

# Assignment 1: Discrete Structures

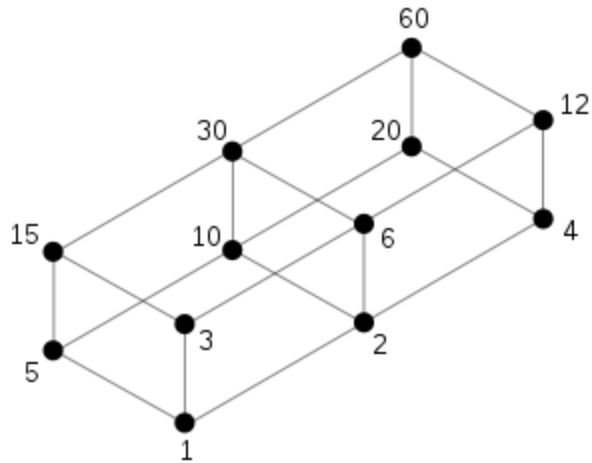
1. The avengers have discovered a new infinity stone. In order to make use of the power of the stone, they need to understand its structure. Early reports suggest that it resembles a divisor lattice. To exploit the power, they need to know the number of edges it possesses in order to make a machine to hold. As the chief engineer of Stark industries, its your job to find the number of edges.

**Input:**

An integer  $n$  representing the number

**Output:**

Print the number of edges that the divisor lattice has with  $n$  at its top.



**Sample Test Case 1**

**Input:**

60

**Output:**

20

**Sample Test Case 2**

**Input:**

1890

**Output:**

72

**Sample Test Case 3**

**Input:**

31

**Output:**

1

2. You decide to make a trip to military communist nation of Arstotzka in the height of the cold war. However, you were detained at the passport booth as you did not respond to 'Papers Please'. The jail warden gives you a chance to escape. The warden being a proud nationalist says that you need to only figure out the alphabet that is followed within the country. Given that they follow the same alphabet as English, but in a different order. From the clues written on the walls of your cell you need to find out the correct ordering of the letters. Note: If there is no well/multiple (look at the sample test case) defined ordering, you need to follow the lexicographic order.

**Note: If an ordering is not possible return -1**

**Input:**

An integer **n** representing the number of hints.

This is followed by **n tuples** in the following format:

a b

This means that a precedes b.

**Output:**

Output the alphabetical ordering within the country.

**Sample Test Case 1**

**Input:**

2

a c

b c

**Output:**

a b c

[a->c and b->c, but a comes before b thus, we get a,b,c]

**Sample Test Case 2**

**Input:**

3

a b

b c

c a

**Output:**

-1

[Here we have a->b and b->c which means a->c, however c->a which is a contradiction]

**Sample Test Case 3**

**Input:**

10

c a

c b

b d

d e

e a

e f

a f

d g

g f

b a

**Output:**

c b d e a g f

3. **Sondmon** has escaped from prison and is currently hiding in the sewer system. Your job as **Spodermon** need to flush him out. The only way to do so, is with the help of the sewer pipes. If a sewer pipe is activated, it will fill all the connected tunnels. In order to defeat him you need to open the minimum number of sewer pipes. How will you go about it. **Note: A sewer pipe is present at every junction.**

**Input:**

Two integers **n,m** : **n** representing the number of junctions and **m** which is the number of connections in the sewer system. The junctions are implicitly labelled from 0 to **n-1**. The next **m** lines contain the connections in the following format.

a b

Which means that we can go from either a to b and from b to a.

**Output:**

Print the number of sewer pipes to be opened.

**Sample Test Case 1**

**Input:**

4 3

0 1

1 2

2 3

**Output:**

1

[Here 0->1->2->3, thus using only one pipe in either of 0,1,2,3 will flood the entire system.]

**Sample Test Case 2**

**Input:**

5 4

0 4

1 2

2 3

3 1

**Output:**

2

**Sample Test Case 3**

**Input:**

5 0

**Output:**

5

[None are connected, thus, we need to open all of the pipes.]

4. In the best Hero Academy in all of Japan, **UA; All Might** has come up with a plan to balance out the teams for the upcoming UA sport tournament. He will form teams of one or more players. Given the number of teams and the strength of each player, you need to form teams so that no team is too weak or too overpowered. [Look at the example to clear any confusion]. The power of a team is equal to the sum of power of its constituents.

**Input:**

Two integers **n,m**: **n** representing the number of teams and **m** represents the number of would be heroes. The next m lines contain the power level of each hero. The heroes are implicitly labelled from 0 to **m-1**. The **i<sup>th</sup>** line gives the power of the **i<sup>th</sup>** hero.

