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Zero Moment Point Control of a Biped Robot Using Feedback Linearization

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Abstract. In this paper we investigate design and implementation of an input-output linearization controller on a biped robot. First, the model of the biped robot is derived by a 3D linear inverted pendulum model (3D-LIPM). Blending the 3D-LIPM and the definition of the zero moment point (ZMP), the mathematical representation of ZMP is obtained. Next, we use state space representation of biped robot dynamics and finally, ZMP input is linearized to output and the controller is applied on a virtual biped robot conforming to NAO robot. Results are illustrated through simulations and local robustness of proposed controller is demonstrated by introducing the noise into the measured states. Furthermore, the proposed controller seems convenient for implementation on real robots.

Keywords: Biped robot, Input-Output linearization, Zero moment point (ZMP).

1 Introduction

BIPEDrobots have better mobility than conventional wheeled robots, especially for moving on rough and uneven terrain. Study of these robots and their stability has been the focus of too many researches in the last decades. Always there are two processes to realize a stable walk for a biped robot. The first one is walking pattern generation and the other one is stabilization via a feedback control algorithm [1].
Current trends in robotics have been shifted towards autonomous mobile robots rather than the conventional fixed-base robots or wheeled ones, which are not capable of obstacle avoidance. Nowadays more research is carried out on the field of autonomous humanoid robots and some real humanoid robots have been therefore introduced [2]. A perfect application for developing humanoid robots that can interact with humans is RoboCup. RoboCup is pursuing the goal which states “By the year 2050, develop a team of fully autonomous humanoid robots to win against the human world cup champion team” [3]. It’s expected that the humanoids be the suitable

replacement for cumbersome human involved activities and also be a good assistant for human beings especially the disabled or aged people.

One of the most important challenges in humanoid robotics is to design a stable gait through walking path generation. Such design is highly dependent on a closed-loop control. Current robots suffer from open-loop control which gives no opportunity to stabilize the walking or enable it to walk on uneven terrains and consequently the walking is not smooth enough and it's statically stable [4]. Static stability states that the projection of humanoid robot's center of mass (CoM) must lie in the convex hull of foot polygon. Since therobots center of mass is in motion all the time while the feetperiodically interact with the ground in a unilateral way, therefore the movement of the CoM cannot be controlled directly, but is governed by its momentum. Although, controlling the CoM to be inside the safe area, makes the walking very slow. In opposition, the dynamic walking stability, as in human being, brings the ZMP concept into account [5] by which the projection of CoM can sometimes leave the convex hull provided that the ZMP doesn't leave the sole polygon.

In fact, according to Vukobratovic's classical notation3, the ZMPis only defined inside the support polygon. ThisZMP definition is equivalent tothe center of pressure(CoP), which naturally is defined inside the robot's foot. If the ZMP is at the support polygon's edge, a small moment would bring about the robot to tip over around thatedge. Nevertheless, applying the criterion of zerotipping moment, result in a point outside the support polygonin this case. Such a point has been proposed as the foot rotationindicator (FRI) point [6] or the fictitious ZMP (FZMP). In fictitious case the distance to the support polygon is anindicator for the magnitude of the unbalanced moment that causesthe instability and therefore is a useful measure for controlling thegait.

Input-output linearization method is a nonlinear control method by which a feedback control law is derived such that the system's output representation is linearized. Although the control law is locally stabilizing [7,8] but it has sufficient robustness required to stabilize the biped robot. Setting appropriate coefficients in linearized differential equation, faster tracking and less error and noise rejection is gained.

Since the precise mathematical representation of a biped robot is burdensome to derive, because of high degree of freedom, a simple model is instead introduced. This simple model is called 3D linear inverted pendulum mode (3D-LIPM) by which a general information of robot such as total center of mass, total angular momentum, total acceleration and etc. is gained. So the controller must compensate for lack of each link's exact information. The results will confirm that input-output linearization can do this.

