STUDY OF NEURAL NETWORK INTERNALS

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STUDY OF NEURAL NETWORK INTERNALS

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by

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March 2017

Dedicated to

My parents, My Wife Divya and My sweet daughter Samartha

Thesis Certificate

This is to certify that the thesis titled **Study of Neural network internals** submitted to the International Institute of Information Technology, Bangalore, for the award of the degree of **Master of Technology** is a bona fide record of the research work done by **Gaurav Kumar Jain**, **EMT2013007**, under my supervision. The contents of this thesis, in full or in parts, have not been submitted to any other Institute or University for the award of any degree or diploma.

Prof. G.Srinivasaraghavan

Bangalore,
The 15th of March, 2017.

STUDY OF NEURAL NETWORK INTERNALS

Abstract

Neural networks generated lot of interest and constantly created new bench marks in recent times which were never imagined. Certain cases it is able to pass human level performance. However it comes with lot of challenges and specifically optimization of these networks poses lot of challenges. In this work we will try to uncover these aspects. Network optimization aspects are studied in great detail in this work and experimentations with many possible configurations are performed on datasets such as MNIST, CIFAR10, CIFAR 100. Experimental results are analysed and on their basis recommendations are suggested which could help in better convergence performance and able to generate robust classifiers.

Acknowledgements

It is famously said that *"Great Teacher Inspires"* and I am very fortunate to blessed with one. His presence, kept motivating me to achieve greater heights, never give up. It is his inspiration that this work has seen light of the day. I have no words to describe the greatness you possess Prof. G.S.Srinivasarghavan, How calmly you handle the situation with me from last 1.5 years and guided me wonderfully in difficult times.

Samsung has given me full support through our masters and specially during thesis they have provided us extra system and GPU computing facility which is very much essential for this type of extensive experimentation work using Neural networks. Thanks Balaji, SVP, Samsung research, Dr. Balvinder Singh, VP, Samsung research for taking the initiative for understanding our requirements for the thesis work and providing us all these facilities.

I would like to thank my family for understanding me and supporting me throughout. My wife Divya has given me unconditional support in every situation I faced, so that I can focus on this work.

I would like to acknowledge great work done by all the worlds greatest minds, which is used as great references for building this work.

— IIITB Bangalore, India

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[1] batch size recommendations

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List of Abbreviations

DNN . . . . . Deep Neural Network

X_r . . . . . Training Data

IIITB . . . . . International Institute of Information Technology Bangalore

CHAPTER 1

INTRODUCTION AND OVERVIEW

Deep neural networks have demonstrated excellent results in many machine learning tasks [REFERENCES:TBD] and became a default choice for machine learning researchers irrespective of the end result they achieved. Computer vision , Natural language and Speech processing tasks have scaled to the next level of accuracies using these networks.

Structure: The core structure of Neural network is the Neurons arranged in a layer manner also known as hidden layer and stack of these layers with interconnected Neurons provides depth to the network. This arrangement of layers is known as network architecture. A simple Neural Network is shown in Figure FC1.1. Several networks are proposed till date for the range of machine learning tasks.[REFERENCES:TBD]

Training: Network architecture remains passive until it gets trained for the specific task for which network parameters are estimated for the desired level of prediction performance on data samples outside training data set, also known as Generalization performance. The most important part is the training strategy which encompass choosing the hyper-parameters to intermediate updates along with learning algorithm. The number of hyper-parameters termed as annoying knobs to be adjusted [Bengio 2012: Practical recommendations] and famously known as nuisance parameters are quite high.

On top of this range of choices for these hyper-parameters make exhaustive search impractical. Once these parameters are chosen, training can be proceed and the algorithm, which is more often than not is Stochastic gradient descent [References] along with Back propagation [Reference: Rumelhart] as a default choice allows network to evolve from untrained to train network. Each training process has stopping criterion, which is more intuition than rules which guides training progress including stopping point.

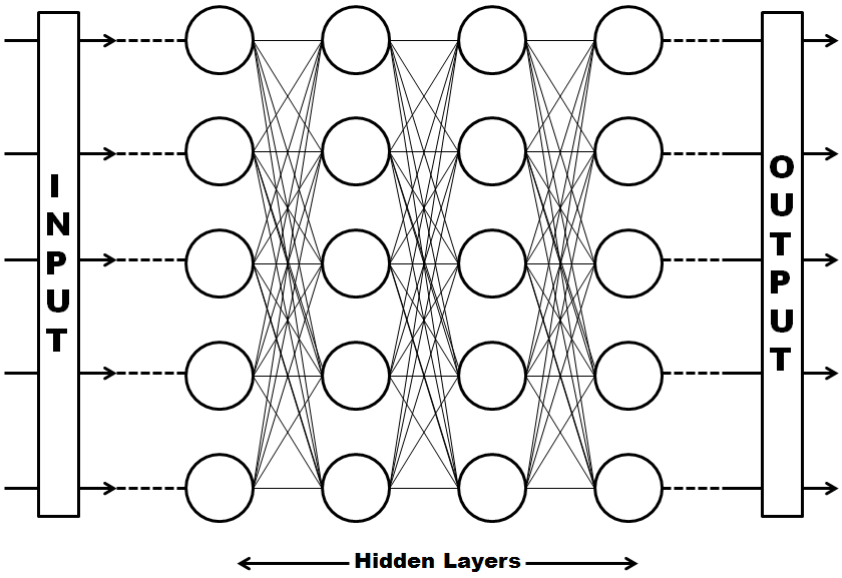


Figure FC1.1: A Simple Neural Network

Four layer fully connected Neural network is shown in FC1.1. All Neurons in successive layers are connected to each other, while within layer they are not connected.
Input feeds the input features and propagation of features undergoes transformation or encoding as it passes via layers. Intermediate layers are known as hidden layers. Output layer gets the final output in desired format.
In case of classification task it could be class vector representing class probabilities, in case of regression task it is the numerical value and in case of unsupervised task it can be input vector itself. Network parameters gets updated via famous technique called back propagation formulating this problem as optimization problem in which loss occured is minimized iteratively.

Considering all these background the **Training Life Cycle** of DNN's has 3 main stages as shown in Figure FC1.2

1. Choosing network structure

2. Selection of hyper parameters and training algorithm

3. Network updates during training and stopping criterion

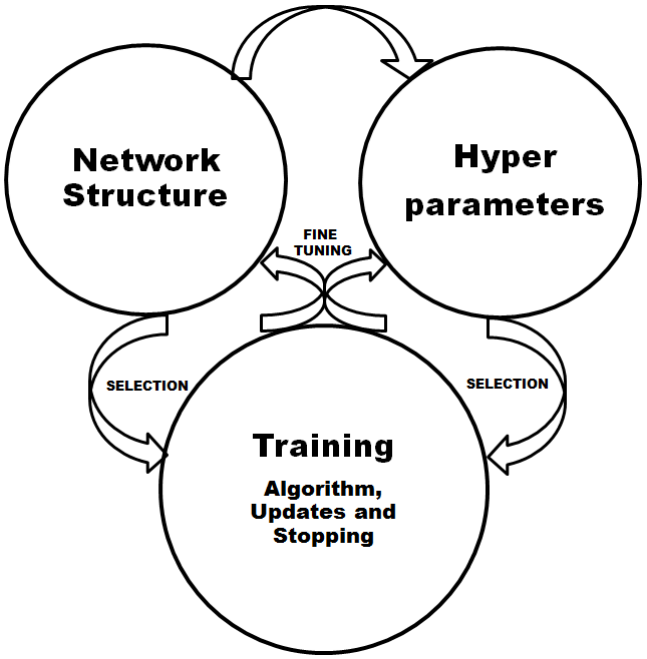


Figure FC1.2: DNN Training Lifecycle

Training lifecycle is shown in FC1.2. Firstly network choice needs to be made, which would be type of network such as CNN, LSTM, ANN, RNN, Auto encoder and so on, once network type is finalized network configuration has to be decided, such as number of layers, number of neurons in layers, their interconnections, filter size in case of CNN and so on.
secondly comes choice of hyperparameters, which starts with some standard initial choices and during learning gets tuned for better performance.
Lastly choice of learning algorithm and update rule for back propagation is to be decided, Here stopping criterion also is crucial which governs the network final learnt parameters.
This also allows fine tuning of network structure or hyperparameters as more and more knowledge about underlying data and its performance in current selected environment is known.

1.1 Chapters Organization

This study is divided into 6 chapters, Chapter 2, discusses **Network architecture**, Chapter 3 discusses **Hyper-parameters**, Chapter 4 discusses **Training the Network**,

Chapter 5 discusses the **Insights and Recommendations** from this study. Chapter 6 **concludes** and poses some **open questions** for future work.

1.1.1 Network types

Chapter 2 details different architectures, which can be seen as small survey on the state of the art networks used in deep learning.We will keep them for study purposes.

As we studied the details and interdependencies among the training parameters,so we have used our own network, we call it **GsNet**. We recommend to use them for comparative study of this kind. Table TC1.1 shows the layers configuration of GsNet-2,GsNet-3 and GsNet-5.

Table TC1.1: GsNet architectures

GsNet-2	**GsNet-3**	**GsNet-5**
conv 3x3x64 pool 2x2	conv 3x3x64 pool 2x2	conv 3x3x64 pool 2x2
conv 3x3x64 pool 2x2	conv 3x3x64 pool 2x2	conv 3x3x64 pool 2x2
	conv 3x3x64 pool 2x2	conv 3x3x64 pool 2x2
		conv 3x3x64 pool 2x2
		conv 3x3x64 pool 2x2
dense,128	dense,128	dense,128
softmax,c	softmax,c	softmax,c

We have used these networks for our experiments and their analysis. This may bring how depth affects learning. Different architectures in practice are described, which can be seen as small survey on state of the art network architectures in chapter 2.

1.1.2 Hyper-parameters

Hyper-parameters are discussed in chapter 3.Main parameters, which we studied are as following

1. Batch Size
2. Optimizations
3. Initializations

Firstly we describe all the different prescribed available techniques for these parameters which can be seen as a small survey of the available studies, experimental results and techniques.

Secondly we present results of almost exhaustive set of parameters configuration. Then best of parameters and configurations are chosen for the next set of experiments.

1.1.3 Training the network

Chapter 4 discusses training the network and study which describes different techniques used in training.We will also explain our training set up which is used for our experiments.

1.1.4 Insights and Recommendations

Chapter 5 provides all insights and analysis of our results. This includes well performing strategies as well as strategies which may didn't perform well. Based on these we will describe our recommendations. Also we will explain novel technique which perform well and provide more stability to the learning system.

1.1.5 Conclusion

Chapter 6 concludes with the summary of our study and future direction of this work.

1.2 Notations used

This section explains the notations used through out this study.

1.2.1 DNN Setting

DNN goal is to approximate a target function g^* for the unknown distribution input $X^* = (x_1, x_2, ...x_d)^T \in \mathbb{R}^d$. The target value is $Y^* = (y_1, y_2, ...y_s) \in \mathbb{R}^s$. if θ^* is the parameters associated, then

$$Y^* = g^*(X^*, \theta^*) \qquad \text{(Eqn 1.1)}$$

X^* represents entire input data for the underlying input distribution. Getting X^* is almost impossible, so generally g^* is approximate using the representative input X of size N which is sampled from X^* and hoped to have same distribution as the original input distribution. Let Y is the target value for X.

So DNN problem reduces to approximating g^* using (X,Y) of size N. $\theta = (\theta_1, \theta_2 ... \theta_m) \in \mathbb{R}^m$ represents the parameters which gives best approximation for target function. Finally the DNN has to learn the best θ such that $g(X,\theta) \sim g^*$.

g represents a chained function in context of DNN as it flows from input to output via hidden layers as shown in FC1.1. g as chained function flowing via hidden layers

can be written as

$$g(X) = g^K(g^{K-1}(g^{K-2}.....(g^2(g^1(X))).....)) \quad \text{(Eqn 1.2)}$$

where K represents total number of hidden layers and $\{g^k, k = 1....K\}$ is output of k^{th} layer.Let $\hat{Y} = g(X)$ then lets define a loss function $\mathscr{L}(\hat{Y}, Y)$ as the cost it incurs using $g(X)$ to approximate $g^*(X)$ and hence it is also known as cost function.

The gradient of $\mathscr{L}$ with respect to θ is denoted as $\nabla_{\theta_t}\mathscr{L}(\theta_t)$ at t^{th} iteration. For simplicity we denote this as ∇_{θ_t}, where θ_t denotes the network parameters at iteration/time t.

Network Parameters

1.3 Experimental set up

We have performed exhaustive set of experiment using standard datasets on Nvidia-Tesla K80 GPU.Regularly results are analyzed to reduce the experiment space and become basis for the next set of experiments.

1.3.1 Databases

Following is the list of datasets used in the experiments:

1. MNIST [1]
2. CIFAR-10 [2]
3. CIFAR 100 [2]

1.3.2 MNIST

MNIST dataset has 60000 training samples and 10000 testing samples.It is database of handwritten digits. Sample examples are shown in fig.**??**

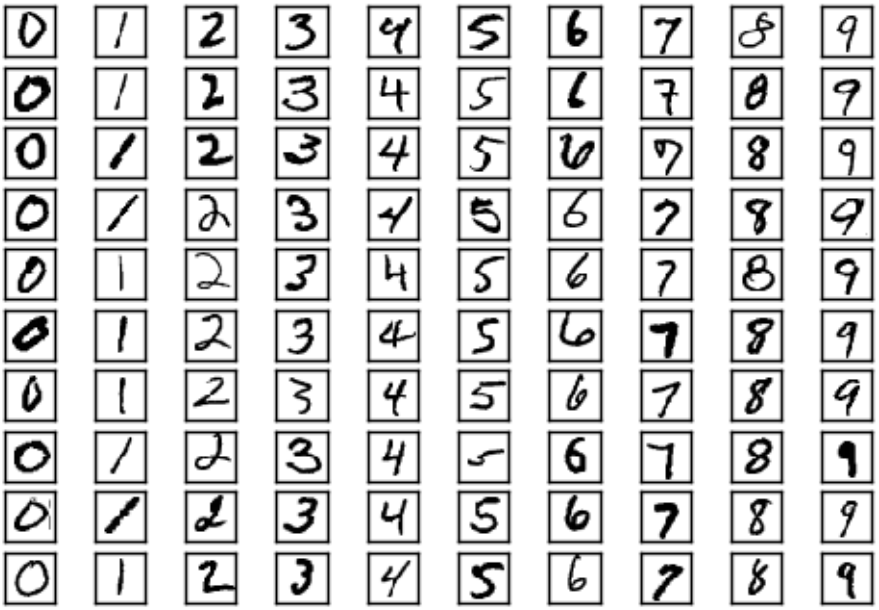


Figure FC1.3: MNIST dataset input images example

MNIST sample images having handwritten digits from 0-9, image size is 28x28 and images are grayscale, shown samples are randomly chosen, 10 for each class.

