I_2 - 2020 - 01

① PREGUNTA NORMAL (FACIL)

$X \sim \text{NORMAL}(\mu, \sigma)$

$$\boxed{\begin{aligned} x_{30\%} &= \mu + \sigma \cdot \Phi^{-1}(0.3) \\ x_{50\%} &= \mu \end{aligned}}$$

pnorm(24, μ, σ) ó 1 - pnorm(24, μ, σ)

② PREGUNTA LOG-NORMAL (FACIL)

$X \sim \text{Log-NORMAL}(\lambda, \zeta)$

$\lambda = \ln(x_{50\%})$

$\zeta = \sqrt{\ln(1 + \delta_x^2)}$

plnorm(8, λ, ζ) ó 1 - plnorm(8, λ, ζ)

③ METODO DELTA ("DIFICIL")

$\left.\begin{aligned} X &\sim \text{Exp}(\nu) \\ Y &\sim \text{Exp}(\nu) \end{aligned}\right\}$ indep ; $Z = \dfrac{X}{X+Y} = g(x,y)$; ¿ δ_Z ?

$\dfrac{d}{dx} g = \dfrac{y}{(x+y)^2}$; $\dfrac{d}{dy} = \dfrac{-x}{(x+y)^2}$

$$Z \approx \frac{\cancel{\mu}}{2\cancel{\mu}} + (X-\cancel{\mu})\cdot\frac{\cancel{\mu}}{(2\cancel{\mu})^2} + (Y-\cancel{\mu})\cdot\left(-\frac{\cancel{\mu}}{(2\cancel{\mu})^2}\right)$$

$$= \frac{1}{2} + (X-\cancel{\mu})\cdot\frac{1}{4\cancel{\mu}} + (Y-\cancel{\mu})\cdot\left(-\frac{1}{4\cancel{\mu}}\right)$$

$$E(Z) = \frac{1}{2}$$

$$Var(Z) \overset{\text{indep}}{=} \frac{\sigma_X^2}{16\cancel{\mu}^2} + \frac{\sigma_Y^2}{16\cancel{\mu}^2} = 2\cdot\frac{1}{\gamma^2}\cdot\frac{1}{16\cancel{\mu}^2}$$

$$= 2\cdot\frac{1}{\gamma^2}\cdot\frac{\gamma^2}{16} = \frac{1}{8}$$

$$\longrightarrow \delta_Z = \frac{\sqrt{\frac{1}{8}}}{\frac{1}{2}} = \frac{\frac{1}{2\sqrt{2}}}{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$ //

(4) PREGUNTA WEIBULL (MODERADA)

$$X \sim Weibull(\eta, \beta) \longrightarrow Y = \left(\frac{X}{\eta}\right)^\beta \sim ¿f_Y?$$

$$F_Y(y) = P(Y \leq y) = P(X \leq y^{1/\beta}\cdot\eta)$$

$$= 1 - e^{-\left(\frac{y^{1/\beta} \cdot \lambda}{\lambda}\right)^{1/\beta}} = 1 - e^{-y}$$

$\longrightarrow Y \sim Exp(1) \longrightarrow \min\{Y_1, \dots, Y_n\} \sim Exp(n)$ //

5 MODELO BINOMIAL (MODERADA)

$FP = P(+ \mid SANO)$

$FN = P(- \mid ENFERMO)$

$q = P(ENFERMO)$

$p = P(+) = P(+ \mid E)P(E) + P(+ \mid S)P(S)$

$= (1 - FN) \cdot q + FP \cdot (1 - q)$

$X \sim BINOMIAL(n = 10, p)$

$pbinom(x, n, p)$ o $1 - pbinom(x, n, p)$

6 PREGUNTA HIPERGEOMETRICA

A: "MESA FOCO CONTAGIO"

X: # DE COVID + EN LA MESA

ASIENTOS DISPONIBLES EN LA MESA

ASISTENTE COVID +

$X \sim HIPERGEOMETRICA(n, N, m)$

OTRAS PERSONAS

USTED NO SE CUENTA

$$1 - P_X(0) = 1 - \frac{\binom{m}{0}\binom{N-m}{n-0}}{\binom{N}{n}}$$

$$= 1 - \text{dhyper}(0, m = m, n = N-m, k = n)$$

⑦ PREGUNTA GEOMETRICA

p: PROB TEMPERATURA SOSPECHA COVID.

