Homework 5

Description

The `Auto` dataset from the ISLR2 package contains information on various automobile models from the 1970s and 1980s, providing a useful context for exploring relationships between vehicle characteristics and fuel efficiency. It includes **392 observations** on **nine variables**, such as `mpg` (miles per gallon), `horsepower`, `weight`, `acceleration`, `displacement`, `cylinders`, and `year`. These variables are a mix of quantitative and categorical data, with the `name` column identifying each car model. The dataset is particularly valuable for regression analysis due to its real-world relevance and the presence of nonlinear relationships, multicollinearity, and opportunities for transformation—making it ideal for studying how predictor variables influence fuel efficiency.

```r
library(here)                                          ## File Path Management
library(ISLR2)                                         ## Data Extraction
library(dplyr)                                         ## Data Transformation
library(tidyr)                                         ## Data Transformation
library(ggplot2)                                       ## Data Visualization
library(broom)                                         ## Data Analysis
source(here("R","assessment_regression.R"))
```

Question: *Fit a linear regression model to predict `mpg` using `horsepower`. How well does the model fit the data, and what does the residual plot suggest about the relationship between the two variables?*

Summary Statistics

```
auto_df <- Auto %>% drop_na()

# Summary statistics: mean, sd, min, max
summary_stats <- auto_df %>%
  select(mpg, horsepower) %>%
  summarise(across(everything(), list(
    mean = mean,
    sd   = sd,
    min  = min,
    max  = max
  ), .names = "{.col}_{.fn}")) %>%
  pivot_longer(
    everything(),
    names_to = c("variable", "statistic"),
    names_sep = "_",
    values_to = "value"
  )

summary_stats

# A tibble: 8 × 3
  variable   statistic  value
  <chr>      <chr>      <dbl>
1 mpg        mean       23.4
2 mpg        sd          7.81
3 mpg        min         9
4 mpg        max        46.6
5 horsepower mean      104.
6 horsepower sd         38.5
7 horsepower min        46
8 horsepower max       230
```

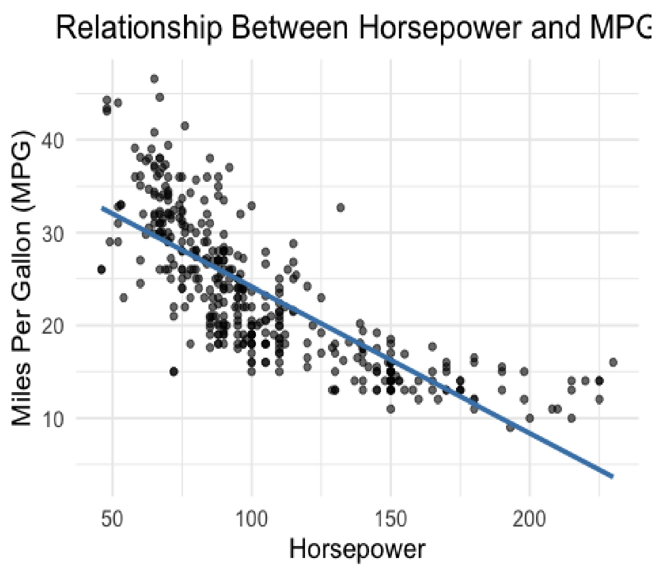
The summary statistics show that `mpg` ranges from 9.0 to 46.6 with a mean of 23.4, indicating a wide spread in fuel efficiency among cars. `Horsepower` has a mean of 104 with a standard deviation of 38.5, ranging from 46 to 230, suggesting substantial variation in engine strength across the dataset.

```
 # Compute correlation
cor_val <- auto_df %>%
  summarise(correlation = cor(mpg, horsepower))

cor_val

  correlation
1  -0.7784268
```

The correlation between `mpg` and `horsepower` is approximately -0.78, indicating a strong negative linear relationship: as horsepower increases, fuel efficiency tends to decrease.

Visualization

```
ggplot(auto_df, aes(x = horsepower, y = mpg)) +
  geom_point(alpha = 0.6) +
  geom_smooth(method = "lm", se = FALSE, color = "steelblue", linewidth = 1.2
) +
  labs(
    title = "Relationship Between Horsepower and MPG",
    x = "Horsepower",
    y = "Miles Per Gallon (MPG)"
  ) +
  theme_minimal(base_size = 14) +
  theme(plot.title = element_text(hjust = 0.5))
```



While the scatterplot initially suggests that transforming the response variable `mpg` might help, the pattern more strongly supports transforming the predictor `horsepower` to better linearize the relationship. This approach helps achieve more constant variance and a better-fitting linear model.

