Matlab tutorial

$v = [0 \quad 1 \quad 2 \quad 3]$ row vector

$u = [9; \quad 10; \quad 11; \quad 12; \quad 13]$ column vector

$u(2)$ to access the second element

$u(2{:}\,4)$ slice out a vector

$w = v'$ (transpose a vector)

$x = -1{:}\,.1{:}\,1$

$y = \text{linspace}(-1, 1, 11)$

What is length of a vector?

Plotting in matlab

$x = \mathrm{linspace}(0, 2{*}\mathrm{pi}, 40)$

$y = \sin(x)$

$\mathrm{plot}(x, y)$

User-defined anonymous functions

f=@x 2*x.^2 -3*x + 1 $\qquad f(x) = 2x^2 - 3x + 1$

z=f(2) evaluate function at x=2

Vectorization

x=-2:.1:2

y=f(x)

Writing functions in matlab

```
function y = myfun(x)
         y = 2*x.^2 - 3*x +1;
end
```

x = -3:.2:3; Vector of x-values

y = myfun(x); Vector of y-values

plot(x,y)

- Starts with the word "function"
- Input and output
- Output, name of function in first line
- Program should assign value to output variables

Built-in constants and special variables

ans: default name for results

eps: smallest number for which $1 + \mathrm{eps} > 1$

inf: Infinity

NaN: Not a number

i or j: $\sqrt{-1}$

pi: π

realmin: smallest usable positive number

realmax: largest usable positive number

Remember NaN==NaN gives ans=0

i.e. different NaNs are not equal

e.g. 0/0 = NaN, 2*NaN=NaN

Matrices in matlab

A=[2 -1 0; -1 2 -1; 0 -1 1]

A(2,3) ans=-1

A(:,2) prints second column

Print a 2×2 submatrix

A(2:3,2:3)

Two division operators

/ : Right division

\ : Left division

X*A=B $\Rightarrow X = A/B$

A*X=B $\Rightarrow X = A\backslash B$

$A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}, \; B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$A/B = A \times \mathrm{inv}(B) = \begin{bmatrix} -1 & 4 \\ 0 & 2 \end{bmatrix}$

$A \backslash B = \mathrm{inv}(A) \times B \; (\text{used to solve AX} = B)$

$A \backslash B = \begin{bmatrix} -2 & -1 \\ 2.5 & 1.5 \end{bmatrix}$

Element-wise operations

.*:Element-wise multiplication

./:Element-wise division

.^:Element-wise exponentiation

$C_{\mathrm{ij}} = A_{\mathrm{ij}} B_{\mathrm{ij}}$

$A = [1 \quad 2 \quad 3; \quad 4 \quad 5 \quad 6];$

$B = [7 \quad 8 \quad 9; \quad 0 \quad 1 \quad 2]$

$C = A. * B = [7 \quad 16 \quad 27; \;\; 0 \quad 5 \quad 12]$

$D = [-2 \quad -1 \quad 0 \quad 1 \quad 2].\hat{}2$

$\quad = [(-2)^2 \quad (-1)^2 \quad 0 \quad (1)^2 \quad (2)^2]$

$F = [-2 \quad -1 \quad 0 \quad 1 \quad 2]\hat{}2$

What is the result?

Comparison operators

< Less than,

> Greater than

<= Less than or equal to

>= Greater than or equal to

== Equal to

~= Not equal to

A=[1 2 3; 4 5 6]; B=[7 8 9; 0 1 2]

A > B

ans= [0 0 0; 1 1 1]

Logical operators:

& AND; | OR; ~ NOT

(A>B) | (B > 5)

Function with multiple inputs

```
function y = myfun(x,p)
         y = 2*x.^p-3*x+1
         end
```

Function with multiple outputs

```
function [x2, x3, x4] = mypow(x)
x2 = x.^2
x3 = x.^3
x4 = x.^4
end
```

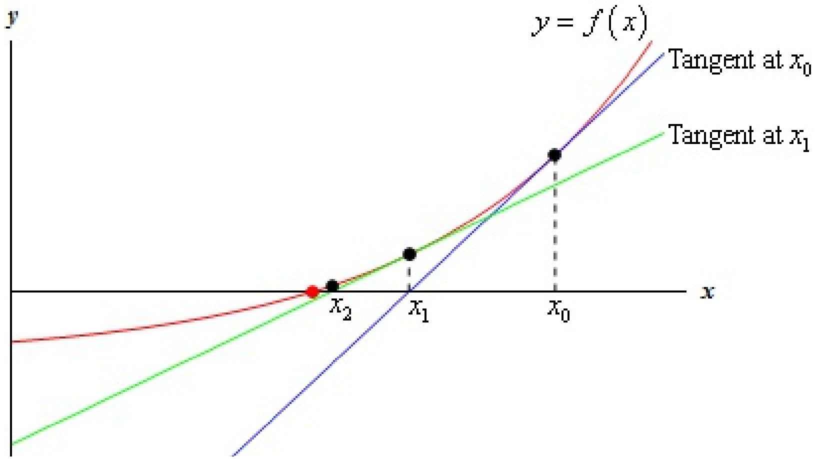
Newton-Raphson Method (Newton iterations)

$f(x) = 0$

Let the actual solution be x^*.

