PROJECT: TRANSIENT THERMAL ANALYSIS OF A CAR BRAKE SYSTEM

MA203: NUMERICAL METHODS

Gaurav Budhwani
2211085
Chemical Engineering

Hitesh Kumar
22110098
Computer Science and Engineering

Aditya Arvind Mehta
22110150
Chemical Engineering

Pawan Seth
22110192
Material Science and Engineering

Shaury Kumar Patel
22110241
Mechanical Engineering

Professor: Dilip Srinivas Sundaram
and
Akshaa Vatwani

October 4, 2023

Contents

1 Problem Statement

Brake systems in automobiles are crucial for ensuring vehicle safety and control, as they play a key role in converting kinetic energy into heat during braking events. The transient thermal behaviour of car brake systems is a critical aspect of their design and performance. This project aims to analyse the thermal dynamics within a car's brake system under various operating conditions.
Understanding the transient thermal behaviour of car brake systems is essential for optimising their design, preventing overheating, and ensuring consistent braking performance. The project seeks to address real-world challenges associated with thermal management in brake systems.
The project will involve a combination of computational modelling, numerical simulations, and data analysis to gain insights into the transient thermal behaviour of car brake systems. The Finite Difference Method [4] will provide a robust framework for simulating heat conduction within the brake components. The outcomes of this project will contribute to advancing our understanding of transient thermal dynamics in car brake systems. The findings may lead to improved brake system designs, enhanced safety, and the prevention of issues related to overheating during diverse driving conditions.

2 Objectives

This code aims to perform numerical simulation and visualisation of temperature distribution within a two - dimensional car brake system. This simulation aims to provide insights into how heat is distributed and dissipated within the brake's components while braking. The code written allows us to investigate the thermal behaviour over time. Simulating the temperature changes over time and visualising the results contributes to the development of more efficient and reliable braking solutions, ultimately enhancing the vehicle's performance and safety.

2.1 Specific Objectives are as follows:

- Comparing the numerical model with the physical model of the brake system.
- Making Suitable Assumptions.
- Deriving the governing equations from first principles.
- Setting the Initial Boundary Conditions.
- Choosing the correct set of parameters and values.
- Adopting a suitable method for Numerical Analysis.
- Writing the Computer Program for solving the equation and obtaining solutions for the above method for Numerical Analysis.

- Visualisation of Temperature Profiles, initially and finally.
- Temperature Evolution at Specific Locations.
- Analysing Temperature at Centre Point w.r.t. Frictional Heat Generation (Q).

3 Physical Model

The physical model is like a basic map that shows the main parts of a car's brake system. It's a simple but complete representation that includes the important stuff like brake rotors, brake pads, and the things around them.
This model is designed in a way that's easy to understand. It uses two-dimensional coordinates (x, y) to help us see how heat moves through the brake system when you press the brakes. It's like looking at a picture to figure out where the heat goes.
Imagine you're playing with building blocks. When you stack them, they might get warm. The model helps us figure out where the warmth goes in the brake system when you use it. This is important because it helps us make the brakes work well and stay safe. In simple terms, this model is like a helpful tool that shows us how the brakes do their job, which is to stop the car when you need them to.

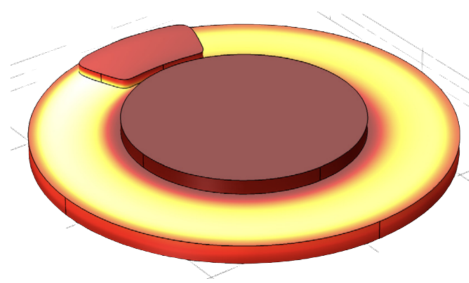

Figure 1: Frictional Heat Generation in a Brake Disc [1]

4 Assumptions

To ensure the problem's tractability, we make the following assumptions:

- Steady-state conditions before braking events.
- Uniform initial temperature across brake components.
- Negligible heat conduction in the axial direction of the brake components.
- Simplified geometry for numerical efficiency.

- This analysis is done when the brakes have been applied, but the automobile has not stopped, and due to the friction, the heat is generated, and its propagation is shown in the 3D plot.
- The temperature variation across a tiny piece of the brake disc is similar to the temperature variation across the brake disc; therefore, instead of applying the Numerical analysis on the complete brake disc, we have taken one small area, which can be considered as a thin metal plate/sheet.

