Angular Kinetics
Moment of Inertia

Anthony A. Gatti, PhD
Stanford University
Biomechanical Analysis of Human Movement
Queen’s University (KNPE 254)

Warmup

What is the angular equivalent of displacement?

- Position
- Angular displacement
- Angle
- None of the above

Total Results: 0

What is the angular equivalent of displacement?

Position

Angular displacement

Angle

None of the above

What is the angular equivalent of displacement?

Position

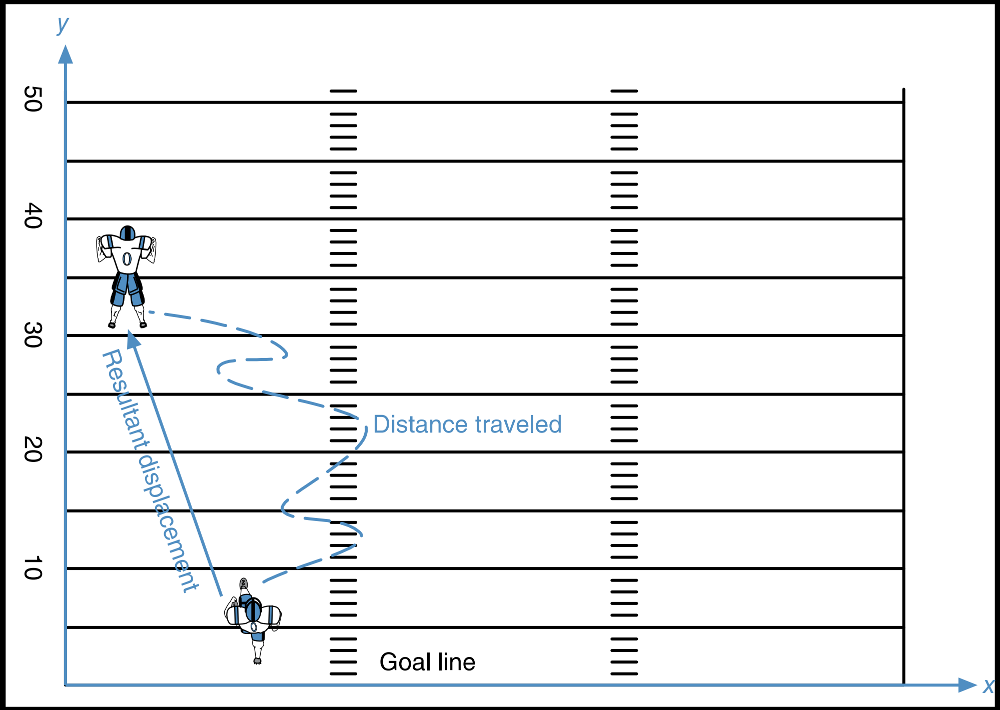
Angular displacement

Angle

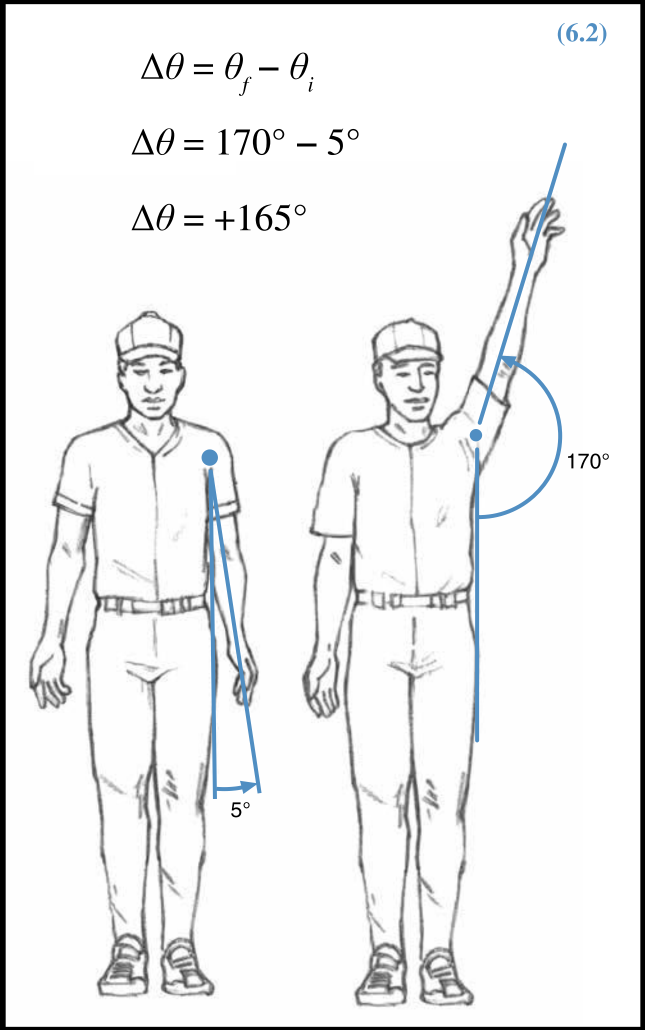
None of the above

(Angular) Displacement

Linear Displacement



Angular Displacement



What is the angular equivalent of acceleration?

RPM	Omega	Angular Acceleration	Circumfrential Acceleration	None of the above

(Angular) Acceleration

Average
Acceleration

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

Average
Angular Acceleration

$$\bar{\alpha} = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$$


POPCORN
MOVIE TIME


I'M SO EXCITED!!
CAN'T WAIT!!!

What did you notice in the video?

What did you wonder about?

Top

Powered by Poll Everywhere

What did you notice/wondera bout in the video?

- There’s a solid and a hollow cylinder
- Are they the same weight?
- They are on a ramp
  - Why are they on a ramp!?
- It’s a race!
  - Who’s going to win?

Linear Inertia (mass)

A body's tendency/ability to resist change in motion

$$F = ma$$

$F = Force$
$\mathrm{m} = mass$
$a = acceleration$

Linear Inertia (mass)

A body's tendency/ability to resist change in motion

$$F = ma$$

$$\frac{F}{m} = a$$

$$a = \frac{F}{m}$$

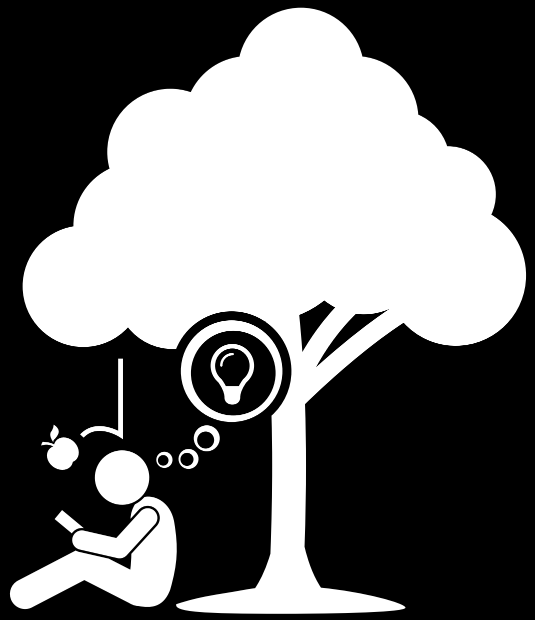
$$a \propto F \qquad constant\ m$$

$$a \propto \frac{1}{m} \qquad constant\ F$$

Angular Inertia (Moment of Inertia)

A body's tendency/ability to resist change in angular motion

$$\tau = I\alpha$$



$\tau = Torque$
$I = moment\ of\ inertia$
$\alpha = angular\ acceleration$

Angular Inertia (Moment of Inertia)

