Matrix Project

EE1390

Gaurav Gijare - EP17BTECH11007
Aditya Patel - EP17BTECH11002

Indian Institute of Technology, Hyderabad

February 14, 2019

Outline

1. Matrix Analysis
   - Geometry Question
   - Matrix transformation of the question
   - Solution in form of matrix
   - Solution Figure

Outline

Geometric Question

Question 1

Q1. Tangent and normal are drawn at $\mathbf{P} = \begin{pmatrix} -16 \\ 16 \end{pmatrix}$ on the parabola $\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 16 & 0 \end{pmatrix} \mathbf{x} = 0$ which intersect the axis of the parabola at **A** and **B** respectively. If **C** is the centre of the circle through the points **P**, **A** and **B**, find **tan CPB**.

Outline

Matrix transformation of the question

The general equation of a conic in matrix form is:

$$\mathbf{x}^T \mathbf{V}\ \mathbf{x} + 2\mathbf{U}^T\ \mathbf{x} + \mathbf{F} = 0$$

Comparing with the given equation of parabola:

$$\mathbf{x}^T \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 16 & 0 \end{pmatrix} \mathbf{x} = 0$$

Matrix transformation of the question

We get,

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{U} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$\mathbf{F} = \begin{bmatrix} 0 \end{bmatrix}$$

Outline

Solution using matrices

The equation of a tangent to the parabola at point $\mathbf{P}$ is:

$$\begin{pmatrix} \mathbf{P}^T & 1 \end{pmatrix} \begin{pmatrix} \mathbf{V} & \mathbf{U} \\ \mathbf{U}^T & \mathbf{F} \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ 1 \end{pmatrix} = 0$$

It can also be expressed as:

$$\left(\mathbf{P}^T\mathbf{V} + \mathbf{U}^T\right)\mathbf{x} + \mathbf{P}^T\mathbf{U} + \mathbf{F} = 0$$

Therefore, the equation of tangent becomes:

$$\left[\begin{pmatrix} -16 & 16 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 8 & 0 \end{pmatrix}\right]\mathbf{x} + \begin{pmatrix} -16 & 16 \end{pmatrix} \begin{pmatrix} 8 \\ 0 \end{pmatrix} = 0$$

Solution using matrices

Comparing with the equation of line, the normal vector of tangent will be:

$$\mathbf{n}^T = \left[\begin{pmatrix} -16 & 16 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 8 & 0 \end{pmatrix}\right] \implies \mathbf{n} = \begin{pmatrix} 8 & 16 \end{pmatrix}$$

Therefore, the direction vector for the tangent will be

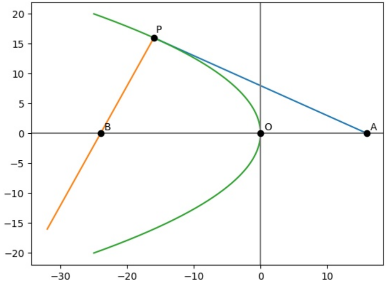
$$\mathbf{m}_{tangent} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 8 \\ 16 \end{pmatrix} = \begin{pmatrix} -16 \\ 8 \end{pmatrix}$$

Therefore, the equation of normal will be:

$$\mathbf{X} = \begin{pmatrix} -16 \\ 16 \end{pmatrix} + \lambda \begin{pmatrix} 8 \\ 16 \end{pmatrix}$$

Solution using matrices

Now, we have equation of tangent, normal as well as parabola.
Lets visualize it:



Solution using matrices

Now, we have to draw a circle which passes through **B**, **P** and **A**:

- As normal and tangent at **P** are perpendicular to each other, seg **AB** will be the diameter of the circle with centre **C**.

Solution using matrices

Now, we have to draw a circle which passes through **B**, **P** and **A**:

- As normal and tangent at **P** are perpendicular to each other, seg **AB** will be the diameter of the circle with centre **C**.
- The x-intercepts $\mathbf{A} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$

Solution using matrices

Now, we have to draw a circle which passes through **B**, **P** and **A**:

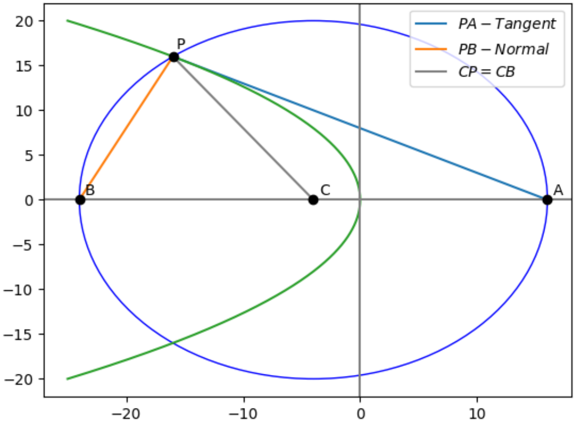
- As normal and tangent at **P** are perpendicular to each other, seg **AB** will be the diameter of the circle with centre **C**.
- The x-intercepts $\mathbf{A} = \begin{pmatrix} 16 \\ 0 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$
- Centre of circle = mid-point of **A** and **B**
  $\implies \mathbf{C} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

Solution using matrices

Now, we have to draw a circle which passes through **B**, **P** and **A**:

- As normal and tangent at **P** are perpendicular to each other, seg **AB** will be the diameter of the circle with centre **C**.
- The x-intercepts $\mathbf{A} = \begin{pmatrix} 16 \\ 0 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} -24 \\ 0 \end{pmatrix}$
- Centre of circle = mid-point of **A** and **B**
  $\implies \mathbf{C} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$
- Radius of the circle = distance between points **C** and **A**
  $\implies \mathbf{R} = 20$

We have to find Tan $\angle$**CPB** :



Solution using matrices

- $\mathbf{CB} = \mathbf{CP} = \mathbf{CA} = \mathbf{R}$

Solution using matrices

- $\mathbf{CB} = \mathbf{CP} = \mathbf{CA} = \mathbf{R}$
- $\angle\mathbf{CPB} = \angle\mathbf{CBP} \implies Tan\ \angle\mathbf{CPB} = Tan\ \angle\mathbf{CBP}$

Solution using matrices

- $\mathbf{CB} = \mathbf{CP} = \mathbf{CA} = \mathbf{R}$
- $\angle\mathbf{CPB} = \angle\mathbf{CBP} \implies Tan\ \angle\mathbf{CPB} = Tan\ \angle\mathbf{CBP}$
- But $Tan\ \angle\mathbf{CBP}$ = slope of normal $\mathbf{BP}$

Solution using matrices

- $\mathbf{CB} = \mathbf{CP} = \mathbf{CA} = \mathbf{R}$
- $\angle\mathbf{CPB} = \angle\mathbf{CBP} \implies Tan\ \angle\mathbf{CPB} = Tan\ \angle\mathbf{CBP}$
- But $Tan\ \angle\mathbf{CBP}$ = slope of normal **BP**
- $Tan\ \angle\mathbf{CPB} = \frac{16}{8} = 2$

Outline

Solution Figure