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Evolutionary Dynamics

# Assignment #01

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## 1.1 Problem 1: Logistic difference equation

We can find stable points from

$$f(x,a) = ax(1-x) = x \tag{1.1}$$

as the roots of the quadratic equation:

$$ax^2 - (a-1)x = x(ax - (a-1)) = 0 \tag{1.2}$$

which are:

$$x_1 = 0 \tag{1.3}$$

$$x_2 = \frac{(a-1)}{a} \tag{1.4}$$

The fixed point $x_2$ is non-negative if $a \geq 1$. One can analyze the local stability of the difference equation 1.1 by examining the partial derivative of $f$ with respect to $x$ evaluated at each fixed point $x^*$:

$$f' = -ax - a(x-1) \tag{1.5}$$

Substituting $x_1$ and $x_2$ into 1.5 yields:

$$f'(x_1) = a \tag{1.6}$$

$$f'(x_2) = 2 - a \tag{1.7}$$

One finds that if $a > 1 \rightarrow f'(x_1) > 1$ with $x_1 = 0$, then $x^*$ is repelling, while if $1 < a < 3 \rightarrow 1 > f'(x_2) > -1$ with $x_2 = (a-1)/a$ is stable (attractor).

### 1.1.1 Point stability at different values of $a$

When $a = 0.9$ then $x_1 = 0.9, x_2 = 1.1$
When $a = 2.1$ then $x_1 = 2.1, x_2 = -0.1$

For $a$ values in excess of 3.57, the orbits $x(t, x0) = x0, x1, x2, ...$ depend crucially on the initial condition $x0$. Slight variations in $x0$ result in dramatically different orbits, an important characteristic of chaos.

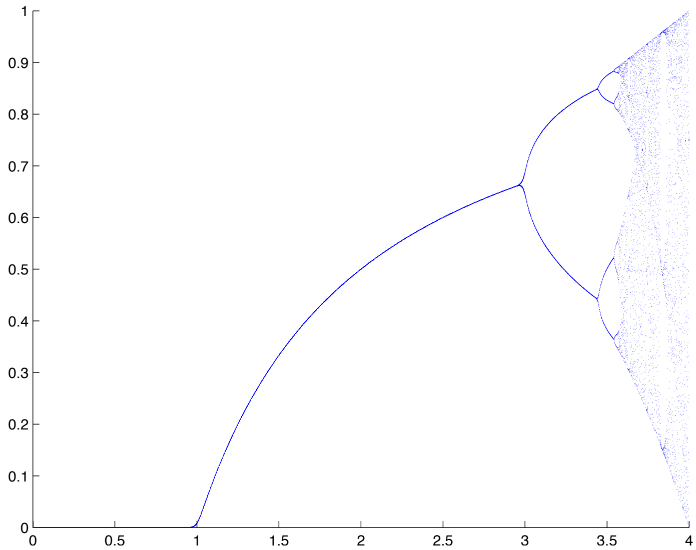


Figure 1.1: Logistic Map Bifurcation Diagram

## 2.1 Problem 2: Logistic growth in continuous time

We have to solve the equation

$$\frac{dx}{dt} = rx(1 - \frac{x}{K}) = r(x - \frac{x^2}{K}) = \frac{rx(K-x)}{K} \tag{2.8}$$

using the separation of variables

$$\frac{K}{x(K-x)}dx = rdt \tag{2.9}$$

decomposing the right part with partial fractions

$$\frac{K}{x(K-x)} = \frac{A}{x} + \frac{B}{K-x} \tag{2.10}$$

We find A and B

$$A = \frac{K - Bx}{K - x} \tag{2.11}$$

$$B = \frac{K - A(K-x)}{x} \tag{2.12}$$

Supposing $A = 1$, and according to above, $B$ is also equal to 1, so our partial fraction decomposition is

$$\frac{K}{x(K-x)} = \frac{1}{x} + \frac{1}{K-x} \tag{2.13}$$

Now we have to take the integral from both parts:

$$\int \frac{1}{x} + \frac{1}{K-x} dx = \int r dt \tag{2.14}$$

$$\int \frac{1}{x} + \int \frac{1}{K-x} dx = r \int dt \tag{2.15}$$

$$\ln x - \ln K - x = rt + x_0 \tag{2.16}$$

$$\ln \frac{x}{K-x} = rt + x_0 \tag{2.17}$$

$$\frac{x}{K-x} = x_0 e^{rt} \tag{2.18}$$

Finally the solution

$$x(t) = x_0 K e^{rt} \frac{1}{K + x_0(e^{rt} - 1)} \tag{2.19}$$

### 2.1.1 Determining the stability of equilibria

To determine the stability of the two equilibria points found solving the equation 2.8 in 0

$$x_1 = 0 \tag{2.20}$$

$$x_2 = K \tag{2.21}$$

one has to derive it for $x$:

$$f' = -r(\frac{x}{K} - 1) - \frac{rx}{K} \tag{2.22}$$

Substituting $x_1$ and $x_2$ into 2.22 yields:

$$f'(x_1) = -1 \tag{2.23}$$

$$f'(x_2) = -r \tag{2.24}$$

One finds that if $f'(x_1)$ with $x_1 = 0$, then $x^*$ is always stable, while if $forr < 0 \rightarrow f'(x_2) > 0$ with $x_2 = -r\ x^*$ is repelling.

## 3.1 Problem 3: Hardy-Weinberg equilibrium

## 4.1 Problem 4: Sequence alphabets

## 5.1 Problem 5: Random sequences

## 6.1 Problem 6: Quasispecies