A body's tendency/ability to resist change in angular motion

$$\tau = I\alpha$$

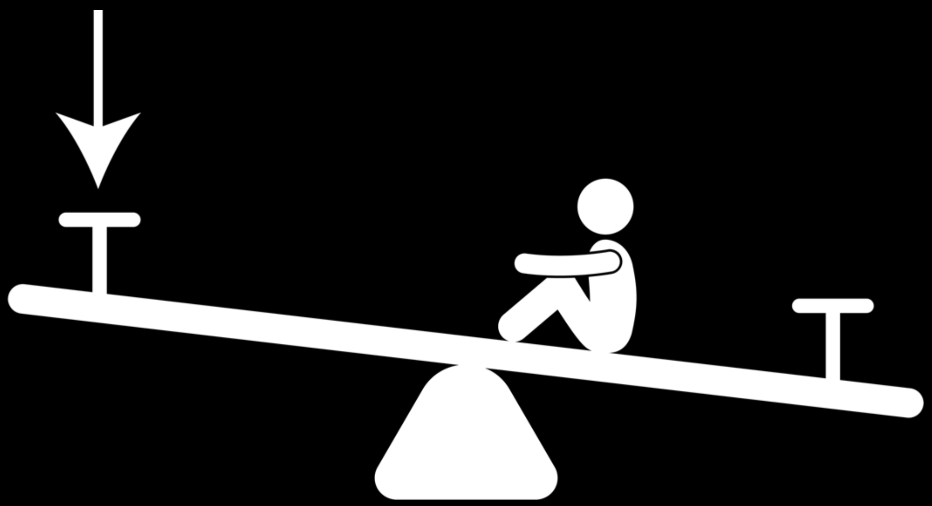
$$\alpha \propto \tau \qquad constant\ I$$

$$\frac{\tau}{I} = \alpha$$

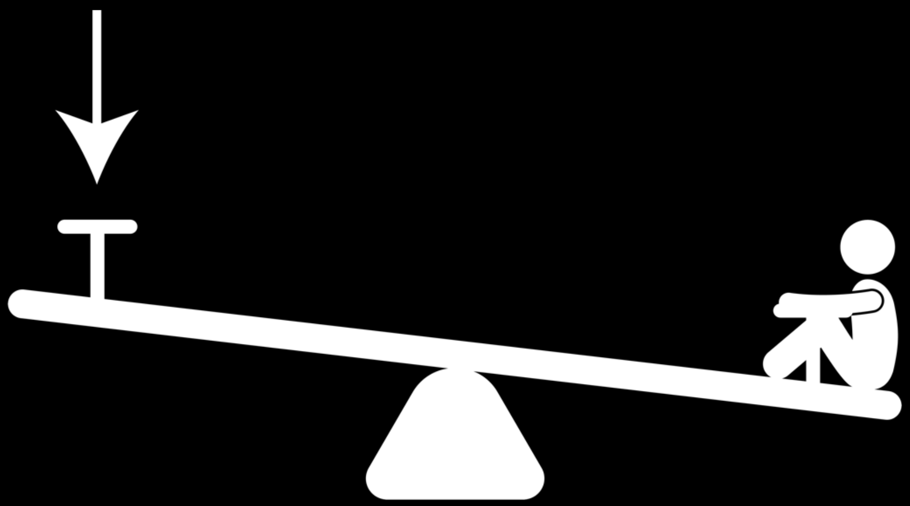
$$\alpha \propto \frac{1}{I} \qquad constant\ \tau$$

$$\alpha = \frac{\tau}{I}$$

Moment of Inertia (I) in the real world

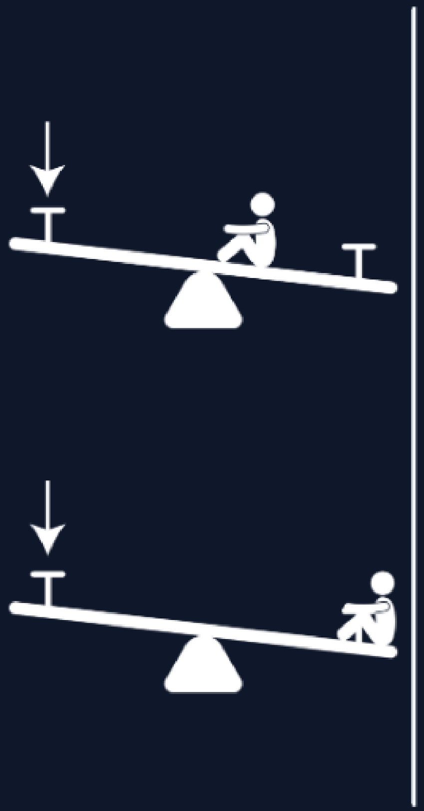


Close to axis of rotation



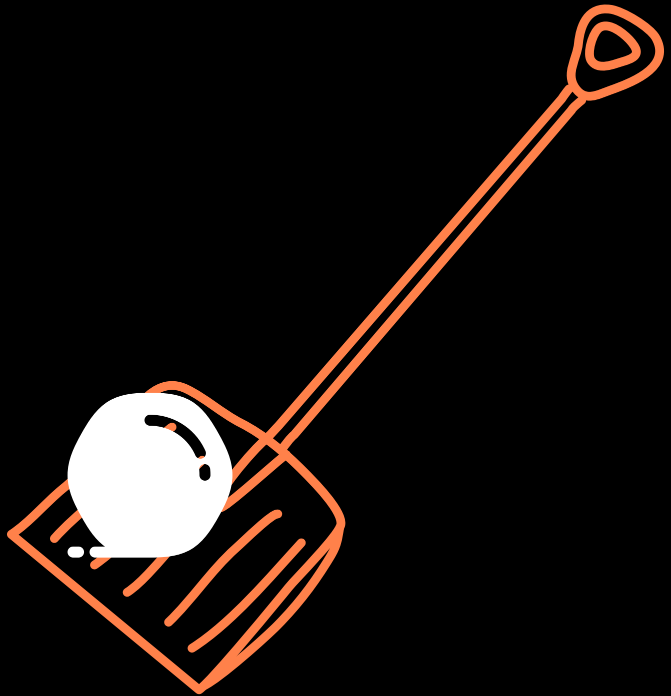
Far from axis of rotation

Which will resist rotating most?



