

Vocabulary Fun

- ▶ The varying coefficients are sometimes called **random effects**, since they are associated with distributional statements like $\alpha_j = N(\mathbf{Z}_j\gamma, \sigma_\alpha^2)$.
- ▶ The term **fixed effects** is more nebulous with different meanings from different authors:
 - ▷ coefficients that are constant across individuals (most common definition)
 - ▷ nuisance coefficients that are uninteresting but included
 - ▷ coefficients in population models
 - ▷ realized random variables
 - ▷ MLE values assuming infinite group-level variance(see the discussion in G&H, p.245).
- ▶ Sometimes these models labeled as **mixed effects** models.
- ▶ Prescription: use **multilevel models** or **hierarchical models** with appropriate descriptor or detailed specification.

Vocabulary Overview

- For the data matrices, \mathbf{X}_i for individual i in cluster j , and \mathbf{Z}_j for cluster j , there are five canonical models that we will look at:

“Completely Pooled”

$$y_i = \beta_0 + \beta_1 \mathbf{X}_i + \gamma \mathbf{Z} + e_i$$

“Fixed Effect”

$$y_i = \beta_{j0[i]} + \beta_1 \mathbf{X}_i + e_i$$

“Random Effect”

$$y_i = \beta_{j0[i]} + \beta_1 \mathbf{X}_i + \gamma \mathbf{Z}_j + e_i$$

“Random Intercept and Random Slope”

$$y_i = \beta_{j0[i]} + \beta_{j1[i]} \mathbf{X}_i + e_i$$

“Completely Unpooled”

$$y_{j[i]} = \beta_{j0} + (\beta_{j1} \mathbf{X}_{j[i]} + \gamma \mathbf{Z}_j) + e_{j[i]}$$

- This is produced by replacing the previous γ coefficient names with common regression-style language.
- “Fixed” and “random” can differ in definition by literature (Kreft and De Leeuw 1988, Section 1.3.3, Gelman 2005), and better notation is “random intercepts” for “fixed effect,” and “varying-intercept, varying-slope” for “random intercept and random slope.”
- Best to conceptualize these specifications as members of a larger multilevel family where indices are *turned-on* or *turned-off* systematically depending on the hierarchical purpose.