$$f_i(x) \approx g_i(x) = f_i(x^k) + \nabla f_i|_{x^k} \cdot (x - x^k) + \frac{\rho_i}{2} \sum_{j=1}^{n} \left(\frac{x_j - x_j^k}{\sigma_j} \right)^2 \qquad i \in \{0, 1, \ldots, m\}$$

$$\begin{aligned}
L(x,\lambda) =& g_0(x) + \sum_{i=1}^{m} \lambda_i g_i(x) \\
=& f_0(x^k) + \nabla f_0|_{x^k} \cdot (x - x^k) + \frac{\rho_0}{2} \sum_{j=1}^{n} \left(\frac{x_j - x_j^k}{\sigma_j} \right)^2 \\
&+ \sum_{i=1}^{m} \lambda_i \left(f_i(x^k) + \nabla f_i|_{x^k} \cdot (x - x^k) + \frac{\rho_i}{2} \sum_{j=1}^{n} \left(\frac{x_j - x_j^k}{\sigma_j} \right)^2 \right) \\
=& f_0(x^k) + \sum_{i=1}^{m} \lambda_i f_i(x^k) \\
&+ \sum_{j=1}^{n} \left(\left(\left.\frac{\partial f_0}{\partial x_j}\right|_{x^k} + \sum_{i=1}^{m} \lambda_i \left.\frac{\partial f_i}{\partial x_j}\right|_{x^k} \right) (x_j - x_j^k) + \frac{1}{2} \left(\rho_0 + \sum_{i=1}^{m} \lambda_i \rho_i \right) \left(\frac{x_j - x_j^k}{\sigma_j} \right)^2 \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial L}{\partial x_j} = \left.\frac{\partial f_0}{\partial x_j}\right|_{x^k} + \sum_{i=1}^{m} \lambda_i \left.\frac{\partial f_i}{\partial x_j}\right|_{x^k} + \left(\rho_0 + \sum_{i=1}^{m} \lambda_i \rho_i \right) \frac{x_j - x_j^k}{\sigma_j^2} = 0 \\
\implies \qquad \hat{x}_j = x_j^k - \frac{\left.\frac{\partial f_0}{\partial x_j}\right|_{x^k} + \sum_{i=1}^{m} \lambda_i \left.\frac{\partial f_i}{\partial x_j}\right|_{x^k}}{\rho_0 + \sum_{i=1}^{m} \lambda_i \rho_i} \sigma_j^2 \\
\implies \qquad \hat{x} = x^k - \left(\nabla f_0|_{x^k} + J|_{x^k}^T \lambda \right) ./ (\rho_0 + \lambda \cdot \rho) .\times \sigma.^2 \\
x_j^{\min} = \max\{x_j^k - \sigma_j, \text{lower_bound}_j\} \\
x_j^{\max} = \min\{x_j^k + \sigma_j, \text{upper_bound}_j\} \\
x_j^* = x_j(\lambda) = \begin{cases} \hat{x}_j & \text{if } x_j^{\min} < \hat{x}_j < x_j^{\max} \\ x_j^{\min} & \text{if } \hat{x}_j \le x_j^{\min} \\ x_j^{\max} & \text{if } x_j^{\max} \le \hat{x}_j \end{cases} \\
x^* = \min.\{\max.\{\hat{x}, x^{\min}\}, x^{\max}\}
\end{aligned}$$

$$h(\lambda) = \min_{x \in T} L(x,\lambda) = L(x^*, \lambda) = g_0(x^*) + \sum_{i=1}^{m} \lambda_i g_i(x^*) = g_0(x^*) + \lambda \cdot g(x^*)$$

$$\frac{\partial h}{\partial \lambda} = g(x^*)$$

$$\max_{\lambda \succeq 0} h(\lambda)$$