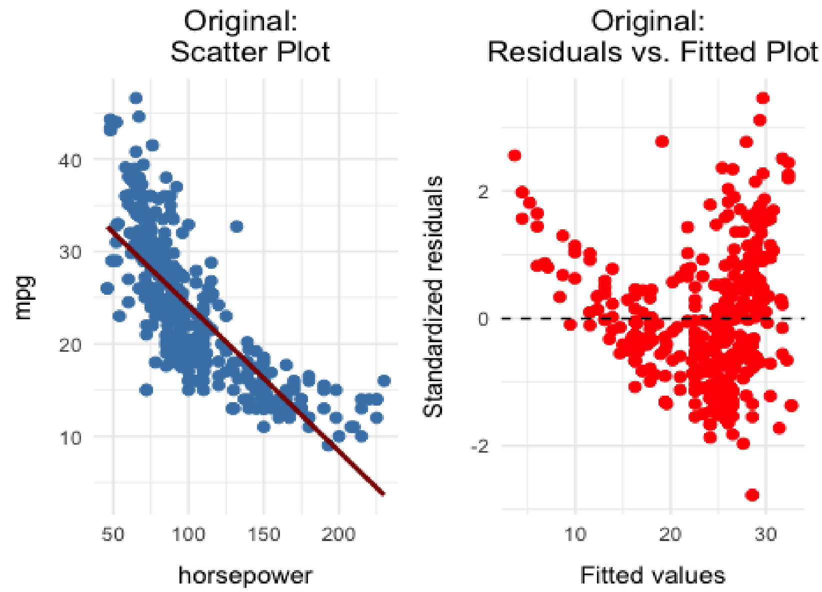
Analysis

Regular Model

```
model_0 <- lm(mpg ~ horsepower, data = auto_df)
summary(model_0)


Call:
lm(formula = mpg ~ horsepower, data = auto_df)

Residuals:
     Min       1Q   Median       3Q      Max 
-13.5710  -3.2592  -0.3435   2.7630  16.9240 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 39.935861   0.717499   55.66   <2e-16 ***
horsepower  -0.157845   0.006446  -24.49   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.906 on 390 degrees of freedom
Multiple R-squared:  0.6059,    Adjusted R-squared:  0.6049 
F-statistic: 599.7 on 1 and 390 DF,  p-value: < 2.2e-16
```

The intercept of approximately 39.94 suggests that a car with 0 horsepower is expected to achieve 39.94 mpg—though not realistic, this sets the baseline for interpretation. The slope of -0.158 indicates that, on average, each additional unit of horsepower is associated with a decrease of 0.158 mpg in fuel efficiency.

```
plot_scatter_resid(model_0,auto_df ,"Original")
```



The scatterplot reveals a nonlinear pattern between `horsepower` and `mpg`, with diminishing drops in mpg at higher horsepower. The residual plot shows a curved pattern, indicating nonlinearity and non-constant variance—violating linear model assumptions.

Transformations

```r
mod_1_auto_df <- auto_df %>%
  mutate(
    log_hp = log(horsepower),
    sqrt_hp  = sqrt(horsepower)
  )
```

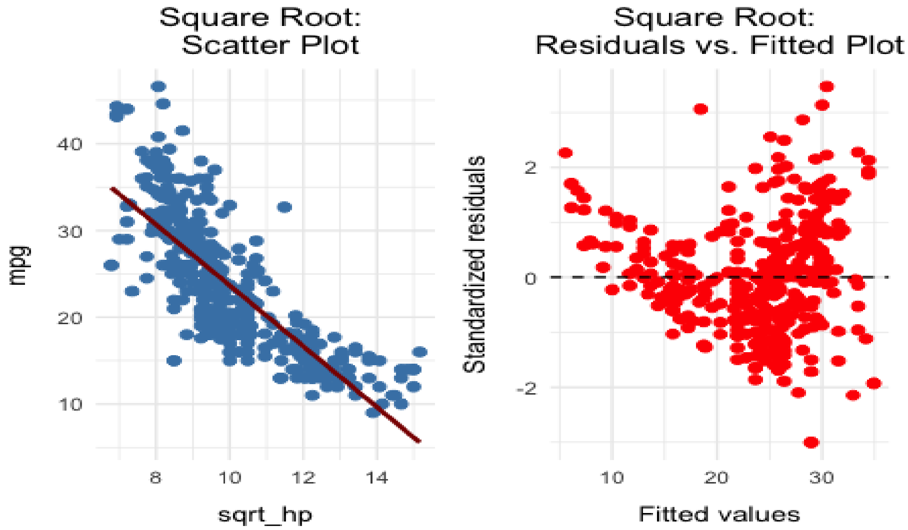
We selected `log(horsepower)` and `sqrt(horsepower)` as ad hoc transformations based on the curvature observed in the residual plots. Although a formal power transformation like Box-Tidwell could have been applied to the predictor, we opted not to pursue it, as it is beyond the course scope.

