Assignment 2.

1. Solve a system of equations $H\,x = [\,1 \;\; 1 \;\; \dots \;\; 1\,]^T$ where H is a 20×20 Hilbert matrix using Gaussian elimination/ LU decomposition. Now solve $H\,x = [\,0.99 \;\; 0.99 \;\; \dots \;\; 0.99\,]$. Discuss the accuracy of the results.

Hint: Hilbert matrix: The $n \times n$ Hilbert matrices are defined by

$$H(i,j) = 1/(i+j-1), 1 \leqslant i, j \leqslant n.$$

A 3×3 Hilbert matrix is given as $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$. For a 5×5 Hilbert matrix, execute in MATLAB »H=hilb(5).

2. Write a MATLAB function $t = \mathrm{tr}(A)$ which computes the trace of a given matrix A. The trace of matrix is given by the sum of its diagonal elements. Test to make sure that A is a square matrix.

3. Using MATLAB commad `diag`, build a tridiagonal matrix T as follows:

» `a=ones(4,1);`

»`b=5*ones(5,1);`

»`c=-ones(4,1);`

»`T=diag(a,-1)+diag(b)+diag(c,1);`

Write an LU decomposition function to factor the matrix T. Verify that $T = \mathrm{LU}$.