1.3.3 CIFAR10

MNIST dataset has50000 training samples and 10000 testing samples.It is database of different objects present in the images. Sample examples are shown in fig.**??**

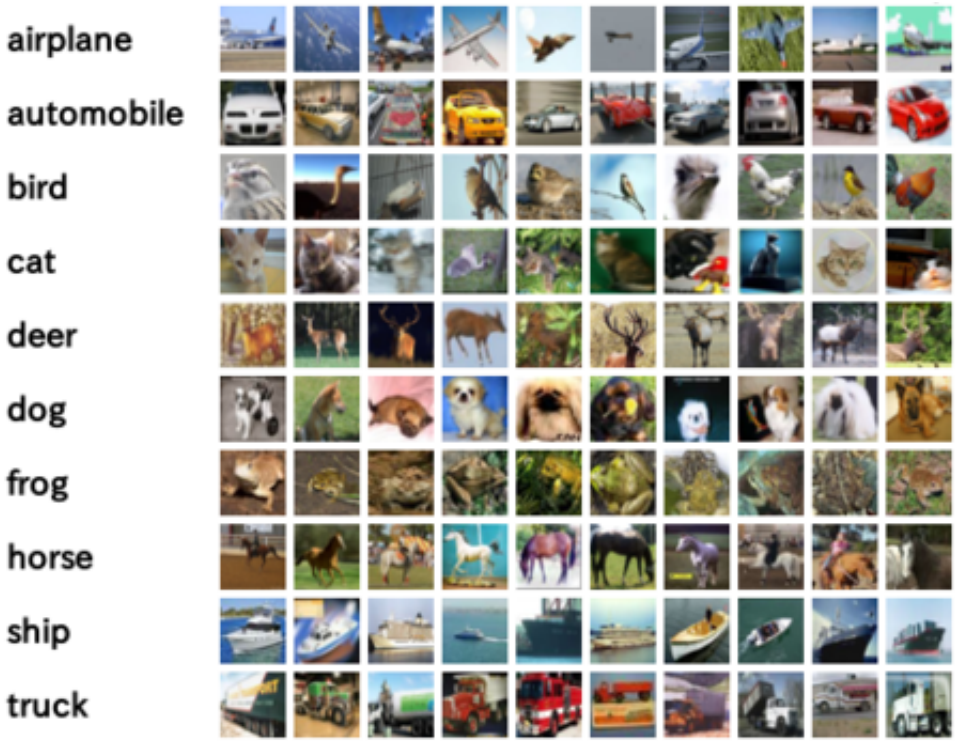


Figure FC1.4: CIFAR-10 dataset input images example

CIFAR-10 sample images having different objects present in the images, image size is 32x32 and images are color

1.3.4 CIFAR100

CIFAR100 dataset has 50000 training samples and 10000 testing samples.It is database of 100 different object classes. Sample examples are shown in fig.**??**

Figure FC1.5: CIFAR-100 dataset input images example

CIFAR-100 sample images consist of database with 100 different object classes, image size is 32x32 and images are color

1.3.5 Software

Keras [3] is mainly used for almost all the experiments. Theano [4] is used as main backend for Keras.Python is used as main programming language.

CHAPTER 2

NETWORK STRUCTURE

This chapter discusses different type of networks in practice, their usage and prominent network architectures. We will discuss only few networks which have achieved significant success in recent times. Primarily we will discuss CNN(Convolutional Neural network) as that is the main network used through out this work.

2.1 Networks

The introduction to Neural network has started long ago with Frank Rosenblatt, famous MLP(Multi Layer Perceptron) [5].Working of neural nets today are precisely captured by Rosenblatt. It talks about pathways to connect to output so that for particular input associated pathway gets activated and produces corresponding output. Following that several networks are suggested directed for specific tasks. For example for image to extract neighborhood relationship, convolution neural networks are suggested. For time series data Recurrent neural networks are suggested. LSTM is recent state of the art for handling time series tasks.

Auto Encoders are suggested as unsupervised nets, which generates low level representation of inputs using only input data. Restricted boltzmann machines are another type of network which focuses on convergence by lowering the energy. In Hopfield

network every neuron connects to every other neuron in the network.

Other types of networks are Radial Basis Network(RBN), Gated Recurrent Unit(GRU), Deep belief network(DBN), Generative adversial network(GAN).

2.1.1 Perceptron

The perceptron has been introduced to handle perceptual recognition, generalization and hence the name Perceptron. In this landmark paper Rosenblatt nicely maps the neurons development in human being to perceptron. For instance connection of nervous system are assumed as random and in neural nets at start mostly random initializations are used. The original system has said to be capable of plasticity, which allows other neuron output to change over time seeing stimulus applied. And if same or similar stimuli is seen large number of times, will tend to form pathways to same sets of responding cells. This almost sums up the current neural nets, however the network construction, random initializations may differ a lot. The perceptron is shown in fig FC2.1

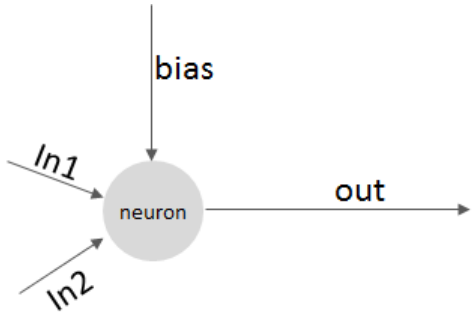


Figure FC2.1: Perceptron

Regarded as starting point of neural network evolution perceptron is simplest of neural networks. It has no hidden layers, just inputs with weights and bias term added together with non linear transformation or activation function produces output. Perceptron was limited in their power due to their non-ability to handle non linear target functions, such as XOR.

2.1.2 CNN

CNN famously known as Convolutional Neural Network are revolutionary architecture which has provided significant results in many image based learning tasks. CNN takes advantage of neighborhood relationships between data and so image data is natural choice for these networks. see figure FC2.2.

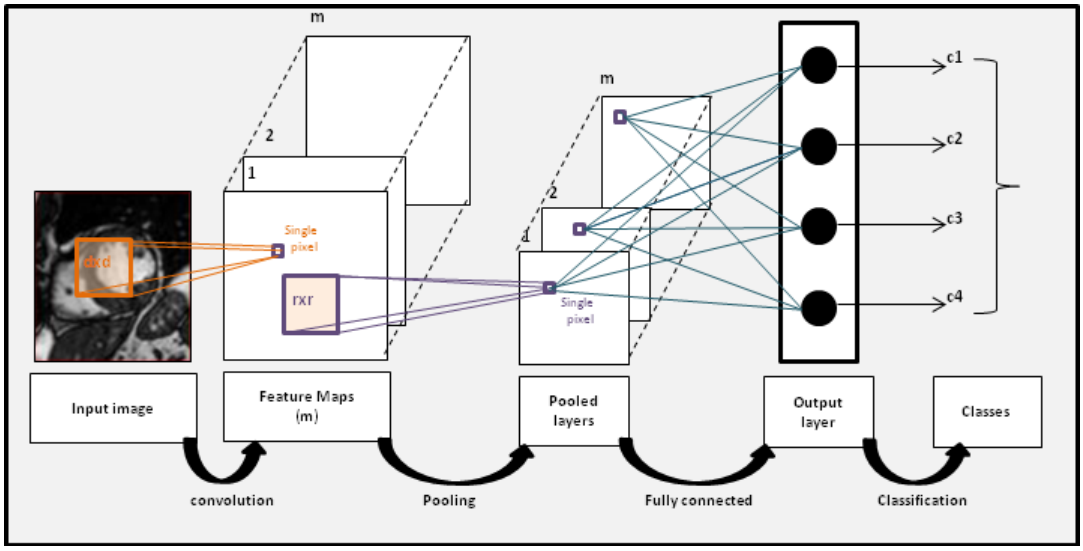


Figure FC2.2: CNN

CNN composition is shown in figure FC2.2, This is most common network configuration, however there are many different variations suggested from time to time. CNN works well on data which is having neighborhood relationship such as image has spatial relationship between pixels. Input undergoes convolution to get low level representation with shared weights for the convolution. Generally convolution filter size is 3x3. Immediately after convolution layer there would be pooling layer, which reduces the data representation further. Deepness comes by repeating these layers. At last flatten layer reduces representation to number of classes which activates one of its neuron or provide the output probabilities of image belonging to output class vector.

2.1.3 RNN

Recurrent neural nets are having connections to same hidden layer neurons, and thus capable of storing time series data or feedback to be used with next set of input in sequence.

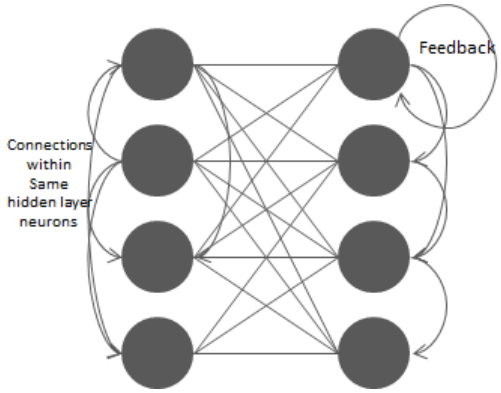


Figure FC2.3: RNN

Recurrent neural network are useful in case of sequential data, where data at current instance is dependent on previous data instances. Most popular RNN models are language models, where it generates probabilities of existence of any sentence based on underlying language it trained with. Another important application of RNNs are generative models, where in given one word it generates next word. This is possible by RNN having feedback to same neuron or hidden layers. The feedback give this network immense power as unfolding this network it will act as several deep layers which is dependent on RNN ability to store previous information.

2.1.4 LSTM

Long Short Term Memory(LSTM) is one type of RNN network which uses LSTM cells.

2.1.5 Auto Encoders

Auto encoders helps in learning representation by using input as output and just keep encoded layers and grow network as deep as possible. Auto encoder has delivered significant results for various tasks including image classification tasks. Auto encoder LSTM is variant of LSTM with Auto encoding capabilities.

CHAPTER 3

HYPER-PARAMETERS

This chapter introduces hyper-parameters. They are not directly related to the machine learning algorithm parameters, but responsible for overall algorithm evolution. For example learning rate dictates the update strength per iteration, choice of initializer can lead to slow or fast convergence, choice of optimizers provide way to update learning parameters(θ).

Other than direct algorithm parameters, we will consider everything else as hyper-parameter. Mainly we will consider following hyper-parameters, which we will discuss and explain the experiments performed with different choices of these parameters and their effect in overall convergence.

1. **Initializer**
2. **Optimizer**
3. **Batch size**
4. **Total parameters**
5. **Number of Epochs**

3.1 Initializer

Initial network condition is described as *"At birth the construction of the most important networks is largely random, subject to a minimum number of constraints"* in [5] suggests that random initializations can be used to initialize network parameters and constraints could be range of these parameters.

Uniform initialization, assigns initial weights from $U[-r,\ r]$, where U is uniform distribution, Mostly r is used as small value ~ 0.1. Another random initialization scheme is **Normal Initialization**, based on sampling initial weights from normal distribution, $N(0,1)$. These are simple methods which are getting used.

Uniform initialization using fan in [6] suggests, using fan-in to determine standard deviation σ_i and sampling initial weights from $N(0,\sigma_l)$. value of σ_l is dependent on fan-in, which is number of inputs to a hidden unit. It is suggested to be chosen as per Eqn 3.1

$$\sigma_l = \mathbf{m}^{-1/2}, where,\ \mathbf{m}\ is\ the\ fan\ in \qquad \text{(Eqn 3.1)}$$

Normalized initialization [7] suggest to use uniform weight initialization as per Eqn 3.2

$$U[-\frac{\sqrt{6}}{\sqrt{n_l+n_{l+1}}}, \frac{\sqrt{6}}{\sqrt{n_l+n_{l+1}}}] \qquad \text{(Eqn 3.2)}$$

where n_l and n_{l+1} are total number of units in l^{th} and $(l+1)^{th}$ hidden layer respectively. This work on paradigm of maintaining activation variances in feed forward direction and back propagated gradient variances in both the directions.

For very deep models and to support activation ReLU/PReLU [8] suggests to use slight modification in considering the variance from [7] and the **initialization scheme becomes zero-mean Gaussian with standard deviation as** $\sqrt{2/n_l}$, where n_l is number of hidden units in l^{th} layer.

Orthogonal random initializations [9] suggests simple initialization scheme where in initial

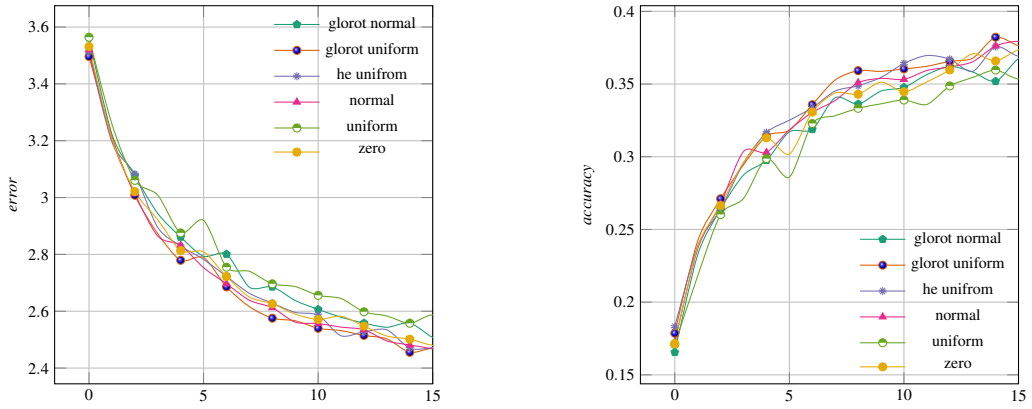
weights are chosen from the random orthogonal matrix satisfying $W^T W = I$. This yield depth independent learning times, which means as depth increases learning time remains same as oppose to suggested initializations in [6] [7]

3.1.1 Experiments

In this section we analyze the effect of different initializers based on our experiment results.

