

Project 11: Ladner Fischer Adder

A Comprehensive Study of Advanced Digital Circuits

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1 Introduction

The Ladner-Fischer adder is an efficient parallel prefix adder designed to compute binary addition with logarithmic depth. It aims to optimize the trade-off between circuit area and delay. By using a balanced structure, the adder reduces the number of logic levels required for carry propagation, making it faster than traditional adders like the ripple-carry adder. This makes the Ladner-Fischer adder particularly useful in high-performance computing where both speed and resource efficiency are critical.

2 Key Concepts

- **Parallel Prefix Operation:** The adder uses a parallel prefix approach to compute carries across multiple bits simultaneously, leading to faster addition.
- **Logarithmic Depth:** The depth of the carry propagation is logarithmic in terms of the number of bits, which minimizes delay.
- **Carry Propagation:** It computes the carry bits using generate and propagate functions, ensuring efficient and accurate carry propagation.
- **Balanced Structure:** The adder is designed to balance between logic depth (delay) and area complexity, optimizing performance and resource utilization.
- **Scalability:** The Ladner-Fischer adder is highly scalable, making it ideal for use in wide-bit addition required in high-performance digital systems.

3 Steps in Ladner Fischer Adder

- **Generate and Propagate Signals:**
For each bit position i , the generate (G_i) and propagate (P_i) signals are computed from the input operands A_i and B_i :

$$G_i = A_i \cdot B_i$$

$$P_i = A_i \oplus B_i$$

- **Group Prefix Computation:**
Using a parallel prefix network, group generate ($G_{i:j}$) and propagate ($P_{i:j}$) signals are computed over sections of bits. This is done recursively:

$$G_{i:j} = G_i \vee (P_i \cdot G_{i+1:j})$$

$$P_{i:j} = P_i \cdot P_{i+1:j}$$

These group signals determine the carry information for multiple bit positions.

- **Carry Propagation:**
Carry signals are propagated in a logarithmic fashion across the bit positions, with each level of the prefix tree reducing the number of bits that need to compute their carry.
- **Final Sum Calculation:**
After all the carries have been computed, the final sum bits are calculated using the propagate signals and the generated carries:

$$S_i = P_i \oplus C_{i-1}$$

where C_{i-1} is the carry from the previous bit.

- **Output the Result:**
Once all the sum bits are computed, the final result of the addition is produced.

4 Why to Choose It

- **Faster Computation:**

The logarithmic depth of the carry propagation reduces the critical path delay, making it significantly faster than traditional adders like the ripple-carry adder.

- **Scalability:**

The Ladner-Fischer adder is scalable, supporting wide-bit adders efficiently, making it suitable for processors and large-scale arithmetic units.

- **Balanced Area vs. Speed Trade-off:**

It provides an optimal trade-off between circuit area and delay by carefully balancing the number of logic gates required and the number of stages for carry computation.

- **Parallelism:**

The parallel prefix computation allows multiple operations to happen simultaneously, which leads to faster results, especially for large bit-widths.

- **Efficient Carry Propagation:**

Its structured approach to generate and propagate signals ensures efficient and accurate carry computation over a wide range of bits.

5 SystemVerilog Code

Listing 1: Ladner Fischer Adder RTL Code

```
1 module ladner_fischer_adder #(parameter WIDTH = 16) (  
2     input  logic [WIDTH-1:0] A, B,      // Input operands  
3     input  logic          Cin,         // Carry-in  
4     output logic [WIDTH-1:0] Sum,       // Sum output  
5     output logic          Cout        // Carry-out  
6 );  
7  
8     logic [WIDTH-1:0] G, P;            // Generate and propagate  
9     logic [WIDTH-1:0] carry;          // Carry signals  
10  
11    // Generate and propagate signals  
12    assign G = A & B;                  // Generate: G[i] = A[i] & B[i]  
13    assign P = A ^ B;                  // Propagate: P[i] = A[i] ^ B[i]  
14  
15    // Carry lookahead logic using Ladner-Fischer structure  
16    assign carry[0] = Cin;  
17  
18    genvar i;  
19    generate  
20        for (i = 1; i < WIDTH; i = i + 1) begin  
21            assign carry[i] = G[i-1] (P[i-1] & carry[i-1]);  
22        end  
23    endgenerate  
24  
25    // Sum and carry-out  
26    assign Sum = P ^ carry;  
27    assign Cout = G[WIDTH-1] (P[WIDTH-1] & carry[WIDTH-1]);  
28  
29 endmodule
```