5 Governing Equation

The Transient Heat Equation [2] is given by

$$\rho c \frac{\partial T}{\partial t} = \nabla(k \Delta T) + Q \tag{1}$$

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial(k \frac{\partial T}{\partial x})}{\partial x} + \frac{\partial(k \frac{\partial T}{\partial y})}{\partial y} + Q \tag{2}$$

- ρ: This symbolizes the density of the brake materials. In simpler terms, it signifies how much mass is packed into a given volume of the brake components. This factor is crucial because it affects how quickly the brake parts heat up or cool down when you apply the brakes.
- **c**: This is the specific heat of the material used in the brake.
- **T**: Temperature (in degrees Celsius) is what we're trying to understand. It reveals how hot or cold the brake components become as time progresses during braking.
- **k**: Thermal conductivity, denoted by 'k', characterizes the ability of the brake materials to transfer heat. Think of it as how well the heat can travel through the brake components. High thermal conductivity means heat spreads quickly, while low conductivity means it's slower.
- **Q**: This term represents the frictional heat generated during braking. It's like the heat produced when you rub your hands together. Q depends on the force (F) applied on the brakes and the velocity (ν) of the braking surface, along with the coefficient of friction (μ) for the surface. It tells us how much heat is created during the braking process.
- ∇: This is the gradient operator which represents the spatial derivatives.

6 Boundary Conditions

For this analysis, we establish specific conditions at the edges or boundaries of our brake system.

6.1 Initial Condition:

Initially, at y = 0, the temperature is set at 400°C, signifying the starting point for our analysis. At all other points, except the boundary conditions mentioned below, the temperature is a uniform 25°C.

$$T = 400\ ^\circ\text{C, at } y = 0$$
$$\text{At any other point } T = 25\ ^\circ\text{C}$$

6.2 Insulated Boundaries:

Dirichlet conditions are considered where the temperature conditions at the boundaries are predefined. These conditions replicate scenarios where the brake system's edges are thermally isolated, which prevents heat from escaping easily. At y = L_y, x = 0, and x = L_x, the temperatures are fixed at 25°C, mimicking the effect of insulated boundaries.

$$T = 400\ ^\circ\text{C, at } y = 0$$
$$T = 25\ ^\circ\text{C, at } y = L_y$$
$$T = 25\ ^\circ\text{C, at } x = 0 \text{ and } x = L_x$$

6.3 Heat Generation at Brake Components:

This parameter represents the heat produced due to the friction between the brake components during braking. It's calculated using the coefficient of friction (μ), the normal force applied on the brakes (F), and the velocity of the braking surface (ν). This equation helps us quantify the amount of heat generated during the braking process.

Q = frictional heat generation during braking.

Q = μ.F.ν, where μ = 0.55 (coefficient of friction of the surface)

F = 660 - 1000 N (Normal force applied on the brakes)

V = 40 - 60 km/h (Velocity of braking surface)

7 Parameters

7.1 Source term

The source term corresponds to the Frictional Heat Generation during braking which is denoted by Q . It quantifies the conversion of kinetic energy into thermal energy due to friction between the brake components. Since, Q is directly proportional to Force and Velocity, higher will be the velocity or force, more is the heat generation.

7.2 Boundary conditions for the equation

- T_0 is the initial uniform temperature.
- L_x and L_y represents the dimensions of the brake components.
- N_x and N_y represent the number of spatial grid points in the x and y directions, respectively.

7.3 Other Parameters

- ρ, c, k are material properties specific to each brake component.

Table 1: Parameters Related to the Problem [3]

Variable	Value	Unit
Q	1000-5000	W/m^2
c	460-1200	J/Kg.C
α	1.17	cm^2/s
R	8.314	J/mol.C
k	400	J/m.s.C

8 Solution Methadology

8.1 Discretization using Finite Difference Method [4]:

The spatial domain is discretized into a grid with nodes at ix, jy, where x and y are the spatial step sizes. The temporal domain is discretized with a time step t. Central differences are used for spatial derivatives:

$$\frac{\partial T}{\partial x} \approx \frac{T_{i+1,j} - T_{i-1,j}}{2\Delta x} \tag{3}$$

$$\frac{\partial T}{\partial y} \approx \frac{T_{i,j+1} - T_{i,j-1}}{2\Delta y} \tag{4}$$