3.1.1.1 CIFAR100

Two Layer, opti=adagrad, batch size=1xc

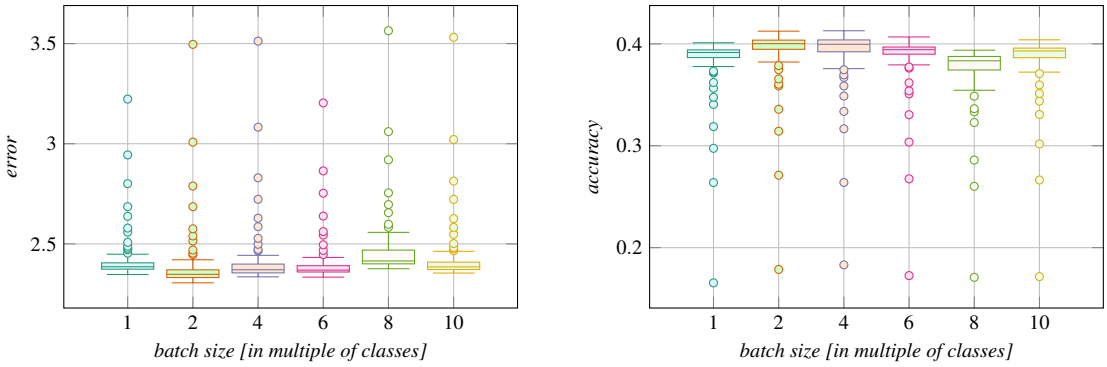


(a) Error for initial epochs:INIT

(b) Accuracy for initial epcohs

Figure FC3.1: *Different batch results for starting 15 epochs*

to write

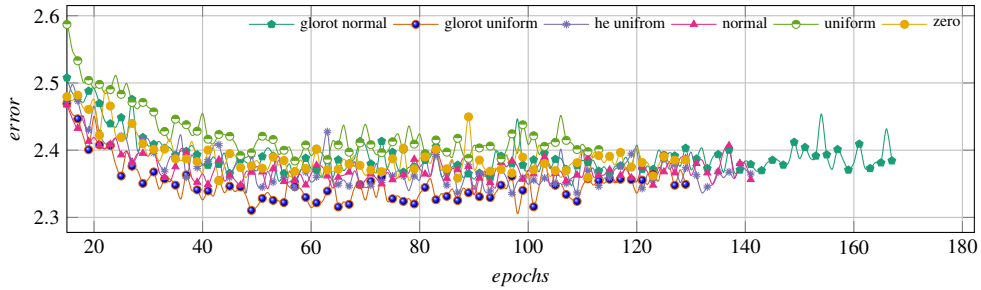


(a) Error variation

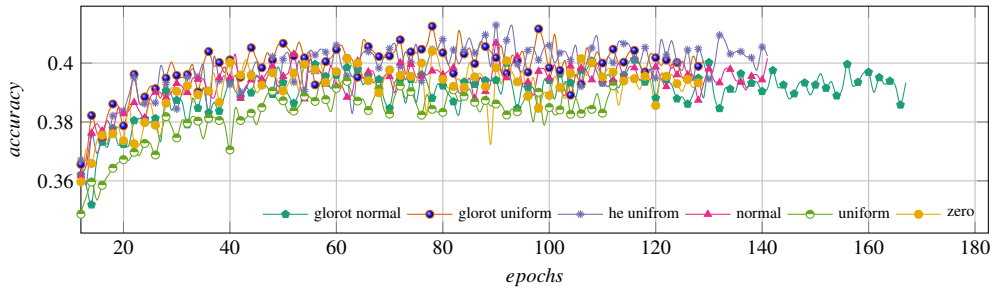
(b) Accuracy variation

Figure FC3.2: *accuracy and error plot for full training epochs*

to write



(a) Error for later epochs

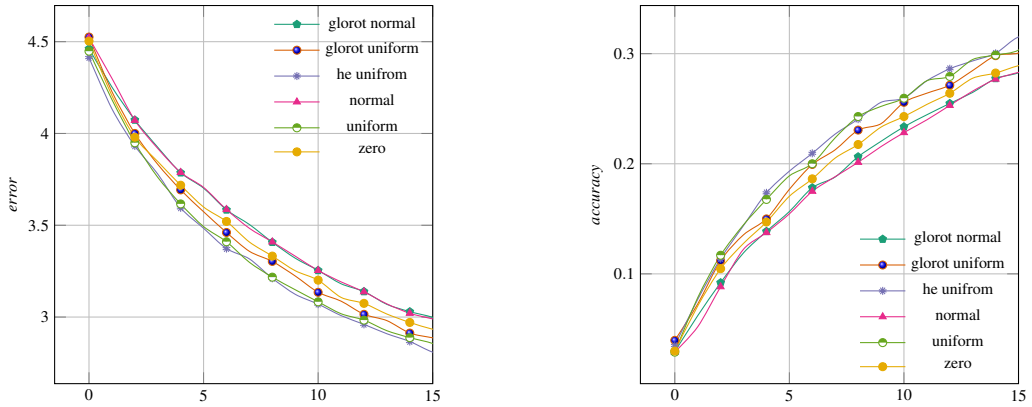


(b) Accuracy for later epochs

Figure FC3.3: *Different batch results for later epochs*

to write

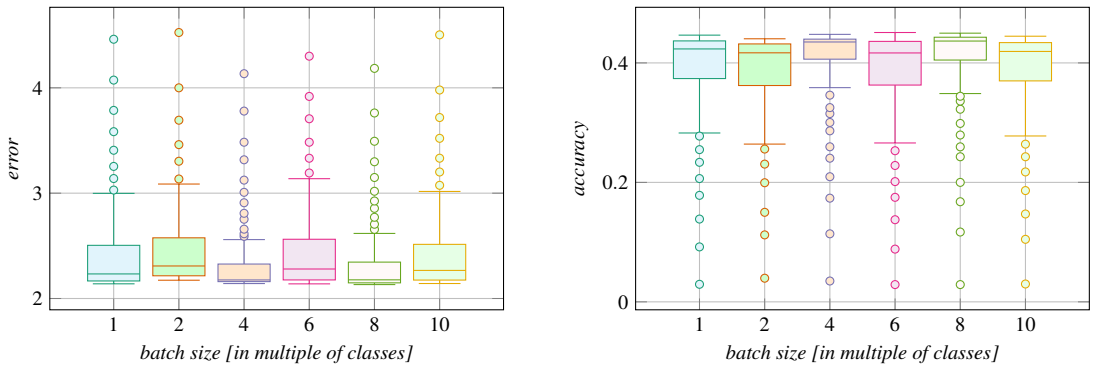
Two Layer, opti=SGD with nesterov momentum, batch size=1xc



(a) Error for initial epochs:INIT (b) Accuracy for initial epcohs

Figure FC3.4: *Different batch results for starting 15 epochs*

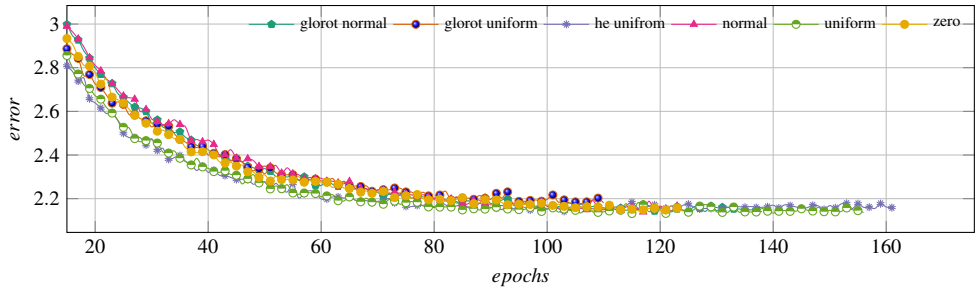
to write



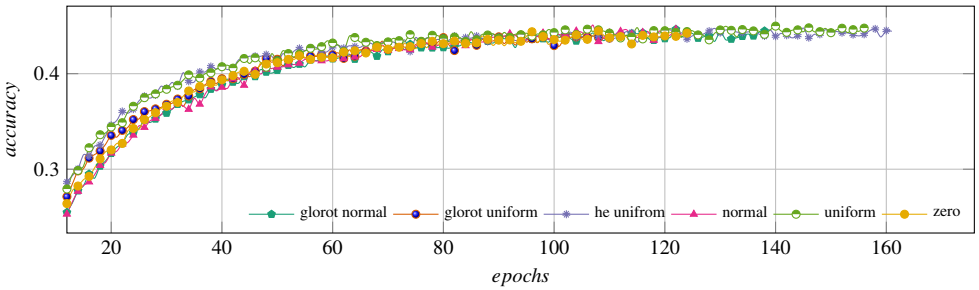
(a) Error variation (b) Accuracy variation

Figure FC3.5: *accuracy and error plot for full training epochs*

to write



(a) Error for later epochs



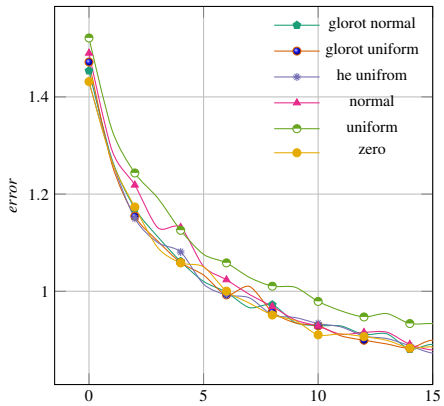
(b) Accuracy for later epochs

Figure FC3.6: *Different batch results for later epochs*

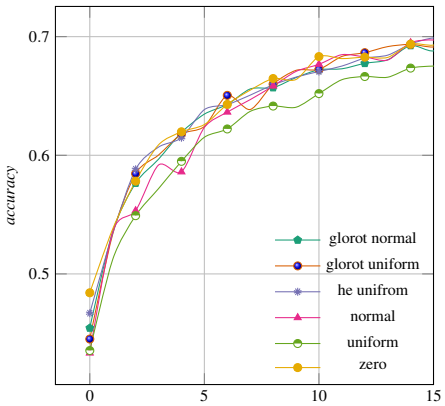
to write

3.1.1.2 CIFAR-10

Five Layer, opti=adagrad, batch size=2xc



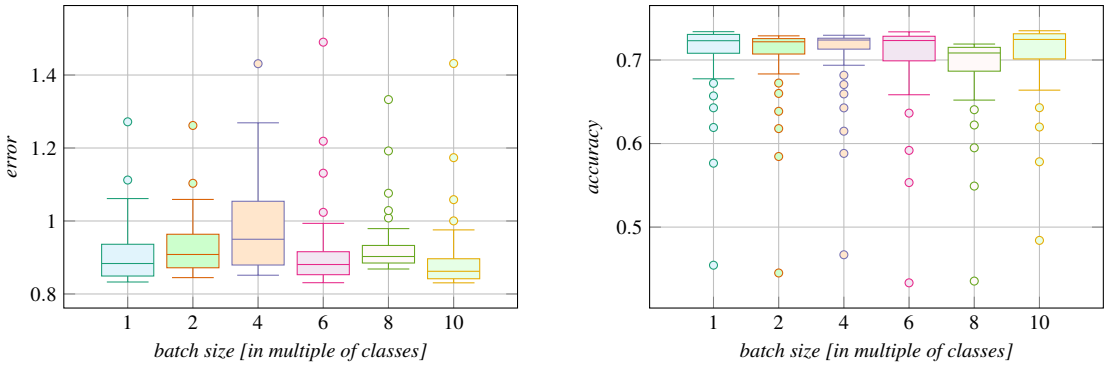
(a) Error for initiaNITCIFAR10



(b) Accuracy for initial epcohs

Figure FC3.7: *Different batch results for starting 15 epochs*

to write

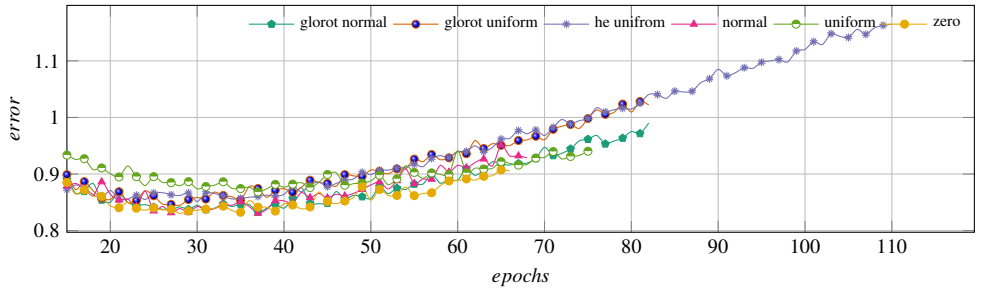


(a) Error variation

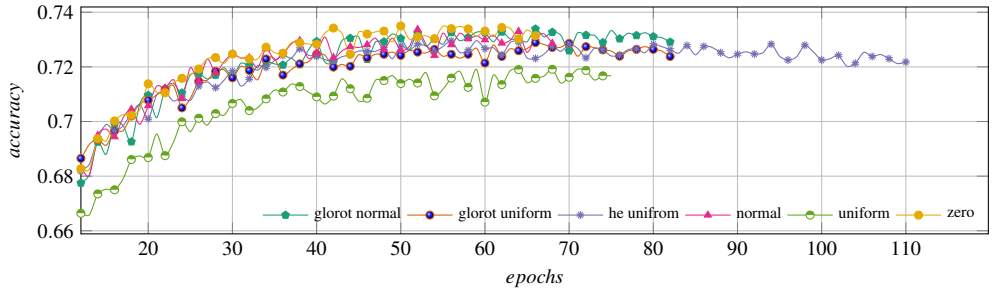
(b) Accuracy variation

Figure FC3.8: *accuracy and error plot for full training epochs*

to write



(a) Error for later epochs



(b) Accuracy for later epochs

Figure FC3.9: *Different batch results for later epochs*

to write

3.1.1.3 MNIST

Two Layer, opti=nadam, batch size=1xc

3.2 Optimizers

Optimizers are main learning algorithm or routine which is responsible for changes in the network as and when update takes place. The updates are not limited to parameters update alone, even hyperparameters can also get updated as per underlying optimizer routines.

Here we will discuss different optimizers, mainly gradient/sub gradient methods,their analysis and effect from experimental results. Detailed survey of these methods can be found at [10] and [11]

GD(Gradient descent) is one of the important and robust optimization algorithm. The gradient of the function to be optimized is computed with respect to the parameters. In deep learning setting the function to be optimized usually is loss function $\mathscr{L}$ and parameters are θ_t at t^{th} iteration. The gradient of $\mathscr{L}$ with respect to θ is denoted as $\nabla_{\theta_{t-1}}\mathscr{L}(\theta_{t-1})$, which gets computed at $(t-1)$ iteration and θ_t is updated as per Eqn 3.3

$$\theta_t = \theta_{t-1} - \eta(\nabla_{\theta_{t-1}}\mathscr{L}(\theta_{t-1})), \textit{where } \eta \textit{ is known as learning rate} \quad \text{(Eqn 3.3)}$$

In deep learning due to large training size gradient descent, also known as **batch learning** is almost impossible. So for large scale learning reduced size is considered and network parameters are updated, this batch will not be used till new repetition of dataset starts. This repetition is known as an **epoch**. The size of batch used in one single update is known as **minibatch**. The minibatch and epochs are considered later in this chapter. We will see batch learning in chapter 4.

The strategy to update network parameters after every $minibatch\ size = 1$ is known as **SGD (Stochastic Gradient Descent)**, which is also known as online learning. Commonly $minibatch\ size \gg 1$ is used in deep learning with large datasets. We will use SGD for minibatch or online learning.

Eqn 3.3 is applicable as it is to SGD also, only difference is computation of loss gradient is

restricted to the current minibatch instead of full training dataset. We will discuss its details in chapter 4. Another optimizers explained ahead are variants of SGD, which is essentially differs in the way network parameters get adjusted as learning progresses.