Square Root of hp

```r
# Linear model with sqrt(horsepower)
model_2 <- lm(mpg ~ sqrt_hp, data = mod_1_auto_df)
summary(model_2)


Call:
lm(formula = mpg ~ sqrt_hp, data = mod_1_auto_df)

Residuals:
     Min       1Q   Median       3Q      Max 
-13.9768  -3.2239  -0.2252   2.6881  16.1411 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   58.705      1.349   43.52   <2e-16 ***
sqrt_hp       -3.503      0.132  -26.54   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.665 on 390 degrees of freedom
Multiple R-squared:  0.6437,    Adjusted R-squared:  0.6428 
F-statistic: 704.6 on 1 and 390 DF,  p-value: < 2.2e-16
```

With the square root transformation applied to `horsepower`, the intercept of 58.71 represents the expected `mpg` when `sqrt(horsepower)` is zero. The slope of -3.50 means that for each unit increase in the square root of horsepower, the expected fuel efficiency decreases by 3.50 mpg, on average.

```r
plot_scatter_resid(model_2,mod_1_auto_df ,"Square Root")
```



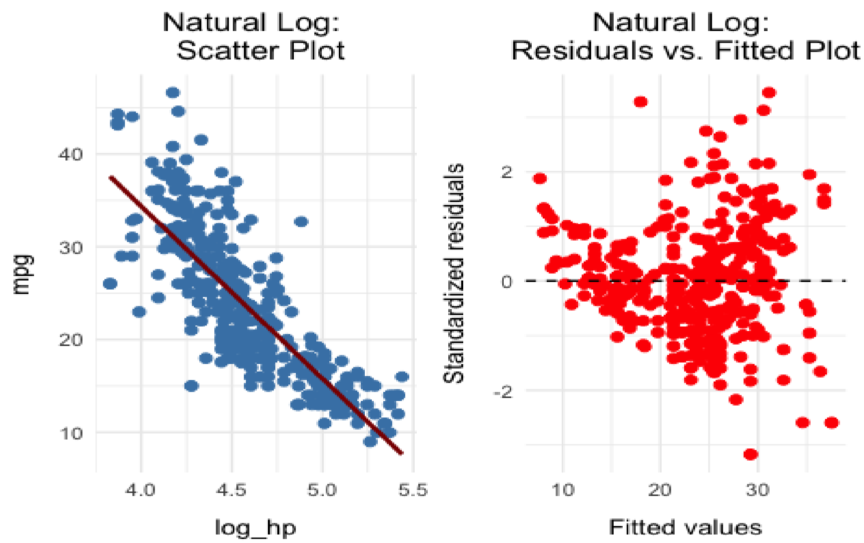
After the square root transformation, the scatterplot shows a more linear trend and the residual plot reveals reduced curvature. This suggests a better fit compared to the original model, although some mild heteroscedasticity remains.

Natural Log of hp

```r
# Linear model with log10(horsepower)
model_1 <- lm(mpg ~ log_hp, data = mod_1_auto_df)
summary(model_1)


Call:
lm(formula = mpg ~ log_hp, data = mod_1_auto_df)

Residuals:
     Min       1Q   Median       3Q      Max 
-14.2299  -2.7818  -0.2322   2.6661  15.4695 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 108.6997     3.0496   35.64   <2e-16 ***
log_hp      -18.5822     0.6629  -28.03   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.501 on 390 degrees of freedom
Multiple R-squared:  0.6683,    Adjusted R-squared:  0.6675 
F-statistic: 785.9 on 1 and 390 DF,  p-value: < 2.2e-16
```

Applying the natural log to `horsepower`, the intercept of 108.70 indicates the expected `mpg` when `log(horsepower)` is zero. The slope of -18.58 implies that, on average, a one-unit increase in `log(horsepower)` corresponds to a decrease of 18.58 mpg in fuel efficiency.

```
plot_scatter_resid(model_1,mod_1_auto_df ,"Natural Log")
```



After the square root transformation, the scatterplot shows a more linear trend and the residual plot reveals reduced curvature. This suggests a better fit compared to the original model, although some mild heteroscedasticity remains.

Interpretation of Results

```
summary_model_0 <- summarize_reg_model(model_0,'original')
summary_model_1 <- summarize_reg_model(model_1,'log_hp')
summary_model_2 <- summarize_reg_model(model_2,'sqrt_hp')


bind_rows(
  summary_model_0,
  summary_model_1,
  summary_model_2
)

      type     RSS  RSE   R2 Adj_R2     AIC     BIC
1 original 9385.92 4.91 0.61   0.60 1246.88 1250.85
2   log_hp 7899.93 4.50 0.67   0.67 1179.31 1183.28
3  sqrt_hp 8486.62 4.66 0.64   0.64 1207.39 1211.37
```

Among the three models, the log-transformed model (`log_hp`) has the lowest residual standard error (4.50), highest R^2 (0.67), and lowest AIC/BIC values, indicating it provides the best overall fit. The transformation successfully linearizes the relationship and reduces model error, making it the preferred choice.

Box-Cox Transformation

A Box-Cox transformation should not be used here because it is only applicable to the **response variable** (y), and `horsepower` is the **predictor**. Applying Box-Cox to x violates its assumptions and intended use.