Homework

Problems on UD

2.1

(a) antecedent: it's raining **conclusion:** I'll stay home
(b) antecedent: the baby cries **conclusion:** I wake up
(c) antecedent: I wake up **conclusion:** the fire alarm goes off
(d) antecedent: x is odd **conclusion:** x is prime
(e) antecedent: x is prime **conclusion:** x is odd
(f) antecedent: you can come to the party **conclusion:** you have an invitation
(g) antecedent: the bell rings **conclusion:** I leave the house

2.5

Truth Table:

P	Q	R	¬R	(¬R∨Q)	P → (¬R∨Q)	(P → (¬R∨Q))∧R
T	T	T	F	T	T	T
T	T	F	T	T	T	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
T	F	F	T	T	T	F
F	T	F	T	T	T	F
F	F	T	F	F	T	T
F	F	F	F	T	T	F

This statement form is neither a tautology nor a contradiction.

2.6

(a) I won't do my homework or I won't pass this class.
(b) Seven isn't an integer or seven isn't even.

(c) T is continuous and T isn't bounded.
(d) I can neither eat dinner nor go to the show.
(e) x is odd and x isn't prime.
(f) x is prime and x isn't odd.
(g) I am not home and Sam won't answer the phone or I am not home and Sam won't tell you how to reach me.
(h) The stars are green and the world isn't eleven feet wide or the white horse is shining and the world isn't eleven feet wide.

2.7

(a) $\neg(\neg P)\leftrightarrow P$
(b) $\neg(P\vee Q)\leftrightarrow\neg P\wedge\neg Q$
(c) $\neg(P\wedge Q)\leftrightarrow\neg P\vee\neg Q$
(d) $P\rightarrow Q\leftrightarrow\neg P\vee Q$

2.8

Answer: $(P\wedge Q)\vee R$
Process: There are only two situations.The first one is $P\wedge(Q\vee R)$,which contradicts the 5_{th} row of the truth table,so it's obvious that the correct answer is $(P\wedge Q)\vee R$.

2.10

(a) If it is sunny,then it snows.
(b) If it is sunny,then it doesn't snow and it is sunny.

2.11

(a) Suppose that *A* means Arnie is a truth-teller,*B* means each person living on this island is either a truth-teller or a liar.Because *B* is true,so $A\rightarrow B$ is a tautology,which means that no matter *A* is true or false,what Arnie says is always true,so Arnie must be a truth-teller.
(b) Suppose that *A* means Arnie is a truth-teller,*B* means Barnie is a truth-teller.So we can paraphrase Arnie's sentence into $A\rightarrow B$.Then we can write down the truth table.

A	B	$A\rightarrow B$	$A\leftrightarrow(A\rightarrow B)$
T	T	T	T
T	F	F	F
F	T	T	F

A	B	A→B	A↔(A→B)
F	F	T	F

So Arnie and Barnie are both truth-tellers.

3.2

(a) contrapositive: If you don't live in a white house,then you aren't the President of the United States.

converse: If you live in a white house,then you are the President of the United States.

(b) contrapositive: If you don't need eggs,then you are not going to bake a souffle.

converse: If you need eggs,you are going to bake a souffle.

(c) contrapositive: If x isn't an integer,then x isn't a real number.

converse: If x is an integer,then x is a real number.

(d) contrapositive: If $x^2 \geq 0$,then x isn't a real number.

converse: If $x^2 < 0$,then x is a real number.

3.6

C	B	Y	B∨C	C∧B	(C∧B)→ (C∧B)∧Y	B∨Y	(B∨Y)→ (B∨Y)∧C	¬(C∧Y)	B∨C∨Y
T	T	T	T	T	T	T	T	F	T
T	T	F	T	T	F	T	T	T	T
T	F	T	T	F	T	T	T	F	T
F	T	T	T	F	T	T	F	T	T
T	F	F	T	F	T	F	T	T	T
F	T	F	T	F	T	T	F	T	T
F	F	T	F	F	T	T	F	T	T
F	F	F	F	F	T	F	T	T	F

All the highlighted statement forms must be true,so Matilda eats only cereal on Monday.

3.7

(a) *A*="the coat is green";*B*="the moon is full";*C*="the cow jumps over the moon"

$A \to (B \vee C)$

(b) $(\neg B \wedge \neg C) \rightarrow \neg A$

If the moon isn't full and the cow doesn't jump over it,then the coat isn't green.