Second-order central differences are used for the Laplacian terms:

$$\frac{\partial^2 T}{\partial x^2} \approx \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{2\Delta x^2} \tag{5}$$

$$\frac{\partial^2 T}{\partial y^2} \approx \frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{2\Delta y^2} \tag{6}$$

8.2 Numerical Algorithm-Forward Euler Method

The Forward Euler Method is used for accurate time-stepping:

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \frac{1}{\rho c}\left(\frac{T_{i+1,j}^{n} - 2T_{i,j}^{n} + T_{i-1,j}^{n}}{\Delta x^2} + \frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{\Delta y^2}\right) + \frac{Q_{i,j}^{n}}{\rho c} \tag{7}$$

Where n is the step index.T

9 Analytical Solution

Obtaining an analytical solution for the transient heat conduction equation with complex boundary conditions is challenging and often not feasible. Analytical solutions are generally limited to more straightforward cases with specific boundary conditions.

10 Non-Dimensionalization of Governing Equations

- Non-dimensionalization involves introducing dimensionless variables to simplify and analyse the governing equations.
- Introduce dimensionless variables:
  - $$\bar{T} = \frac{T - t_0}{\Delta T} \tag{8}$$
  - $$\bar{x} = \frac{x}{L_x} \tag{9}$$
  - $$\bar{y} = \frac{y}{L_y} \tag{10}$$
  - $$\bar{t} = \frac{t}{t_{simulation}} \tag{11}$$
- The non-Dimensionalized heat conduction equation becomes:

  $$\frac{\partial \bar{T}}{\partial t} = \alpha\left(\frac{\partial^2 \bar{T}}{\partial x^2} + \frac{\partial^2 \bar{T}}{\partial y^2}\right) + \frac{Q}{\rho c \Delta T} \tag{12}$$

  where α is thermal diffusivity m^2/s

11 Numerical Solution

- The temperature distribution within the brake components is simulated over time using the Finite Difference Method [4].
- The time-stepping loop iterates through each time step, updating the temperature array based on the discretized heat conduction equation.

12 Algorithm Used

- A time-stepping loop is implemented to update the temperature array using the Forward Euler method iteratively.
- Nested loops traverse the interior grid points, applying finite difference calculations to update the temperature at each point.

12.1 Visualisation

- The code includes visualisations of temperature evolution over time using 3D surface plots and contour plots.
- Temperature profiles are plotted initially, at specific time steps, and finally.

12.2 Temperature Evolution at Specific Locations

- The code tracks the temperature evolution at specific locations, such as the centre and corner of the brake components.
- Plots depict how temperatures change at these locations over the simulation period.

12.3 Heat Generation(Q) Sensitivity Analysis

- The code conducts a sensitivity analysis by simulating temperature distribution for different values of the heat generation term Q.
- Centre temperatures are plotted for various Q values, providing insights into the impact of heat generation on temperature evolution.

12.4 Python Code for Algorithm

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# Defining all the parameters
Lx = 0.04      # Length of the brake comp. in the x-dir.
Ly = 0.01      # "  "  "  "  "  "  "  "  "  "  "  y-dir.
Nx = 50        # No. of spatial grid pts. in the x-dir.
Ny = 10        # "  "  "  "  "  "  "  "  "  "  "  y-dir.
alpha = 1.17e-4
t_simulation = 1.0  # Total simulation time (in sec.)
dx = Lx / Nx  # Length of a single grid point in x-dir.
dy = Ly / Ny  # "  "  "  "  "  "  "  "  "  "  "  y-dir.
dt = 0.001    # Time step
Q = 5000.0
c = 460
ro = 7870
T_high = 400


# Creating a grid for spatial coord. (x, y)
x = np.linspace(0, Lx, Nx)
y = np.linspace(0, Ly, Ny)
X, Y = np.meshgrid(x, y)

# Initializing the temp. array with uniform initial temp.
T0 = 25.0
T = np.ones((Nx, Ny)) * T0

# Dirichlit Boundary Conditions
T[:, 0] = T_high  # High temp. at one end
T[:, -1] = T0 # Low temp. at the other end

# Creating a list to store snapshots of temp. over time
temp_ss = []

```

```python
# A for loop for performing Finite Difference Method
num_time_steps = int(t_simulation / dt)
for step in range(num_time_steps):
    T_new = T.copy()
    for i in range(1, Nx - 1):
        for j in range(1, Ny - 1):
            # Updating temp. using the transient heat eqn.
            T_new[i, j] = T[i, j] +  dt * alpha * ((T[i+1, j] - 2 * T[i, j]
                    + T[i-1, j]) / dx**2 + (T[i, j+1] - 2 * T[i, j]
                    + T[i, j-1]) / dy**2
                    + Q / (alpha * ro * c * (T_high - T0)))
    T = T_new
    temp_ss.append(T.copy())