Moment of Inertia (I) in the real world

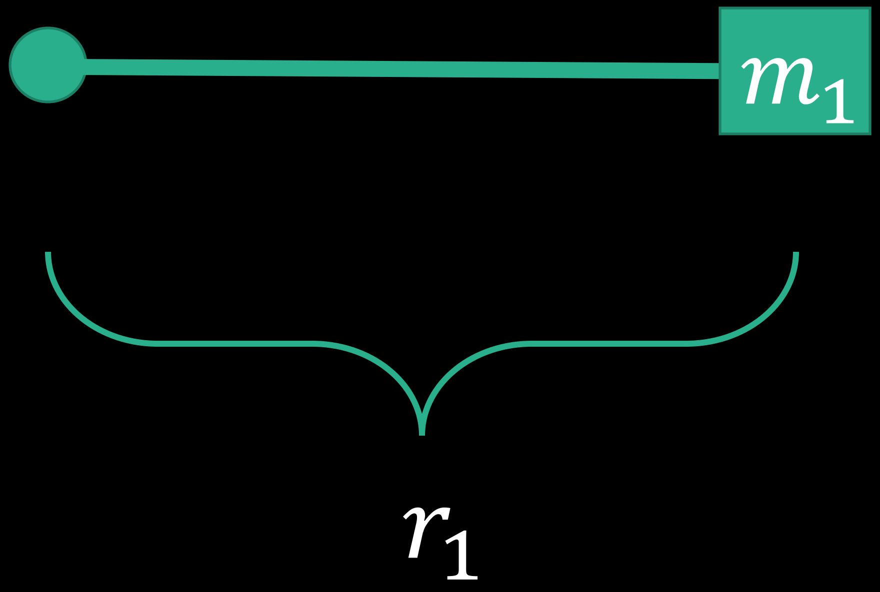
Large mass



Small mass

Calculating Moment of Inertia

$$I = mr^2$$



$I = moment\ of\ inertia$
$m = mass$
$r = radius$

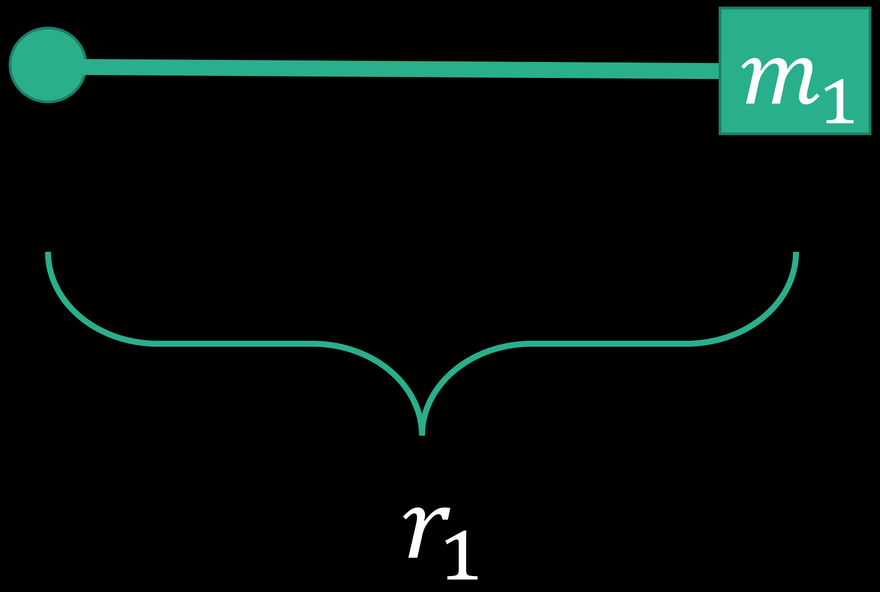
Calculating Moment of Inertia

$$I = mr^2$$

$$I = 10 \times 1^2$$

$$I = 10\,kg{\cdot}m^2$$

$m_1 = 10kg$
$r_1 = 1m$



Calculating Moment of Inertia

$$I = mr^2$$

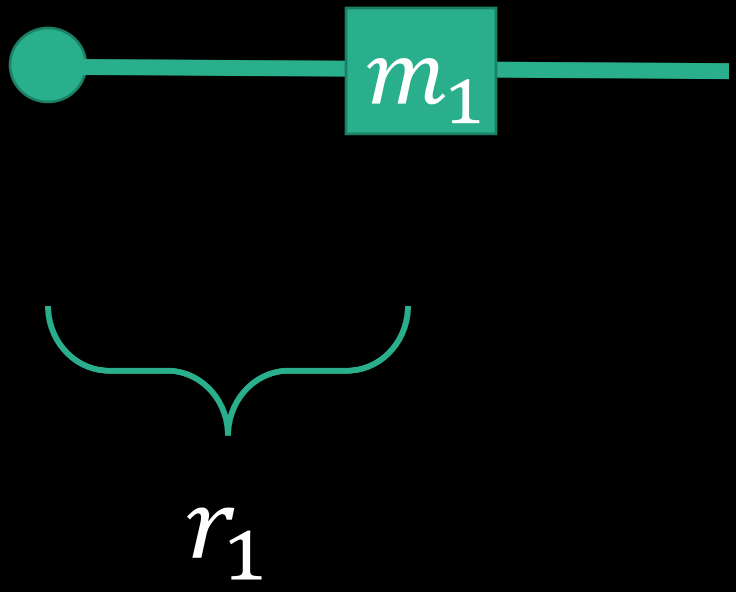
$$I = 10 \times 0.5^2$$

$$I = 10 \times 0.25$$

$$I = 2.5\ kg{\cdot}m^2$$

