Math 102-F5   2020-Sep-2 (Wed)

9.2 Rates Ratios Proportions

9.1's webwork due this Sunday
Read 9.3 befor next class (~~Monday~~).
            Wednesday

No class labor day

Warmup: 8:06 am

79) Solve for b_2.

$$A = \frac{1}{2}h(b_1 + \boxed{b_2})$$

$$2A = h(b_1 + b_2) \qquad \text{multiply } 2$$

$$\frac{2A}{h} = b_1 + b_2 \qquad \text{divide } h$$

$$\boxed{\frac{2A}{h} - b_1} = b_2 \qquad \text{subtract } b_1$$

"solving literal equations"

72. $P = R - C$; C (business)

73. $A = P + Prt$; t (business)

74. $S = V_0 t - 16t^2$; V_0 (physics)

75. $T = fm - gm$; f (engineering)

76. $P = \dfrac{R - C}{n}$; R (business)

77. $R = \dfrac{C - S}{t}$; S (business)

78. $V = \dfrac{1}{3}\pi r^2 h$; h (geometry)

79. $A = \dfrac{1}{2}h(b_1 + b_2)$; b_2 (geometry)

80. $a_n = a_1 + (n - 1)d$; d (mathematics)

81. $S = 2\pi r^2 + 2\pi rh$; h (geometry)

■ In Exercises 82 and 83, solve the equation for y.

82. $2x - y = 4$ **83.** $4x + 3y = 6$

■ In Exercises 84 and 85, solve the equation for x.

84. $ax + by + c = 0$ **85.** $y - y_1 = m(x - x_1)$

EXTENSIONS

■ In Exercises 86 and 87, solve the equation.

86. $3(4x + 2) = 7 - 4(1 - 3x)$

87. $4(x + 5) = 30 - (10 - 4x)$

88. Use the numbers 5, 10, and 15 to make equations by filling in the boxes: $x + \square = \square - \square$. Each equation must use all three numbers.

    a. What is the largest possible solution of these equations?

    b. What is the smallest possible solution of these equations?

89. Solve the equation $ax + b = cx + d$ for x. Is your solution valid for all numbers a, b, c, and d? Explain.

■ **Writing Formulas** When we know there is an explicit relationship between two quantities, often we can write a formula to express the relationship.

    For example, suppose that a toll of $3.75 is collected from each vehicle that crosses a particular bridge. Let A be the total amount of money collected, and let c be the number of vehicles that cross the bridge on a given day. Then,

$$A = \$3.75c$$

is a formula that expresses the total amount of money collected from vehicles on any given day.

In Exercises 90 to 93, write a formula for the situation. Include as part of your answer a list of variables that were used, and state what each variable represents.

90. Write a formula to represent the total cost to rent a copier from a company that charges $325 per month plus $0.08 per copy made.

91. Suppose you buy a used car with 30,000 mi on it. You expect to drive the car about 750 mi per month. Write a formula to represent the total number of miles the car has been driven after you have owned it for m months.

92. A parking garage charges $7.50 for the first hour and $5.25 for each additional hour. Write a formula to represent the parking charge for parking in this garage for h hours. Assume h is a counting number greater than 1.

93. Write a formula to represent the total cost to rent a car from a company that rents cars for $29.95 per day plus 50¢ for every mile driven over 100 mi. Assume the car will be driven more than 100 mi.

SECTION 9.2 Rate, Ratio, and Proportion

Rates

The word *rate* is used frequently in our everyday lives. It is used in such contexts as unemployment rate, tax rate, interest rate, hourly rate, infant mortality rate, school dropout rate, inflation rate, and postage rate.

    A **rate** is a comparison of two quantities and can be written as a fraction. For instance, if a car travels 135 mi on 6 gal of gas, then the miles-to-gallons rate is written

$$\frac{135 \text{ mi}}{6 \text{ gal}}$$

Note that the units (miles and gallons) are written as part of the rate.