6 Testbench

Listing 2: Ladner Fischer Adder Testbench

```
1 module tb_ladner_fischer_adder();
2
3     parameter WIDTH = 16;
4
5     logic [WIDTH-1:0] A, B;    // Test inputs
6     logic Cin;                // Carry-in
7     logic [WIDTH-1:0] Sum;    // Test output sum
8     logic Cout;               // Carry-out
9
10    // Instantiate the Ladner-Fischer Adder
11    ladner_fischer_adder #(WIDTH) dut (
12        .A(A),
13        .B(B),
14        .Cin(Cin),
15        .Sum(Sum),
16        .Cout(Cout)
17    );
18
19    initial begin
20        // Test Case 1
21        A = 16'h1234; B = 16'h5678; Cin = 1'b0;
22        #10;
23        $display("A = %h, B = %h, Cin = %b, Sum = %h, Cout = %b", A,
24            B, Cin, Sum, Cout);
25
26        // Test Case 2
27        A = 16'hFFFF; B = 16'h0001; Cin = 1'b0;
28        #10;
29        $display("A = %h, B = %h, Cin = %b, Sum = %h, Cout = %b", A,
30            B, Cin, Sum, Cout);
31
32        // Test Case 3
33        A = 16'hAAAA; B = 16'h5555; Cin = 1'b1;
34        #10;
35        $display("A = %h, B = %h, Cin = %b, Sum = %h, Cout = %b", A,
36            B, Cin, Sum, Cout);
37
38        // Add more test cases as needed
39
40        $stop;
41    end
42 endmodule
```

7 Results

7.1 Schematic

[h]