(c) $(B \vee C) \rightarrow A$

$A \rightarrow (B \vee C)$

If the coat is green,then the moon is full or the cow jumps over it.

(d) $A \rightarrow (\neg B \wedge \neg C)$

$(\neg B \wedge \neg C) \rightarrow A$

If the moon isn't full and the cow doesn't jump over it,then the coat is green.

(e) $A \rightarrow (B \vee C) \leftrightarrow (\neg B \wedge \neg C) \rightarrow \neg A$

3.8

(a)

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

P	¬P	Q	Q∨¬P	P→(Q∨¬P)
T	F	T	T	T
T	F	F	F	F
F	T	T	T	T
F	T	F	T	T

(b)

P→Q and P→(Q∨¬P) are equivalent statement forms.

3.9

	S	G	A	F
ch	T	T	F	T
fl	F	T	T	T
su	T	F	T	T

The French recipe.

3.10

The contrapositive of "if 3n is odd,then n is odd" is "if n isn't odd,then 3n isn't odd".To prove the original statement form,we only need to prove its contrapositive.Below is the proof.
Because n is an integer and n isn't odd,n must be even.Suppose n=2k($k \in Z$),so 3n=6k,it is obvious that 3n(6k) is divisible by 2,namely 3n is even.So "if n isn't odd,then 3n isn't odd" is proved.Thus,the original statement is proved to be true.

3.11

The contrapositive of the statement to be proved is "if$\sqrt{2x}$ is an integer,then x isn't odd".Now let's prove the contrapositive.
Since$\sqrt{2x}$ is an integer,we might as well suppose it as n($n \in Z$),so 2x=n^2,namely x=$\frac{n^2}{2}$,apparently x is either even integer or a fraction,so the contrapositive proves true,namely the original statement proves true.

4.1

(a) $\forall x, (\exists y, (x = 2y))$
(b) $\forall y, (\exists x, (x = 2y))$
(c) $\forall x, \forall y, (x = 2y)$
(d) $\exists x, (\exists y, (x = 2y))$
(e) $\exists x, \exists y, (x = 2y)$

4.5

(a) There exists an $x \in R$,such that $x^2 \leq 0.$
(b) There exists an odd integer such that it is zero.
(c) If I am hungry,then I won't eat chocolate.
(d) There exists a girl such that she likes every boy.
(e) For all $x \in R$,we have $g(x) \leq 0.$
(f) There exists an $x \in R$ such that $xy \neq 1$for all $y \in R.$
(g) For all $y \in R$ there is an $x \in R$ such that $xy \neq 0.$
(h) If $x \neq 0$,then for all $y \in R$,we have $xy \neq 1.$
(i) If $x > 0$,then there exists a $y \in R$ such that $xy^2 < 0.$
(j) There exists an $\epsilon > 0$ and an $n > N$ such that if x is a real number with $|x - 1| < \delta$, then $|x^2 - 1| \geq \epsilon$ for all $\delta > 0.$
(k) There exists a real number M such that $|f(n)| \leq M$ for all real number N.

4.7

(a) $\exists x, ((x \in Z \wedge (\forall y, (y \in Z \rightarrow x \neq 7y))) \wedge (\forall z, (z \in Z \rightarrow x \neq 2z)))$.
(b) For all x, if $x \in Z$ and for all $y \in Z$ we have $x \neq 7y$,then there exists an $z \in Z$ such that $x = 2z$.
(c) The negation is true.For the original statement,considering that if $x = 5$, then for all $y \in Z$ we have $x \neq 7y$, but there doesn't exist a $z \in Z$ such that $5 = 2z$, namely $x = 2z$, so the original statement is false.

4.9

If we put too much emphasis on some trivial details,it would be ridiculous.

4.13

(a) true If I love Bill,I must love Sam,since I don't love Sam,so I don't love Bill
(b) false Even though Susie didn't go to the ball in the red dress,it doesn't mean that I didn't stay home.
(c) true If t is positive,then there exists a real number m such that $m > t$, since all the m is less than t, t is not positive.
(d) true Since my name isn't Igor,then the first half of the statement must be true.
(e)false Even though there is no blue house on the street,there still could be a black house on the street.
(f) true If $x > 5$, then $y \leq \frac{1}{5}$, since $y = 1$,then $x \leq 5$.
(g) false Even if $n < M$, there is still such possibility that $n^2 > M^2$.
(h) false If $y > x$ and $y \leq 0$, it is also true that $y \leq z$.

Extra Problem

Does this way lead to you knights' village?