# Visualization of temp. variation over t with const. temp. axis scaling
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
ax.clear()
ax.plot_surface(X, Y, temp_ss[1].T, cmap='hot')
ax.set_xlabel('X-axis')
ax.set_ylabel('Y-axis')
ax.set_zlabel('Temperature (°C)')
ax.set_title(f'Temperature Distribution at Time Step 0')
plt.pause(5)
for i, temp_snapshot in enumerate(temp_ss):
    if i % 10 == 0:
        ax.clear()
        ax.plot_surface(X, Y, temp_snapshot.T, cmap='hot')
        ax.set_xlabel('X-axis')
        ax.set_ylabel('Y-axis')
        ax.set_zlabel('Temperature (°C)')
        ax.set_title(f'Temperature Distribution at Time Step {i}')
        plt.pause(0.01)
plt.show()

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(12, 5))
# Plot initial temp. profile
ax1.contourf(X, Y, temp_ss[0].T, cmap='hot')
ax1.set_xlabel('X-axis')
ax1.set_ylabel('Y-axis')
ax1.set_title('Initial Temperature Distribution')

# Plot final temp. profile
ax2.contourf(X, Y, temp_ss[-1].T, cmap='hot')
ax2.set_xlabel('X-axis')
ax2.set_ylabel('Y-axis')
ax2.set_title(f'Final Temperature Distribution')

plt.show()
```

```python
center_temp = []  # Store temperature at the center (Nx/2, Ny/2)
corner_temp = []  # Store temperature at one of the corners (0, 0)

for step in temp_ss:
    center_temp.append(step[Nx//2, Ny//2])
    corner_temp.append(step[0, 0])

plt.figure(figsize=(8, 6))
plt.plot(np.arange(num_time_steps), center_temp, label='Center Temperature')
plt.plot(np.arange(num_time_steps), corner_temp, label='Corner Temperature')
plt.xlabel('Time Step')
plt.ylabel('Temperature (°C)')
plt.legend()
plt.title('Temperature Evolution at Specific Locations')
plt.grid()
plt.show()

Q_values = [2000.0, 4000.0, 6000.0]  # Diff. values of Q
plt.figure(figsize=(10, 6))

for Q_val in Q_values:
    # Simulate temp. distribution for each Q value
    T = np.ones((Nx, Ny)) * T0
    temperature_snapshots_Q = []

    for step in range(num_time_steps):
        T_new = T.copy()
        for i in range(1, Nx - 1):
            for j in range(1, Ny - 1):
                T_new[i, j] = T[i, j] +  dt * alpha * ((T[i+1, j] - 2
                * T[i, j] + T[i-1, j]) / dx**2 + (T[i, j+1] - 2 * T[i, j]
                + T[i, j-1]) / dy**2 + Q_val
                / (alpha * ro * c * (T_high-T0)))
        T = T_new
        temperature_snapshots_Q.append(T.copy())
    # Plot temp. variation for each Q value
    center_temperatures = [step[Nx // 2, Ny // 2] for step in temperature_snapshots_Q]
    plt.plot(np.arange(num_time_steps), center_temperatures, label=f'Q={Q_val}')