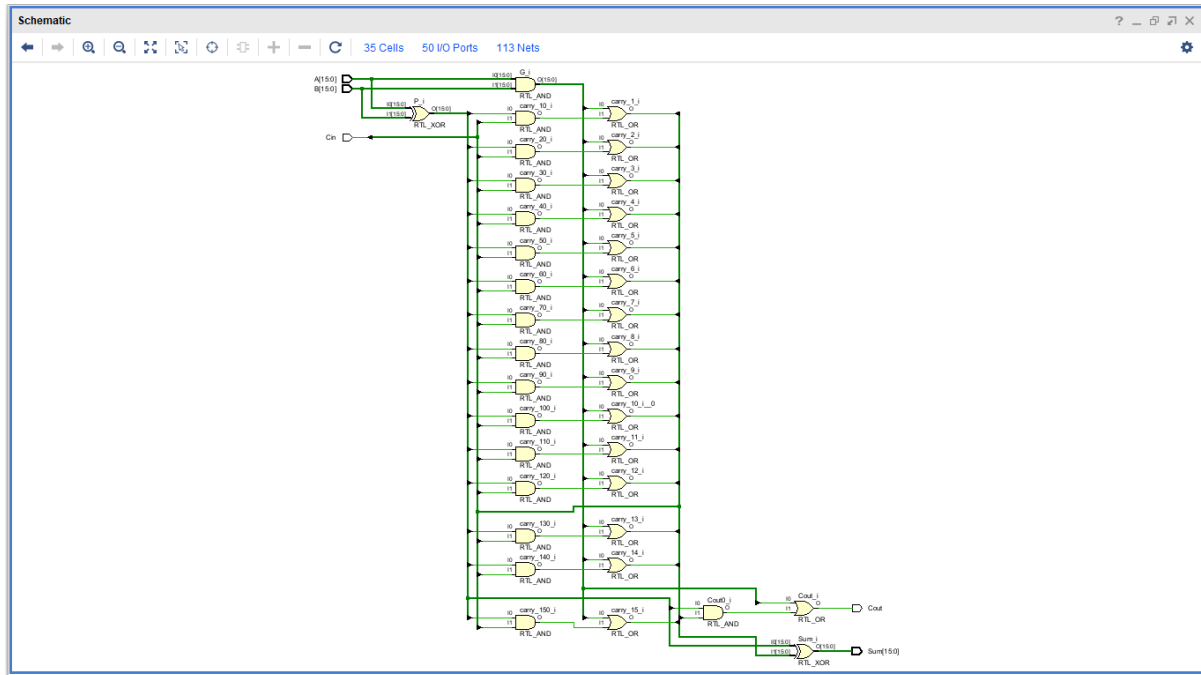


Figure 1: Schematic of Ladner Fischer Adder

8 Conclusion

The Ladner-Fischer adder is a sophisticated parallel prefix adder known for its efficiency in binary addition. Its design leverages a parallel prefix network to achieve logarithmic depth in carry propagation, resulting in significantly faster computation compared to traditional adders like the ripple-carry adder. The key benefits of the Ladner-Fischer adder include:

- **Speed:** The logarithmic carry propagation depth reduces delay, making it highly suitable for high-speed applications.
- **Scalability:** Its efficient design supports wide-bit additions, making it ideal for modern processors and large-scale digital systems.
- **Balanced Performance:** By optimizing the trade-off between circuit area and delay, it provides a balanced performance, avoiding excessive resource consumption.

Overall, the Ladner-Fischer adder is a powerful choice for high-performance computing, digital signal processing, and other applications requiring efficient and rapid addition operations. Its structured approach to carry computation and parallelism makes it a valuable asset in modern digital arithmetic.

9 References

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3. T. Givargis and L. S. Peh, *Introduction to Digital Systems: Design and Synthesis*, Wiley, 2005.
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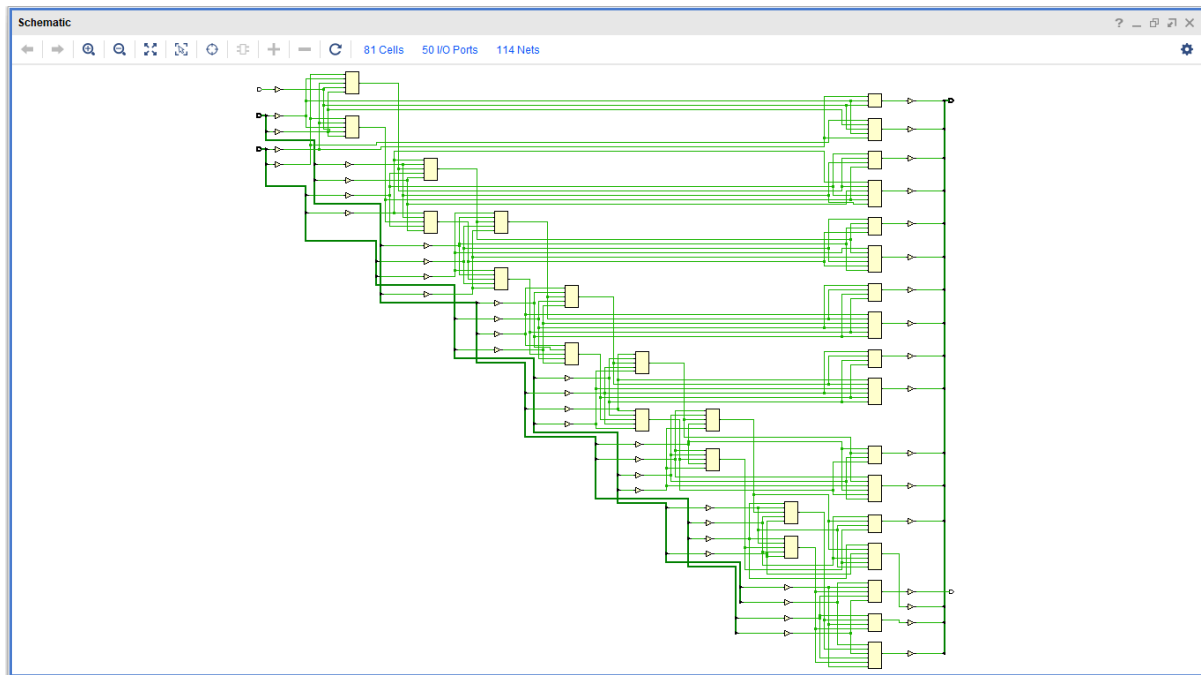


