Automata and Formal Languages
Assignment 2

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Question 1

1) L_1 = { w | w is a binary string with equal number of zeros and ones }

Solution:

Suppose there is a DFA for L_1 with p states.

• Choose $x = 0^p 1^p$ to pump; clearly x is in L_1.

• The Pumping Lemma yields x = u v w, |v| ≥ 1, |uv| ≤ p, and u vm w in L_1 for every m.

• Since |uv| ≤ p, uv consists of 0s only.

• Since |v| ≥ 1, v contains at least one 0.

• Then u v v w is supposed to be in L_1, but it isn't.
(Contradiction)

So, L_1 is not regular.

Question 2

2) L_2 = { w | w is a binary string of the form 0^n1^n , where n≥1}
Note: L_1 is not the same as L_2

Solution:

Suppose there is a DFA for L1 with p states.

• We pick a particular word x in L1 and pump it to get a contradiction.

• Choose x = 0 p 1 p, where p is the number of states.

• Then the Pumping Lemma says that x can be written as u v w, with |v| ≥ 1, so that u v v w is also in L1. – We're using m = 2 here.

• We get a contradiction, by considering three cases:

– v consists of 0s only: Then u v v w contains at least one extra 0, the same 1s, can't match.

– v consists of 1s only: At least one extra 1, can't match.

- v consists of a mix of 0s and 1s: Then u v v w contains a 1 before a 0, so u v v w can't be in L1.

Question 3

3) L_3 = { w | w is a binary string of the form 0^n10^n, where n≥1}

Solution:

Assume L_3 is regular.

From the pumping lemma there exists an n such that every w ∈ L such that |w| ≥ n can be represented as x y z with |y| ≥ 0 and |xy| ≤ n. Let us choose 0^n10^n. Its length is 2n+1 ≥ n. Since the length of xy cannot exceed n, y must be of the form 0^p for some p > 0.

From the pumping lemma $0^{n-p}10^n$ must also be in L_3 but it is not of the right form.

Hence the language is not regular.

Question 4

4) L_4 = {w | w is a binary string of the form 0^m1^n, where m < n and m,n≥0 integers}

Solution:

Assume L_4 to be regular.

break w into xyz, in which y ≥ 0 and |xy| ≤ n.

We know that |xy| ≤ n and y 6 y ≥ 0. Since xy comes at the front of w, we know that x and y consist of only 0's, and that y must contain at least one 0.

The Pumping Lemma tells us that xz is in L if L is regular, however, xz has n 1's, since all of the 1's of w are in z. However, xz also has fewer than n 0's, because we have lost the 0's of y. Since y y ≥ 0, we know that there can be no more than n − 1 0's among x and z. We have assumed L_4 to be a regular language, but have proved that xz is not in L_4. Therefore, we have contradicted our assumption that L is regular.

Question 5

5) L_5 = { w | w is a binary string of the form 0^n, where n is a perfect square (i.e., $n=i^2$, for i≥1)}

Solution:

Step I: Assume the set L is regular. Let n be the number of states of the FA accepting the set L.

Step II: Let $w = a^{n^2}$. $|w| = n^2$ which is greater than n, the number of states of the FA accepting L. By using the pumping lemma, we can write w = xyz with $|xy| \le n$ and $|y| > 0$

Step III: Take i = 2. So, the string will become xy^2z.

$$|xy^2z| = |x| + 2|y| + |z| > |x| + |y| + |z| \text{ as } |y| > 0.$$

From step II, we know $|w| = |xyz| = |x| + |y| + |z| = n^2$.
So, $|xy^2z| > n^2$.
Again, $|xy^2z| = |x| + 2|y| + |z| = |x| + |y| + |z| + |y| = n^2 + |y|$.
As $|xy| \le n$, $|y| \le n$.
Therefore, $|xy^2z| \le n^2 + n$.
From the previous derivations, we can write

$$n^2 < |xy^2z| \le n^2 + n < n^2 + n + n + 1$$
$$n^2 < |xy^2z| < (n+1)^2.$$

Hence, $|xy^2z|$ lies between n^2 and $(n+1)^2$. They are the square of two consecutive positive integers. In between the square of two consecutive positive integers, no square of positive integer belongs. But a^{i^2}, where $i \ge 1$ is a perfect square of an integer. So, the string derived from it, i.e., $|xy^2z|$ is also a square of an integer, which lies between the square of two consecutive positive integers. This is not possible.

So, $xy^2z \notin L$. This is a contradiction.
So, $L = L = \{a^{i^2} \mid i \ge 1\}$ is not regular.