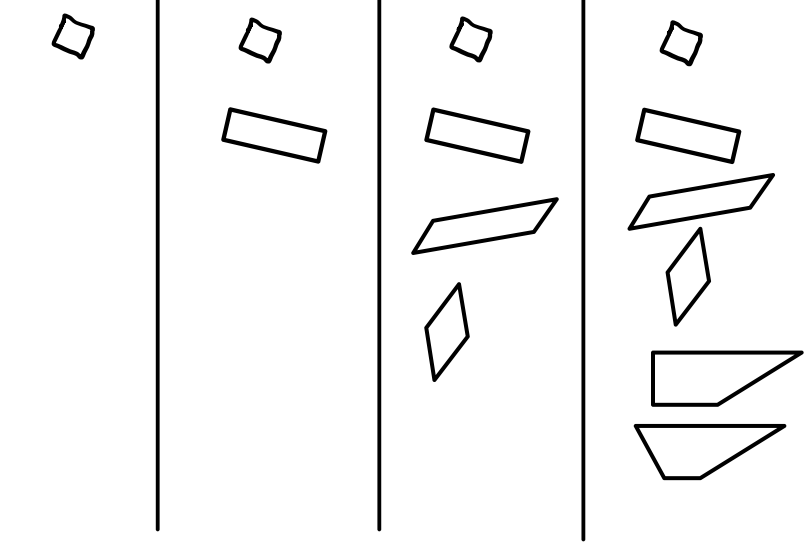
1. Students sometimes get confused about the relationship between squares and rectangles. Explain this relationship in your own words, using our (short) definitions of these shapes.

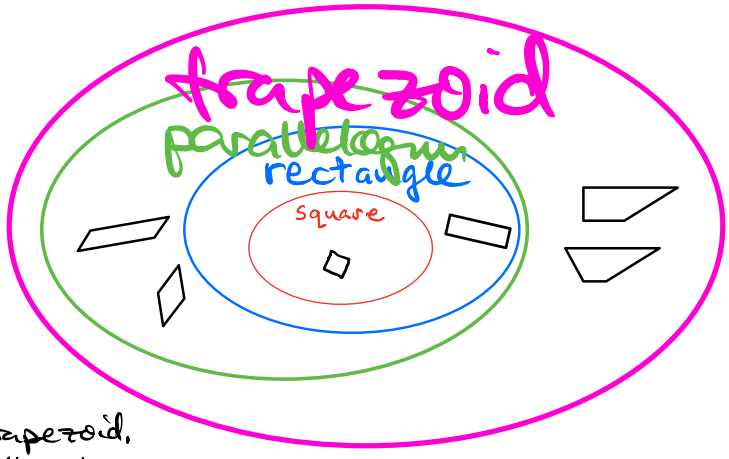
2. Draw a Venn diagram or other clear diagram showing the relationships among the categories of squares, rectangles, parallelograms, and trapezoids. Explain how you use properties of shapes to determine relationships among the categories.

See figure above.

let's do it like Figure 10.68 p 48

square	rectangle	parallelogram	trapezoid (original definition, so at least one pair parallel sides)





let's make Trapezoid the biggest circle because everything here is a trapezoid, but not every trapezoid is the others.

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PROBLEMS FOR SECTION 10.4

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2. Draw a Venn diagram or other clear diagram showing the relationships among the categories of squares, rectangles, parallelograms, and trapezoids. Explain how you use properties of shapes to determine relationships among the categories. See figure above.

3. Draw a Venn diagram or other clear diagram that shows the relationships among the categories of quadrilaterals, squares, rectangles, parallelograms, rhombuses, and trapezoids. Explain briefly. See problem 2 and Practice Exercise 1.

quadrilaterals square rectangles parallelograms rhombuses trapezoids

↓

everything here is a quadrilateral