Figure 2: Synthesis of Ladner Fischer Adder

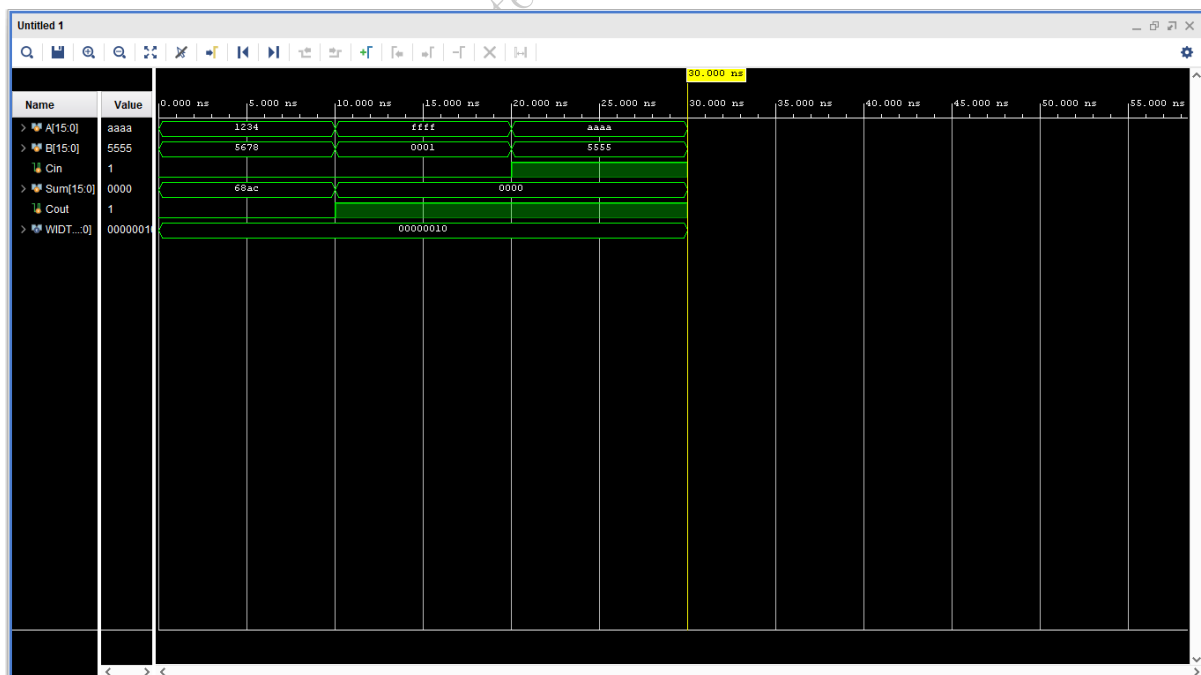


Figure 3: Simulation of Parallel Adder