Homework no 2

Given equations of motion

$$\begin{cases} x_1 = aX_1 + bX_2 + cX_3 \\ x_2 = dX_1 + eX_2 + fX_3 \\ x_3 = gX_1 + hX_2 + iX_3 \end{cases} \tag{1}$$

and stress tensor

$$\mathbf{T} = \begin{bmatrix} \alpha & \beta & \kappa \\ \beta & \alpha + \beta & \beta + \kappa \\ \kappa & \beta + \kappa & \alpha + \beta + \kappa \end{bmatrix} \text{MPa.}$$

Choose values for constants $a, ..., i$ and α, β, κ from table according to your variant (see file „VARIANTS_FOR_HW2_2021.pdf").

Some answers can be integers and some can be decimal fractions. In case of decimal fractions please give all answers to four decimal places, i.e., for example like 15.9741, 0.3459, 1.4500, 0.5000 etc.

1. Calculate deformation gradient $\mathbf{F} = \frac{\partial \mathbf{X}}{\partial \mathbf{x}}$ (in tensor notation $F_{ij} = \frac{\partial x_i}{\partial X_j}$).
   /max 5 points/

2. Calculate the 2nd Piola-Kirchhoff stress tensor $\mathbf{S} = J\,\mathbf{F}^{-1} \cdot \mathbf{T} \cdot \mathbf{F}^{-T}$.
   /max 5 + 5 + 10 = 20 points/

3. Calculate von Mieses stress for stress tensor $\mathbf{T}$. /max 3 + 5 + 5 = 13 points/

4. Take $E = 70$ GPa and $\nu = 0.3$ and calculate strain tensor $\boldsymbol{\epsilon}$ according to Hooke's law. Show that the relation between deviators $\hat{T}_{ij} = 2\,G\,\hat{\epsilon}_{ij}$ is satisfied. /max 7 + 3 + 5 + 5 = 20 points/

5. Take $E = 70$ GPa and $\nu = 0.3$ and calculate stiffness tensor component

$$C_{ijkl} = \frac{E}{(1+\nu)} \left[\frac{1}{2} (\delta_{ik}\,\delta_{jl} + \delta_{jk}\,\delta_{il}) + \frac{\nu}{(1-2\nu)} \delta_{ij}\,\delta_{kl} \right].$$

Choose values for $ijkl$ from table according to your variant (see file „VARIANTS_FOR_HW2_2021.pdf"). /max 8 points/