Earned Run Average

One measure of a pitcher's success is earned run average. **Earned run average (ERA)** is the number of earned runs a pitcher gives up for every nine innings pitched. The definition of an earned run is somewhat complicated, but basically an earned run is a run that is scored as a result of hits and base running that involves no errors on the part of the pitcher's team. If the opposing team scores a run on an error (for example, a fly ball that should have been caught in the outfield was fumbled), then that run is not an earned run.

A proportion is used to calculate a pitcher's ERA. Remember that the statistic involves the number of earned runs per *nine innings*. The answer is always rounded to the nearest hundredth. Here is an example.

During the 2015 baseball season, Clayton Kershaw gave up 55 earned runs and pitched 232.2 innings for the Los Angeles Dodgers. To calculate Clayton Kershaw's ERA, let $x =$ the number of earned runs for every nine innings pitched. Write a proportion and then solve it for x.

$$\frac{55 \text{ earned runs}}{232.2 \text{ innings}} = \frac{x}{9 \text{ innings}}$$
$$55 \cdot 9 = 232.2 \cdot x$$
$$495 = 232.2x$$
$$\frac{495}{232.2} = \frac{232.2x}{232.2}$$
$$2.13 \approx x$$

Clayton Kershaw

Clayton Kershaw's ERA for the 2015 season was 2.13.

Year	Player, club	ERA
2005	Roger Clemens, Houston	1.87
2006	Johan Santana, Minnesota	2.77
2007	Jake Peavy, San Diego	2.54
2008	Johan Santana, New York	2.53
2009	Zack Greinke, Kansas City	2.16
2010	Felix Hernandez, Seattle	2.27
2011	Clayton Kershaw, Los Angeles	2.28
2012	Clayton Kershaw, Los Angeles	2.53
2013	Clayton Kershaw, Los Angeles	1.83
2014	Clayton Kershaw, Los Angeles	1.77
2015	Zack Greinke, Los Angeles	1.66

EXCURSION EXERCISES

1. In 1979, his rookie year, Jeff Reardon pitched 21 innings for the New York Mets and gave up four earned runs. Calculate Reardon's ERA for 1979.
2. Roger Clemens's first year with the Boston Red Sox was 1984. During that season, he pitched 133.1 innings and gave up 64 earned runs. Calculate Clemens's ERA for 1984.
3. In 1987, Nolan Ryan had the lowest ERA of any pitcher in the major leagues. He gave up 65 earned runs and pitched 211.2 innings for the Houston Astros. Calculate Ryan's ERA for 1987.
4. During the 2015 season, Jake Arrieta of the Baltimore Orioles pitched 229 innings and had an ERA of 1.77. How many earned runs did he give up during the season?
5. Find the necessary statistics for a pitcher on your "home team," and calculate that pitcher's ERA.

EXERCISE SET 9.2

1. Provide two examples of situations in which unit rates are used.
2. Provide two examples of situations in which ratios are used.

■ In Exercises 3 to 8, write the expression as a unit rate.
3. 582 mi in 12 h
4. 138 mi on 6 gal of gasoline
5. 544 words typed in 8 min
6. 100 m in 8 s
7. $9100 for 350 shares of stock
8. 1000 ft² of wall covered with 2.5 gal of paint
9. **Wages** A machinist earns $682.50 for working a 35-hour week. What is the machinist's hourly rate of pay?

518

8) $\dfrac{1000 \text{ ft}^2}{2.5 \text{ gal}} = \boxed{400 \text{ ft}^2/\text{gal}}$

3) $\dfrac{582 \text{ mi}}{12 \text{ h}} = \boxed{48.5 \text{ mph}}$

9) $\dfrac{682.50 \text{ \$}}{35 \text{ h}} = \boxed{19.50 \text{ \$/h}}$

14. Wages You have a choice of receiving a wage of \$34,000/year, \$2840/month, \$650 per week, or \$16.50/h. Which pay choice would you take? Assume a 40-hour work week and 52 weeks of work per year.

