Homework 2

diamonds Analysis

The `diamonds` dataset, available in the **ggplot2** package, contains information on **53,940 diamonds**, detailing both their physical attributes and market prices. Each row represents a single diamond, with variables such as `price` (in US dollars), `carat` (weight), `cut` (quality rating from Fair to Ideal), `color` (ranging from J [worst] to D [best]), and `clarity` (from I1 [worst] to IF [best]). Additional numerical features include `depth` (the total depth percentage), `table` (top width relative to widest point), and the physical dimensions `x`, `y`, and `z` (measured in millimeters).

To make our analysis more manageable and visually interpretable, we use a random sample of 300 diamonds from the dataset. This subset still captures the diversity and real-world complexity of the full dataset, allowing us to explore key relationships among variables without the computational overhead of analyzing all 53,940 observations.

```r
# Load packages
library(dplyr)            ## Data Transformation   --- T
library(ggplot2)          ## Data Visualization    --- V & Data Extraction
library(ggfortify)        ## Data Visualization    --- V
library(broom)            ## Data Analysis         --- A
library(lmtest)           ## Data Analysis         --- A


# Use a sample to simplify visualization
set.seed(123)
diamonds_sample <- diamonds %>%
  sample_n(300)

# Fit multiple regression model
diamond_model_1 <- lm(price ~ carat, data = diamonds_sample)
diamond_model_2 <- lm(price ~ carat + x, data = diamonds_sample)
```

```r
diamond_model_3 <- lm(price ~ carat + x + y, data = diamonds_sample)
```

```r
summarize_reg_model <- function(model,model_description) {
  # Get number of observations and predictors
  n <- length(residuals(model))
  p <- length(coefficients(model)) - 1

  # 0) RSS
  RSS <- round(sum(residuals(model)^2),2)

  # 1) RSE
  RSE <- round(sqrt(RSS / (n - p - 1)),2)

  # 2) R-Squared and Adjusted R-Squared
  R2 <- round(summary(model)$r.squared,2)
  adj_R2 <- round(summary(model)$adj.r.squared,2)

  # 3) AIC
  aic_val <- round(AIC(model),2)

  # 4) BIC
  bic_val <- round(BIC(model),2)

  # Create summary dataframe
  mlr_metrics <- data.frame(
    type = model_description,
    RSS = RSS,
    RSE = RSE,
    R2 = R2,
    Adj_R2 = adj_R2,
    AIC = aic_val,
    BIC = bic_val
  )

  return(mlr_metrics)
}
```

Summary of Model 1: price ~ carat

```r
summary(diamond_model_1)
```

```

Call:
lm(formula = price ~ carat, data = diamonds_sample)

Residuals:
    Min      1Q  Median      3Q     Max 
-4762.0  -923.7   -38.7   616.7  6985.1 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -2413.2      183.3  -13.16   <2e-16 ***
carat         7959.3      195.5   40.72   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1564 on 298 degrees of freedom
Multiple R-squared:  0.8476,    Adjusted R-squared:  0.8471 
F-statistic:  1658 on 1 and 298 DF,  p-value: < 2.2e-16
```

Summary of Model 2: price ~ carat + x

```r
summary(diamond_model_2)
```

```
Call:
lm(formula = price ~ carat + x, data = diamonds_sample)

Residuals:
    Min      1Q  Median      3Q     Max 
-4991.1  -668.3   -35.6   282.8  6912.9 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   5164.1     1545.6   3.341 0.000941 ***
carat        12552.7      949.7  13.218  < 2e-16 ***
x            -1957.4      396.7  -4.935 1.34e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1506 on 297 degrees of freedom
Multiple R-squared:  0.8592,	Adjusted R-squared:  0.8582 
F-statistic:   906 on 2 and 297 DF,  p-value: < 2.2e-16
```

```r
anova(diamond_model_1,diamond_model_2)
```

```
Analysis of Variance Table

Model 1: price ~ carat
Model 2: price ~ carat + x
  Res.Df       RSS Df Sum of Sq      F   Pr(>F)    
1    298 729068344                                 
2    297 673820646  1  55247698 24.352 1.34e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Summary of Model 3: price ~ carat + x + y

```r
summary(diamond_model_3)
```

```
Call:
lm(formula = price ~ carat + x + y, data = diamonds_sample)

