LexBFS and its applications

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Overview

What's in a graph ?

Graph

We consider non-oriented, simple and **connected** graphs

INCLUDE GRAPHS EXAMPLES AND COUNTEREXAMPLES

Generic Search

```
for i in [1, ..., n]:
    if i == 1:
        u = any vertex
    else:
        u = any unvisited marked vertex
    visit(u)
    for v in neighbours(u):
        mark(v)
```

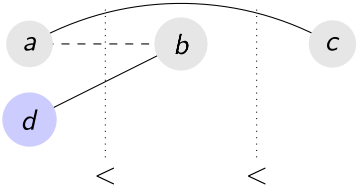
Another Characterization

Let's number vertices in the order they are visited.

Theorem

An order σ corresponds to a Generic Search if and only if

$$\forall a <_\sigma b <_\sigma c, ac \in E \text{ and } ab \notin E, \exists d <_\sigma b \text{ st } db \in E$$



DFS

INCLUDE GRAPH EXAMPLE

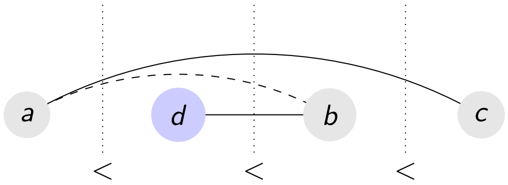
```
for i in [1, ..., n]:
    if i == 1:
        u = any vertex
    else:
        u = any unvisited vertex w/ max label
    visit(u)
    for v in neighbours(u):
        label[v] = i
```

Another Characterization

Theorem

An order σ corresponds to a DFS if and only if

$$\forall a <_\sigma b <_\sigma c, ac \in E \text{ and } ab \notin E, \exists a <_\sigma d <_\sigma b \text{ st } db \in E$$



BFS

```
for i in [n, ..., 1]:
    if i == n:
        u = any vertex
    else:
        u = any unvisited vertex w/ max label
    visit(u)
    for v in neighbours(u):
        if v has no label:
            label[v] = i
```

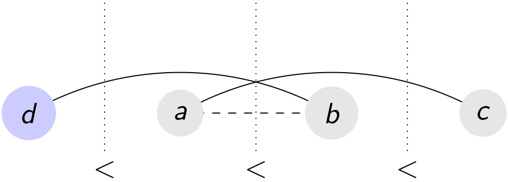
INCLUDE GRAPH EXAMPLE

Another Characterization

Theorem

An order σ corresponds to a BFS if and only if

$$\forall a <_\sigma b <_\sigma c, ac \in E \text{ and } ab \notin E, \exists d <_\sigma a \text{ st } db \in E$$



Let's rewrite BFS

```
for i in [n, ..., 1]:
    if i == n:
        u = any vertex
    else:
        u = any unvisited vertex
            w/ max first element of label
    visit(u)
    for v in neighbours(u):
        label[v].append(i)
```

Here is LexBFS

```
for i in [n, ..., 1]:
    if i == n:
        u = any vertex
    else:
        u = any unvisited vertex
            w/ max lexicographical label
    visit(u)
    for v in neighbours(u):
        label[v].append(i)
```

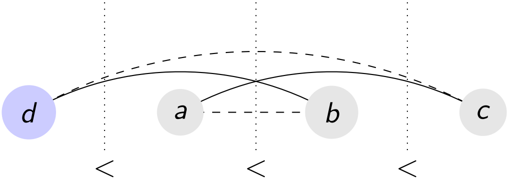
INCLUDE GRAPH EXAMPLE

Another Characterization

Theorem

An order σ corresponds to a LexBFS if and only if

$$\forall a <_\sigma b <_\sigma c, ac \in E \text{ and } ab \notin E, \exists d <_\sigma a \text{ st } db \in E \text{ and } dc \notin E$$



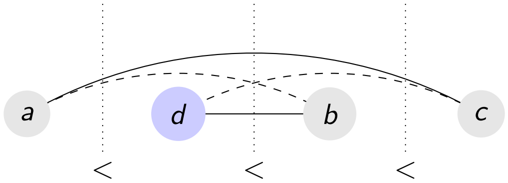
Another Characterization

Iterating on [1, ..., n] and prepending, we obtain LexDFS

Theorem

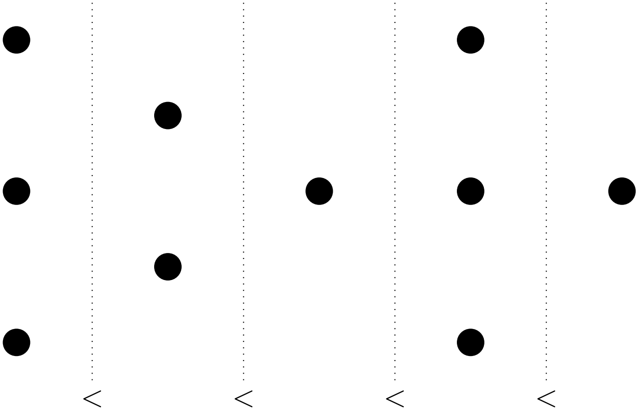
An order σ corresponds to a LexDFS if and only if

$$\forall a <_\sigma b <_\sigma c, ac \in E, ab \notin E, \exists a <_\sigma d <_\sigma b \text{ st } db \in E, dc \notin E$$



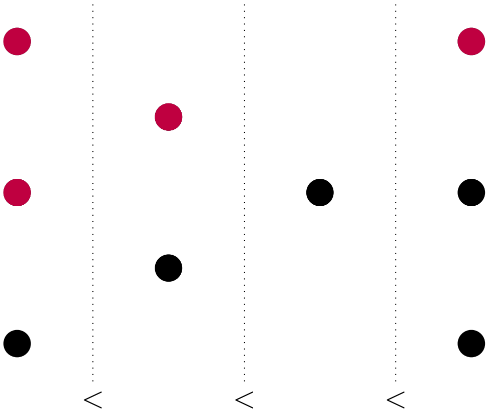
How to implement LexBFS: Pivoting

Let's order vertices by label :



How to implement LexBFS: Pivoting

We remove one of the maximum vertex :



How to implement LexBFS: Pivoting

Now, we split each bucket