This paper is organized as follows: first a brief model of the biped robot is presented and it's transformed into state space form. Then the input-output linearization controller is implemented in part three. Section four deals with the results of the proposed controller.

2 Biped Robot Modeling

2.1 Three Dimensional Inverted Pendulums

The common way of biped robot modeling as mentioned above is using 3D-LIPM. As the robot experiences the single supporting phase, the swinging leg can be modeled by an inverted pendulum whose massless rod connects the supporting foot to the CoM of the robot. This simple model is illustrated in Fig.1.

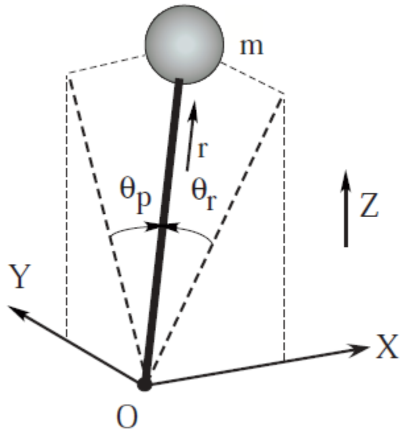

Fig.1. Three Dimensional Inverted Pendulum

θ_r (<0) and θ_p (>0) are the angles between the CoM and XZ plane and YZ plane, respectively. The signs are chosen so that the rule of right hand is satisfied. r is the length of the pendulum. Let (τ_r,τ_p,f) be the set of actuators' torque and force, associated with the set of joint variables (θ_r,θ_p,r). These inputs define the governing equations of motion of the 3D pendulum as follows

$$m\left(-z\ddot{y}+y\ddot{z}\right)=\frac{D}{C_r}\tau_r-mgy \qquad (1)$$

$$m\left(z\ddot{x}-x\ddot{z}\right)=\frac{D}{C_p}\tau_p+mgx \qquad (2)$$

Where $C_r \equiv \cos\theta_r$, $C_p \equiv \cos\theta_p$, $D \equiv \sqrt{C_r^2+C_p^2-1}$, m is the mass and g is the gravity.

2.2 Three Dimensional Linear Inverted Pendulum Model

Since the inverted pendulum has vast possibilities for movements, it's desired to select a particular motion corresponds to biped robot's motion. This motion is obtained by restricting the CoM to be in a plane defined by the given normal vector $(k_x,k_y,-1)$ and z intersection z_c; i.e.

$$z = k_x x + k_y y + z_c \tag{3}$$

In order to walk on a rugged terrain the coordinate must be chosen so that the normal vector of plane, $(k_x,k_y,-1)$, matches the slope of the terrain.
The second derivation of the constraint **(3)** is

$$\ddot{z} = k_x \ddot{x} + k_y \ddot{y} \tag{4}$$

Substituting this equation into equations (**1**) and (**2**) the dynamics of 3D-LIPM is obtained as

$$\ddot{y} = \frac{g}{z_c} y - \frac{k_x}{z_c}(x\ddot{y} - \ddot{x}y) - \frac{1}{mz_c} u_r \tag{5}$$

$$\ddot{x} = \frac{g}{z_c} x + \frac{k_y}{z_c}(x\ddot{y} - \ddot{x}y) + \frac{1}{mz_c} u_p \tag{6}$$

Where u_r and u_p are new virtual inputs and are introduced to compensate input nonlinearity

$$u_r = \frac{D}{C_r}\tau_r \tag{7}$$

$$u_p = \frac{D}{C_p}\tau_p \tag{8}$$

When the robot is walking on the flat terrain, the resultant constraint **(3)** is simply converted to $z=z_c$ which makes the Eqs. (5) and (6) as

$$\ddot{y} = \frac{g}{z_c} y - \frac{1}{mz_c} u_r \tag{9}$$

$$\ddot{x} = \frac{g}{z_c} x + \frac{1}{mz_c} u_p \tag{10}$$

2.3 Control through input-output linearization

In order to proceed with the control, first the equations of motion need to be mentioned in the state space form. By introducing the states in the form of

$$\dot{X} = AX + BU \tag{11}$$

$$
\mathrm{Y} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathrm{x} \\ \dot{\mathrm{x}} \\ \mathrm{y} \\ \dot{\mathrm{y}} \end{bmatrix} = \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix} \tag{12}
$$