plt.xlabel('Time Step')
plt.ylabel('Center Temperature (°C)')
plt.legend()
plt.title('Temperature vs. Heat Generation (Q)')
plt.grid()
plt.show()
```

13 Results and Discussions

The code provided us with several results related to the thermal behaviour of the car's brake system.

1. Temperature Distribution over time: The code simulates and visualises how the temperature distribution on the brakes evolves over time. We can observe how temperature changes at different locations in the brake system.
   The initial temperature distribution graph shows the temperature difference between the hot and cold ends of the system. The final temperature distribution represents steady - state temperature distributions, showing how the system reached thermal equilibrium.
   Figures below shows us the Temperature Distribution at various points at different time steps:

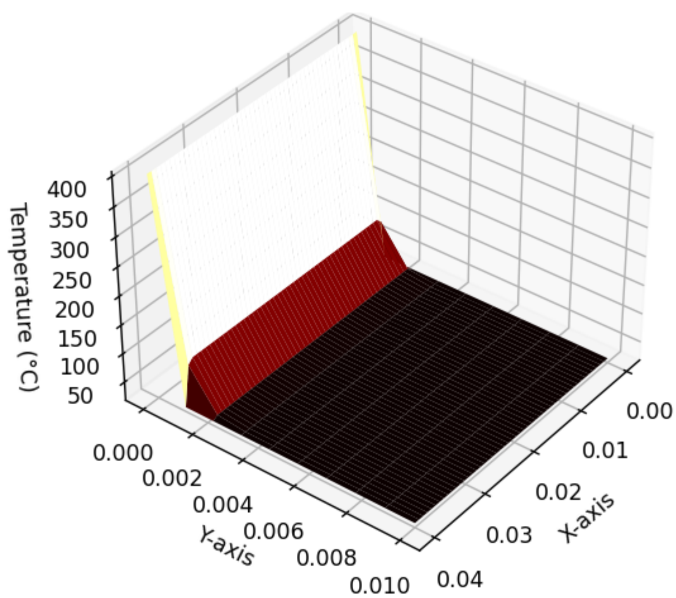

Figure 2: 3D Plot of Temperature Variation with Time

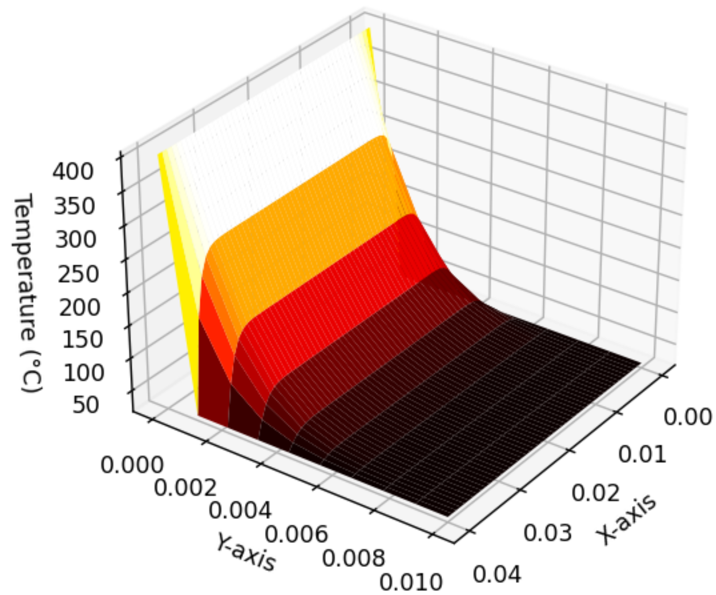

Figure 3: 3D Plot of Temperature Variation with Time

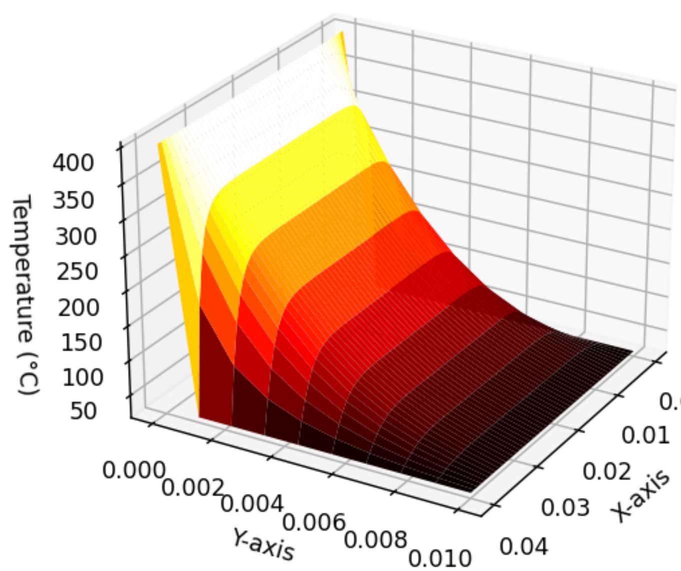

Figure 4: 3D Plot of Temperature Variation with Time

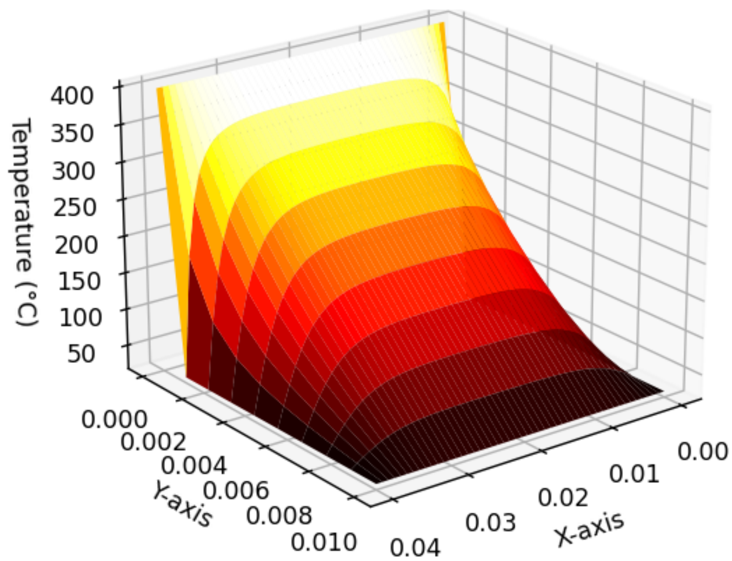

Figure 5: 3D Plot of Temperature Variation with Time

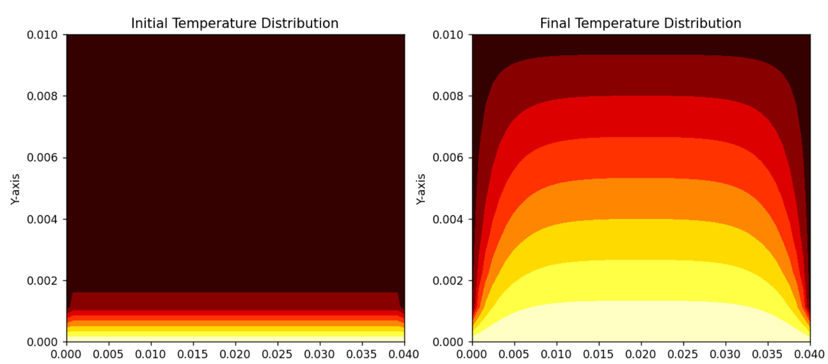

Figure 6: Temperature Distribution over the material grid at Initial and Final Condition

2. Evolution of Temperature at specific locations:
   By tracking the temperature changes at specific locations, we can discuss how different regions of the brake respond to heating and cooling. This information can be crucial for any thermal damage to the system. The centre temperature curve typically exhibits a gradual increase as heat spreads from the hot end to the centre and becomes constant after some time at near $300°C$. The corner temperature remains constant i.e. relatively stable at $400°C$.

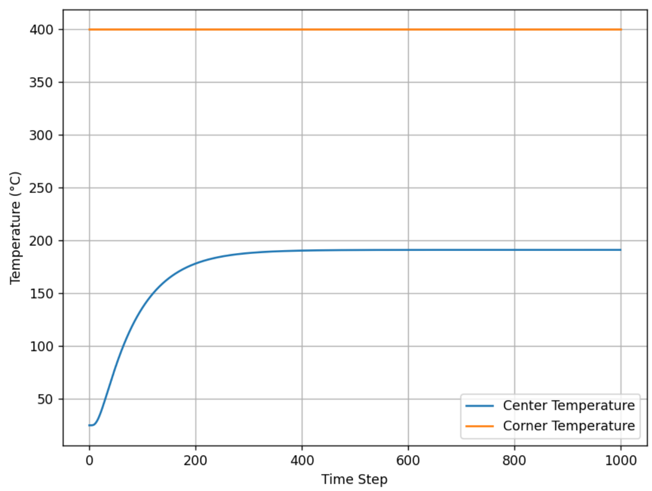

Figure 7: Evolution of Temperature at Specific Locations

