Homework no 1

1. Stress tensor
$$\boldsymbol{\sigma} = \begin{bmatrix} a & b & c \\ b & ac & ab \\ c & ab & abc \end{bmatrix}.$$
Choose values for constants a, b, c from table according to your variant (see file „VARIANTS_FOR_HW1_2021.pdf"). Here ab, ac and abc mean multiplication of corresponding constants.

   Please give all answers to four decimal places, i.e., for example like 15.9741, 0.3459, 1.4500, 0.5000 etc.

   (a) Calculate principal stresses σ_1, σ_2 and σ_3 and principal directions $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_3$. */max 5 + 5 = 10 points/*
   IMPORTANT!

      i. For calculation of principal stresses and directions mathematical software is recommended to use.
      ii. Principal values must be ordered $\sigma_1 \geq \sigma_2 \geq \sigma_3$.
      iii. Vectors $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_3$ must be unit vectors and they must form right hand triplet, i.e. $\mathbf{N}_3 = \mathbf{N}_1 \times \mathbf{N}_2$.

   (b) Draw $\mathbf{N}_1$, $\mathbf{N}_2$ and $\mathbf{N}_3$ at point (0; 0; 0) in case of Cartesian coordinates. */max 4 points/*

   (c) Calculate invariants I^σ, II^σ and III^σ for stress tensor $\boldsymbol{\sigma}$. */max 3 * 2 = 6 points/*

   (d) Calculate maximal shear stress $\tau_{\max}$ and present it in a figure together with extremal normal stresses. */max 3 + 5 = 8 points/*

2. Take $E = 110$ GPa and given stress tensor $100\boldsymbol{\sigma}$ MPa. Use Hooke's law to calculate two separate strain tensors:

   (a) one ($\boldsymbol{\varepsilon}_{0.33}$) for $\nu = 0.33$ (metals) */max 10 points/*

   (b) the second ($\boldsymbol{\varepsilon}_{0.5}$) for $\nu = 0.5$ (incompressibles) */max 10 points/*

3. In file „VARIANTS_FOR_HW1_2021.pdf" two stress tensors $\boldsymbol{\sigma}_I$ and $\boldsymbol{\sigma}_{II}$ are given by tables. Either $\boldsymbol{\sigma}_I$ or $\boldsymbol{\sigma}_{II}$ is equivalent to $\boldsymbol{\sigma}$ differing only by a coordinate transformation. Which one is equivalent to $\boldsymbol{\sigma}$ and which one represents different stress state? */max 5 + 5 + 2 = 12 points/*