EECS 101: HOMEWORK #8

Due: March 16, 2017

1. Consider a pattern classification system where we want to determine if a coin is head up or tail up by measuring the brightness of the coin. The coin is equally likely to appear head up or tail up. We use a two bit imaging system, so that measured brightness takes on one of the four values in the range 0-3. By supervised learning, we determine that for a head we have the following probabilities of observing each brightness

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P(0) = 0.1,  P(1) = 0.5,  P(2) = 0.3,  P(3) = 0.1
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and for a tail, we have the following probabilities of observing each brightness

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P(0) = 0.0,  P(1) = 0.1,  P(2) = 0.6,  P(3) = 0.3
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a) Is perfect classification possible in this system? Explain your answer.

b) Suppose our system is presented with an unknown sample and we observe a brightness. For each of the four brightness levels, what is our best guess as to whether we have a head or tail? For each of these guesses, what is the probability of error?

2. Consider a pattern classification problem where we would like to discriminate between two materials M_1 and M_2 using a measured color vector (R, G, B) of the unknown material. Assume that the *a priori* probability of M_1 is $3/7$ and that the *a priori* probability of M_2 is $4/7$. Suppose that the probability density for M_1 is uniform over the cube $50 \leq R \leq 90, 30 \leq G \leq 70, 40 \leq B \leq 80$. Suppose that the probability density for M_2 is uniform over the cube $70 \leq R \leq 120, 30 \leq G \leq 80, 50 \leq B \leq 100$.

a) What is the conditional pdf $p(R, G, B|M_1)$ as a function of (R, G, B)?

b) What is the conditional pdf $p(R, G, B|M_2)$ as a function of (R, G, B)?

c) What is the *a posteriori* probability $P(M_1|R, G, B)$ as a function of (R, G, B)?

d) What is the *a posteriori* probability $P(M_2|R, G, B)$ as a function of (R, G, B)?

e) What is the best guess for what material we are looking at as a function of (R, G, B)?

f) What is the probability of error as a function of (R, G, B) if we take the guess in part e)?