J. Probability. (Some of Ch 3.1-3.5 pp 176-208)

J1.

P(event) = size(event) / size(sample space)

probability — proportion

J2. Example (Die). From 3.2 p 182.

Example 3.2

The Random Experiment

list of equally likely outcomes, all of them

A fair, six-sided die is rolled. Describe the sample space S, identify each of the following events with a subset of S and compute its probability (an outcome is the number of dots that show up).

Sample space $S = \{1, 2, 3, 4, 5, 6\}$ — A, B

A or B is the union (everything together)
A and B is the intersection (overlap)

some collection of outcome(s)

a. Event T = the outcome is two.
b. Event A = the outcome is an even number.
c. Event B = the outcome is less than four.
d. The complement of A. = not A = A'
e. A GIVEN B
f. B GIVEN A

save for lesson L

g. A AND B
h. A OR B
i. A OR B'
j. Event N = the outcome is a prime number. (has exactly 2 factors)
k. Event I = the outcome is seven.

$S = \{1, 2, 3, 4, 5, 6\}$

a. $T = \{2\}$ $P(T) = 1/6$

b. $A = \{2, 4, 6\}$ $P(A) = 3/6$

c. $B = \{1, 2, 3\}$ $P(B) = 3/6$

d. not $A = \{1, 3, 5\}$ $P(\text{not } A) = 3/6$

g. A and B = the outcome is even and less than 4 $= \{2\}$
$P(A \text{ and } B) = 1/6$

h. A or B = the outcome is even or less than 4
$= \{1, 2, 3, 4, 6\}$
$P(A \text{ or } B) = 5/6$

i. A or not B = the outcome is even or not less than 4
$= \{2, 4, 5, 6\}$
$P(A \text{ or not } B) = 4/6$

j. $N = \{2, 3, 5\}$ $P(N) = 3/6$

k. $I = \{\ \}$ $P(I) = 0/6 = 0$

J3. Definition.

Events A, B are mutually exclusive if $P(A \text{ and } B) = 0$.

That means A, B cannot occur at the same time.

For example roll a fair die.

T = roll a two $= \{2\}$ T and F $= \{\ \}$ $P(T \text{ and } F) = 0$

F = roll a five $= \{5\}$ T and A $= \{2\}$ $P(T \text{ and } A) = 1/6$

A = roll an even $= \{2, 4, 6\}$ F and A $= \{\ \}$ $P(F \text{ and } A) = 0$

So, T, F are mutually exclusive; F, A are mutually exclusive.

J4. Example. Based on 3.2, p 178.

A fair, six-sided die is rolled. Let X be the number you get.

Find the probability of the following events

And write your answer using random variable notation

X

a. Event T = the outcome is two.

b. Event A = the outcome is an even number.

c. Event B = the outcome is less than four.

h. A OR B

a. $T = \{2\}$

$P(T) = \text{size}(T)/\text{size}(S)$

$= 1/6$

$\boxed{P(X=2) = 1/6}$

b. $\boxed{P(X \text{ is even}) = 3/6}$

1 2 3 4 5 6

c. $\boxed{P(X<4) = 3/6}$

1 2 3 4 5 6

h. $\boxed{P(X \text{ is even or } <4) = 5/6}$

1 2 3 4 5 6

J5. Fact. As we see in J2 and J3,
(Recall J1. that P(A) always means size(A)/size(S))

For finite S, Size = count

For finite S, $P(A) = \frac{\text{count}(A)}{\text{count}(S)}$

J6. Example (Cards). Based on 3.5 p182.

You have a fair, well-shuffled deck of 52 cards. It consists of four suits. The suits are clubs, diamonds, hearts, and spades. There are 13 cards in each suit consisting of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, *J* (jack), *Q* (queen), and *K* (king) of that suit. *S* = spades, *H* = Hearts, *D* = Diamonds, *C* = Clubs.

a standard deck of cards

Let K = the event you draw a king.
Let D = the event you draw a Diamond.

Find P(K)
P(not K)
P(D)
P(K and D)
P(K or D)

Sample space S = {

d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	dj	dq	dk
c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	cj	cq	ck
h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	hj	hq	hk
ℓ1	ℓ2	ℓ3	ℓ4	ℓ5	ℓ6	ℓ7	ℓ8	ℓ9	ℓ10	ℓj	ℓq	ℓk

}

P(K)?

{

d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	dj	dq	dk
c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	cj	cq	ck
h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	hj	hq	hk
ℓ1	ℓ2	ℓ3	ℓ4	ℓ5	ℓ6	ℓ7	ℓ8	ℓ9	ℓ10	ℓj	ℓq	ℓk

}

K = {dk, ck, hk, ℓk} $P(K) = 4/52 = \boxed{1/13}$

P(not K)?

