Matrix Theory Assignment 2

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***Abstract*—This document contains the solution to calculate the shortest distance between 2 lines L_1 and L_2.**

Download all python codes from

```
https://github.com/gaurav−1205/EE5609−MatrixTheory/A2
```

and latex-tikz codes from

```
https://github.com/gaurav−1205/EE5609−MatrixTheory/A2/latex
```

1 Problem

Find the shortest distance between the lines

$$L_1 : \mathbf{x} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \tag{1.0.1}$$

and

$$L_2 : \mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \tag{1.0.2}$$

2 Solution

In the given problem,

$$\mathbf{A}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \mathbf{m}_1 = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}, \mathbf{A}_2 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, \mathbf{m}_2 = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \tag{2.0.1}$$

The solution for the lines to intersect is -

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} \tag{2.0.2}$$

$$\implies \lambda_1 \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} - \lambda_2 \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\implies \begin{pmatrix} 2 & 3 \\ -1 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \tag{2.0.4}$$

The augmented matrix will be -

$$\begin{pmatrix} 2 & 3 & 1 \\ -1 & -5 & 0 \\ 1 & 2 & -1 \end{pmatrix} \tag{2.0.5}$$

Row reducing the augmented matrix,

$$\begin{pmatrix} 2 & 3 & 1 \\ -1 & -5 & 0 \\ 1 & 2 & -1 \end{pmatrix} \xleftrightarrow{R_3 \leftrightarrow R_1} \begin{pmatrix} 1 & 2 & -1 \\ -1 & -5 & 0 \\ 2 & 3 & 1 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_3 = 2R_1 - R_3]{R_2 = R_1 + R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & -3 & -1 \\ 0 & 1 & -3 \end{pmatrix} \xleftrightarrow{R_2 \leftrightarrow R_3} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & -3 & -1 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_3 = 3R_2 + R_3} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & -3 \\ 0 & 0 & -10 \end{pmatrix} \tag{2.0.8}$$

This matrix has rank = 3 implying that the lines don't intersect. It is important to note that the lines are not parallel. Instead, they lie on parallel planes. Such lines are called *skew* lines. The figure 0 is generated using the following code.

```
latex/figures/line_dist_skew.py
```

The normal to both the lines (and corresponding planes) is

$$\mathbf{n} = \mathbf{m}_1 \times \mathbf{m}_2 \tag{2.0.9}$$

The equation of the second plane is then obtained as

$$\mathbf{n}^T \mathbf{x} = \mathbf{n}^T \mathbf{A}_2 \tag{2.0.10}$$

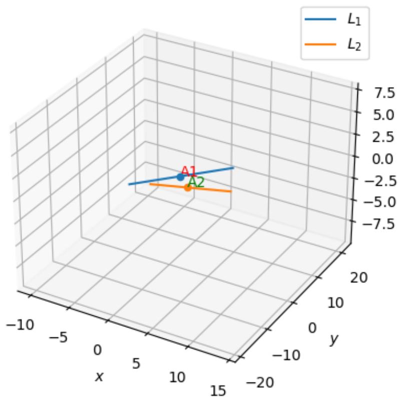

Fig. 0

The distance from $\mathbf{A}_1$ to the above line is then obtained as

$$\frac{|\mathbf{n}^T(\mathbf{A}_2 - \mathbf{A}_1)|}{||\mathbf{n}||} = \frac{|(\mathbf{A}_2 - \mathbf{A}_1)^T(\mathbf{m}_1 \times \mathbf{m}_2)|}{||\mathbf{m}_1 \times \mathbf{m}_2||} \tag{2.0.11}$$

$$= \frac{\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 3 \\ -1 \\ -7 \end{pmatrix}}{\sqrt{9+1+49}} \tag{2.0.12}$$

$$= \frac{10}{\sqrt{59}} \tag{2.0.13}$$

Therefore, the shortest distance between the 2 lines is **1.302** units.