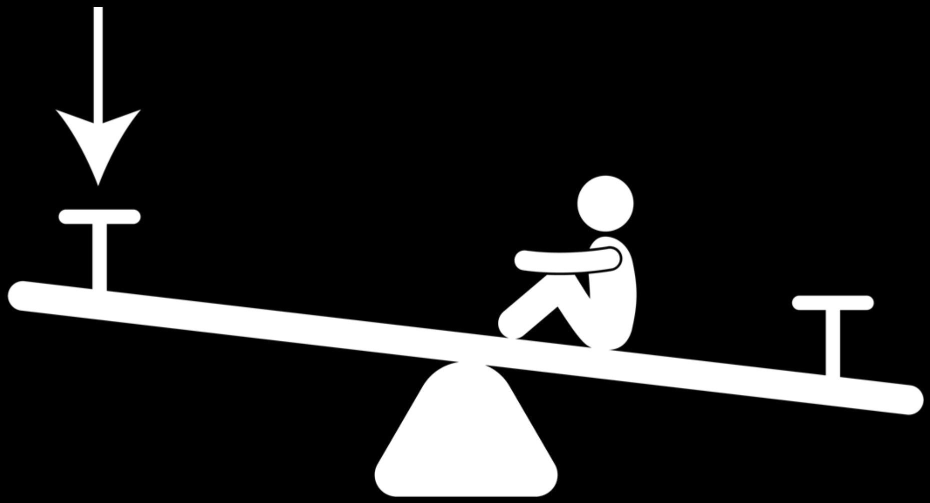
$m_1 = 10kg$

$r_1 = 0.5m$

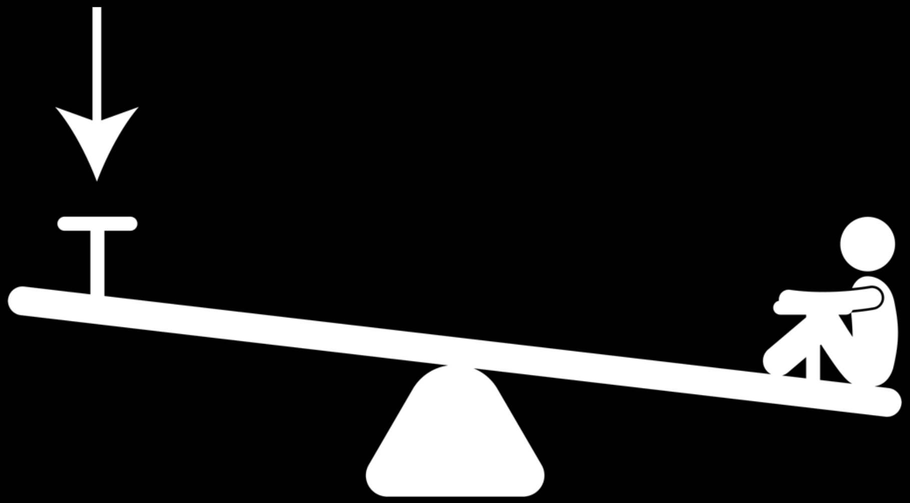


Calculating Moment of Inertia

$$I = mr^2$$



Small Moment of Inertia



Big Moment of Inertia

Calculating Moment of Inertia

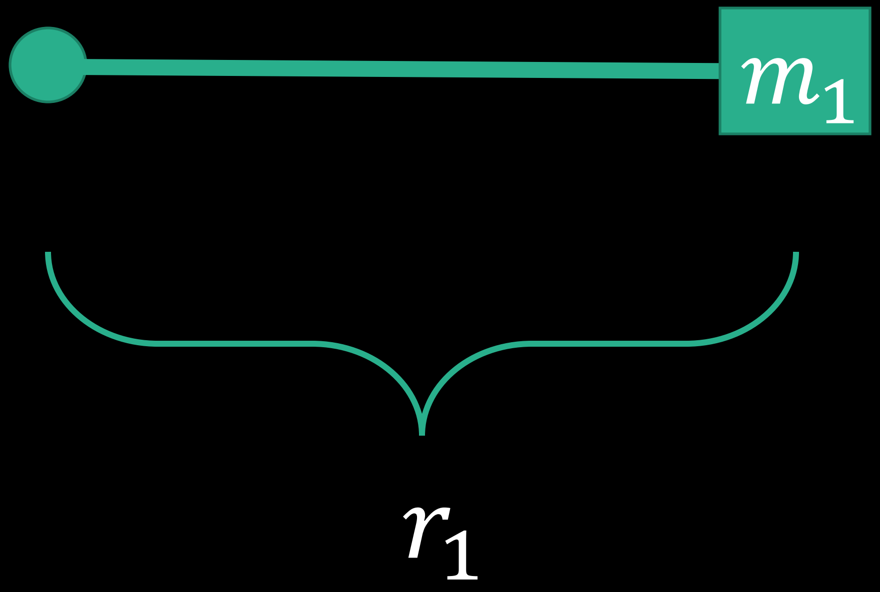
$$I = mr^2$$

$$I = 10 \times 1^2$$

$$I = 10 \ kg{\cdot}m^2$$

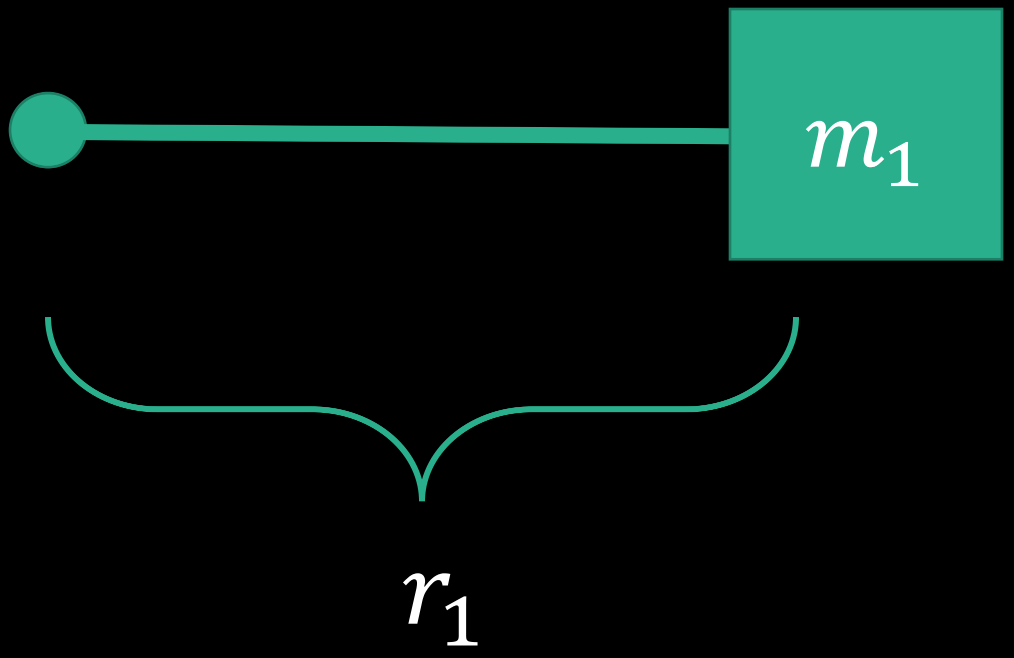
$m_1 = 10 \ kg$

$r_1 = 1 \ m$



Calculating Moment of Inertia

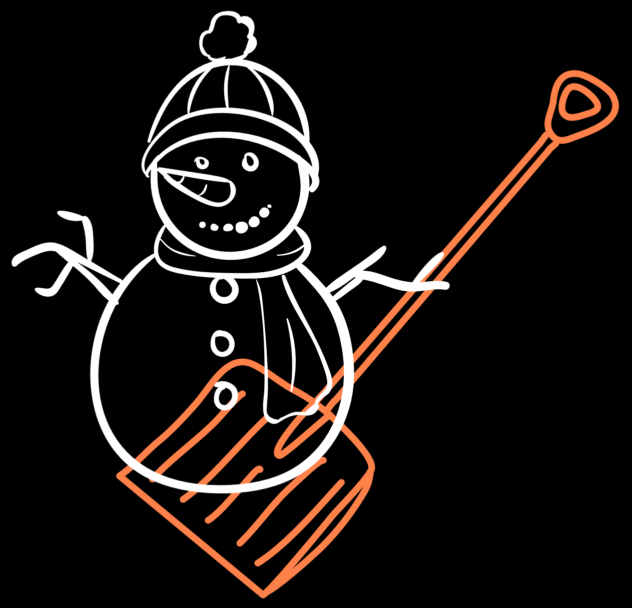
$$I = mr^2$$
$$I = 20 \times 1^2$$
$$I = 20\ kg{\cdot}m^2$$