Residuals:
    Min      1Q  Median      3Q     Max 
-4738.8  -636.6   -54.0   338.9  6875.0 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   4539.7     1566.0   2.899  0.00403 ** 
carat        12269.7      954.1  12.859  < 2e-16 ***
x            -5542.4     1766.8  -3.137  0.00188 ** 
y             3733.1     1793.4   2.082  0.03823 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1498 on 296 degrees of freedom
Multiple R-squared:  0.8612,	Adjusted R-squared:  0.8598 
F-statistic: 612.2 on 3 and 296 DF,  p-value: < 2.2e-16
```

```r
anova(diamond_model_2,diamond_model_3)
```

```
Analysis of Variance Table

Model 1: price ~ carat + x
Model 2: price ~ carat + x + y
  Res.Df       RSS Df Sum of Sq      F  Pr(>F)  
1    297 673820646                              
2    296 664098768  1   9721879 4.3332 0.03823 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

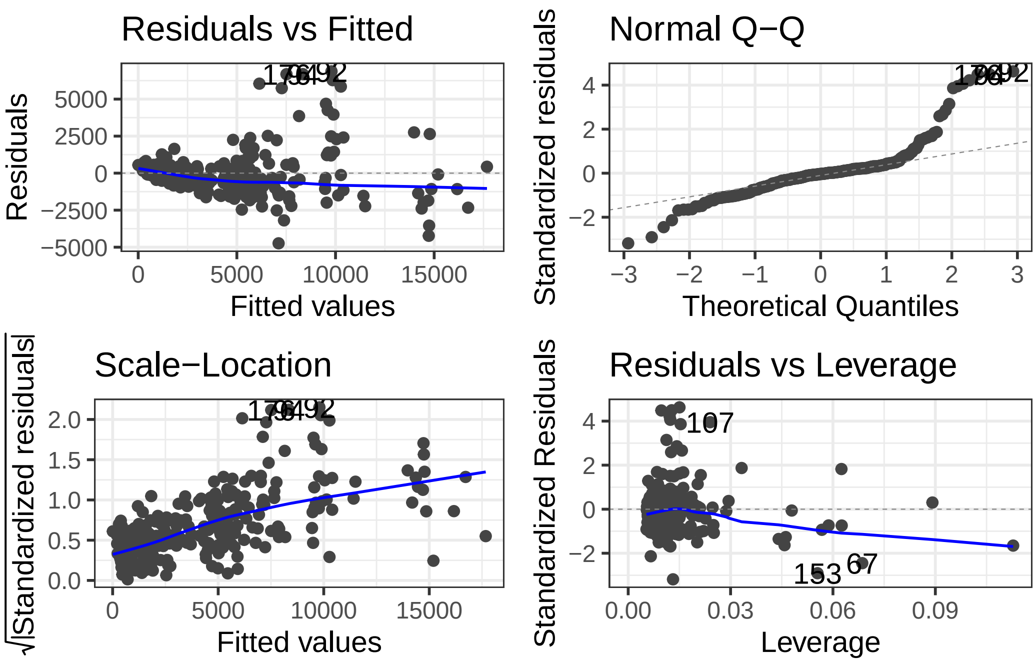
```r
summary_diamond_model_1 <- summarize_reg_model(diamond_model_1, model_description = "Model
summary_diamond_model_2 <- summarize_reg_model(diamond_model_2, model_description = "Model
summary_diamond_model_3 <- summarize_reg_model(diamond_model_3, model_description = "Model

bind_rows(
  summary_diamond_model_1,
  summary_diamond_model_2,
  summary_diamond_model_3
  )
```

```
                              type       RSS     RSE   R2 Adj_R2     AIC     BIC
1          Model 1: Price ~ Carat 729068344 1564.14 0.85   0.85 5268.41 5279.52
2      Model 2: Price ~ Carat + x 673820646 1506.24 0.86   0.86 5246.77 5261.59
3 Model 3: Price ~ Carat + x + y 664098768 1497.86 0.86   0.86 5244.41 5262.93
```

