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IDS 561 - Assignment 5

Given $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \\ 4 & -8 & 7 \end{bmatrix}$

SVD of a matrix $A = U \Sigma V^T$

Step 1 :- Find $A^T A$

$$A^T A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 3 & -8 \\ 2 & -2 & 7 \end{bmatrix} \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \\ 4 & -8 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & -20 & 30 \\ -20 & 77 & -58 \\ 30 & -58 & 57 \end{bmatrix}$$

Step 2 :- Find eigenvalues and eigenvectors of $A^T A$

For a $n \times n$ matrix W, a non-zero vector x is eigenvector of W if,

$W . x = 1 . x$

i.e $(W - 1I) x = 0$

i.e $(W - \lambda I) x = 0$

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where $(W - \lambda I) = \begin{bmatrix} 29-\lambda & -20 & 30 \\ -20 & 77-\lambda & -58 \\ 30 & -58 & 57-\lambda \end{bmatrix}$

$$= (29-\lambda)\begin{vmatrix} 77-\lambda & -58 \\ -58 & 57-\lambda \end{vmatrix} - (-20)\begin{vmatrix} -20 & -58 \\ 30 & 57-\lambda \end{vmatrix}$$

$$+ 30 \begin{vmatrix} -20 & 77-\lambda \\ 30 & -58 \end{vmatrix}$$

$$(W - \lambda I) = \lambda^3 - 163\lambda^2 + 3611\lambda - 7225 = 0$$

Solving for the equation, we get

$\lambda_1 = 2.22$, $\lambda_2 = 23.74$, $\lambda_3 = 137.03$

where $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues of A^TA.

Now,

For $\lambda_1 = 2.22$, eigenvector is obtained by subtracting λ_1 from A^TA and solving for 0.

i.e $\left[\begin{array}{ccc|c} 29-\lambda_1 & -20 & 30 & 0 \\ -20 & 77-\lambda_1 & -58 & 0 \\ 30 & -58 & 57-\lambda_1 & 0 \end{array}\right]$ — (1)

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Solving (1), we get eigenvector $V_1 = \begin{bmatrix} -0.50 \\ 0.44 \\ 0.74 \end{bmatrix}$

~~For~~ Similarly for $\lambda_2 = 23.74$ we get

$$\left[\begin{array}{ccc|c} 29-\lambda_2 & -20 & 30 & 0 \\ -20 & 77-\lambda_2 & -58 & 0 \\ 30 & -58 & 57-\lambda_2 & 0 \end{array}\right] \quad -(2)$$

Solving (2), we get eigenvector $V_2 = \begin{bmatrix} 0.81 \\ 0.55 \\ 0.22 \end{bmatrix}$

Similarly for $\lambda_3 = 137.03$ we get eigenvector $V_3 = \begin{bmatrix} 0.31 \\ -0.71 \\ 0.63 \end{bmatrix}$

~~Thus~~ Step 3 :- Find V and Σ

Thus, based on the eigenvectors V_1, V_2, V_3 we get

$$V = \begin{bmatrix} -0.50 & 0.81 & 0.31 \\ 0.44 & 0.55 & -0.71 \\ 0.74 & 0.22 & 0.63 \end{bmatrix}$$

☆ and $V^T = \begin{bmatrix} -0.50 & 0.44 & 0.74 \\ 0.81 & 0.55 & \cancel{-0.71}\ 0.22 \\ 0.31 & -0.71 & 0.63 \end{bmatrix}$

☆ Also, $\Sigma = \begin{bmatrix} \sqrt{\lambda_1} & 0 & 0 \\ 0 & \sqrt{\lambda_2} & 0 \\ 0 & 0 & \sqrt{\lambda_3} \end{bmatrix} = \begin{bmatrix} 1.49 & 0 & 0 \\ 0 & 4.67 & 0 \\ 0 & 0 & 11.71 \end{bmatrix}$

Step 4:- Find U

we know that, $A = U \Sigma V^T$

Hence $A\,V = U \cdot \Sigma$

where $A.V = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \\ 4 & -8 & 7 \end{bmatrix} \begin{bmatrix} -0.50 & 0.81 & 0.31 \\ 0.44 & 0.55 & -0.71 \\ 0.74 & 0.22 & 0.63 \end{bmatrix}$

$$= \begin{bmatrix} 0.86 & 3.97 & 0.77 \\ -1.16 & 2.63 & -2.77 \\ -0.34 & 0.38 & 11.33 \end{bmatrix} = U.\Sigma \quad -(4)$$

Since, ~~eqn~~ (4) is $U.\Sigma$ we divide each column by the $\sqrt{\text{eigenvalues}}$ to get the matrix U.

☆ Hence, $U = \begin{bmatrix} 0.57 & 0.62 & 0.07 \\ -0.78 & 0.58 & -0.24 \\ -0.23 & 0.08 & 0.97 \end{bmatrix}$