Homework 1 Solutions

Question 5

Question: Write out the null and alternative hypotheses for the global F-test in Model 2 in words and symbols. What does this test tell us?

Full Credit Answer

In Words:

The global F-test for Model 2 tests whether the overall regression model is useful. Specifically, it tests whether at least one of the predictors (`hp`, `wt`) contributes significantly to explaining the variation in `mpg`.

- **Null hypothesis (H):** None of the predictors explain variation in `mpg`. That is, all regression coefficients (excluding the intercept) are zero.
- **Alternative hypothesis (H):** At least one predictor has a non-zero coefficient and contributes to the model.

In Symbols:

$$H_0 : \beta_1 = \beta_2 = 0$$

- **H :** At least one   0 (for i $=$ 1 or 2)

$$H_1 : \textbf{At least one } \beta_i \neq \textbf{ (for i = 1 or 2)}$$

Grading Rubric (Total: 2.25 points)

Component	Criteria	Points
A. Hypotheses in Words	Clear and correct description of what H_0 and H_1 mean in context of model utility	**0.75**
B. Hypotheses in Symbols	Symbols correctly reflect all slopes $= 0$ (H_0) vs at least one $\neq 0$ (H_1)	**0.75**
C. Interpretation of the F-test	Explains that the F-test assesses overall model usefulness / whether at least one predictor matters	**0.75**

Example Partial Credit Scenarios

- **2.0 pts:** Fully correct words and symbols, but vague or incomplete interpretation of what the F-test means.
- **1.5 pts:** Only includes the hypotheses (words and symbols) but omits the interpretation.
- **1.0 pt or less:** Misstates the meaning of hypotheses (e.g., confuses individual t-tests with global F-test).

Question 8

Question Based on the partial F-test comparing Model 2 and Model 3, should we include drat in the model? Justify your answer using statistical evidence and practical reasoning.

Answer Key

Short Answer:
No, we should likely **exclude drat** from the model.

Justification Using Statistical Evidence:

- The partial F-test comparing Model 2 (`mpg ~ hp + wt`) and Model 3 (`mpg ~ hp + wt + drat`) yielded:

  - **F = 1.73, p-value = 0.1988**
  - This p-value is well above 0.05, indicating that **drat does not significantly improve** the model's explanatory power.
  - The **coefficient for drat** in Model 3 was not statistically significant (p = 0.198755).

Justification Using Practical Reasoning: - Since `drat` does not contribute statistically and the change in adjusted R^2, AIC, and BIC between Models 2 and 3 is **minimal**, there is no strong evidence of practical gain.

- Including unnecessary variables can lead to overfitting and reduced model interpretability.

Grading Rubric (Total: 2.25 points)

Component	Criteria	Points
A. Statistical Justification	Correctly interprets p-value from partial F-test and coefficient p-value	**1.00**
B. Practical Justification	Explains why statistical insignificance also implies lack of practical importance	**0.75**
C. Conclusion	States a clear and justifiable recommendation (e.g., exclude `drat`)	**0.50**

Example Responses for Grading

Score	Description
2.25	Fully correct interpretation of p-value, reference to lack of improvement, and practical reasoning
2.0	Misses a small point (e.g., forgets coefficient p-value or doesn't mention model simplicity)
1.5	Only gives statistical justification, no mention of practical relevance
1.0 or less	Misinterprets the F-test or gives unsupported recommendation