Assumptions of Model 3

```r
autoplot(diamond_model_3) + ## from ggfortify
  theme_bw()
```



```r
shapiro.test(resid(diamond_model_3)) ## From base R
```

```
	Shapiro-Wilk normality test

data:  resid(diamond_model_3)
W = 0.81926, p-value < 2.2e-16
```

```r
dwtest(diamond_model_3)   ## From lmtest package
```

```
	Durbin-Watson test

data:  diamond_model_3
DW = 2.1397, p-value = 0.8884
alternative hypothesis: true autocorrelation is greater than 0
```

```r
bptest(diamond_model_3) ## From lmtest package
```

```
	studentized Breusch-Pagan test

data:  diamond_model_3
BP = 46.049, df = 3, p-value = 5.537e-10
```

`mtcars` Analysis

The `mtcars` dataset is a classic and widely used dataset in R that contains specifications and performance data for 32 different car models from the 1974 Motor Trend magazine. Each row represents a unique vehicle, and each column records a specific attribute related to engine performance, design, or efficiency. Some key variables include `mpg` (miles per gallon), `hp` (gross horsepower), `wt` (weight in 1000 lbs), `drat` (rear axle ratio), and `qsec` (quarter-mile time). Additionally, the dataset includes categorical variables encoded as numeric values, such as `cyl` (number of cylinders), `am` (transmission type), and `gear` (number of forward gears).

This dataset is frequently used in regression modeling and statistical learning due to its compact size, real-world relevance, and mixture of quantitative and categorical variables. Analysts often model fuel efficiency (`mpg`) as a function of other variables to understand how engine power, vehicle weight, or gear ratios impact gas mileage. With its balance of complexity and interpretability, `mtcars` serves as a great playground for developing skills in exploratory data analysis, model selection, variable interpretation, and diagnostics in both teaching and applied settings.

```r
# Fit multiple regression models
mtcars_model_1 <- lm(mpg ~ drat, data = mtcars)
mtcars_model_2 <- lm(mpg ~ drat + wt, data = mtcars)
mtcars_model_3 <- lm(mpg ~ drat + wt + hp, data = mtcars)
```

Summary of Model 1: mpg ~ drat

```r
summary(mtcars_model_1)
```

```

Call:
lm(formula = mpg ~ drat, data = mtcars)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.0775 -2.6803 -0.2095  2.2976  9.0225 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   -7.525      5.477  -1.374     0.18    
drat           7.678      1.507   5.096 1.78e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.485 on 30 degrees of freedom
Multiple R-squared:  0.464,	Adjusted R-squared:  0.4461 
F-statistic: 25.97 on 1 and 30 DF,  p-value: 1.776e-05
```

Summary of Model 2: mpg ~ drat + wt

```r
summary(mtcars_model_2)
```

```
Call:
lm(formula = mpg ~ drat + wt, data = mtcars)

Residuals:
    Min      1Q  Median      3Q     Max 
-5.4159 -2.0452  0.0136  1.7704  6.7466 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   30.290      7.318   4.139 0.000274 ***
drat           1.442      1.459   0.989 0.330854    
wt            -4.783      0.797  -6.001 1.59e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.047 on 29 degrees of freedom
Multiple R-squared:  0.7609,	Adjusted R-squared:  0.7444 
F-statistic: 46.14 on 2 and 29 DF,  p-value: 9.761e-10
```

```r
anova(mtcars_model_1, mtcars_model_2)
```

```
Analysis of Variance Table

Model 1: mpg ~ drat
Model 2: mpg ~ drat + wt
  Res.Df    RSS Df Sum of Sq     F    Pr(>F)    
1     30 603.57                                 
2     29 269.24  1    334.33 36.01 1.589e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Summary of Model 3: mpg ~ drat + wt + hp

```r
summary(mtcars_model_3)
```

```

Call:
lm(formula = mpg ~ drat + wt + hp, data = mtcars)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.3598 -1.8374 -0.5099  0.9681  5.7078 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 29.394934   6.156303   4.775 5.13e-05 ***
drat         1.615049   1.226983   1.316 0.198755    
wt          -3.227954   0.796398  -4.053 0.000364 ***
hp          -0.032230   0.008925  -3.611 0.001178 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.561 on 28 degrees of freedom
Multiple R-squared:  0.8369,	Adjusted R-squared:  0.8194 
F-statistic: 47.88 on 3 and 28 DF,  p-value: 3.768e-11
```