And [8] then

$$
\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \mathrm{g}/z_c & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \mathrm{g}/z_c & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1/mz_c & 0 \\ 0 & 0 \\ 0 & -1/mz_c \end{bmatrix} \begin{bmatrix} u_p \\ u_r \end{bmatrix} \tag{13}
$$

To linearize the input-output equation, the output equation is differentiated with respect to time until the input appears on the right hand side:

$$
\begin{gathered}
\begin{bmatrix} e_x \\ e_y \end{bmatrix} = Y_r - Y = \begin{bmatrix} x_r - x \\ y_r - y \end{bmatrix} \\
\begin{bmatrix} \dot{e}_x \\ \dot{e}_y \end{bmatrix} = \dot{Y}_r - \dot{Y} = \begin{bmatrix} \dot{x}_r - \dot{x} \\ \dot{y}_r - \dot{y} \end{bmatrix} \\
\begin{bmatrix} \ddot{e}_x \\ \ddot{e}_y \end{bmatrix} = \ddot{Y}_r - \ddot{Y} = \begin{bmatrix} \ddot{x}_r - \ddot{x} \\ \ddot{y}_r - \ddot{y} \end{bmatrix} = \begin{bmatrix} \ddot{x}_r - \mathrm{g}x/z_c - u_p/mz_c \\ \ddot{y}_r - \mathrm{g}y/z_c - u_r/mz_c \end{bmatrix}
\end{gathered} \tag{14}
$$

Thus a stabilizing control law is constructed such that the dynamics of the system becomes linear. By applying the input as:

$$
\begin{bmatrix} u_p \\ u_r \end{bmatrix} = \begin{bmatrix} mz_c\left(-k_{dx}\dot{e}_x - k_{px}e_x + \mathrm{g}x/z_c\right) \\ mz_c\left(-k_{dy}\dot{e}_y - k_{py}e_y + \mathrm{g}y/z_c\right) \end{bmatrix} \tag{15}
$$

and using a linear (i.e. PD controller), output tracking can be achieved.

$$
\begin{bmatrix} \ddot{e}_x + k_{dx}\dot{e}_x + k_{px}e_x \\ \ddot{e}_y + k_{dy}\dot{e}_y + k_{py}e_y \end{bmatrix} = 0 \tag{16}
$$

By proper selection of $k_{px}, k_{dx}, k_{py}, k_{dy}$, a desirable error dynamics can be achieved.

3 Controller layout and simulation results

Now the feedback control law (15) is applied on a model with real properties (A 58cm tall NAO humanoid robot with z_c=25cm [9]). The robots model is depicted in Fig. 2. Since the behavior of the robot in sagittal plane is same as that in the frontal plane, only one plane is considered to be controlled. The results of control are shown in Fig. 3 through Fig. 6. It can be seen that the commanded ZMP is tracked thoroughly (Fig. 3 and Fig. 4). Now the robustness of the controller is checked by applying noise on measured states. This situation are illustrated in Fig. 5 and Fig. 6 where a nearly strong change can be seen in ZMP trajectory.