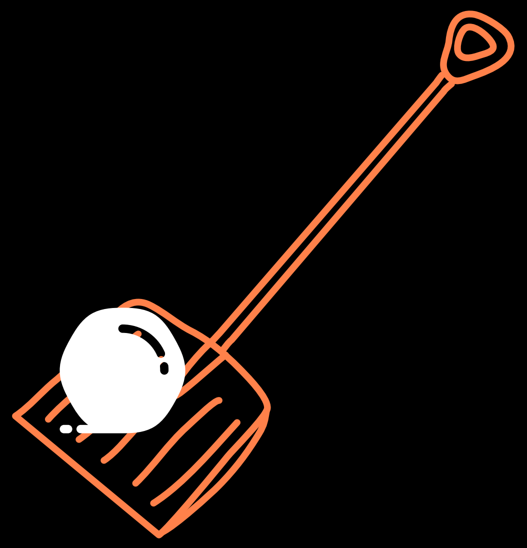
$m_1 = 20\ kg$
$r_1 = 1\ m$

Calculating Moment of Inertia

$$I = mr^2$$



Big Moment of Inertia



Small Moment of Inertia

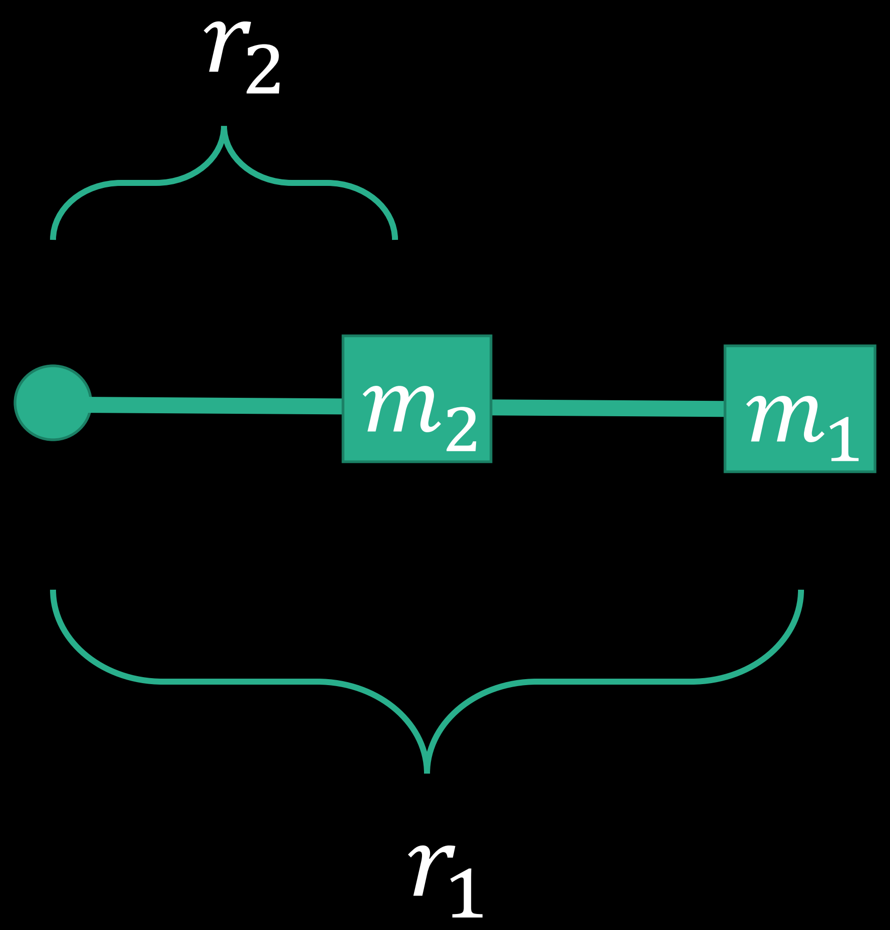
Calculating Moment of Inertia

$$I = \sum \quad m_i {r_i}^2$$

$I = moment\ of\ inertia$
$m = mass$
$r = radius$

Calculating Moment of Inertia

$$I = \sum m_i {r_i}^2$$

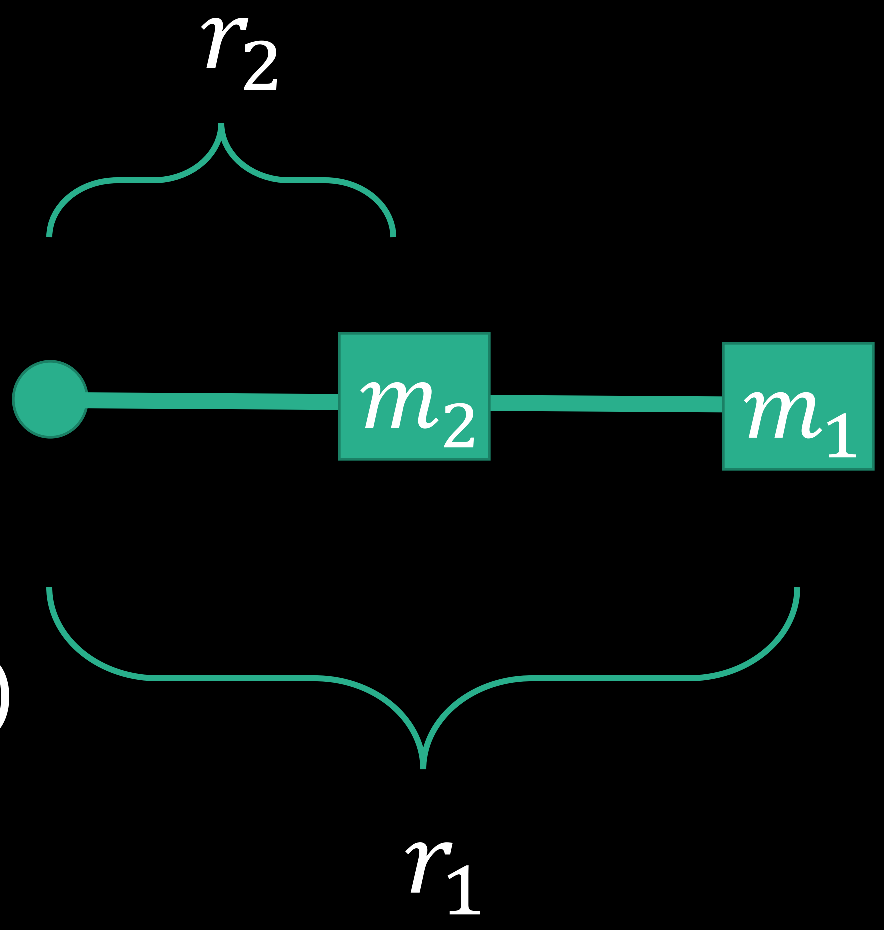


$I = moment\ of\ inertia$
$m = mass$
$r = radius$

$m_1 = m_2 = 10\ kg$
$r_1 = 1\ m$
$r_2 = 0.5\ m$

Calculating Moment of Inertia

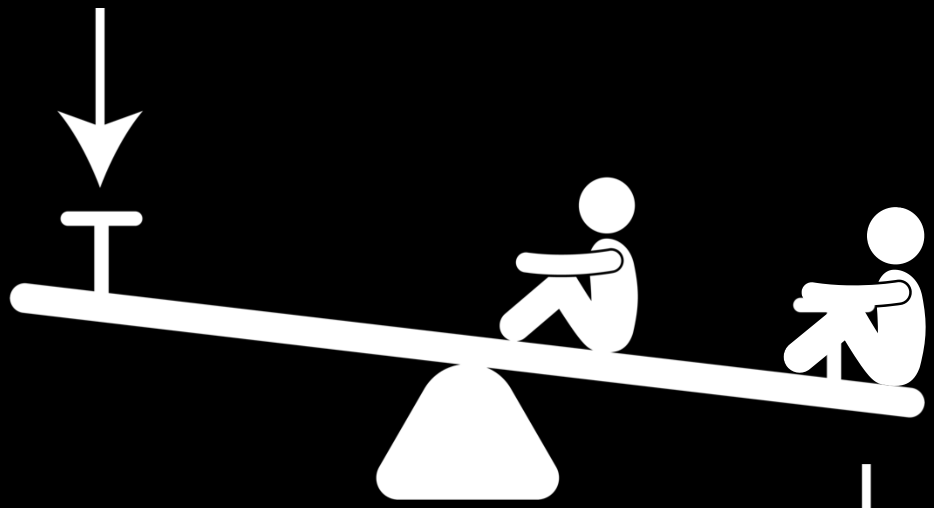
$$I = \sum \quad m_i {r_i}^2$$



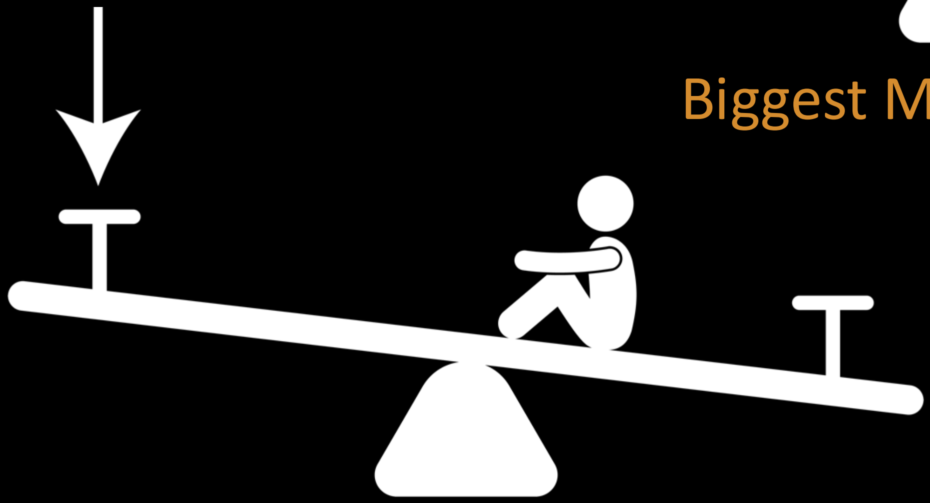
$$I = m_1 {r_1}^2 + m_2 {r_2}^2$$
$$I = 10\,(1^2) + 10(0.5^2)$$
$$I = 10 + 2.5$$
$$I = 12.5\ kg{\cdot}m^2$$