{

d1	d2	d3	d4	d5	d6	d7	d8	d9	d10	dj	dq	dk
c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	cj	cq	ck
h1	h2	h3	h4	h5	h6	h7	h8	h9	h10	hj	hq	hk
ℓ1	ℓ2	ℓ3	ℓ4	ℓ5	ℓ6	ℓ7	ℓ8	ℓ9	ℓ10	ℓj	ℓq	ℓk

}

$P(\text{not } K) = 48/52 = \boxed{12/13}$

(we will revisit in Lesson K → notice 12/13 is 1 − 1/13)

$P(D)$?

$$\left\{\begin{array}{ccccccccccccc} d1 & d2 & d3 & d4 & d5 & d6 & d7 & d8 & d9 & d10 & dj & dq & dk \\ c1 & c2 & c3 & c4 & c5 & c6 & c7 & c8 & c9 & c10 & cj & cq & ck \\ h1 & h2 & h3 & h4 & h5 & h6 & h7 & h8 & h9 & h10 & hj & hq & hk \\ l1 & l2 & l3 & l4 & l5 & l6 & l7 & l8 & l9 & l10 & lj & lq & lk \end{array}\right\}$$

$$P(D) = 13/52 = \boxed{1/4}$$

$P(K \text{ and } D)$?

$$\left\{\begin{array}{ccccccccccccc} d1 & d2 & d3 & d4 & d5 & d6 & d7 & d8 & d9 & d10 & dj & dq & dk \\ c1 & c2 & c3 & c4 & c5 & c6 & c7 & c8 & c9 & c10 & cj & cq & ck \\ h1 & h2 & h3 & h4 & h5 & h6 & h7 & h8 & h9 & h10 & hj & hq & hk \\ l1 & l2 & l3 & l4 & l5 & l6 & l7 & l8 & l9 & l10 & lj & lq & lk \end{array}\right\}$$

$$K \text{ and } D = \{dk\}$$

$$P(K \text{ and } D) = \boxed{1/52}$$

$P(K \text{ or } D)$?

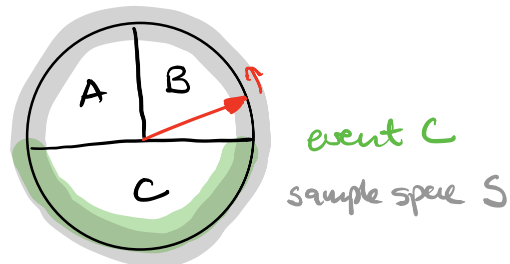
$$K \text{ or } D =$$

$$\left\{\begin{array}{ccccccccccccc} d1 & d2 & d3 & d4 & d5 & d6 & d7 & d8 & d9 & d10 & dj & dq & dk \\ & & & & & & & & & & & & ck \\ & & & & & & & & & & & & hk \\ & & & & & & & & & & & & lk \end{array}\right\}$$

$$P(K \text{ or } D) = \boxed{16/52}$$

J7. Example (Spinner, Line, Darts)

Spin this spinner.
Let C be the event
you land in area C.



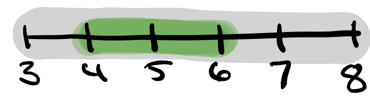
Find $P(C)$

$P(C) = \frac{1}{2}$

(Notice size of C, S means length)

Choose a random real number between
between 3 and 8 and call it X.

Find $P(4 < X < 6) = \boxed{\frac{2}{5}}$

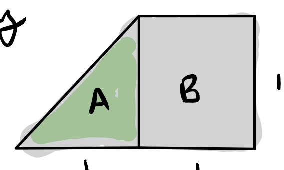


$P(\text{event}) = \frac{\text{size (event)}}{\text{size (samplespace)}}$

For one-dimensional sample space (an interval), size = length.

Throw a dart randomly
onto the shape.

Let A = the event
you land in A.



Total area
(area of S)
is 0.5 + 1

area(S) = 1.5

area(A) = 0.5

Find $P(A) = \frac{0.5}{1.5} = \frac{5}{15} = \boxed{\frac{1}{3}}$

For 2D sample space, size = area.

J8. Fact.

For infinite S, then size means length or area

(1D or 2D)
instead of just a finite list

J9. Example (Probability table).

Roll an unfair die where

event =	roll 1	roll 2	roll 3	roll 4	roll 5	roll 6
P(event) =	0.2	0.15	0.2	0.1	0.1	0.25

Find P(roll an even number).