Classical Momentum [12] remembers previous gradient update vector and fraction of it is added to the next parameter update. The momentum term is computed as per Eqn 3.4 and update takes place as per Eqn 3.5.

$$m_t = \mu m_{t-1} + \nabla_{\theta_{t-1}} \mathscr{L}(\theta_{t-1}), \tag{Eqn 3.4}$$

$$\theta_t = \theta_{t-1} - \eta m_t \tag{Eqn 3.5}$$

where μ is known as momentum term

NAG (Nesterov accelerated gradient) is accelerated gradient descent which converges faster than classical momentum or SGD. The gradient is calculated on possible future update without using gradient and then using it to update network parameter. Acceleration is achieved as it can be seen as looking into future as this can prevent slows gradient update to move uphill and accelerate if it is moving downhill. The updates are calculated as per Eqn 3.6

$$\begin{aligned} m_t &= \mu m_{t-1} + \eta \nabla_{\theta_{t-1}} \mathscr{L}(\theta_{t-1} - \mu m_{t-1}) \\ \theta_t &= \theta_{t-1} - m_t \end{aligned} \tag{Eqn 3.6}$$

Adagrad (Adaptive subgradient descent) [13] adapts the learning rate as per the parameters updates. So this optimizer adjusts learning rate hyper parameter and falls under category where it updates parameters as well as hyper parameters. Basic premise of Adagrad is to have larger updates for less frequent parameters and smaller updates for frequent ones.

The updated learning rate for each parameter thus varies based on their earlier gradient updates individually. The updates take place as per Eqn 3.7

$$
\begin{aligned}
n_t &= n_{t-1} + (\nabla_{\theta_{t-1}} \mathscr{L}(\theta_{t-1}))^2 \\
\theta_t &= \theta_{t-1} - \eta \frac{\nabla_{\theta_{t-1}} \mathscr{L}(\theta_{t-1})}{\sqrt{n_t + \varepsilon}}
\end{aligned}
\qquad \text{(Eqn 3.7)}
$$

Adadelta [14] counters the effect of increasing norm n_t of Adagrad as that can reduce learning rate monotonically, which can be easily seen in Eqn 3.7, where n_t increases as iteration progresses. Adadelta restricts the size of gradients which get accumulated to a sliding window of fixed size. The sum of gradients are maintained as running average $E[g^2]_t$ of previous gradient average squared $E[g^2]_{t-1}$ and current gradient g_t squared as per Eqn 3.8

$$
E[g^2]_t = \rho E[g^2]_{t-1} + (1-\rho){g_t}^2, \textit{where } \rho \textit{ is a decay constant} \qquad \text{(Eqn 3.8)}
$$

Other than maintaining the running average of gradients sqaured, running average $E[\nabla\theta^2]$ of previous parameter updates squared are also maintained and this is compute in similar way of running average gradient computation of Eqn 3.8. It is given in Eqn 3.9

$$
\begin{aligned}
\nabla\theta_t &= -g_t \frac{\sqrt{E[\theta^2]_{t-1} + \varepsilon}}{\sqrt{E[g^2]_t + \varepsilon}} \\
E[\nabla\theta^2]_t &= \rho E[\nabla\theta^2]_{t-1} + (1-\rho){\nabla\theta_t}^2,
\end{aligned}
\qquad \text{(Eqn 3.9)}
$$

Now updates of Adadelta takes place as per Eqn 3.10

$$
\theta_t = \theta_{t-1} - \nabla\theta_t \qquad \text{(Eqn 3.10)}
$$

if instead of using running average of parameters update, we use η in Eqn 3.9 to calculate $\nabla\theta_t$ given in Eqn 3.11 and update happens as per Eqn 3.10, then this optimizer is known as **RMSprop**.value of ρ as proposed by the author is 0.95.

$$\nabla\theta_t = -g_t \frac{\eta}{\sqrt{E[g^2]_t + \varepsilon}} \tag{Eqn 3.11}$$

Adam(Adaptive moment estimation) [15] combines momentum and norm based optimizer. It computes first and second moment estimate as per Eqn 3.12. $\hat{m_t}$ and $\hat{v_t}$ are known as first and second moment estimates respectively,

$$\begin{aligned} \hat{m_t} &= \frac{\beta_1 m_{t-1} + (1-\beta_1) g_t}{(1-{\beta_1}^t)} \\ \hat{v_t} &= \frac{\beta_2 v_{t-1} + (1-\beta_2) {g_t}^2}{(1-{\beta_2}^t)} \end{aligned} \tag{Eqn 3.12}$$

Adam updates then takes place as per Eqn 3.13

$$\theta_t = \theta_{t-1} - \eta \frac{\hat{m_t}}{\sqrt{\hat{v_t} + \varepsilon}} \tag{Eqn 3.13}$$

Adamax [15] uses same updates as Adam other than it uses l_∞ norm instead of l_2 norm.

3.3 Number of Epochs

Number of epochs is the parameter which governs how many time full dataset will undergo training progress. As in minibatch learning batch size $\ll$ full training data. To see full data one time several minibatch undergoes update but not used later till full training data is used. One epoch thus sees full data set once and then it starts the process again.

Number of epochs are required to average out noise which is introduced due to stochastic minibatch updates. We have used this parameters in conjunction with Early stopping, which means set epoch value to a large number and use stopping criterion automatically based on certain performance parameter. In ur experiments we have used validation accuracy as performance parameter to monitor for maximum patience of 50 epochs, which means if there is no

improvement from last 50 epochs on validation accuracy, training halts automatically.

3.4 Batch Size

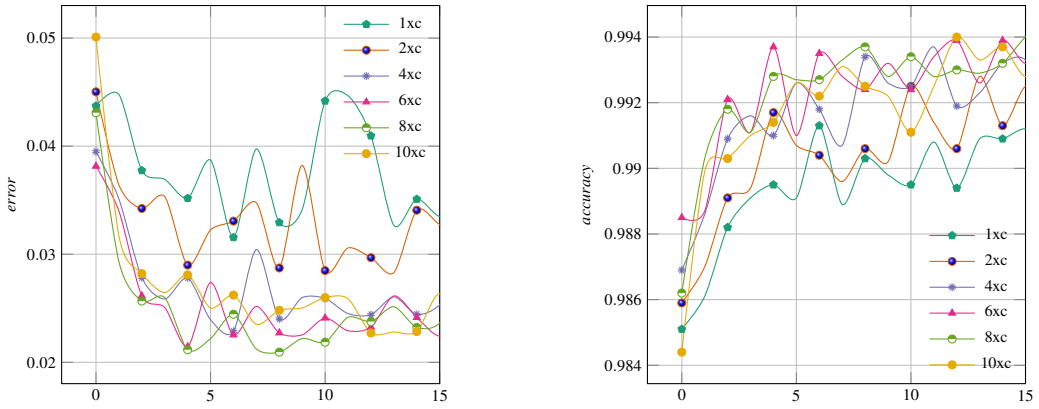
Batch size or minibatch size is the total number of samples chosen from the dataset, which are part of single network update. Often this parameter is chosen arbitarily based on memory availability o the system or capability o underlying mechanism. There is not much study available on the batch size recommendations.As a general rule [16] suggests to use batch size=32, as values above 10 can take advantage of fast matrix multiplications over vector matrix products. [17] in context of RBM(Restricted Boltzmann Machines) also suggest to use batch size greater than 10 for speed up, but strongly against making the size too big when using stochastic gradient descent.

We can understand this bit more in context of network updates where each mini batch is responsible for a single update, so total number of updates in an epoch depends on the size of mini batch. Bigger the size less number of updates it has in an epoch. So weight updates decreases as batch size increases. [17] suggest to use batch size equal to number of classes in case of uniform class data with small number of classes. Also it suggest to have sample from each class in a batch. However there is no experimental evidences provided which supports the suggestions for different networks. Also study of mini batch size with respect to classes seems interesting. So our major work here is experiments the relationship of this kind which is largely neglected and provide a recommendations for their choice.

3.4.1 Experiments

3.4.1.1 MNIST

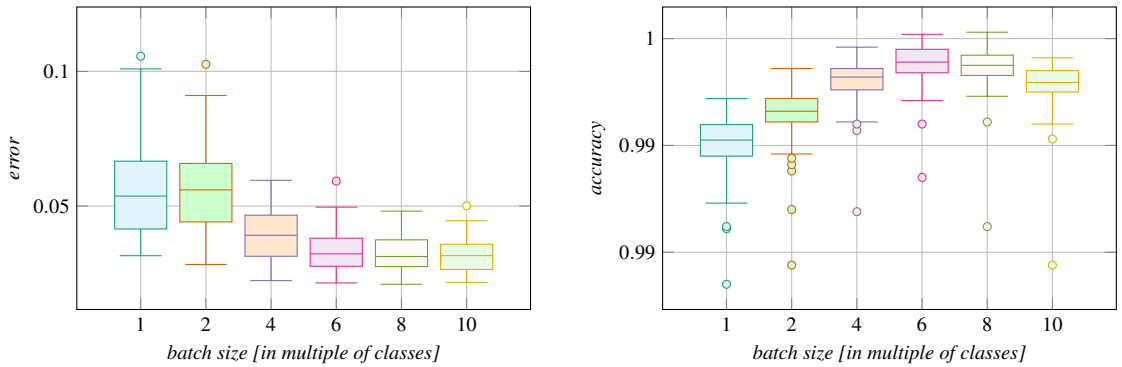
Two Layer, opti=nadam, init=hessian uniform This section provides comprehensive study on the choice of mini batch size, its relationship with number of classes in classification task.



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.10: *Different batch results for starting 15 epochs*

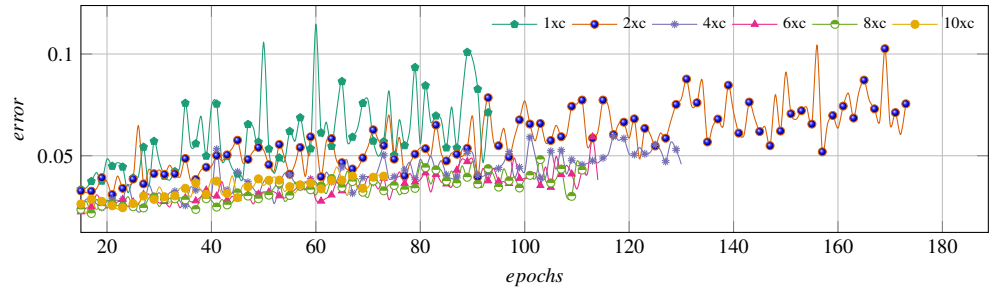
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



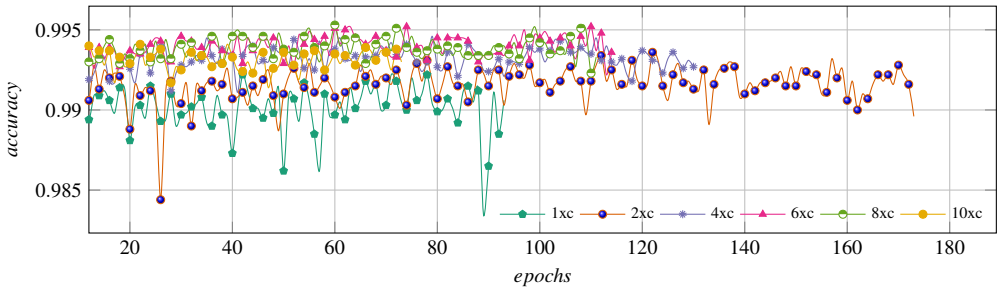
(a) Error variation (b) Accuracy variation