X: # EVALUACIONES HASTA 1ª SOSPECHA

$$P(X \leq a \mid X > b) = \frac{P(b < X \leq a)}{1 - P(X \leq b)}$$

$$= \frac{F_X(a) - F_X(b)}{1 - F_X(b)}$$

$F_X(x) = \text{pgeom}(x-1, p)$ //

⑧ PREGUNTA POISSON

X_t: # DE TEST + EN t HORAS

$X_t \sim \text{Poisson}(\nu \cdot t)$

$\lambda = \nu \cdot 24 \longrightarrow 1 - \text{ppois}(3, \nu)$

⑨ TEO PROB TOTALES ("DIFICIL")

X: PROB CONTAGIO $\sim$ Beta (1,1)

Y: # DE CONTAGIADOS ENTRE 100 "n"
VISTOS × VENTANA

$Y \mid X = x \sim \text{BINOMIAL}(n, x)$

¿ $P_Y(k)$?

$$P_Y(y) = \int_0^1 \binom{n}{y} x^y (1-x)^{n-y} \cdot 1 \, dx$$

$$= \binom{n}{y} B(y+1, n-y+1) \int_0^1 \frac{x^y (1-x)^{n-y}}{B(y+1, n-y+1)} dx$$

$$= \frac{n!}{y!\,(n-y)!} \cdot \frac{y! \times (n-y)!}{(n+1)!} = \frac{1}{n+1}$$

⑩ NORMAL BIVARIADA (MODERADA)

$X: I_1$; $Y: I_2$

$$Y \mid X = x \sim N\Big(\underbrace{\mu_y + (x - \mu_x)\,\rho\,\frac{\sigma_y}{\sigma_x}}_{\mu} \, ; \, \underbrace{\sigma_y \sqrt{1-\rho^2}}_{\sigma}\Big)$$

ENUNCIADO: x, μ_x, μ_y, σ_x, σ_y y ρ

$1 - pnorm(4, \mu, \sigma)$

(11.1) CONJUNTAS

$X \sim N(30; 10)$
$Y \sim N(45; 20)$ } indep

$Z = X + Y \sim N(\mu_Z, \sigma_Z)$

$\mu_Z = 30 + 45$

$\sigma_Z^2 = 10^2 + 20^2 - 2 \cdot \underbrace{0.3 \cdot 10 \cdot 20}_{Cov(X,Y)}$

$1 - pnorm(70, \mu_Z, \sigma_Z)$

(11.2) CONJUNTA

$X_1, \ldots, X_n$ iid $\mu_X = 2.5$ y $\sigma_X = 1$

$\sum X_i \overset{aprox}{\sim} N(n \cdot \mu_X ; \sqrt{n}\, \sigma_X)$

CORRECCION x CONTINUIDAD.

$P(\sum X_i \geq 240) = 1 - pnorm(240.5, n\mu_X, \sqrt{n}\,\sigma_X)$

12 ESPERANZA ITERADA

$T \mid X = x \sim \text{Gama}(k, 1/x)$

$X \sim \text{Poisso}(\nu)$

¿ $\delta_T = \dfrac{\sigma_T}{\mu_T}$?

$\mu_T = E(T) = E(E(T \mid X)) = k\,E(X) = k \cdot \nu$

$\sigma_T^2 = Var(T) = E(Var(T \mid X)) + Var(E(T \mid X))$

$= E(k\,x^2) + Var(k \cdot x)$

$= k\,E(x^2) + k^2\,Var(x)$

$= k \cdot (\nu + \nu^2) + k^2 \cdot \nu$

$$\delta_T = \frac{\sqrt{k \cdot \nu \{ 1 + \nu + k \}}}{k \cdot \nu} = \sqrt{\frac{1 + \nu + k}{k \cdot \nu}}$$