**Output:**

Print the power level of the most strongest team present.

**Example:**

If the heroes have the powers given below :

10 20 30 40 50 60 70 80 90

They can be divided in the following manner for 3 teams:

10 20 30 40 50 | 60 70 | 80 90

Here the maximum value is 170, as no team is above 170.

**Sample Test Case 1**

**Input:**

5 9

10

20

30

40

50

60

70

80

90

**Output:**

110

**Explanation:**

10 20 30 40 | 50 60 | 70 | 80 | 90

**Note:** the input may not be sorted order.

5. Winter is coming and Jon Snow wants to protect his home from the White Walkers. He has a very small army already divided into battalions. To prepare for any exigency, he wants to know how to quickly divide these battalions further. Since he knows nothing, he needs your help: Given a partitioned set, your job is to print all the **immediate refinements** of it.

**Useful definitions:**

A **partition** of a set is a grouping of the members of the set into non-empty subsets (Recall posets). For example, a possible partition of the set  $\{1, 2, 3\}$  is  $\{\{1\}, \{2, 3\}\}$ .

A **refinement** of a partition of a set is another partition of the same set such that each group of the new partition is a subset of some group of the coarser partition. So a possible refinement of  $\{\{1\}, \{2, 3\}\}$  is  $\{\{1\}, \{2\}, \{3\}\}$ .

An **immediate refinement** of a partition is a refinement which has exactly one more group than the coarser partition.

For example,  $\{\{1\}, \{2\}, \{3\}\}$  which has 3 groups is an immediate refinement of  $\{\{1\}, \{2, 3\}\}$  which has 2 groups.

**Input format:**

A number **N** representing the number of groups in the input partition. Then, for each of these groups, a number **K** representing the number of elements in the group, followed by **K** numbers representing the elements. For example:

```
3 // number of groups of the partition
1 // number of elements in the first group
100 //element of the first group
2 //number of elements in the second group
150 200 //elements of the second group
3 //number of elements in the third group
201 202 203 //elements of the third group,
```

This represents the partition  $\{\{100\}, \{150, 200\}, \{201, 202, 203\}\}$ .

**Output format:**

Print each possible immediate refinement, one group in one line.

**Example:**

Input(  $\{\{1, 2, 3\}\}$  ):

1

3

1 2 3

Output(  $\{\{1,3\}, \{2\}\}, \{\{1,2\}, \{3\}\}, \{\{1\}, \{2, 3\}\}$  ):

1 3

2

\*\*\*\*\*

1 2

3

\*\*\*\*\*

1

2 3

\*\*\*\*\*

Decoration is optional, but you must make it easy to distinguish between the various immediate refinements.

### Sample Test Case 1

Input:

2  
2  
1 2  
2  
3 4

Output:

1  
2  
3 4

\*\*\*\*\*

1 2  
3  
4

\*\*\*\*\*

6. As an **undead** who has escaped the **Northern Undead Asylum**, you meet the Crestfallen warrior at firelink shrine. He gives you a quest to journey all of Lordran. You need to find all the places accessible from firelink shrine, conveniently marked 0.

**Input:**

Two integers **n,m**: **n** representing the number of locations and **m** represents the number of paths connecting two locations. The next m lines are in the following format:

a b

Where b can be reached from a and not vice-versa, as this relation is not **symmetric** but it is **transitive**.

**Note: Nodes are labelled 0 to n-1**

**Output:**

Print all the locations reachable from firelink shrine in a space separated format.

**Sample Test Case 1**

**Input:**

10 8

0 2

0 1

7 1

0 5

6 3

8 9

2 4

2 3

**Output:**

0 2 4 1 5 3

**Sample Test Case 2**

**Input:**

4 4

0 1

3 1

1 2

1 0

**Output:**

0 1 2



7. Kamles is a gamer and wants to know the number of ways to complete Bloodborne. Given the difficulty of the bosses in the game, find the possible number of playthroughs where you defeat a weaker boss before a stronger boss.

**Input:**

Two integers **n,m**: **n** representing the number of bosses and **m** represents the number of relations between the bosses. The next m lines are in the following format:

a b

This shows that boss b is stronger than boss a.

**Note: bosses are labelled form 0 to n-1. If no orderings are possible return -1**

**Output:**

Print the number of playthroughs possible.