Figure FC3.11: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

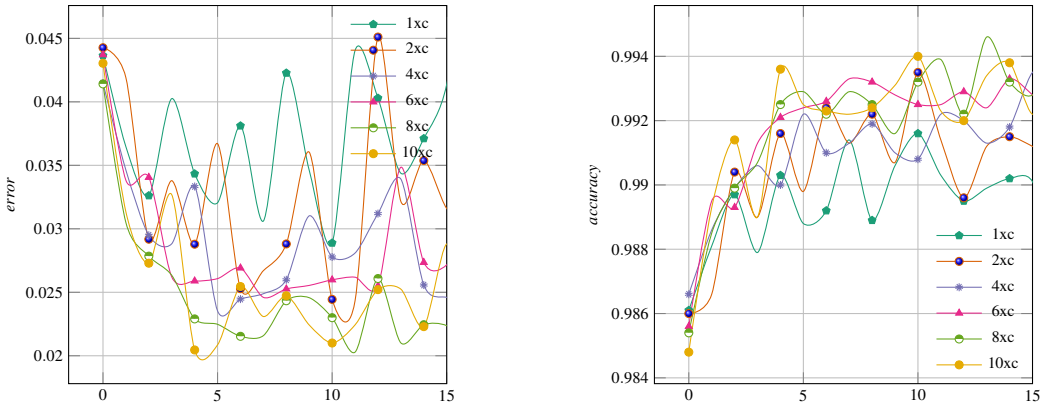


(b) Accuracy for later epochs

Figure FC3.12: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

Two Layer, opti=nadam, init=glorot uniform

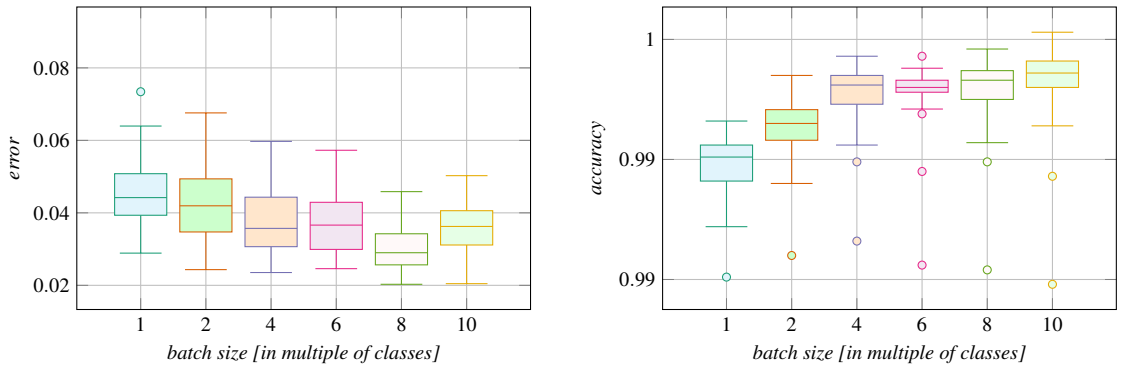


(a) Error for initial epochs

(b) Accuracy for initial epcohs

Figure FC3.13: *Different batch results for starting 15 epochs*

*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*

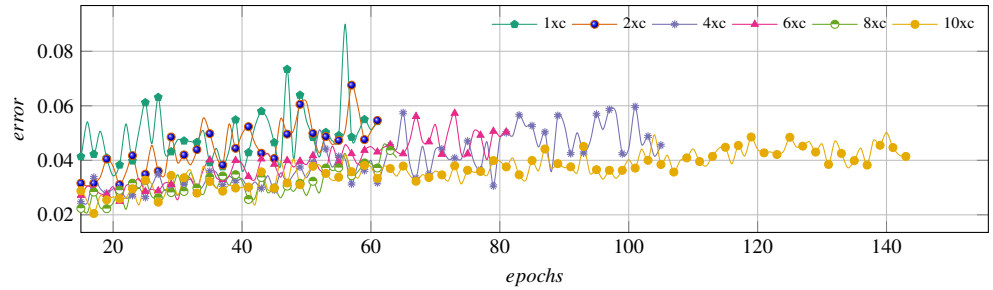


(a) Error variation

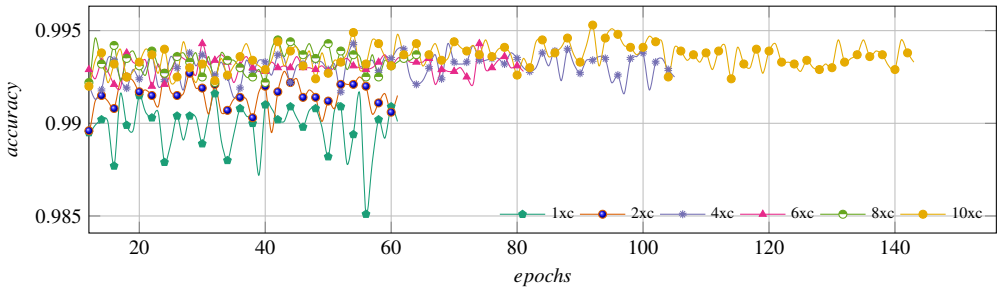
(b) Accuracy variation

Figure FC3.14: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

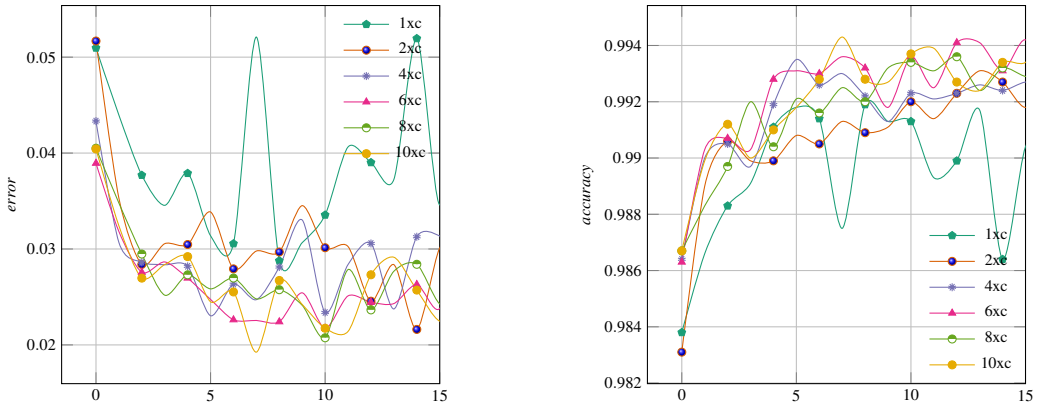


(b) Accuracy for later epochs

Figure FC3.15: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

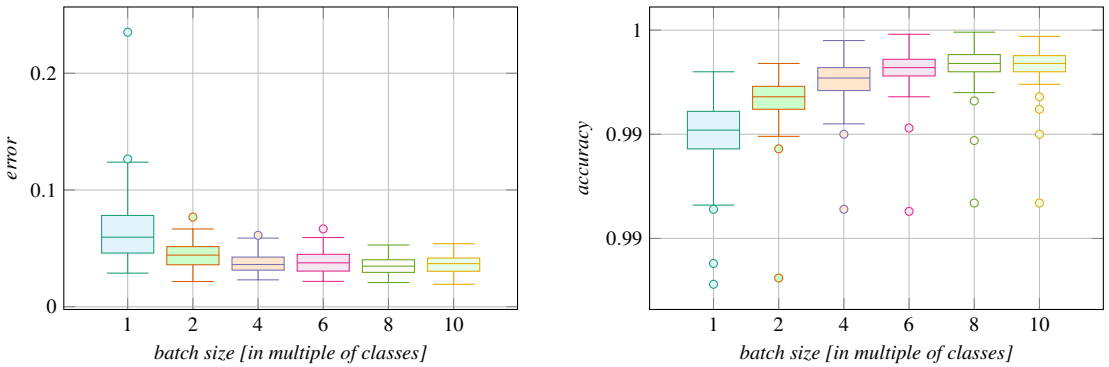
Two Layer, opti=nadam, init=uniform



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.16: *Different batch results for starting 15 epochs*

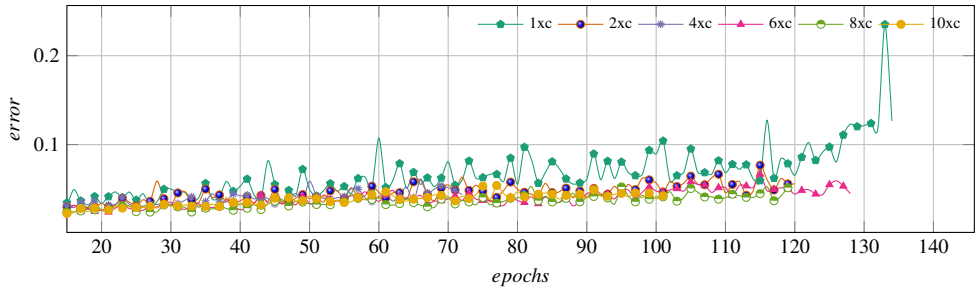
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



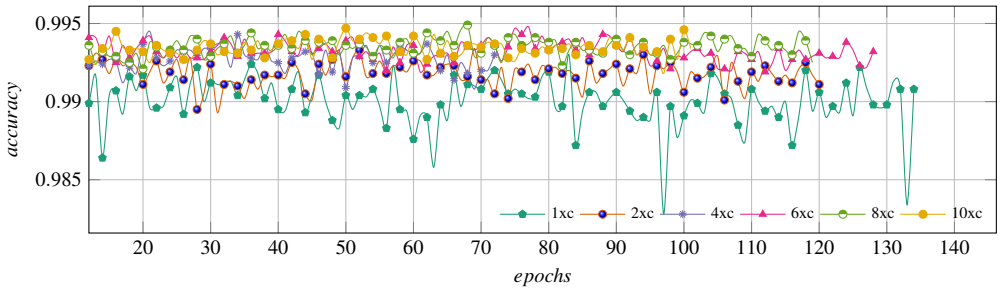
(a) Error variation (b) Accuracy variation

Figure FC3.17: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

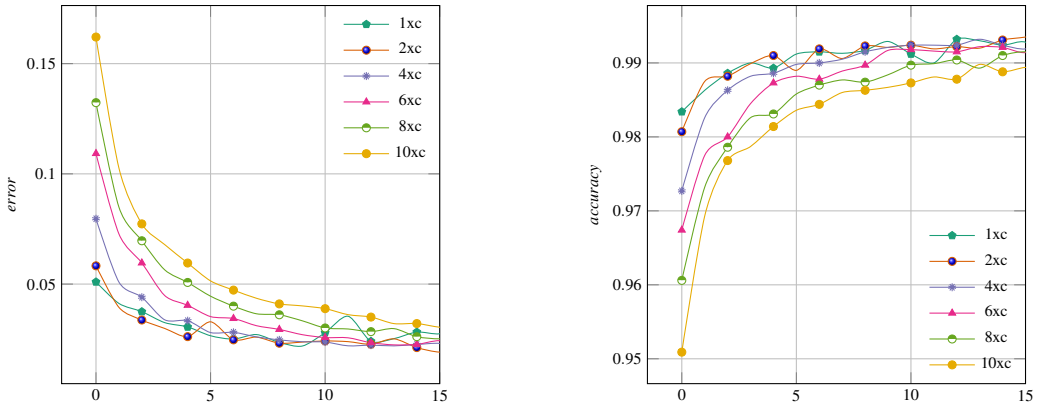


(b) Accuracy for later epochs

Figure FC3.18: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

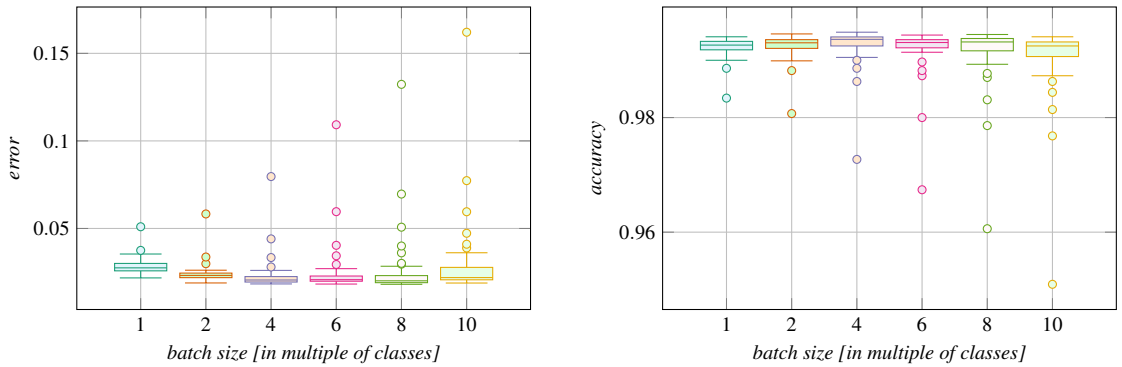
Two Layer, opti=sgd with momentum, init=hessian uniform



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.19: *Different batch results for starting 15 epochs*

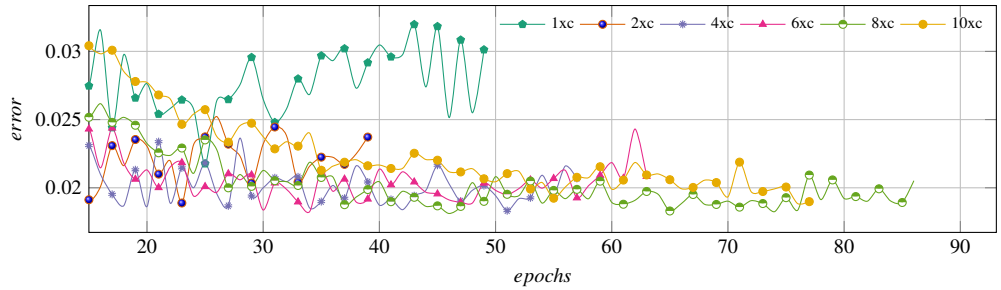
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



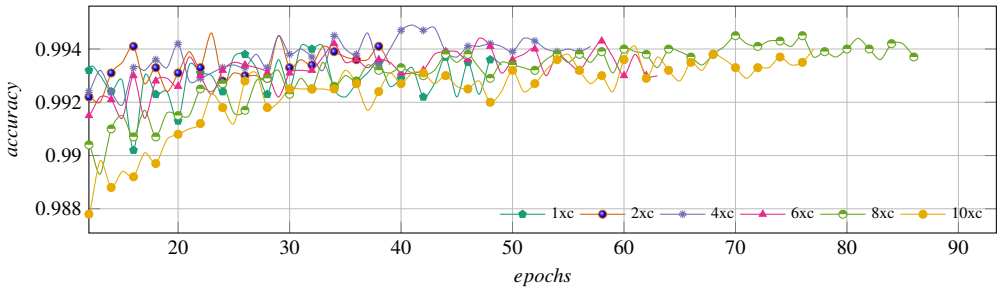
(a) Error variation (b) Accuracy variation

Figure FC3.20: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

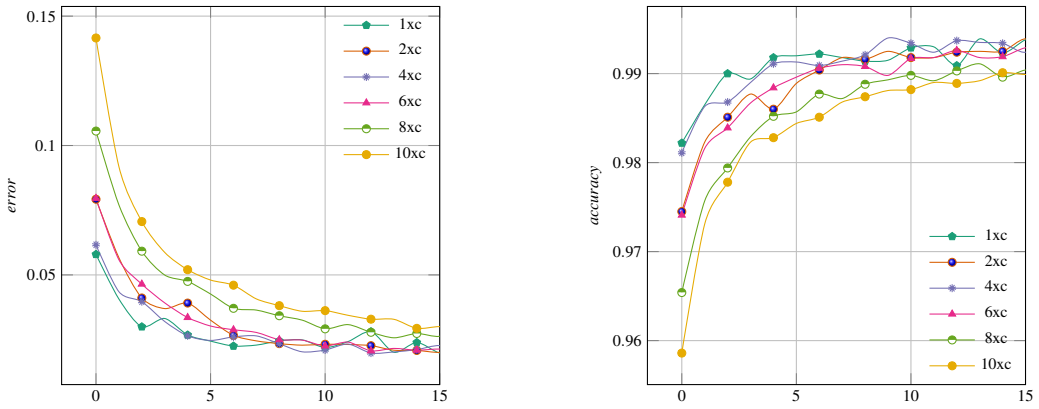


(b) Accuracy for later epochs

Figure FC3.21: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

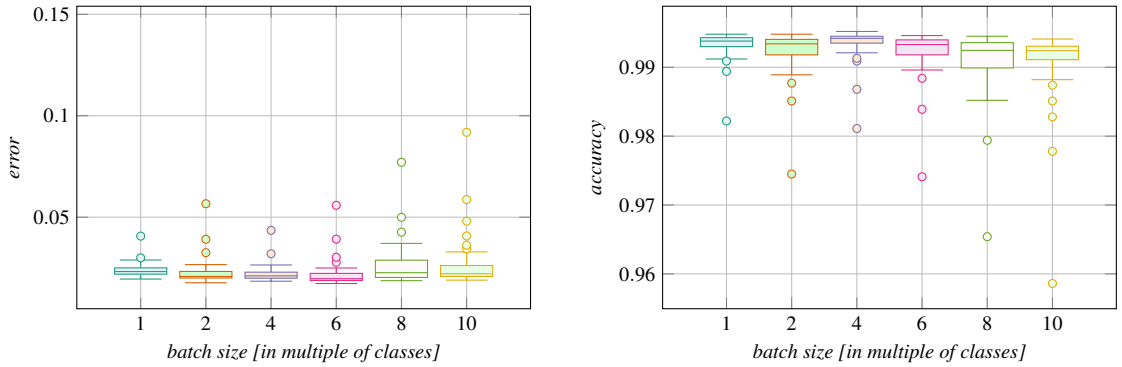
Two Layer, opti=sgd with momentum, init=uniform



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.22: *Different batch results for starting 15 epochs*

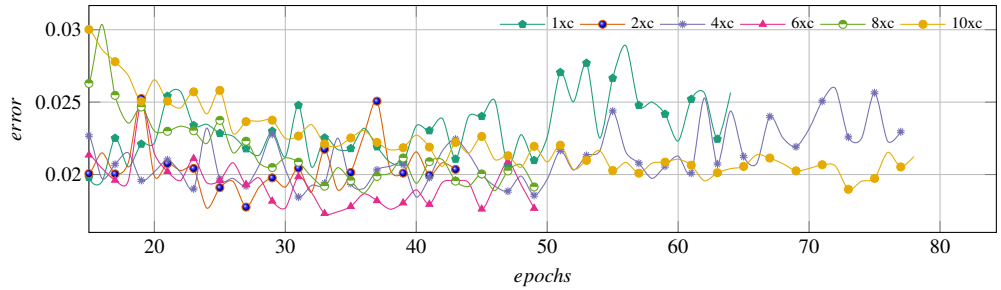
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