$= 0.15 + 0.1 + 0.25$

$= \boxed{0.5}$

(*) The list of events in the table must be mutually exclusive (J3.)

(They are sort of like outcomes, but not equally likely.)

(**) All the probabilities must add to 1.

Check: $0.2 + 0.15 + 0.2 + 0.1 + 0.1 + 0.25 = 1$ ✓✓

J10. Definition. Probability table is a table of probabilities of some mutually exclusive events whose probabilies add to 1.

So instead of being told the sample space of equally likely outcomes, we get a table of "basic" events.

The table "has all the probabilities" means (*) (**)

J11. Example (Contingency table)

From 3.3, p180.

like a probability table

Example 3.3

Table 3.1 describes the distribution of a random sample S of 100 individuals, organized by gender and whether they are right- or left-handed.

these numbers are frequencies (but sometimes they'll write relative freq)

	Right-handed	Left-handed
Males	43	9
Females	44	4

Table 3.1

S

size$(S) = n = 100$

Let's denote the events M = the subject is male, F = the subject is female, R = the subject is right-handed, L = the subject is left-handed. Compute the following probabilities:

a. $P(M)$ $= \text{size}(M)/\text{size}(S) = \dfrac{43+9}{43+9+44+4} = \boxed{52/100}$

b. $P(F)$ $= \dfrac{44+4}{43+9+44+4} = \boxed{48/100}$

c. $P(R)$ $= \dfrac{43+44}{100} = \boxed{\dfrac{87}{100}}$

d. ~~$P(L)$~~

e. $P(M \text{ AND } R)$ $= 43/(43+9+44+4) = \boxed{43/100}$

f. $P(F \text{ AND } L)$ $= 4/(43+9+44+4) = \boxed{4/100}$

g. $P(M \text{ OR } F)$ $= (43+9+44+4)/(43+9+44+4) = 100/100 = \boxed{1}$

h. $P(M \text{ OR } R)$ $= (43+9+44)/(43+9+44+4) = \boxed{96/100}$

i. $P(F \text{ OR } L)$ $= (9+44+4)/(43+9+44+4) = \boxed{57/100}$

j. $P(M')$ $= P(\text{not } M) = (44+4)/(43+9+44+4) = \boxed{48/100}$

J12. In lesson K we will learn how to fill out tables that are missing information when possible using three probability rules.

For example, roll an unfair die, A = roll a 2.
$P(A) = 0.15$. Find $P(\text{not } A)$

$$P(\text{not } A) = 1 - P(A)$$
$$= 1 - 0.15 = \boxed{0.85}$$

J13. Summary. We will see 5 types of one-step probability experiment problems

Remember J1. Probability $P(A) = \text{size}(A)/\text{size}(S)$ proportion

- Finite S: size = count the outcomes
- Infinite S: size = length or area
- Probability table: add up the probabilities
- Contingency table: size = count the outcomes (add)
- Using Three Probability Rules: lesson K

In lesson K we will also learn about multi-step experiments and Tree Diagrams

eg, An urn has 3 green, 2 red balls.
Draw 2 (without replacement).

J14. The Law of Large Numbers (LLN).

Repeat an experiment a large number of times n.

Then <u>relative frequency</u> $\approx$ <u>probability</u>

J15. Example. Roll a fair die.

(a) Express the fact "the probability you roll a 4 is 1/6" using event notation.

(b) Do the same with random variable notation

(c) Suppose you roll a fair die $n=90$ times and get the results in Figure J16. What does The Law of Large Numbers tell you to expect? Confirm it.

(a) Let A = roll a 4

$P(A) = \boxed{1/6}$

$A = \{4\}$

$S = \{1,2,3,4,5,6\}$

(b) Let X = the number you roll.

$P(X=4) = 1/6$

(c) Again A = roll a 4

The LLN says you expect the relative frequency of A to be $\approx$ the probability of A

$A = \{4\}$ $P(A) = \frac{\text{size}(A)}{\text{size}(S)} = \boxed{\frac{1}{6}} = 0.1667$ relative frequency $= 14/90 = 0.1555$

So yes relative frequency $\approx$ probability.

J16. Figure.

5	4	1	6	5	5	1	3	6	2	2	1	2	6	4
2	2	1	4	6	2	1	2	5	2	3	2	5	4	3
4	4	2	2	2	3	3	1	5	3	5	6	3	3	6
5	2	3	1	5	1	4	4	2	4	6	5	3	5	6
3	6	2	6	2	1	1	6	1	6	4	3	2	6	6
4	1	6	1	6	4	1	3	2	5	1	3	4	5	4