— $\dfrac{34\,000 \text{ \$}}{\text{year}} \cdot \dfrac{1 \text{ year}}{52 \text{ week}} \cdot \dfrac{1 \text{ week}}{40 \text{ h}}$

$= \boxed{16.35 \text{ \$/h}}$

— $\dfrac{2840 \text{ \$}}{\text{month}} \cdot \dfrac{1 \text{ month}}{4 \text{ weeks}} \cdot \dfrac{1 \text{ week}}{40 \text{ h}}$

$= \boxed{17.75 \text{ \$/h}}$

— $\dfrac{650 \text{ \$}}{\text{week}} \cdot \dfrac{1 \text{ week}}{40 \text{ h}} = \boxed{16.25 \text{ \$/h}}$

— $\boxed{16.50 \text{ \$/h}}$ so we take the 2840 \$/month

16. 🔵 **Population Density** The table below shows the population and area of three countries. The population density of a country is the number of people per square mile.

a. Which country has the lowest population density?

b. How many more people per square mile are there in India than in the United States? Round to the nearest whole number.

Country	Population	Area (in square miles)
Australia	22,751,000	2,938,000
India	1,251,696,000	1,146,000
United States	321,369,000	3,535,000

17. 🔵 **E-mail** The Radicati Group compiled the following estimates on consumer use of e-mail worldwide.

a. Complete the last column of the table below by calculating the estimated number of messages per day that each user receives. Round to the nearest tenth.

b. The predicted number of messages per person per day in 2019 is how many times the estimated number in 2015? Round to the nearest hundredth.

Year	Number of users (in millions)	Messages per day (in billions)	Messages per person per day
2015	2586	205.6	
2017	2760	225.3	
2019	2943	246.5	

🔵 **Exchange Rates** The table below shows the exchange rates per U.S. dollar for four foreign countries on January 27, 2016. Use this table for Exercises 18 to 21.

Exchange Rates per U.S. Dollar	
Australian dollar	1.4216
Danish krone	6.8628
Indian rupee	67.8697
Mexican peso	18.4398

18. How many Danish kroner are equivalent to $5000?

19. Find the number of Indian rupees that would be exchanged for $45,000.

20. Find the cost, in Mexican pesos, of an order of American computer hardware costing $35,000.

21. Calculate the cost, in Australian dollars, of an American car costing $29,000.

🔵 **Real Estate** Dean Baker, co-director at the Center for Economic and Policy Research in Washington, D.C., suggests that the buy-versus-rent question can be answered using the price-to-rent ratio. Find two houses of similar size and quality in comparable neighborhoods, one for sale and the other for rent. Divide the price of the house for sale by the total cost of the rental for 1 year. If the quotient is higher than 20, renting might be the better option. If the quotient is below 15, buying might be the better option.

22. A house in San Diego, California, is priced at $530,000. The rent on a comparable house is $1800 per month. Find the price-to-rent ratio. Round to the nearest tenth. Does the ratio suggest that you buy or rent a home in San Diego?

23. A house in Orlando, Florida, is priced at $155,000. The rent on a comparable house is $1150 per month. Find the price-to-rent ratio. Round to the nearest tenth. Does the ratio suggest that you buy or rent a home in Orlando?

🔵 **Student–Faculty Ratios** The table below shows the number of full-time men and women undergraduates and the number of full-time faculty at several universities in the Big East. Use this table for Exercises 24 and 25. Round ratios to the nearest whole number. (*Source:* National Center for Education Statistics, nces.ed.gov)

University	Men	Women	Faculty
Georgetown University	3244	3982	1350
Syracuse University	6487	8045	1078
University of Connecticut	8851	8826	2007
West Virginia University	11,434	9429	2044

24. 🔴 Calculate the student–faculty ratio at Syracuse University. Write the ratio using a colon and using the word *to*. What does this ratio mean?