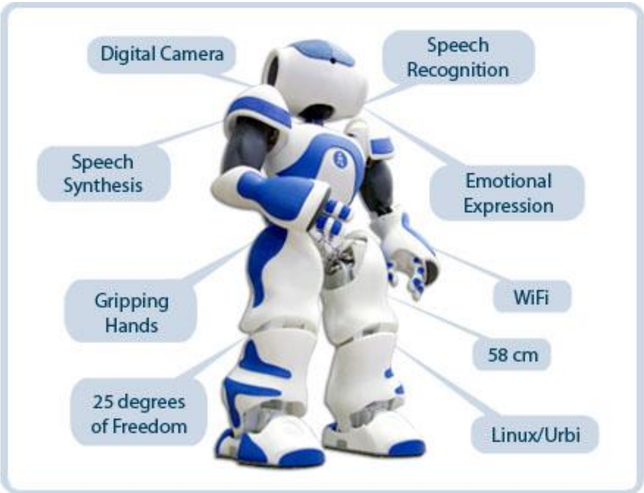

Fig. 2 The NAO Robot

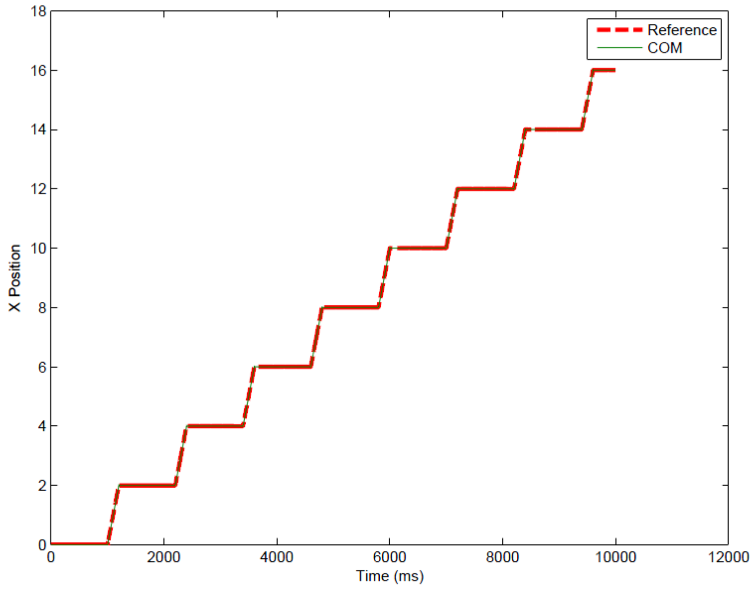

Fig. 3 ZMP tracking by the proposed controller (X Position)

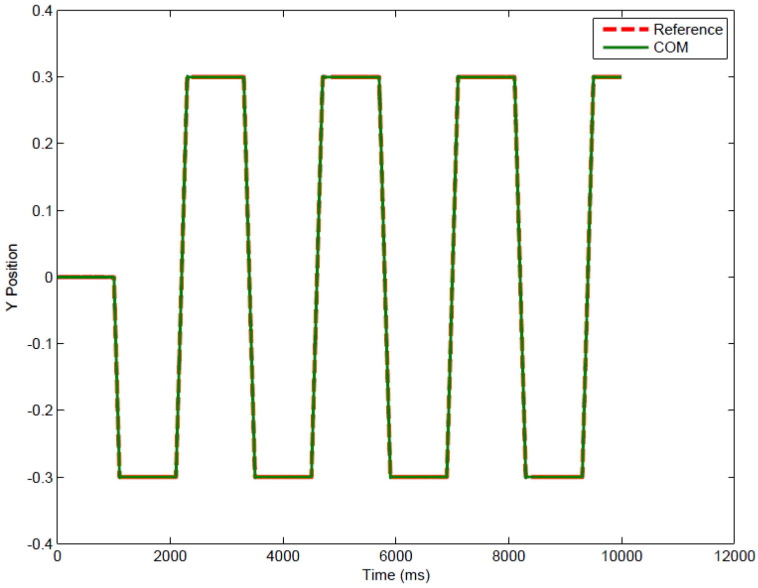

Fig. 4 ZMP tracking by the proposed controller (Y Position)