$$m_1 = m_2 = 10\,kg$$
$$r_1 \quad = 1\,m$$
$$r_2 \quad = 0.5\,m$$

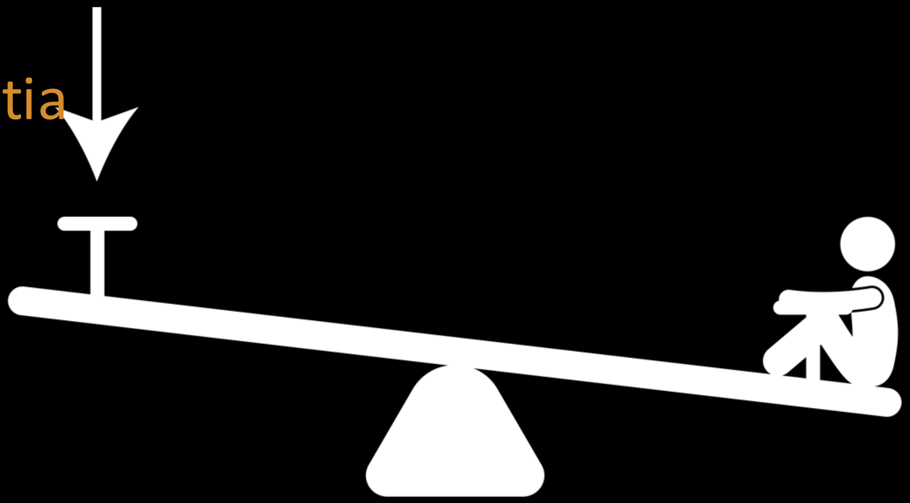
Calculating Moment of Inertia



Biggest Moment of Inertia



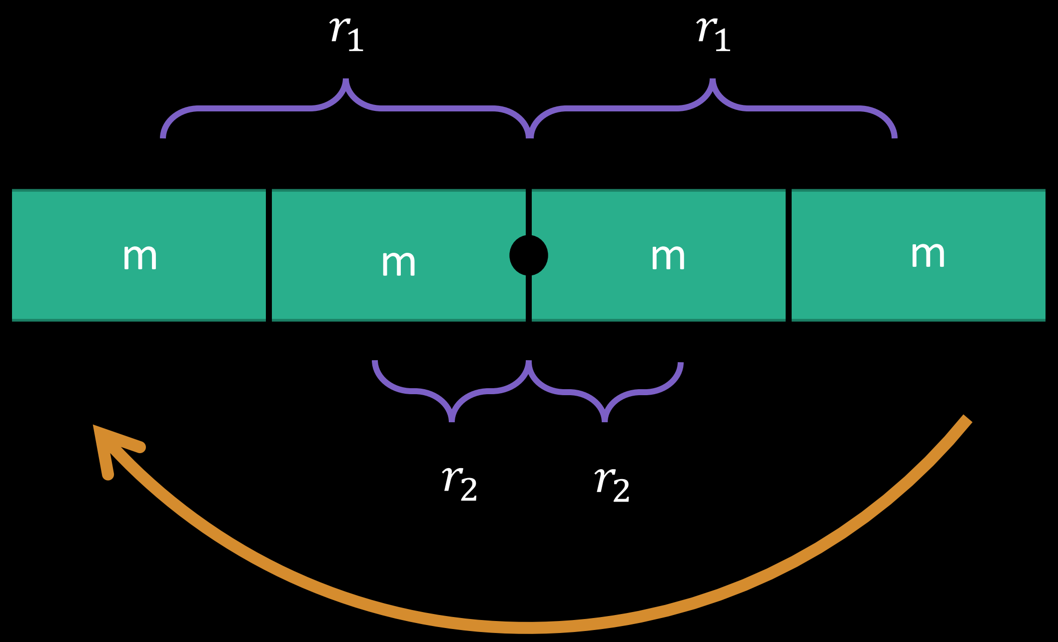
Small Moment of Inertia



Big Moment of Inertia

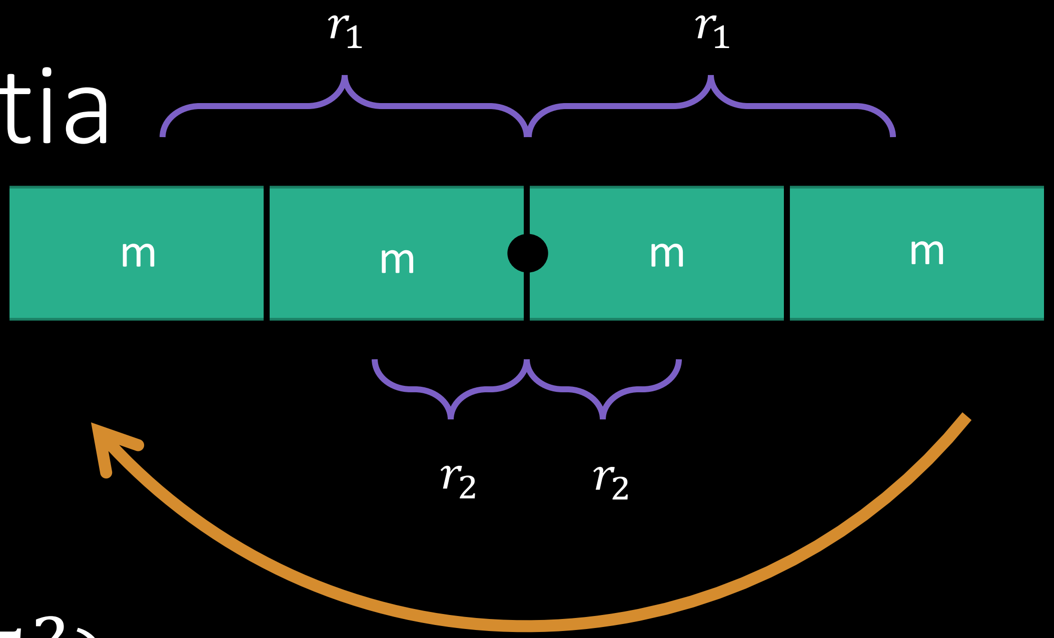
Calculating Moment of Inertia

$$I = \sum m_i {r_i}^2$$



$I = moment\ of\ inertia$
$m = mass$
$r = radius$

Calculating Moment of Inertia



$$I = \sum m_i {r_i}^2$$

$$\frac{I}{2} = m{r_1}^2 + m{r_2}^2$$

$$\frac{I}{2} = 10\,(1^2) + 10(0.5^2)$$

$$\frac{I}{2} = 10 + 2.5$$

$$\frac{I}{2} = 12.5$$

I for one half of the rod

$$I = 25\ kg{\cdot}m^2$$

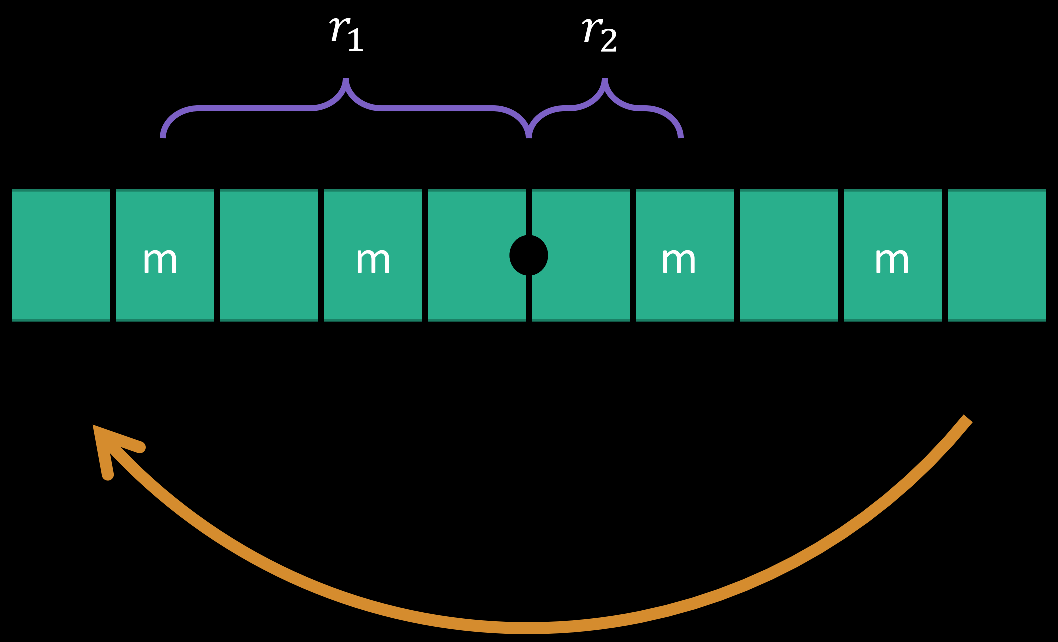
$m = 10\ kg$

$r_1 = 1\ m$

$r_2 = 0.5\ m$

Calculating Moment of Inertia

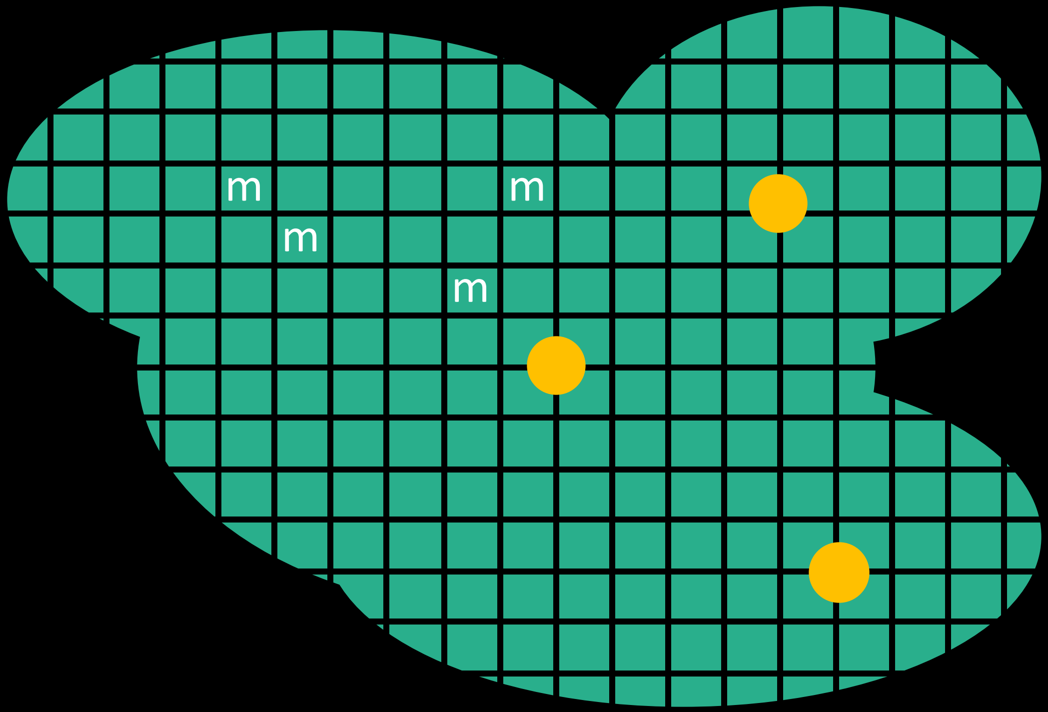
$$I = \sum \quad m_i {r_i}^2$$



$I = moment\ of\ inertia$
$m = mass$
$r = radius$

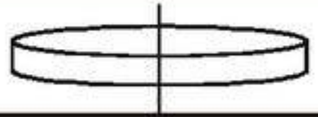
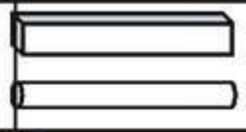
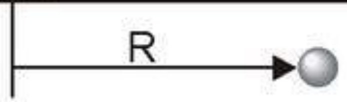
Calculating Moment of Inertia