25. Which school listed has the lowest student–faculty ratio?

▪ In Exercises 26 to 37, solve the proportion. Round to the nearest hundredth.

26. $\dfrac{3}{8} = \dfrac{x}{12}$

27. $\dfrac{3}{y} = \dfrac{7}{40}$

28. $\dfrac{7}{12} = \dfrac{25}{d}$

29. $\dfrac{16}{d} = \dfrac{25}{40}$

30. $\dfrac{15}{45} = \dfrac{72}{c}$

31. $\dfrac{120}{c} = \dfrac{144}{25}$

32. $\dfrac{65}{20} = \dfrac{14}{a}$

33. $\dfrac{4}{a} = \dfrac{9}{5}$

34. $\dfrac{0.5}{2.3} = \dfrac{b}{20}$

35. $\dfrac{1.2}{2.8} = \dfrac{b}{32}$

36. $\dfrac{0.7}{1.2} = \dfrac{6.4}{x}$

37. $\dfrac{2.5}{0.6} = \dfrac{165}{x}$

18] Convert 5000 USD to Danish kroner

$$5000 \; USD \cdot \frac{6.8628 \; kroner}{1 \; USD} = \boxed{34314 \; kroner}$$

units canceled
&
units worked out.

$$5000 \; USD \cdot \frac{1 \; USD}{6.8628 \; kroner}$$

$$= 728.57 \; USD^2/kroner$$

oops, we wanted the USD to cancel

ANS   34314 danish kroner

■ In Exercises 26 to 37, solve the proportion. Round to the nearest hundredth.

26. $\dfrac{3}{8} = \dfrac{x}{12}$ **27.** $\dfrac{3}{y} = \dfrac{7}{40}$ **28.** $\dfrac{7}{12} = \dfrac{25}{d}$

29. $\dfrac{16}{d} = \dfrac{25}{40}$ **30.** $\dfrac{15}{45} = \dfrac{72}{c}$ **31.** $\dfrac{120}{c} = \dfrac{144}{25}$

32. $\dfrac{65}{20} = \dfrac{14}{a}$ **33.** $\dfrac{4}{a} = \dfrac{9}{5}$ **34.** $\dfrac{0.5}{2.3} = \dfrac{b}{20}$

35. $\dfrac{1.2}{2.8} = \dfrac{b}{32}$ **36.** $\dfrac{0.7}{1.2} = \dfrac{6.4}{x}$ **37.** $\dfrac{2.5}{0.6} = \dfrac{165}{x}$

New concept:
proportions
solve by
cross multiplyn'

29) Solve for d

$\dfrac{16}{d} = \dfrac{25}{40}$ (given)

$25d = 640$ (cross multiply)

$d = \boxed{25.6}$ (divide 25)

30) solve $\dfrac{15}{45} = \dfrac{75}{c}$ (given)

$15c = 3375$ (cross multiply)

$c = \boxed{225}$ (divide 15)

Proof of cross multiplyng' for proportions.

Given $\dfrac{a}{b} = \dfrac{c}{d}$, show that $ad = bc$.

$$\dfrac{a}{b} = \dfrac{c}{d} \qquad \text{(Given)}$$

$$a = \dfrac{bc}{d} \qquad \text{(multiply } b\text{)}$$

$(a/b) \bullet b \qquad (c/d) \bullet b$

$$ad = bc \qquad \text{(multiply } d\text{)}$$

38. Gravity The ratio of weight on the moon to weight on Earth is $1:6$. How much would a 174-pound person weigh on the moon?

39. Management A management consulting firm recommends that the ratio of middle-management salaries to management trainee salaries be $5:4$. Using this recommendation, what is the annual middle-management salary if the annual management trainee salary is $52,000?

40. Medication The dosage of a cold medication is 2 mg for every 80 lb of body weight. How many milligrams of this medication are required for a person who weighs 220 lb?

