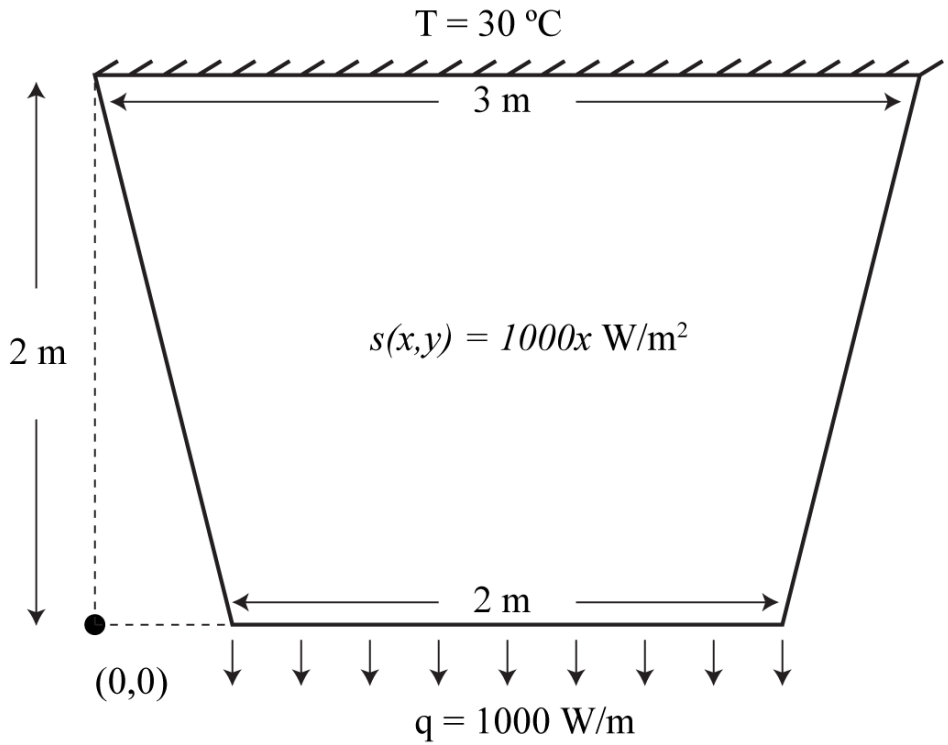
2D heat transfer demo

3/17/2015

Problem description

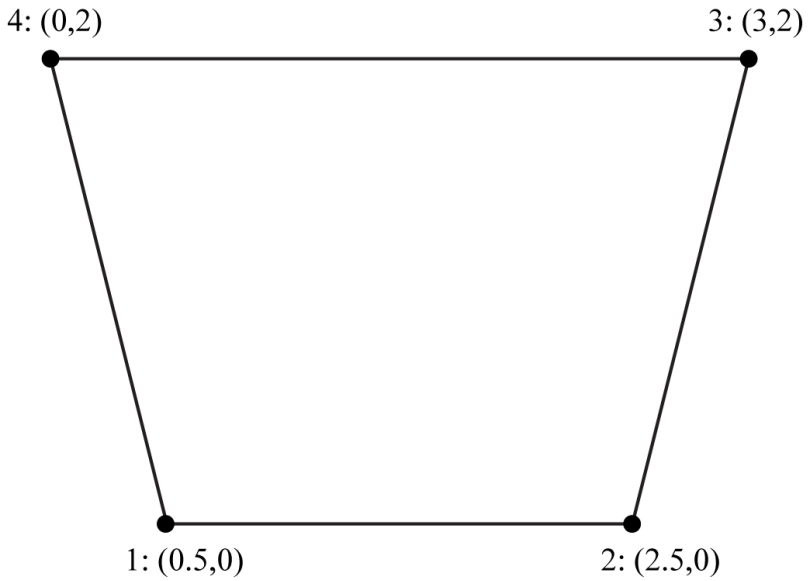
Approximate the temperature and heat flux in the trapezoidal plate using a single quadrilateral element.



Finite element mesh

$$\mathbf{x}^e = \begin{bmatrix} 0.5 & 2.5 & 3 & 0 \\ 0 & 0 & 2 & 2 \end{bmatrix}$$

$$\text{connectivity} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$



Discretization of the weak form

$$\int_{\Omega} \frac{\partial w}{\partial \mathbf{x}} \mathbf{D} \frac{\partial T}{\partial \mathbf{x}} d\Omega = -\int_{\Gamma_q} w \bar{q} d\Gamma + \int_{\Omega} w s d\Omega$$

Weak form

$$\int_{\Omega} \frac{\partial N_I}{\partial \mathbf{x}} \mathbf{D} \frac{\partial N_J}{\partial \mathbf{x}} d\Omega d_J = -\int_{\Gamma_q} N_I \bar{q} d\Gamma + \int_{\Omega} N_I s d\Omega$$

Substitute shape functions and invoke arbitrary w_J

$$K_{IJ} d_J = f_I$$

System of equations

$$K_{IJ} = \int_{\Omega} \frac{\partial N_I}{\partial \mathbf{x}} \mathbf{D} \frac{\partial N_J}{\partial \mathbf{x}} d\Omega$$

Global conductivity matrix

$$f_I = -\int_{\Gamma_q} N_I \bar{q} d\Gamma + \int_{\Omega} N_I s d\Omega$$

Global flux vector