$$I = \sum \; m_i {r_i}^2$$



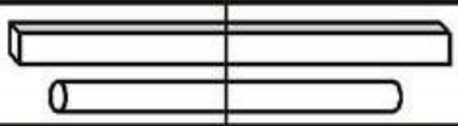
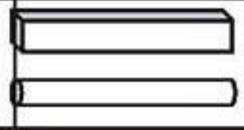
$I = moment\ of\ inertia$
$m = mass$
$r = radius$

Calculating Moment of Inertia

Object	*Drawing*	*Moment of Inertia*
Disk (rotated about center)		$\frac{1}{2}MR^2$
Ring (rotated about center)		MR^2
Rod or plank (rotated about center)		$^1/_{12}ML^2$
Rod or plank (rotated about end)		$^1/_3ML^2$
Sphere		$^2/_5MR^2$
Satellite		MR^2

Moving I - Parallel Axis Theorem

$$I_{new\ axis} = I_{Center\ of\ gravity} + mr^2$$

Rod or plank (rotated about center)		$^1/_{12}ML^2$
Rod or plank (rotated about end)		$^1/_3ML^2$

Moment of Inertia (I) in Sports

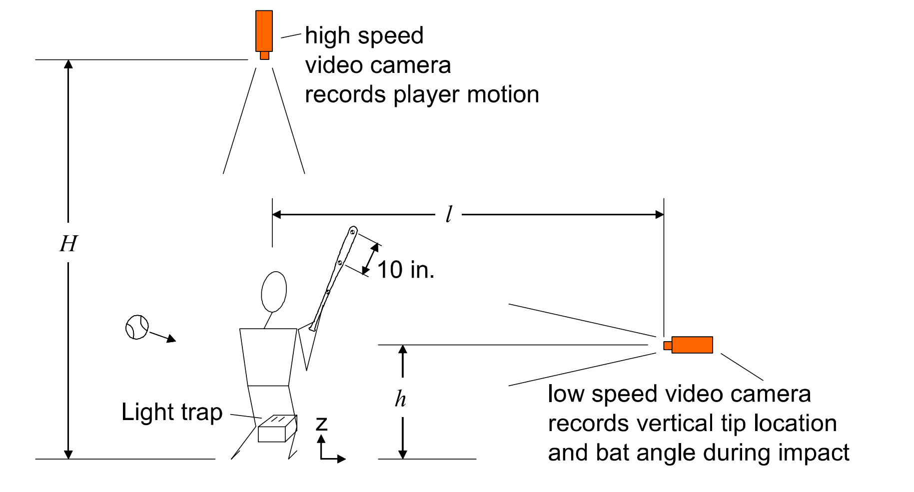


Figure 2. Schematic of video system used to determine the bat speed.

https://www.acs.psu.edu/drussell/bats/bat-moi.html

Moment of Inertia (I) in Sports

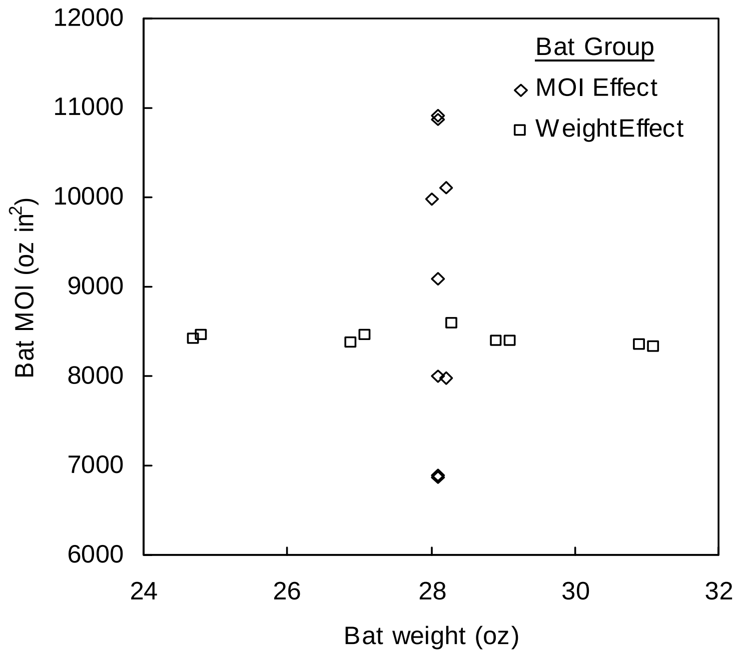


Figure 1. Weight and MOI of the 20 bats

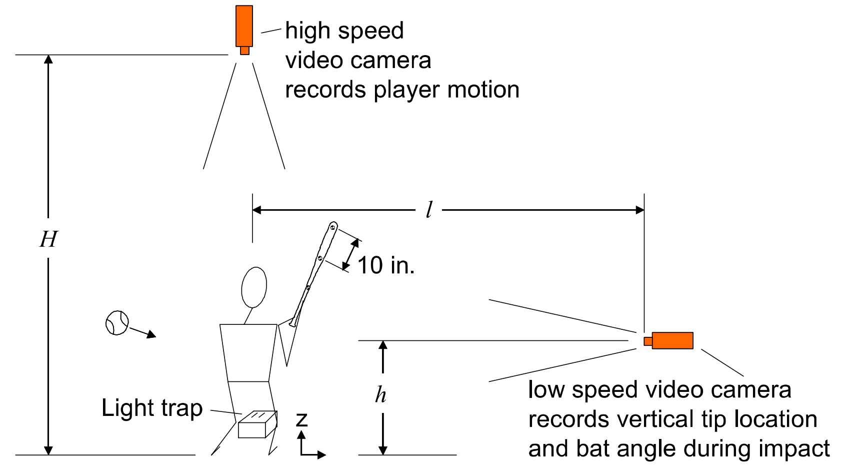


Figure 2. Schematic of video system used to determine the bat speed.