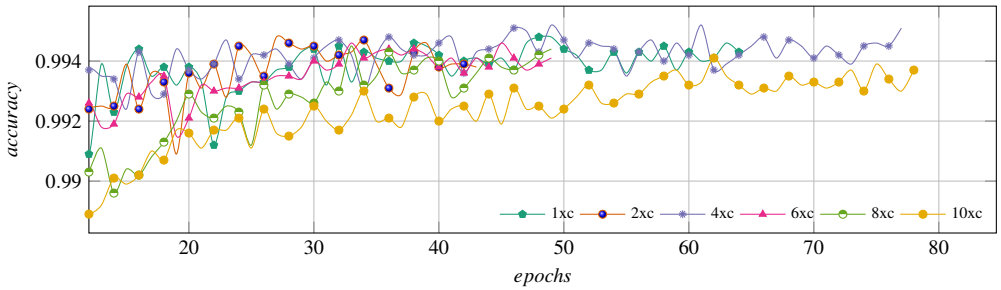
(a) Error variation (b) Accuracy variation

Figure FC3.23: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

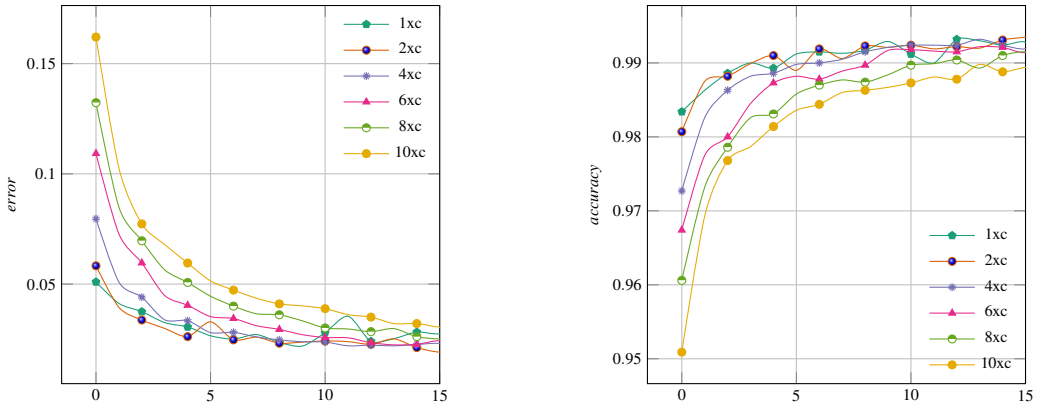


(b) Accuracy for later epochs

Figure FC3.24: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

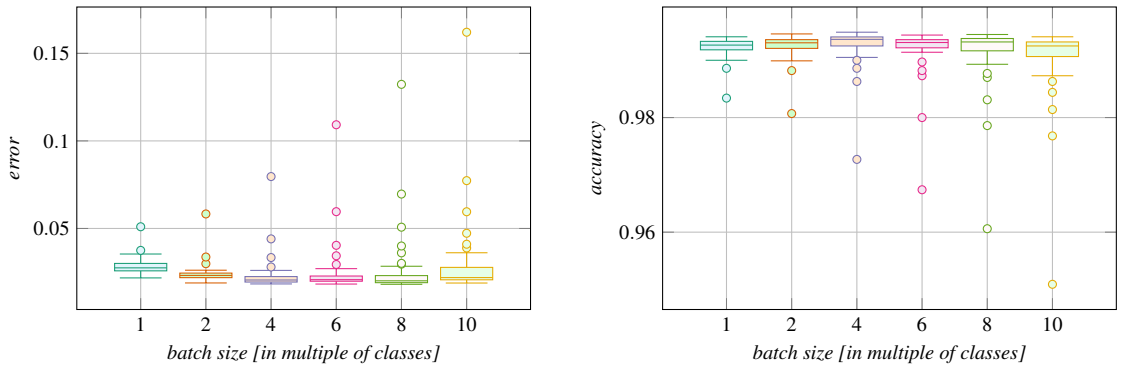
Three Layer, opti=sgd with momentum, init=uniform



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.25: *Different batch results for starting 15 epochs*

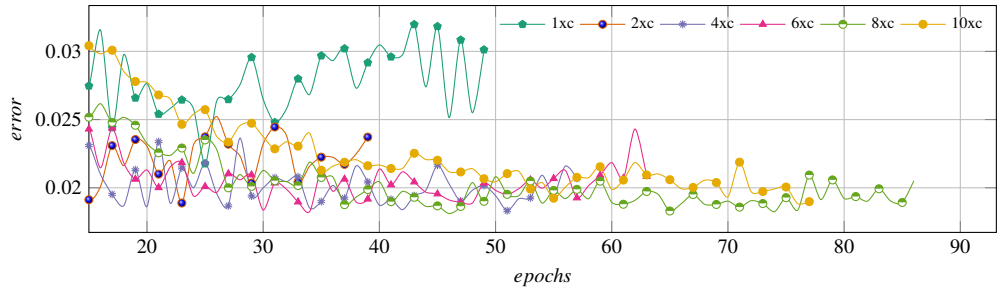
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



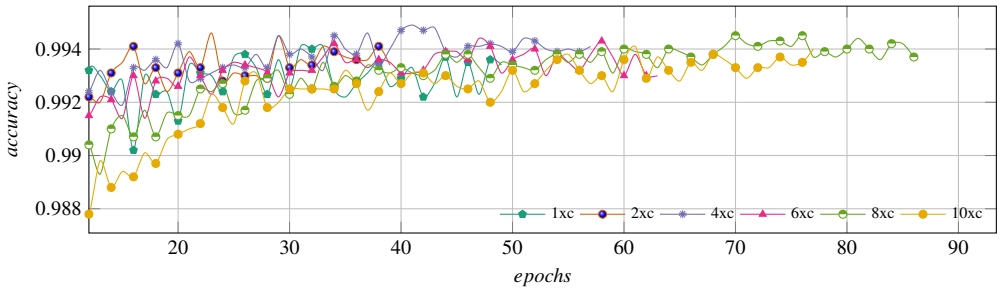
(a) Error variation (b) Accuracy variation

Figure FC3.26: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs



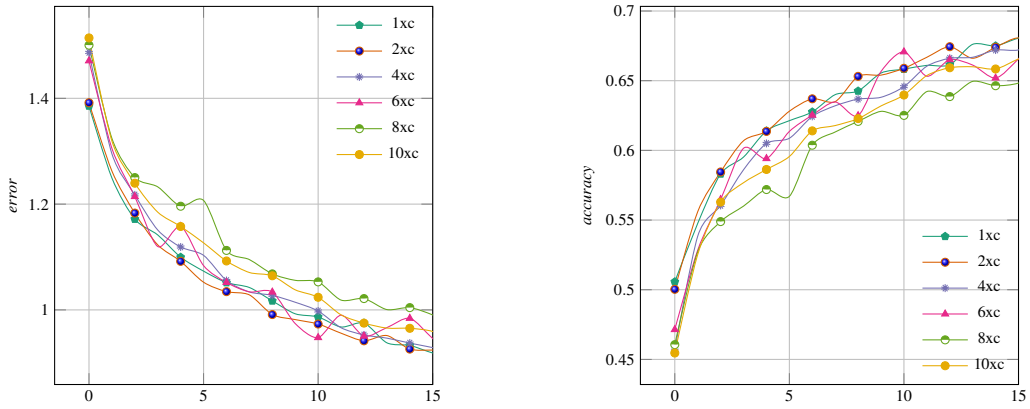
(b) Accuracy for later epochs

Figure FC3.27: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

3.4.1.2 CIFAR10

Three Layer, opti=adagrad, init=uniform

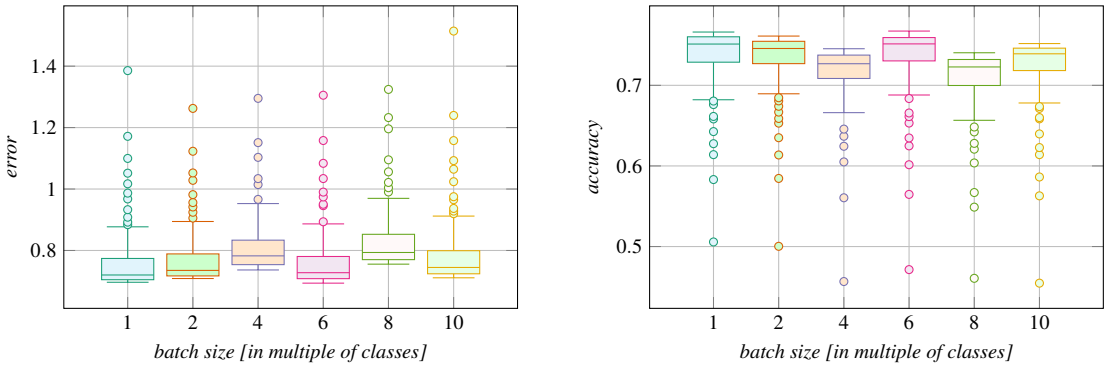


(a) Error for initial epochs

(b) Accuracy for initial epcohs

Figure FC3.28: *Different batch results for starting 15 epochs*

*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*

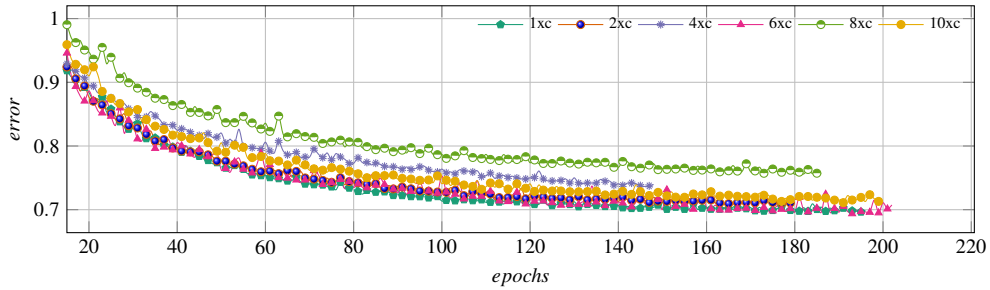


(a) Error variation

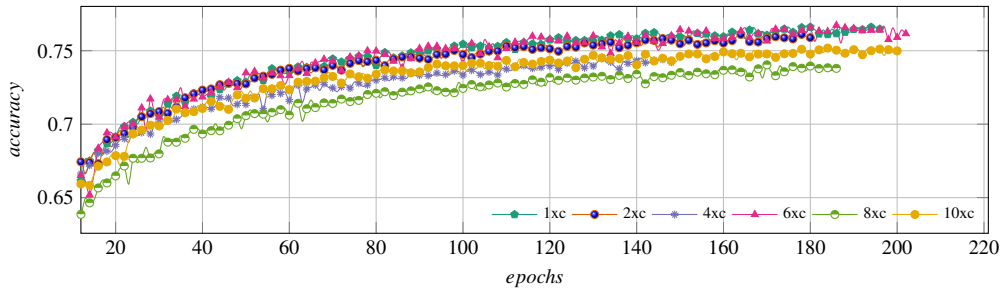
(b) Accuracy variation

Figure FC3.29: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

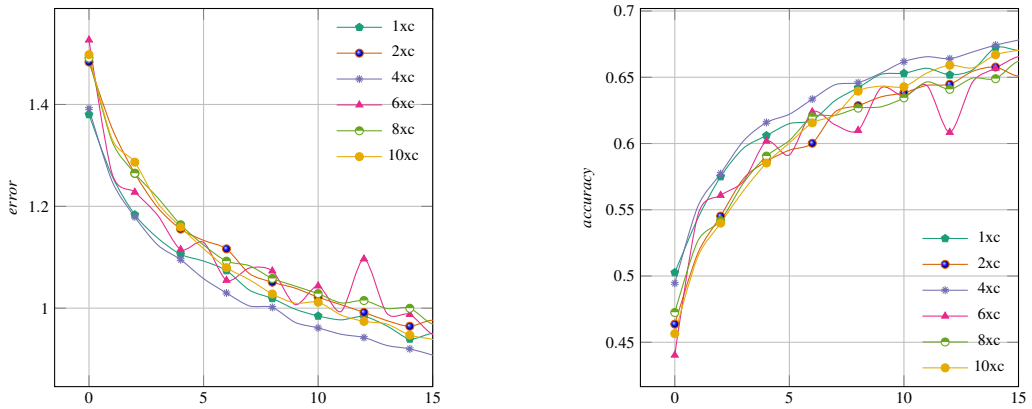


(b) Accuracy for later epochs

Figure FC3.30: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

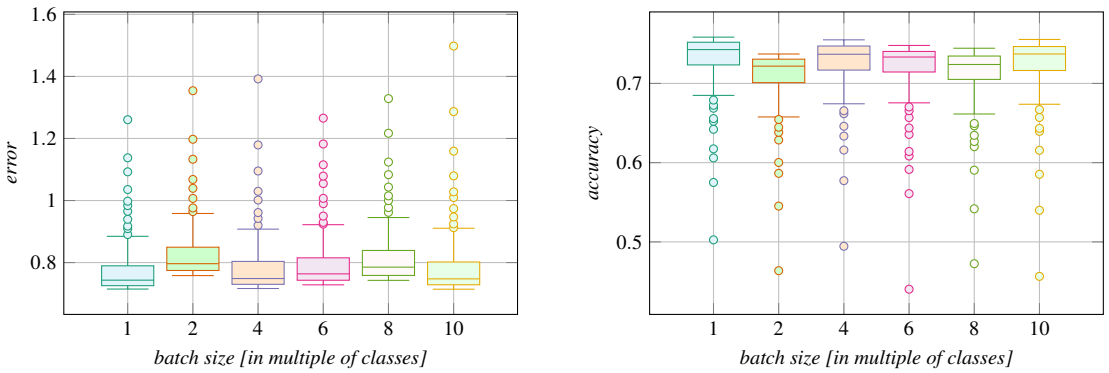
Three Layer, opti=adagrad, init=glorot normal



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.31: *Different batch results for starting 15 epochs*

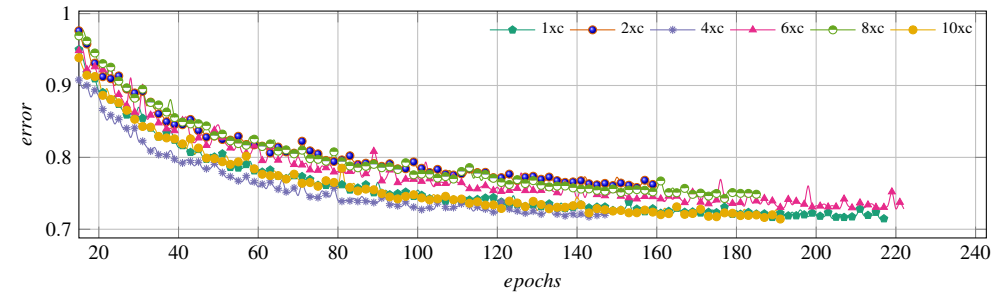
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