3. We can observe how the brake system's temperature changes by changing the rate of heat generation. It illustrates how heat generation directly impacts the temperature within the brake system. So, the higher the value of Q, the faster the temperature increases. We can observe that as the time step reaches 200, the centre temperature is approximately 180 when the Q value is 6000, approximately 130 when Q value is 4000 and approximately 80 when the Q value is 2000.
   This analysis helps us select brake material and design the brake to handle various intensities.
   The figure below shows the same:

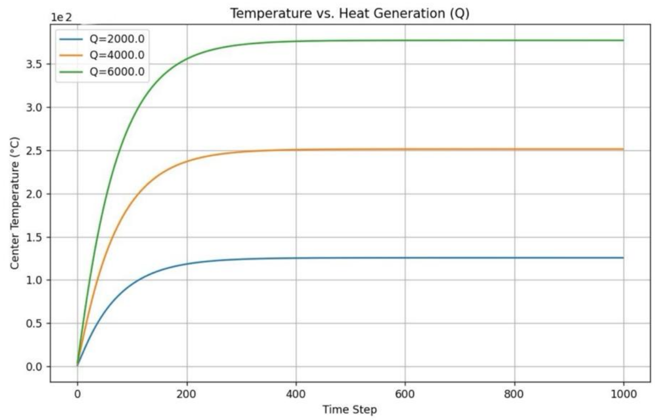

Figure 8: Variation of Temperature at Center Point with different values of Frictinal Heat Generation

14 Error Analysis

- **Material Properties** Material properties (thermal conductivity, density, and specific heat) used in your simulation match the actual properties of the brake material you are analyzing. Consult material data sheets or industry standards to ensure accuracy.

- **Consistency with Physical Laws**The selected parameter values align with fundamental physical laws and principles. For example, the thermal diffusivity (α = k / (ρc)) should follow the relationship between thermal conductivity (k), density (ρ), and specific heat (c).

- **Stability Analysis**The chosen time step size (dt) satisfies stability criteria for the numerical method used. An unstable simulation may produce erroneous results, so stability is crucial.

15 Acknowledgment

We extend our heartfelt appreciation to all those who have contributed to the successful completion of the project titled "Transient Thermal Analysis of car brake ." This endeavor would not have been possible without the combined efforts, expertise, and support of various individuals, my team and labs.

First and foremost, we express our gratitude to Prof. Dilip, our project advisor, whose invaluable guidance, insightful suggestions, and unwavering encouragement have been instrumental in shaping the direction of this project. Your dedication to fostering our learning and research journey has been truly inspiring.We would also want to thank all of the Teaching Assistants for their assistance.