https://www.acs.psu.edu/drussell/bats/bat-moi.html

Moment of Inertia (I) in Sports

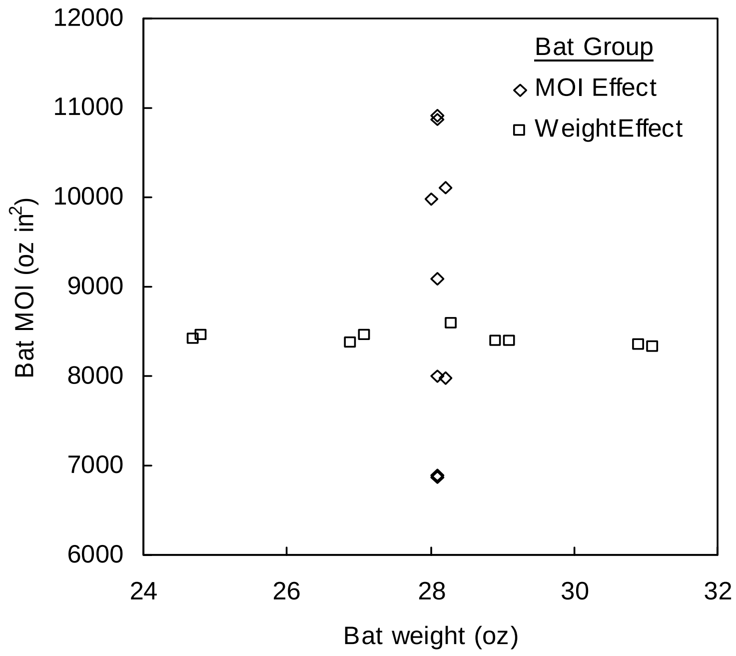


Figure 1. Weight and MOI of the 20 bats

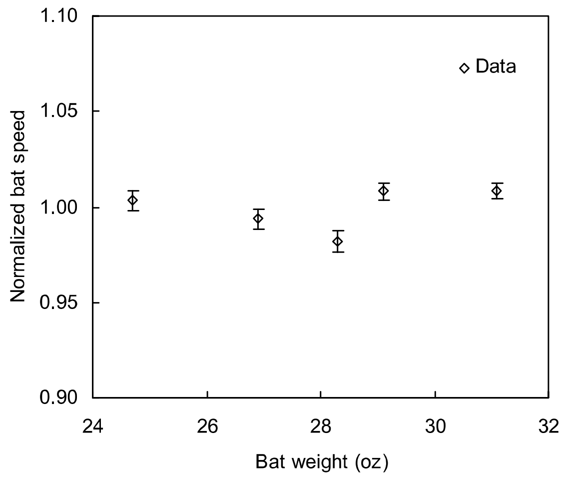


Figure 3. Normalized bat swing speed (6 inches from the tip) as a function of bat mass for bats with constant MOI.

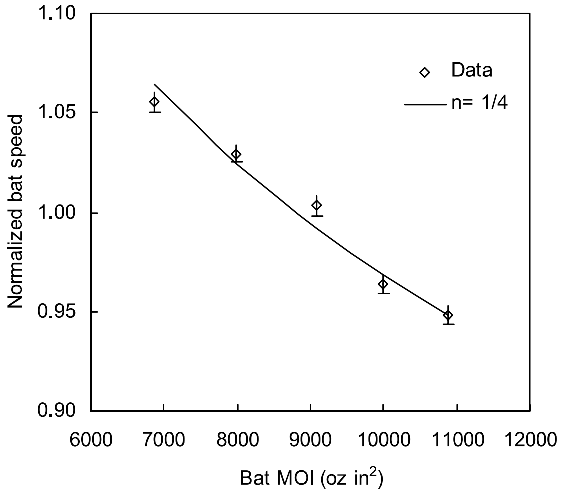


Figure 4. Normalized swing speed as a (6 inches from the tip) function of bat MOI for bats with constant mass.

https://www.acs.psu.edu/drussell/bats/bat-moi.html


I'M SO EXCITED!!
CAN'T WAIT!!!


NCSSM

When poll is active, respond at **pollev.com/queensbiomechanics220**

Text **QUEENSBIOMECHANICS220** to **37607** once to join

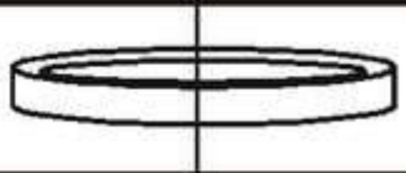
Which one will win?

Tie

Powered by Poll Everywhere

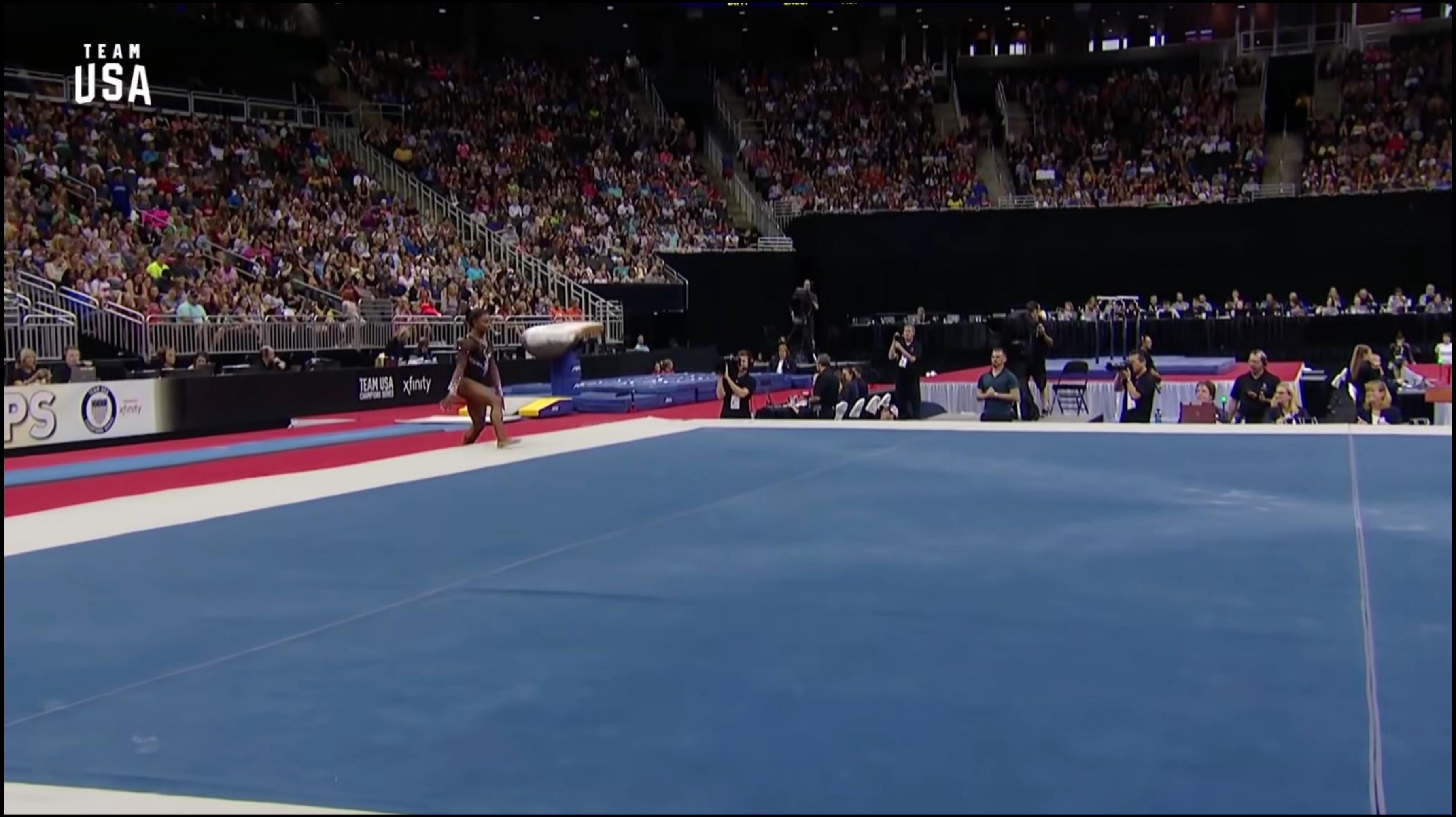

NCSSM

Why did the ring win!?

Object	*Drawing*	*Moment of Inertia*
Disk (rotated about center)		$\frac{1}{2}MR^2$
Ring (rotated about center)		MR^2

- Solid disk has a smaller I
- Therefore, it has a <u>smaller</u> resistance to change in angular motion.

Simone Biles



Simone Biles