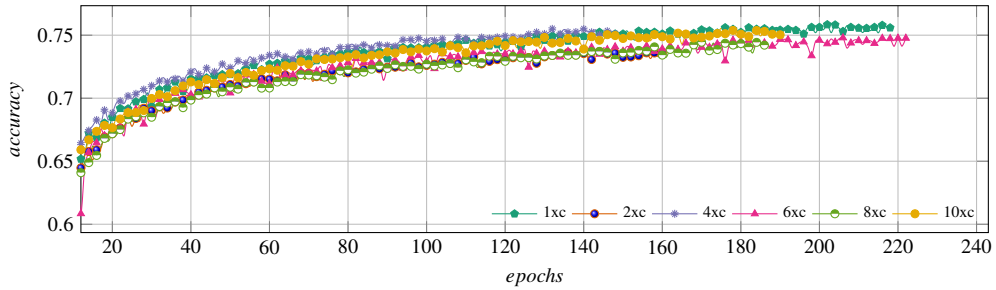
(a) Error variation (b) Accuracy variation

Figure FC3.32: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

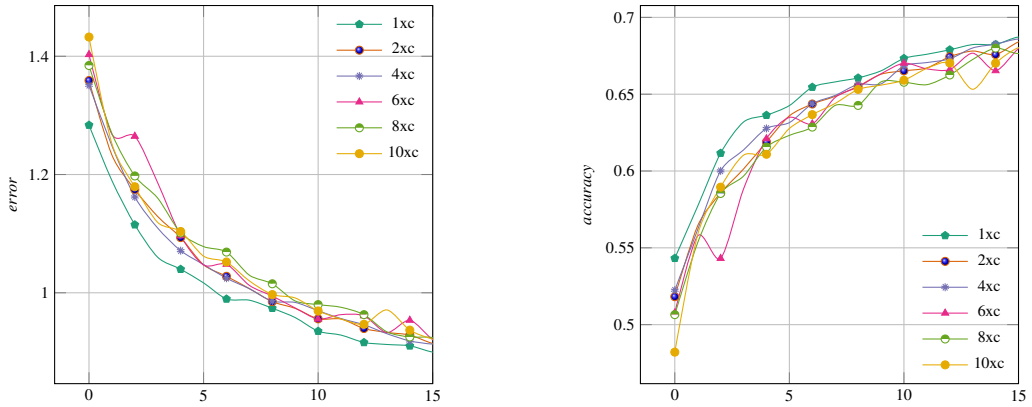


(b) Accuracy for later epochs

Figure FC3.33: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

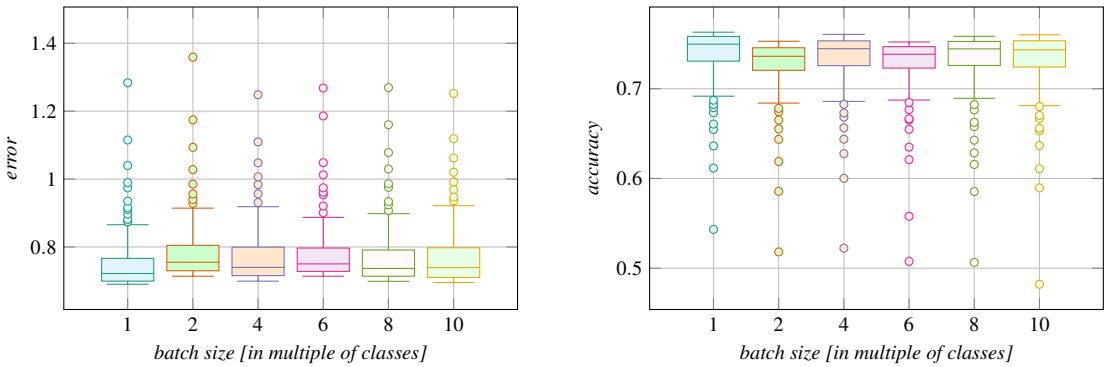
Two Layer, opti=adagrad, init=normal



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.34: *Different batch results for starting 15 epochs*

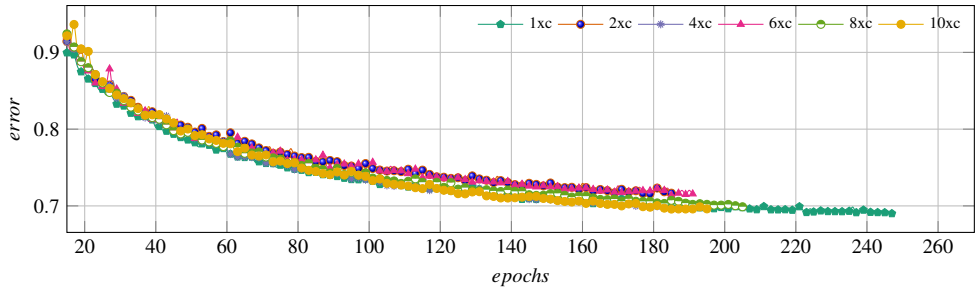
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



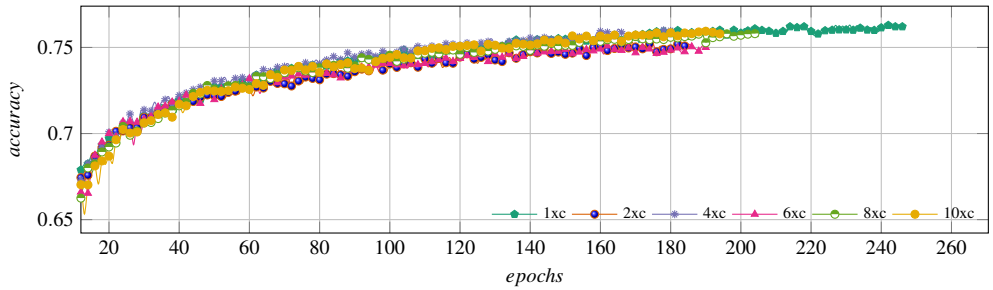
(a) Error variation (b) Accuracy variation

Figure FC3.35: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs



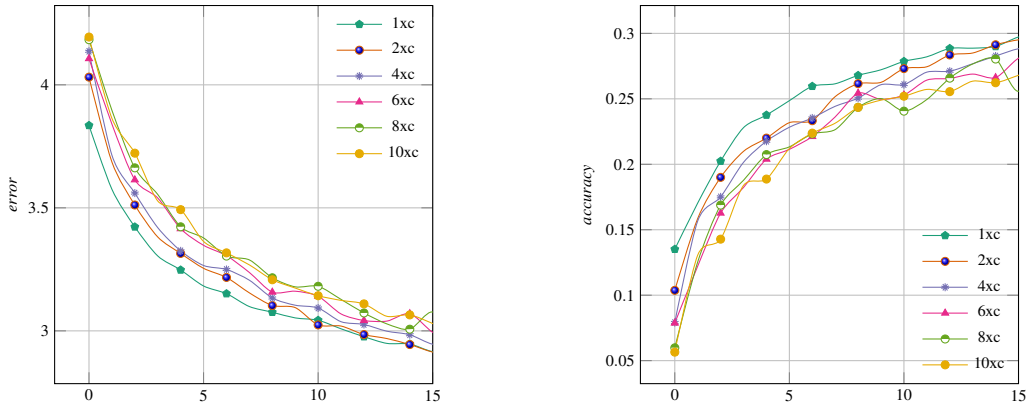
(b) Accuracy for later epochs

Figure FC3.36: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

3.4.1.3 CIFAR100

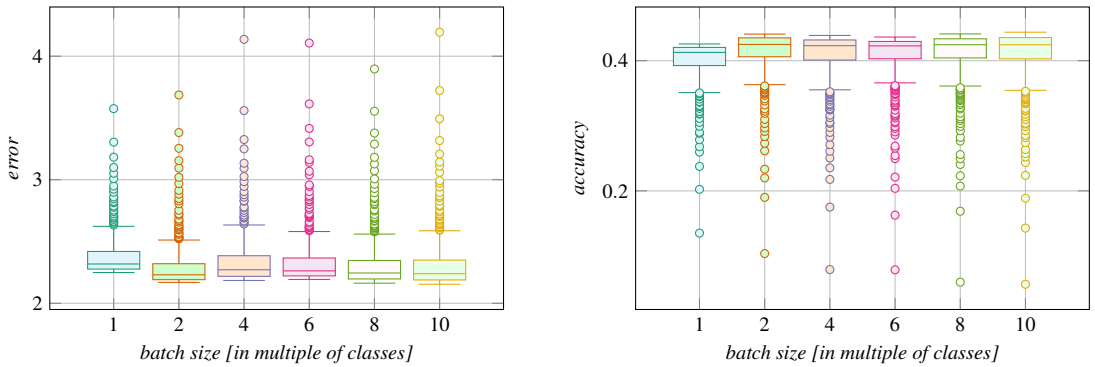
Two Layer, opti=adagrad, init=hessian uniform



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.37: *Different batch results for starting 15 epochs*

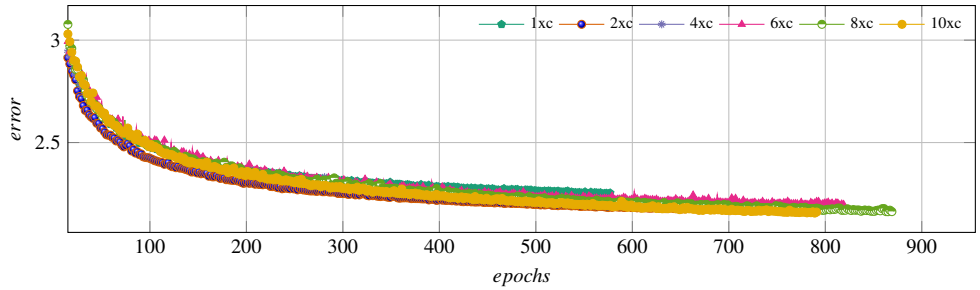
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



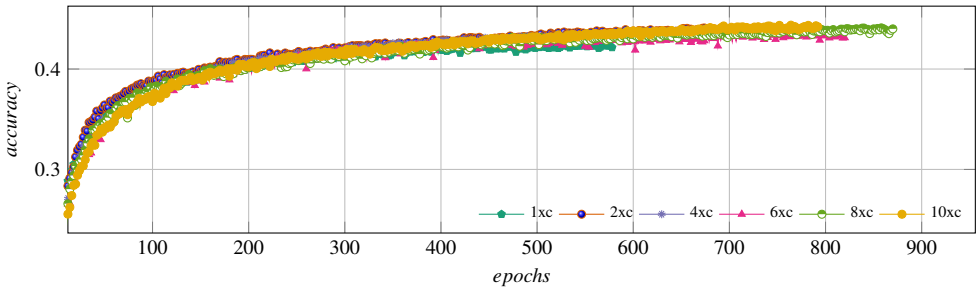
(a) Error variation (b) Accuracy variation

Figure FC3.38: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

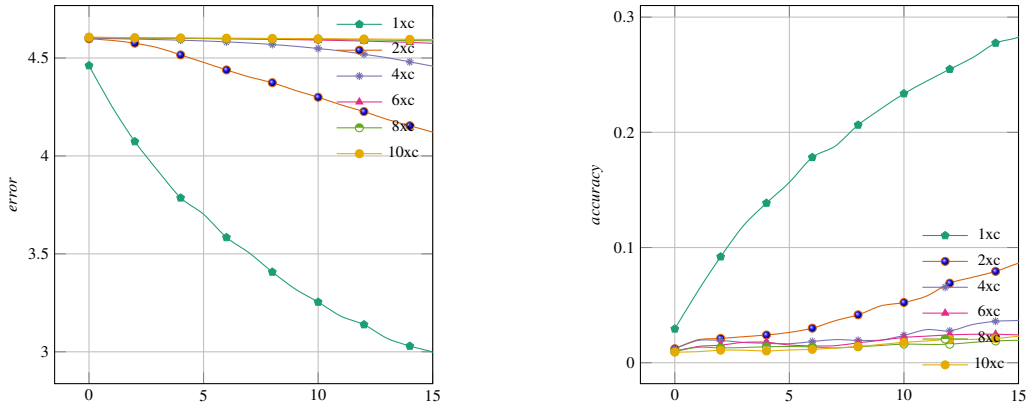


(b) Accuracy for later epochs

Figure FC3.39: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

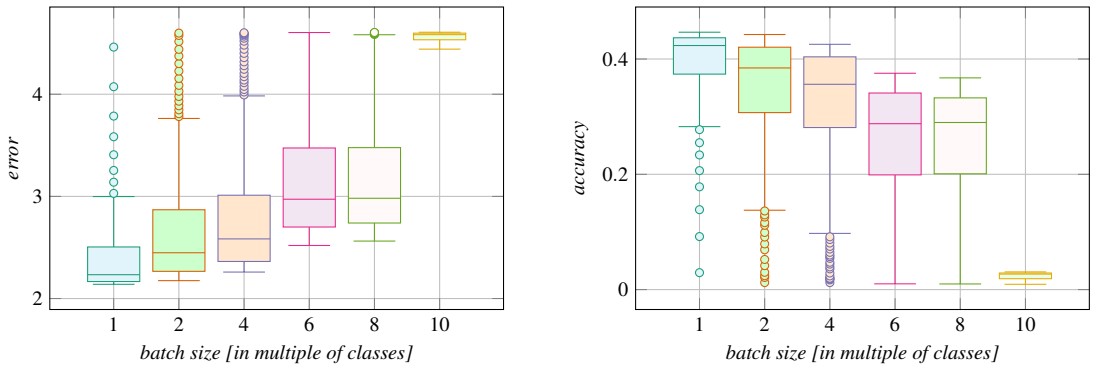
Two Layer, opti=sgd with nesterov momentum, init=glorot normal



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.40: *Different batch results for starting 15 epochs*

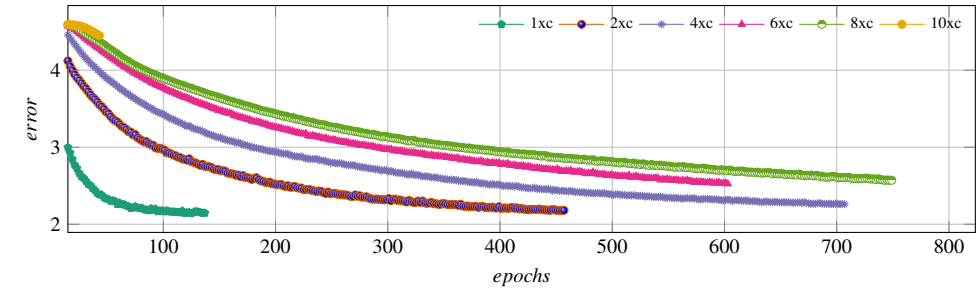
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



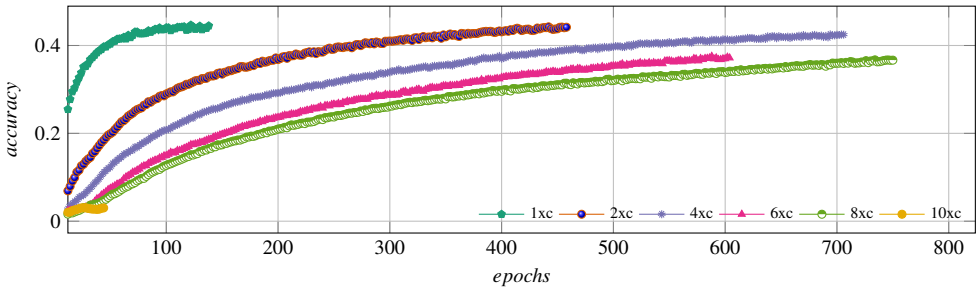
(a) Error variation (b) Accuracy variation