```r
anova(mtcars_model_2, mtcars_model_3)
```

```
Analysis of Variance Table

Model 1: mpg ~ drat + wt
Model 2: mpg ~ drat + wt + hp
  Res.Df    RSS Df Sum of Sq      F   Pr(>F)   
1     29 269.24                                
2     28 183.68  1    85.559 13.043 0.001178 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Model Comparison Table

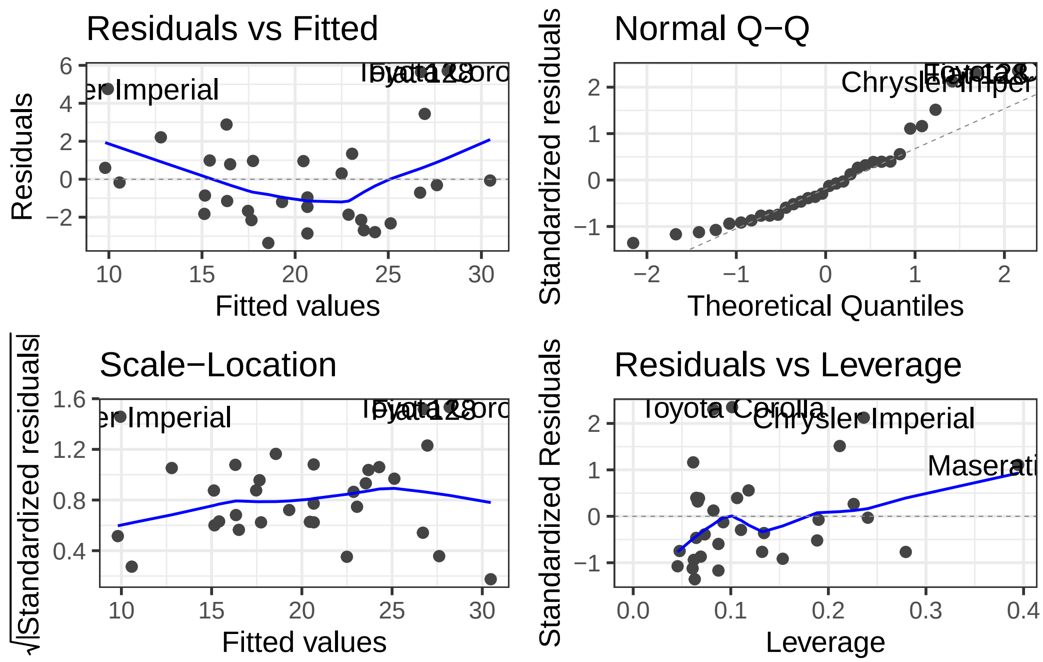
```r
summary_mtcars_model_1 <- summarize_reg_model(mtcars_model_1, model_description = "Model 1
summary_mtcars_model_2 <- summarize_reg_model(mtcars_model_2, model_description = "Model 2
summary_mtcars_model_3 <- summarize_reg_model(mtcars_model_3, model_description = "Model 3

bind_rows(
  summary_mtcars_model_1,
  summary_mtcars_model_2,
  summary_mtcars_model_3
)
```

```
                            type    RSS  RSE   R2 Adj_R2    AIC    BIC
1           Model 1: mpg ~ drat 603.57 4.49 0.46   0.45 190.80 195.20
2      Model 2: mpg ~ drat + wt 269.24 3.05 0.76   0.74 166.97 172.83
3 Model 3: mpg ~ drat + wt + hp 183.68 2.56 0.84   0.82 156.73 164.06
```

Assumptions of Model 3

```r
autoplot(mtcars_model_3) + ## from ggfortify
  theme_bw()
```




```r
shapiro.test(resid(mtcars_model_3)) ## From base R
```

```
    Shapiro-Wilk normality test

data:  resid(mtcars_model_3)
W = 0.91718, p-value = 0.01744
```

```r
dwtest(mtcars_model_3)    ## From lmtest package
```

```
    Durbin-Watson test

data:  mtcars_model_3
DW = 1.706, p-value = 0.1417
alternative hypothesis: true autocorrelation is greater than 0
```

```r
bptest(mtcars_model_3) ## From lmtest package
```

```
	studentized Breusch-Pagan test

data:  mtcars_model_3
BP = 1.5987, df = 3, p-value = 0.6597
```