We are also indebted to the faculty members and staff of IIT Gandhinagar, whose constant encouragement and resources facilitated the seamless execution of this project. The academic environment provided by each department has been an essential catalyst in nurturing our curiosity and fueling our intellectual growth.

We extend our appreciation to our fellow peers and colleagues who provided valuable insights, engaged in stimulating discussions, and shared their diverse perspectives throughout the project's duration. Your collaboration has enriched our understanding and broadened our horizons.

In essence, this project stands as a testament to the combined efforts of a multitude of individuals and entities, all of whom have contributed their expertise, time, and enthusiasm. We are deeply grateful for the opportunity to work on this project and for the support we have received throughout its journey.

Bibliography

[1] "Numerical and experimental analysis of transient temperature field of ventilated disc brake under the condition of hard braking," Numerical and experimental analysis of transient temperature field of ventilated disc brake under the condition of hard braking - ScienceDirect, Sep. 01, 2017. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1290072916314880

[2] "Numerical Methods for Engineers", Seventh Edition, Steven P. Chapra, Raymond P. Canale., [Online]. Available: https://gdcboysang.ac.in/About/Droid/uploads/Numerical%20Methods.pdf

[3] "Thermal analysis of a solid brake disc," Thermal analysis of a solid brake disc - ScienceDirect, Aug. 27, 2011. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1359431111004595

[4] "Structural and Thermal Analysis of Rotor Disc of Disc Brake", International Journal of Innovative Research in Science, Engineering and Technology, Manjunath T V1 , Dr Suresh P M2, [Online]. Available: http://www.kresttechnology.com/krest-academic-projects/krest-mtech-projects/Mechanical/Mech%20Mechanical%202016-2017%20abstract%20base%20paper/base%20papers/7.design%20and%20analysis%20of%20disk%20brake.pdf

[5] "Thermal simulation of Brake disc, CAE in Siemens NX," YouTube, Feb. 10, 2020. [Online]. Available: https://www.youtube.com/watch?v=hKHRZH8DNCU