Figure FC3.41: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs

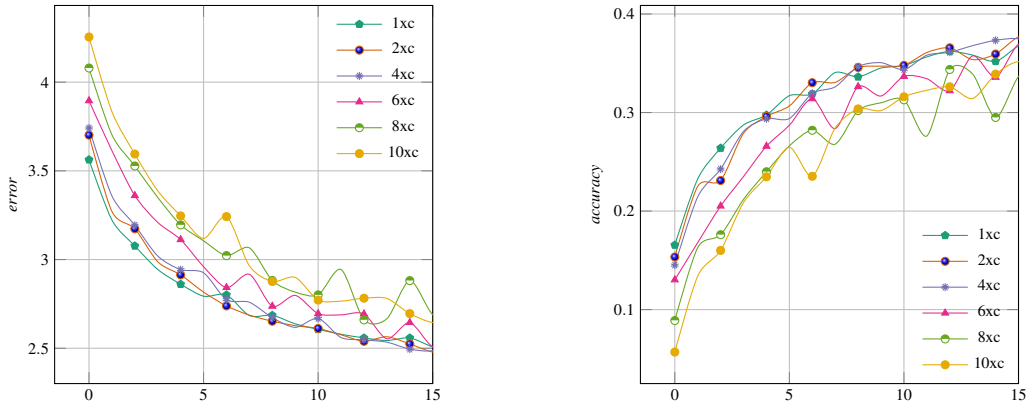


(b) Accuracy for later epochs

Figure FC3.42: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

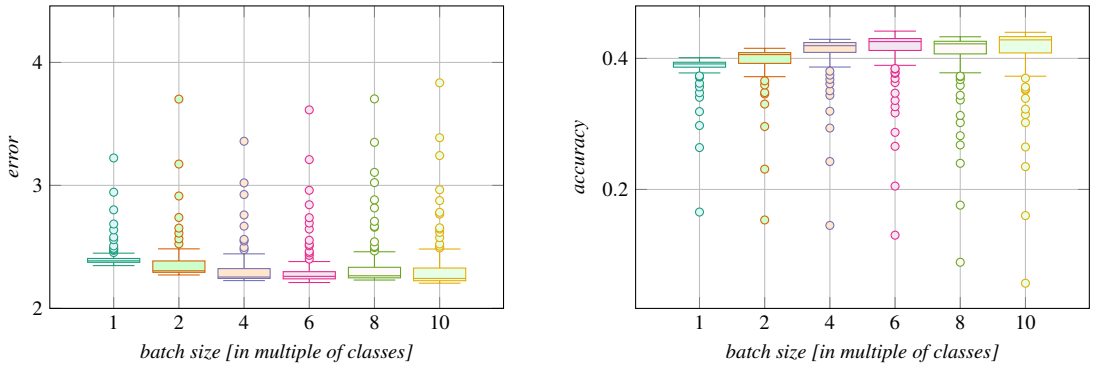
Two Layer, opti=nadam, init=glorot normal



(a) Error for initial epochs (b) Accuracy for initial epcohs

Figure FC3.43: *Different batch results for starting 15 epochs*

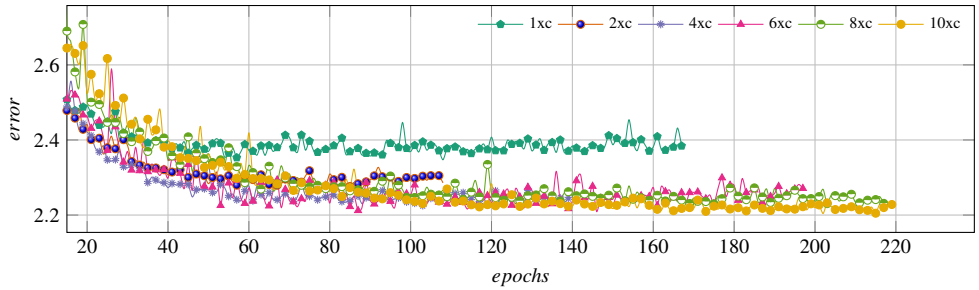
*The initial epochs seen shows that 2x batch size(2x batch size=2*number of classes) started with the lowest error(a) as well as best accuracy(b), surprisingly 8x batch size started quite well inspite of less network updates in an epoch. Then till 15 epochs it is mixed results and no batch size cannot be singled out as a best performer.*



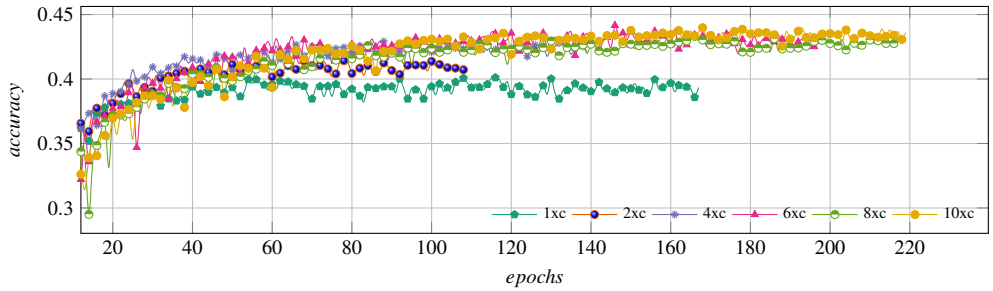
(a) Error variation (b) Accuracy variation

Figure FC3.44: *accuracy and error plot for full training epochs*

4x perform quite well as error remain with in small range, but on high side, 16x and 18x have low error regime as their minimum error but their variance range is on higher side, 10x shows quite good accuracy and has its maximum value is maximum among all other batch sizes, while its error had high variance than 2x. 2x is consistent in both low error with less vairance as well as accuracy. Low error makes 2x a robust classifier.



(a) Error for later epochs



(b) Accuracy for later epochs

Figure FC3.45: *Different batch results for later epochs*

8x started well but in late epochs it could not sustain the start, Another good perfromance is shown by 10x which has reached quite good accuracy but its error become high and entered overfitting regime, 2x performance in error as well as accuracy side remain quite robust.

CHAPTER 4

TRAINING THE NETWORK

For converging to favorable network configuration, network undergoes training. Training consist of network seeing the training data, adjust its parameters to minimize the training loss and finally settling to global possible minimum loss. However this situation is ideal, because in simple convex setting achieving the minimum is not possible as it requires exact small updates to settle to the minimum loss point.

The goal of training is to adjust network parameters in such a way its error performance on samples outside training data remains within the small bound, so it should generalize well. The probability of difference of expected training error and expected testing error bigger than ε is bounded by δ, see Eqn 4.1

$$Pr(E[\mathscr{L}(training)] - E[\mathscr{L}(testing)] > \varepsilon) < \delta \qquad \text{(Eqn 4.1)}$$

Neural network training has tougher challenges to deal than simple convex setting and hence there is no set methodology how to go about tuning the parameters to attain minimum loss possible, or how training should proceed so that network gets attracted in basin of global minimum and not stuck in local minima or saddle points.

These are few challenges which need to dealt with in training neural networks

1. High dimensionality

2. Non convex and existence of saddle points
3. Vanishing gradients
4. Hyper parameters search space
5. stopping criterion

4.1 High Dimensionality

Neural networks often have very high dimensions which is also known as network parameters. Also along with high dimension of network, training data size is huge and it directly affects convergence time. Also high dimensionality affects generalization capability of the network and they tend to over fit to training samples. Generally drop out, regularizers are used to reduce over-fitting Apart from this it needs a huge amount of data for high generalization performance of the network.

Network tend to under fit if parameters are not enough to get the full input feature representation. Balancing out number of network parameters for efficient representation is also very essential, which is not further examined in this work.

4.2 Non Convexity

Non linear transformation of neural networks make it very difficult optimization problem, apart from high dimensionality. Non convexity comes with its own challenges such as existence of more than one minima and saddle points.

4.3 Vanishing gradients

Deep learning has issue of vanishing gradient, which is, while back propagating gradients from output to input they start becoming smaller and smaller,so as number of layers increases this phenomenon led to saturated weights specially near to input layers. This takes training to almost standstill. Once it comes to stand still it is even harder to understand what causes training to stop, is it because of local minima or vanishing gradients.

4.4 Hyper Parameters

Hyper-parameters are the most challenging part of neural network optimization. THis is due to the fact that number of parameters and their range is so high that search space becomes so huge that trying exhaustive possibilities are almost intractable. So we left only with random selection for these parameters. Some recommendations of these values are made from time to time and they almost become default choices for training routines now a days.

In our work we performed almost exhaustive set of experiments and suggested parameters,which performed really well. This could well serve as starting point for any deep learning problems.

4.5 Stopping criterion

It is very hard to determine when to stop the training or how many iterations are enough, it is always said that we should train network long enough, how long is long, is impossible to define and so heuristics are always used for deciding it. This could well be one of the hyperparameters as number of epochs determine when the training should stop. In our work we just used early stopping, which is automatic stopping when performance is not improving.

CHAPTER 5

RECOMMENDATIONS

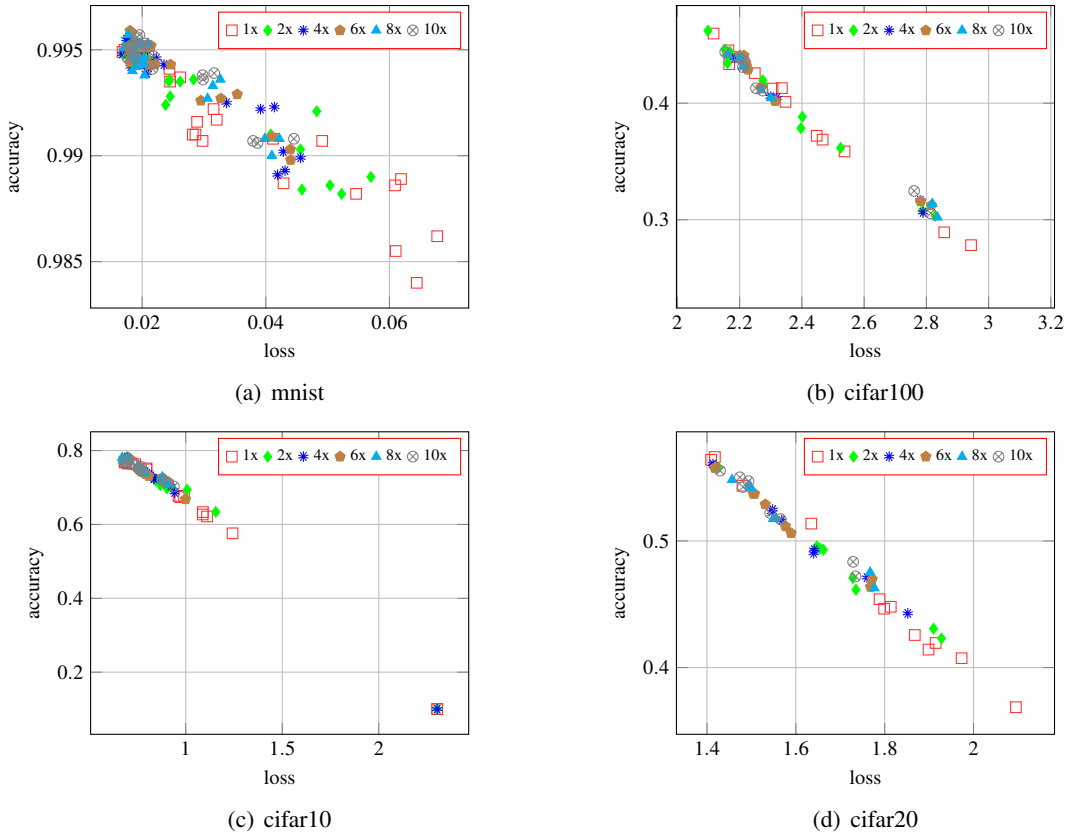
In this chapter major recommendations are detailed. Firstly our experiment process is described and then possible best options ae chosen as recommendation. Based on analysis we devised certain algorithms and discussed their outputs. Lastly some areas for future research is suggested.

5.1 Experiments Process

There are certain hyper-parameters from which we need to choose as explained in 3. For that we choose following set of dictionaries [TBD]

5.1.1 batch size

Chapter 3 has shown lot of results on batch sizes as multiple of class size plays a big role in better accuracy. Here we will see all the experiments performed and their results with respect to class size.



(a) mnist (b) cifar100

(c) cifar10 (d) cifar20

Figure FC5.1: ROC curves for maximum accuracy and minimum loss on different datasets

ROC curve indicates that batch of 4x-8x times class size outperforms other batch sizes. smaller size 1x and 2x . Intuitively this may well be understood that 4x-8x times contains a good representation of each class samples, while bigger batch sizes are prone to noise.

CHAPTER 6

CONCLUSIONS

That’s all folks!

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APPENDIX A

APPENDIX: HOW TO ADD AN APPENDIX

This is Appendix A.

You can have additional appendices too, (*e.g.*, `apdxb.tex`, `apdxc.tex`, *etc.*). These files need to be included in `thesis.tex`.

If you don't need any appendices, delete the appendix related lines from `thesis.tex`.

A.1 Equations

An example mathematical formulae is show in Eqn 1.1.

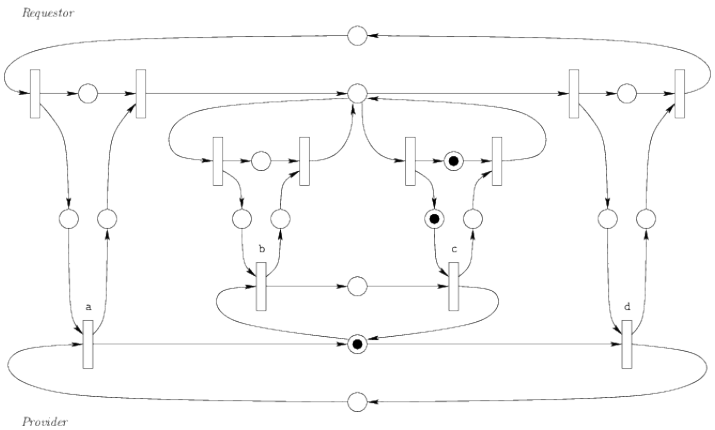

Figure FA1.1: Image of a deadlocked Petri net at 40% scaling.

Table TA1.1: Fall Semester Enrollment

	Undergraduate			Graduate		
	F/T	P/T	Total	F/T	P/T	Total
2004	13,191	2,223	15,414	1,308	879	2,187
2005	13,184	2,143	15,327	1,375	920	2,295
2006	12,809	2,224	15,033	1,373	899	2,272
2007	12,634	2,155	14,789	1,403	899	2,302
2008	12,269	2,208	14,477	1,410	1,005	2,415
2009	12,382	2,323	14,705	1,567	1,106	2,673

$$\sum_{i=0}^{n} i^2 \qquad \text{(Eqn 1.1)}$$

APPENDIX B

APPENDIX: HOW TO ADD ANOTHER ONE

This is Appendix B.