MAE 404/598 Finite Elements in Engineering

Project #2: **Due: April 14, 2015**

Analyze the following heat transfer problem in a thin plate with a center hole, shown in Fig 1. The outer boundary of the plate is fixed at (T = 0 ºC), and a non-uniform heat flux is applied on the surface of the hole. A heat source term is applied over the entire body.

The strong form for the heat equation is

$$\nabla \cdot \mathbf{q} - s = 0 \tag{1}$$

where the heat flux, **q**, is evaluated as

$$\mathbf{q} = -\kappa \nabla T \tag{2}$$

Using the method of manufactured solutions, we first choose an arbitrary solution for the temperature distribution,

$$T(x, y) = 10\left(x - \frac{L}{2}\right)\left(x + \frac{L}{2}\right)\left(y - \frac{L}{2}\right)\left(y + \frac{L}{2}\right) \tag{3}$$

To complete the project, you must first analytically determine the heat source, by substituting (2) and (3) into (1) and solving for *s*. To compute the heat flux on Γ_q, evaluate (2) given (3), and integrate

$$\mathbf{f}_q^e = \int_{\Gamma_q} \mathbf{N}^e \bar{q}\, d\Gamma \quad \text{where } \bar{q} = \mathbf{q} \cdot \mathbf{n}$$

and **n** is the outward pointing normal direction. (*Hint outward means pointing out of the body, thus this direction will point toward the origin).*

Then implement a finite element program in Matlab to approximate the temperature distribution. You should then be able to use exact solution to determine your rate of convergence. The function, **make_mesh.m** is provided to generate a mesh. The **make_mesh** function takes 3 input arguments: nh gives the number of elements along a side (The mesh pictured in Fig 2. was generated with nh = 1), side_len gives the length (L) of a side of the plate, and hole_radius gives the radius of the central hole.

make_mesh returns a structure containing two fields: (mesh.x stores the nodal coordinates in a [2, nn] matrix, and mesh.conn stores the connectivity matrix in a [4, ne] matrix, with nn and ne being the number of nodes and number of elements in the mesh, respectively.

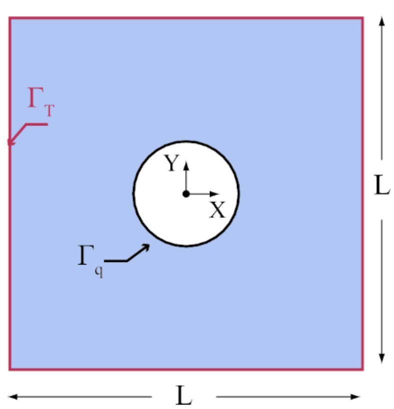

Fig.1. 2D plate with a center hole

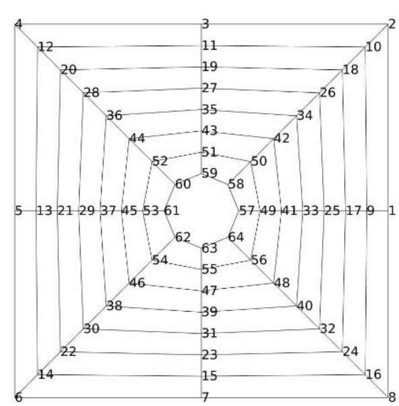

Fig. 2. Mesh with node numbering

Parameter		Value
Plate edge length	L	2 m
Hole radius	r	20 cm
Heat capacity	κ	54 W/(mK)

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Report format
Each group must create one report that is typed and submitted by hard copy either in lecture or at ECG 205 (TA office) before the due date. All group members must be clearly listed on the first page (name and ASU ID).
Each report should have the following sections:
1. Analytical solution (use of symbolic math software is okay).
2. MATLAB solution
3. ABAQUS solution
4. Appendix: MATLAB code(s)

Grading scale:
Points will be awarded for correct completion of each of the tasks listed below with partial credit awarded for incomplete or incorrect attempts. Plots must contain axis labels and units as appropriate for full credit.

The points required for an 'A' depends on the number of people in your group. Final score of each student will be scaled to 100. <u>A group size of 3-4 is encouraged.</u>

Group size	1	2	3	4	5
Points required	**170**	**180**	**190**	**190**	**210**

If all group members are undergraduates – subtract 30 from the required # of points.

General points (20 possible)
- [20] Report is clearly written.

Analytical solution points (40 possible)
- [10] Analytical solution for the heat source field.
- [10] Analytical solution for the heat flux vector field.
- [10] Contour plot of analytical heat source field.
- [10] Quiver plot of the analytical heat flux field.

MATLAB solution points (120 possible)
- [10] Code follows clean and consistent style with comments included as appropriate.
- [10] Contour plot of approximated temperature field for two mesh sizes.
- [10] Plot heat flux at the center of each element (use quiver command) – see demo on Blackboard.
- [10] Analyze the maximum temperature under several mesh sizes – show that it reaches a stable value.
- [20] Plot temperature along a center line ($y=0$, $-L/2 < x < L/2$) and compare the result with theoretical solution.
- [30] Plot L2 temperature error norm vs mesh size on a log-log plot and compute the rate of convergence (*hint*: you can use any element measure (e.g. the distance between nodes 1 & 2), provided that the scaling is proportional.
- [30] Demonstrate that your code passes a patch test. For this, you must generate your own mesh (see Fig 8.13 on pg 203).

ABAQUS solution points (50 possible)
- [20] Show *publication-quality* a plot of the temperature fields for a two different element sizes. Contour plots must have a clear legend, clear font and show the extreme values.
- [20] Compare between solutions using linear and quadratic elements, given the limitation of number of elements, what is the best solution that you can achieve?
- [10] Justify that your Abaqus solution is reasonable.