

Synthetic Biology

# Assignment #01

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## 0.1 The model

The model for a two protein network can be given by the system of equations:

$$\frac{dx}{dt} = f(y) - d_1x \quad (0.1a)$$

$$\frac{dy}{dt} = g(x) - d_2y \quad (0.1b)$$

where  $x$  and  $y$  are the concentration of the two proteins,  $f$  and  $g$  are “repression functions” and  $d_1x$ ,  $d_2y$  are positive constant describing the protein decay.

$$f(y) = \frac{k_1}{1 + [Y]^n}^* \quad (0.2a)$$

$$g(x) = \frac{k_2}{1 + [X]^m}^\dagger \quad (0.2b)$$

The level of repression of  $[X]$  in a “mixed” circuit model is described by a Hill function, which models cooperativity of binding between two different molecular species. Using the conditions for a change in the number of steady states derived from nullclines analysis:

$$d_1[X] = \frac{k_1}{1 + [Y]^n}^* \frac{1}{d_1} = \frac{k'_1}{1 + [Y]^n}^\ddagger \quad (0.3a)$$

$$y_1 = \sqrt[n]{\frac{k'_1}{d_1[X]} - 1} \quad (0.3b)$$

$$d_2[Y] = \frac{k_2}{1 + [X]^m}^\dagger \frac{1}{d_2} = \frac{k'_2}{1 + [X]^m}^\S \quad (0.3c)$$

$$y_2 = \frac{k'_2}{1 + [X]^m} \quad (0.3d)$$

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The equations 0.3a and 0.3d are solved as functions of  $f(x)$  in the form  $y = f(x)$ .

## 0.2 Nullcline plots

The analysis performed in graphs 0.1 shows the existence of multiple steady states, which it is the necessary condition to have a switch. The purpose of the switch is to obtain a consisting change in the concentration of a certain protein. The graph 0.1b shows highly transverse nullclines, the system can tolerate more movements of the nullclines without the disappearance of a steady state.

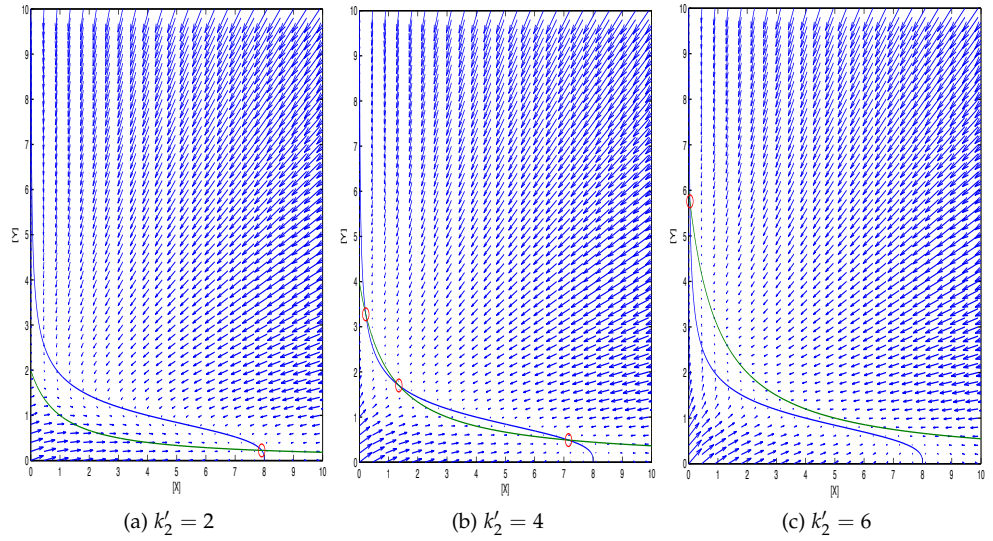


Figure 0.1: Nullclines plots for the parametres  $n = 3$ ,  $k_1' = 8$ ,  $k_2' = \{2, 4, 6\}$

Intersection of functions lines are circled in red. These points represent the steady state at different values of the parameter  $k_2$ . In the graph 0.1a we can distinguish only one point at high concentration of protein Y as well as in the graph 0.1c where the steady state is reached at high concentration of X. The major difference is visible in the graph 0.1b where we clearly confirm a possible switch, due to the presence of three steady state points.

### 0.3 Analytic expressions for $k'_1$ and $k'_2$

According to Matlab *solve* function for the equations:

$$e1 : x = \frac{k'_1}{(1 + y^3)} \quad (0.4a)$$

$$e2 : y = \frac{k_2}{(1 + x)} \quad (0.4b)$$

$$e3 : \delta\left(\frac{k_1}{(1 + y^3)}, y\right) * \delta\left(\frac{k_2}{(1 + x)}, x\right) = 1 \quad (0.4c)$$

we retrieve three solutions:

$$k'_1 = \frac{(3 * x^2)}{(2 * x - 1)} \quad (0.5a)$$

$$k'_2 = \left(\frac{(x + 1)}{(2 * x - 1)}\right)^{\frac{1}{3}} * (x + 1) \quad (0.5b)$$

$$y = \left(\frac{(x + 1)}{(2 * x - 1)}\right)^{\frac{1}{3}} \quad (0.5c)$$

### 0.4 Cooperativity plot

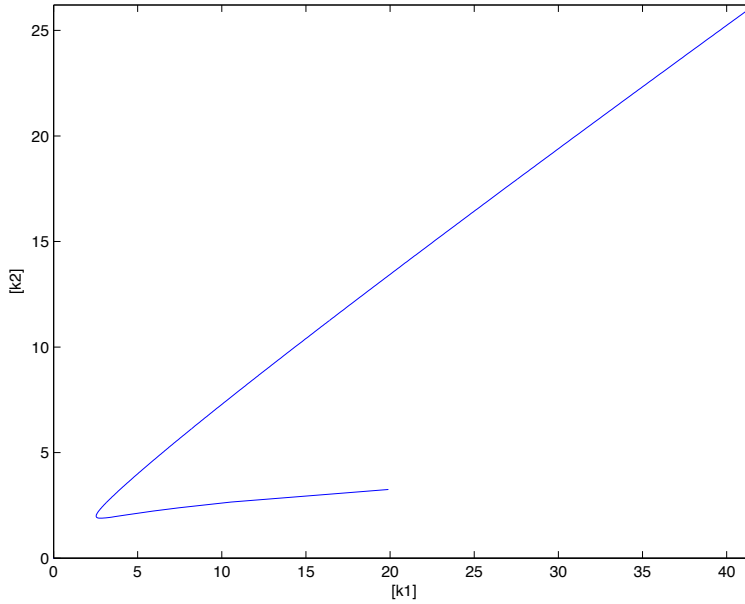


Figure 0.2: Values of the parameter  $k'_1$  and  $k'_2$  needed for a working switch for particular values of the exponent  $n$  in the Hill function.