We start with a guess value x_0.

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$ (Taylor expansion?)



Taylor series

$f(x_{i+1}) = f(x_i) + f'(x_i)(x_{i+1} - x_i) + O(x_{i+1} - x_i)^2$

$\text{If } f(x_{i+1}) = 0$

$0 \approx f(x_i) + f'(x_i)(x_{i+1} - x_i)$

Newton-Raphson formula

Error $E_i = x - x_i$

$E_{i+1} = -\frac{f''(x_i)}{2f'(x_i)} E_i^2$

$\text{Newton}'s \text{ method converges quadratically.}$

```
function x = mynewton(f,f1,x0,N)

%Solve f(x)=0
%Inputs: function f, f1 the derivative of f, x0 - starting guess
%         N number of iterations (steps)
x = x0;
   for i=1:n
    x = x - f(x)/f1(x)
   end
end
```

f=@x x^3 - 5

f1=@x 3x^2

f = @x x^(1/3) f(x*=0)=0 f'(x*=0)=inf

f1 = @x (1/3)*x^(-2/3)

x = mynewton(f,f1,0.1,10)

format long to print more digits

Error at step N: $\epsilon_N = x_N - x^*$

Residual: $r_N = f(x_N) - 0$. We calculate the absolute value of residual to check closeness to the solution: $|r_N| = |f(x_N)|$

```
function x = mynewton(f,f1,x0,tol,N)
  x=x0;
  for i = 1:N
   x = x - f(x)/f1(x)
  end
  r=abs(f(x))
  if r > tol
   warning('Desired accuracy not achieved')
  end
end
```

```
function x= mynewton(f,f1,x0,tol)
  x = x0;
  y = f(x);
  while abs(y) > tol
     x = x  - y/f1(x)
     y = f(x)
  end
end
tol = 10^(-100)
```

Array manipulation

x = linspace(xfirst, xlast, nelem)

x = logspace(xfirst, xlast, nelem)

X=zeros(m,n) returns a matrix of m rows and n columns filled with zeroes

X=eye(n) nxn identity matrix

l = length(X) length of a vector

[m,n] = size(X) determines number of rows m and number of columns n.

a = 1:2:11

A = reshape(a,2,3)

Method of bisection (interval halving method)

Root of $f(x)=0$ is bracketed in the interval [a,b]

Theorem:

If the function $f(x)$ is continuous in $[a,b]$ and

$f(a)f(b)<0$ (i.e. f has values with different signs at a and b)

then a value $c\in(a,b)$ exists such that $f(c)=0$

- Bisection: attempt to locate c where f crosses over zero by checking whether it belongs to either of the two subintervals $[a,x_m]$ or $[x_m,b]$ where $x_m=(a+b)/2$ is the midpoint.

Algorithm (Bisection or binary search)

Algorithm 1

Bisection search on $[a, b]$

procedure BisectionSearch(f,a,b)
    $a_0 \leftarrow a, b_0 \leftarrow b, I_0 \leftarrow [a_0, b_0]$
    for $k = 0, 1, 2, ..., N\,\mathrm{do}$
        $x_m \leftarrow \frac{a_k + b_k}{2}$
        if $x_m = 0$ then
            $x_m\,\mathrm{is\,the\,desired\,solution, return.}$
        else if $f(a_k)f(x_m) < 0$ then
            $I_{k+1} := |a_{k+1}, b_{k+1}| \leftarrow |a_k, x_m|$
        else if $f\,(x_m)f(b_k) < 0$ then
            $I_{k+1} := |a_{k+1}, b_{k+1}| \leftarrow |x_m, b_k|$
        end if
    end for
    return the approximate solution $x_{\mathrm{approx}} = (a_N + b_N)/2$
end procedure

Convergence of bisection method

If $f(x)$ is continuous on $[a,b]$ and $f(a)\cdot f(b)<0$, the bisection mehod generates a sequence $\{p_n\}_{n=1}^{\infty}$ approximating a zero p of $f(x)$ with

$|p_n - p| = \left(\frac{1}{2}\right)^n (b-a)$ where $n \geqslant 1$

Criteria for stopping

Method 1: $|p_n - p_{n-1}| < \epsilon$

Method 2: $\frac{|p_n - p_{n-1}|}{|p_n|} < \epsilon, p_n \neq 0 \,\mathrm{or}\, |f(p_n)| < \epsilon$

Secant Method Solve $f(x)=0$

Take two initial approximations of the root x_0 and x_1

Evaluate $f(x_0)$ and $f(x_1)$

Let the exact root be s

Find the line joining $(x_0, f(x_0))\,\text{and}\,(x_1, f(x_1))$. Let the line intersect the $x-\text{axis}$ at x_2

$$m=\frac{f(x_1)-f(x_0)}{x_1-x_0}=\frac{f(x_2)-f(x_1)}{x_2-x_1}=\frac{0-f(x_1)}{x_2-x_1}$$

$$x_2=x_1-\frac{f(x_1)\,(x_1-x_0)}{f(x_1)-f(x_0)}$$

Iterative: $x_{i+1}=x_i-\dfrac{f(x_i)\,(x_i-x_{i-1})}{f(x_i)-f(x_{i-1})}$