**Sample Test Case 1**

**Input:**

5 7

0 1

0 2

0 3

0 4

1 3

1 4

2 4

**Output:**

5

**Explanation:**

The following 5 orders are possible:

0 1 2 3 4

0 2 1 3 4

0 1 3 2 4

0 1 2 4 3

0 2 1 4 3

**Sample Test Case 2**

**Input:**

5 5

0 1

1 2

2 3

3 4

4 0

**Output:**

-1

**Explanation:**

**This is because:**0->1->2->3->4->0 which is a contradiction as 0 cannot be stronger than 0.

8. **Ash Ketchum** is preparing for the **Johto League** and he wants his pokemons to train in **pairs**. Each of them has a set of skills out of a pool of all possible skills, a pokemon can have. Professor Oak wants to find the best partner for Ash's new pokemon **Giratina** while at the same time, wants to achieve a particular skill level of the combined pair. Help Professor Oak list out all the possible skillsets that in the partner that can achieve this. Given a **set S** of all skills possible, one of its **subsets T** (the required skills in the pair) and one of this **subset's subset U** (Giratina's skills), list all the possible subsets of **S** such that the union of any one of these subsets with **U** gives **T**.

**Input:**

First line has **N**, the number of all possible skills. The next **N** lines have 1 integer each, representing the skill label. The next line has **K**, the number of skills that Giratina has. The next **K** lines have 1 integer each, representing the labels of Giratina's skills. The next line has **L**, the number of skills in the required set. The next **L** lines have 1 integer each, representing the labels of these required skills.

**Output:**

Each possible set, represented by space separated integers (in any order), union of this set with the skill set of Giratina is the required set.

**Sample Test Case 1**

**Input:**

5  
1 2 3 4 5  
2  
1 2  
4  
1 2 3 4

**Output:**

3 4  
1 3 4  
2 3 4  
1 2 3 4

**Explanation:**

$S = \{1, 2, 3, 4, 5\}$ ,  $U = \{1, 2\}$ ,  $T = \{1, 2, 3, 4\}$ . Union of any of  $\{3, 4\}$ ,  $\{1, 3, 4\}$ ,  $\{2, 3, 4\}$  or  $\{1, 2, 3, 4\}$  with  $\{1, 2\}$  yields  $\{1, 2, 3, 4\}$ .

**Sample Test Case 2**

**Input:**

5  
1 2 3 4 5  
1  
4  
2  
3 4

**Output:**

3  
3 4

**Explanation:**

$S = \{1, 2, 3, 4, 5\}$ ,  $U = \{4\}$ ,  $T = \{3, 4\}$ . Union of any of  $\{3\}$ ,  $\{3, 4\}$ , with  $\{4\}$  yields  $\{3, 4\}$ .

9. You need to find the minimum number of chairs to accommodate all the students who enter the mess.

**Input:**

Given an integer n which gives the number of students enter the mess. The next n lines give the entry and exit times of the students.

a b

Where a is the entry time and b is the exit time.

**Output:**

The minimum number of seats required

**Note: If a student leaves the mess and another enters the mess at the same time, this will require another chair as the person enters instantaneously before the other leaves.**

**Note: We use 24 hour time and the day has 1440 minutes**

**Sample Test Case 1**

**Input:**

4

1 4

1 3

3 4

4 5

**Output:**

3

**Explanation:**

At time=3 minutes, 2 people namely 1<sup>st</sup> and the 2<sup>nd</sup> person are sitting and the 3<sup>rd</sup> enters before the 2<sup>nd</sup> leaves.

**Sample Test Case 2**

**Input:**

3

1 2

1 3

4 5

**Output:**

2

10. Pablo Escobar has come up with a new medicine to cure depression and has plans to commercialize it. However, La Quica had noticed that there were fakes in the market, which did not weigh the same as the originals. Unfortunately, while bringing back a **sample** back to the boss, they got mixed with the originals. He then makes use of a weighing scale to make a few comparisons. Given that you are Heisenberg can you figure out the fake.

**Input:**

An integer **n** : Giving the number of comparisons made

The **3n lines** follow have the following data

K //Number of items on each side

$a_1 a_2 \dots a_k b_1 b_2 \dots b_k$  // a is on the left and b is on the right

Symbol // =, >, <

**Output:**

Output the fake sample. If you cannot find it print -1

**Sample Test Case 1:**

**Input**

```
3
2
1 2 3 4
>
1
1 3
=
1
2 3
=
```

**Output**

4

**Sample Test Case 2:**

**Input**

```
2
1
1 3
>
1
2 4
>
```

**Output**

-1

## Helpful Websites:

1. <https://www.hackerearth.com/practice/algorithms/graphs/depth-first-search/tutorial/>
2. <https://www.hackerearth.com/practice/algorithms/graphs/topological-sort/tutorial/>
3. <https://www.hackerearth.com/practice/notes/disjoint-set-union-union-find/>