41. Fuel Consumption If your car can travel 70.5 mi on 3 gal of gasoline, how far can the car travel on 14 gal of gasoline under similar driving conditions?

42. Scale Drawings The scale on the architectural plans for a new house is 1 in. equals 3 ft. Find the length and width of a room that measures 5 in. by 8 in. on the drawing.

43. Scale Drawings The scale on a map is 1.25 in. equals 10 mi. Find the distance between two cities that are 2 in. apart on the map.

44. Art Leonardo da Vinci measured various distances on the human body in order to make accurate drawings. He determined that generally the ratio of the kneeling height of a person to the standing height of that person was $\frac{3}{4}$. Using this ratio, determine the standing height of a person who has a kneeling height of 48 in.

45. Art In one of Leonardo da Vinci's notebooks, he wrote that ". . . from the top to the bottom of the chin is the sixth part of a face, and it is the fifty-fourth part of the man." Suppose the distance from the top to the bottom of the chin of a person is 1.25 in. Using da Vinci's measurements, find the height of the person.

46. Elections A pre-election survey showed that two out of every three eligible voters would cast ballots in the county election. There are 240,000 eligible voters in the county. How many people are expected to vote in the election?

47. Food Waste One study estimated that in the U.S., the average family of four wastes $590 worth of food each year. Estimate the cost of food wasted by **a.** the average family of three and **b.** the average family of five.

48. Lotteries Three people put their money together to buy lottery tickets. The first person put in $25, the second person put in $30, and the third person put in $35. One of their tickets was a winning ticket. If they won $4.5 million, what was the first person's share of the winnings?

49. Nutrition A pancake 4 in. in diameter contains 5 g of fat. How many grams of fat are in a pancake 6 in. in diameter? Explain how you arrived at your answer.

EXTENSIONS

■ In Exercises 50 and 51, assume each denominator is a non-zero real number.

50. Determine whether the statement is true or false.

  a. The quotient $a \div b$ is a ratio.

  b. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{b}{a} = \dfrac{d}{c}$.

  c. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a}{c} = \dfrac{b}{d}$.

  d. If $\dfrac{a}{b} = \dfrac{c}{d}$, then $\dfrac{a}{d} = \dfrac{c}{b}$.

51. If $\dfrac{a}{b} = \dfrac{c}{d}$, show that $\dfrac{a+b}{b} = \dfrac{c+d}{d}$.

43. Scale Drawings The scale on a map is 1.25 in. equals 10 mi. Find the distance between two cities that are 2 in. apart on the map.

Technique One: Proportions

equals sign.

correspond over the equals sign in a proportion

Solution

$$\frac{1.25 \text{ inches}}{10 \text{ miles}} = \frac{2 \text{ inches}}{x}$$

$$1.25 \, x \text{ inches} = 20 \text{ inch-miles} \quad (\text{cross multiply})$$

$$x = \frac{20 \text{ inch-miles}}{1.25 \text{ inches}} \quad \left(\text{divide } 1.25 \text{ inches}\right)$$

$$= \boxed{16 \text{ miles}}$$

Technique Two: Dimensional Analysis
(this is what we did in Q 8, 18)

$$2 \text{ inches} \cdot \frac{10 \text{ miles}}{1.25 \text{ inches}} = \boxed{16 \text{ miles}}$$

multiply

Into calculator:

2 ✕ 10 ÷ 1.25

= 16

49. **Nutrition** ~~A pancake 4 in. in diameter contains 5 g of fat. How many grams of fat are in a pancake 6 in. in diameter? Explain how you arrived at your answer.~~

40. **Medication** The dosage of a cold medication is 2 mg for every 80 lb of body weight. How many milligrams of this medication are required for a person who weighs 220 lb?

$$\frac{2mg}{80\ lb} = \frac{x}{220\ lbs}$$

$$80x = 440 \quad \left(\substack{\text{cross} \\ \text{multiply}}\right)$$

$$x = \boxed{5.5} \quad (\text{divide } 80\ )$$