



Quiz: Hypothesis testing

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You have to answer:

- What would be the null hypothesis (H_0)
- What would the alternative hypothesis (H_1)
- What will be the estimator
- What will be the test statistic
- What will be the null hypothesis distribution

Problem 1

A researcher compares inflammation levels between smokers and non-smokers. The data are highly skewed and contain outliers.

Problem 1 - Solution

A researcher compares inflammation levels between smokers and non-smokers. The data are highly skewed and contain outliers.

H₀: The distributions of the two groups are identical
(Often interpreted as equal medians, but technically distributions)

H₁: The distributions differ

Estimator: Rank sums

Test statistic: Mann–Whitney U

Distribution under H₀: Exact U distribution (small samples), Approximate normal (large samples)

Problem 2

A researcher wants to test whether a new drug changes mean systolic blood pressure. In the general population, the mean is known to be 120 mmHg. She measures blood pressure in 40 treated patients.

Problem 2 - Solution

A researcher wants to test whether a new drug changes mean systolic blood pressure. In the general population, the mean is known to be 120 mmHg. She measures blood pressure in 40 treated patients.

H_0 : $\mu = 120$

H_1 : $\mu \neq 120$ (two-sided)

Estimator: sample mean

Test Statistic: one-sided t-test

Distribution under H_0 : t-distribution with $n-1$ degrees of freedom

Problem 3

A pain score (0–10 scale) is measured in 20 patients after treatment.
The historical median pain score is 5.
The distribution of scores is strongly skewed.

Problem 3 - Solution

A pain score (0–10 scale) is measured in 20 patients after treatment.
The historical median pain score is 5.
The distribution of scores is strongly skewed.

H₀: Median pain score = 5

H₁: Median pain score \neq 5

Estimator: Median of differences (or signed ranks)

Test statistic: Wilcoxon Signed-Rank (W)

Distribution under H₀: Exact distribution of W (small n) or Approximate normal distribution (large n)

Problem 4

A vaccine manufacturer claims the vaccine prevents infection in 90% of cases. In a sample of 150 vaccinated individuals, 120 did not get infected.

Problem 4 - Solution

A vaccine manufacturer claims the vaccine prevents infection in 90% of cases. In a sample of 150 vaccinated individuals, 120 did not get infected.

H_0 : $p = 0.90$

H_1 : $p < 0.90$ (one-sided)

Estimator: sample proportion p

Test statistic: Z-score

Distribution under H_0 : Standard normal (approximate, via CLT)

Problem 5

A manufacturer claims the standard deviation of a machine's output is 5 units.
A quality control analyst measures 20 outputs.

Problem 5 - Solution

A manufacturer claims the standard deviation of a machine's output is 5 units. A quality control analyst measures 20 outputs.

$$H_0: \sigma^2 = 25$$

$$H_1: \sigma^2 \neq 25$$

Estimator: sample variance s^2

Test statistic: χ^2 statistic

Distribution under H_0 : Chi-square with $n-1$ df

Problem 6

A psychologist compares mean anxiety scores between treated and untreated groups. Each group has 25 participants.

Problem 6 - Solution

A psychologist compares mean anxiety scores between treated and untreated groups. Each group has 25 participants.

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Estimator: difference in sample means $\bar{x}_1 - \bar{x}_2$

Test statistic: two-sample t-statistic

Distribution under H_0 : t-distribution with appropriate df

Problem 7

A nutritionist measures cholesterol levels before and after diet intervention in the same 30 patients.

Problem 7 - Solution

A nutritionist measures cholesterol levels before and after diet intervention in the same 30 patients.

H_0 : mean difference = 0

H_1 : mean difference $\neq 0$

Estimator: mean of paired differences

Test statistic: paired t-statistic

Distribution under H_0 : t-distribution with $n-1$ df