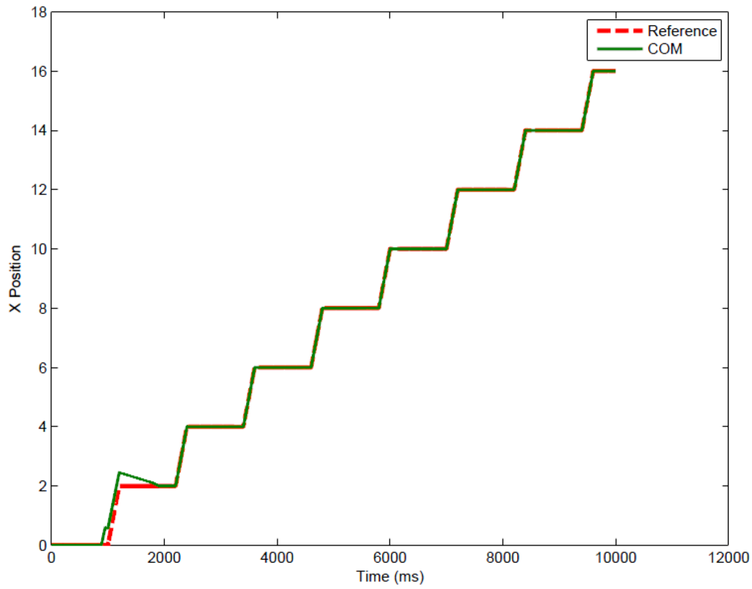

Fig. 5 ZMP Tracking with noise (X Position)

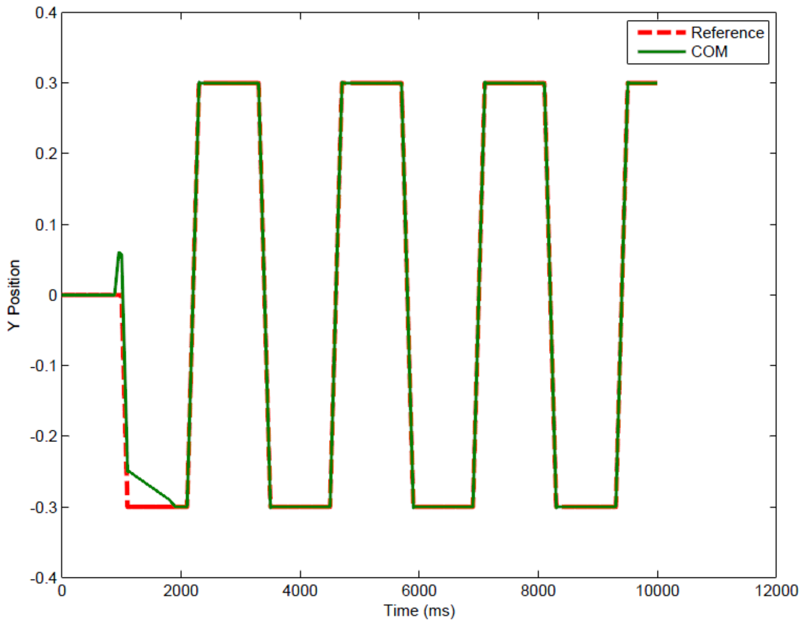

Fig. 6 ZMP Tracking with noise (Y Position)

4 Conclusions

To control the biped robot's ZMP trajectory, input-output linearization method as a novel method in control of biped robot, was employed. It was seen that fast response and low maximum over shoot (%0.5) have been achieved. Simplicity of this method compared to classical optimal methods such as preview and/or LQR methods [10], makes it to be applicable on biped robots. Those classical methods lead in low gain in low frequency commands but the gains of preview become very large in high frequencies and consequently it's very sensitive to sudden changes in ZMP trajectory. Since the input-output linearization method is locally stable, it can compensate changes in biped robot's ZMP trajectory better than those optimal methods. Furthermore, robust performance of the proposed controller was investigated by applying noise on measured states. Afterwards, the next project of the team is to integrate this method into the robot and verify the controller in real environment.

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